Planetary Waves Kinematically Forced by Himalayan Orography

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ABSTRACT

An analysis is made of the planetary-scale response of the atmosphere to the kinematic effects of orographic forcing by, in particular, the Tibetan Plateau–Himalayan Mountain complex. Theoretical scaling arguments are used to deduce a critical mountain height $h_c$ beyond which the component of flow around will dominate that over the orography. The $h_c$ is proportional to the meridional scale of the orography and depends on latitude. For north–south scales appropriate for the Himalayas $h_c \sim 1.5$ km which is much less than the actual height of 3706 m when resolved with four zonal planetary waves, with the implication that the “around” component will dominate.

A steady-state planetary wave model which has a full kinematic nonlinear lower boundary condition is used to simulate the response to the eastern orography whose height has been multiplied by factors ranging from 0.1 to 2.0. Although the mountain configuration was fixed, the locations of the simulated perturbation highs and lows change substantially in such a way that the total flow increasingly adjusts to go around the high orography as the mountain heights are increased. This effect limits the total vertical motion induced by the orography and thus the amplitudes of the forced planetary waves increase at a rate much less than expected from linear theory. Neglected nonlinear terms in the model are shown to be relatively small in all cases. For shallow mountains the maximum response occurs at the latitude of the mountain (35°N) but both the maximum response and the maximum zonal mean poleward heat flux by the simulated waves are shifted poleward to $\sim 55°N$ for orography $>1500$ m high, consistent with the observed location of the wintertime stationary waves in the Northern Hemisphere. Overall, results support the expectations from scaling considerations and show that linear theory may be reasonably applied for Himalayan orography up to $\sim 1$ km high when resolved on planetary scales, but the “around” component dominates the “over” component when the orography exceeds 1.5 km, as is the case in actuality. The around component should also dominate for the Greenland plateau and Antarctica but the effects are more equivocal for the Rockies which are only $\sim 1$ km high when resolved with planetary scales while $h_c \sim 1.5$ km.

1. Introduction

The main purpose of this paper is to consider aspects of the nature of flow past the large-scale earth's orography and, in particular, the Tibetan Plateau–Himalayan mountain complex, and determine the purely kinematic effects of the orography on forcing planetary-scale waves in the atmosphere. Diabatic effects are omitted. The issue is the extent to which the flow goes over or around the orography and how this changes as the height of the orography is increased. To examine these questions we use a planetary wave model to quantify some theoretical expectations based upon scaling arguments. A secondary objective is to provide further insights into the response of the model itself.

Steady-state planetary wave models are formulated by linearizing the governing equations about a zonal mean basic state flow, so that the perturbations are assumed to be small. The linearization carries over into the lower boundary condition (LBC) including the kinematic effects of flow over topography. The full kinematic LBC is

$$w_b = v_b \cdot \nabla h$$  \hspace{1cm} (1)

at $z = h$, where $v_b$ is the horizontal wind at the boundary, $\nabla$ the horizontal del operator and $h$ the orography. In the linearized form this becomes

$$w_z = \tilde{u}_b \frac{\partial h}{\partial z}$$  \hspace{1cm} (2)

where $\tilde{u}_b$ is the zonal mean wind and $\tilde{u}_b$ is assumed zero. A further assumption, namely that the LBC is
applied on a constant pressure surface, is also often invoked. The expression (2) results in upward motion on the upslope, and downward motion on the lee slope with the implication that the mean flow is going over the mountains.

Yet mountain complexes in the Northern Hemisphere (NH) on the planetary wave scale are not small (we will quantify what is meant by “small” later) and the Himalayan complex, in particular, violates this assumption. The kinematic LBC in (1) when expanded becomes

\[ w_b = \bar{u}_b \frac{\partial h}{\partial x} + v_b \cdot \nabla h \]

(3a)

\[ = \bar{u}_b \frac{\partial h}{\partial x} + \left( u_b \frac{\partial h}{\partial x} + v_b \frac{\partial h}{\partial y} \right) \]

(3b)

\[ = w_z + w_E \]

(3c)

where \( \gamma \) is the departure from the zonal mean and \( w_E \) is the eddy-induced vertical motion. The additional term allows for the possibility that the mean flow may prefer to go around rather than over a mountain.

Observational studies (Saltzman and Irsch 1972; Alpert et al. 1983) have shown that \( w_E \) is not negligible compared with \( w_z \). Similarly in atmospheric general circulation models, it has been found that the flow tends to go around rather than over the large-scale Northern Hemisphere mountain complexes (Tokioka and Noda 1986). Chen and Trenberth (1988a,b) were able, for the first time, to incorporate the additional LBC term in a steady state planetary wave model, and confirmed the importance of \( w_E \), especially for the Himalayas. Their model remains mathematically linear, since \( h \) is specified and constant, but the additional term is nonlinear in the sense that the waves become coupled through the LBC and all the waves have to be solved for simultaneously. Chen and Trenberth (1988a) documented the substantial changes that took place with the more complete LBC on simulating orographically forced planetary waves. Chen and Trenberth (1988b) added diabatic heating forcing and found excellent agreement between the model results and observations. However, the Chen–Trenberth model has some limitations since it applies the LBC at a constant pressure level and neglects other nonlinear terms. The importance of the latter is discussed in section 3 and as part of the results of this paper (section 4).

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**FIG. 1.** The Himalayan (eastern) orography as represented in the model truncated at zonal wave 4 and meridional wave 15. The contour interval is 500 m and heights > 1 km are stippled. The outer boundary is at 20°N.
The traditional LBC (2) was first used by Charney and Eliassen (1949) and has been used in most other studies of steady state planetary waves. The main exception has been models formulated with terrain-following ($\sigma$) coordinates (e.g., Nigam et al. 1986; Hoskins and Karoly 1981). There are several differences in the formulation of $\sigma$ coordinate models, such as in the way the static stability is treated, but the main difference is the physical presence of the mountain and application of the LBC at $\sigma = 0$, i.e. at $z = h$. One feature of such models is that they usually assume the basic state $\bar{u} = \bar{u}(y, \sigma)$ in contrast to $\bar{u} = \bar{u}(y, p)$ as in Chen and Trenberth (1988a). In any case, on constant pressure surfaces, $u = 0$ "inside" the mountain or, equivalently, $u' = -\bar{u}$ locally. Thus a substantial part of $w_E$ in (3b) is automatically included in $\sigma$ coordinate models and the orographic forcing then depends greatly on the specification of $u_0$. However, $v_0$ is still assumed to be zero and the waves are not coupled.

In this paper, we explore in much greater detail the tendency for the flow to go over versus around the Himalayan orographic complex (referred to as the

![Diagram](image_url)

**Fig. 2.** For the standard eastern orography, the components of $\omega$ at the lower boundary $\omega_b = \omega_z + \omega_E$. The contour interval is $1 \times 10^{-4}$ mb s$^{-1}$. 

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"Himalayas"). This is done by first isolating the Himalayas, and secondly determining the model response to the Himalayas when they are altered in elevation by factors ranging from 0.1 to 2.0. As will be seen, at lower elevations, the LBC is linear, (2) is a good approximation, and the flow goes over the mountain. But as the elevation is increased over ~1 km, there is an increased tendency for the flow to go around rather than over, and \( w_E \) becomes comparable to \( w_z \). As a consequence, further increases in the orographic height result in only small changes in the planetary waves and the overall response is highly nonlinear.

While the primary purpose of these experiments is to help us understand the atmospheric response to orographic forcing and thereby improve the steady-state planetary wave models, the results may also prove useful for helping to interpret what has happened in the distant past, when the Himalayas were not as high and mountain building was not complete-major uplift of the Rockies, the Alps and the Himalayas occurred during the Tertiary (65 million to ~2 million years ago), e.g. Kutzbach (1985). In addition, huge ice sheets during the last glacial maximum (~18 000 years ago) created additional ice "mountains" which would act to change the more remote climate by changing the forcing of planetary waves.

Section 2 presents some general arguments concerned with whether the flow will preferentially go around or over the mountains. The planetary wave model is outlined in section 3 along with some discussion of the experiments to be performed. The results are presented in section 4 and discussed in section 5.

2. **Scaling considerations**

As noted previously, whether the flow will go over or around a mountain depends upon whether the mountain can be considered small. In all cases here, we are dealing only with planetary scales in the horizontal and "small" refers to the height of the mountain. On smaller horizontal scales a key parameter is \( Nh/u \) where \( N \) is the Brunt–Väisälä frequency. When this quantity becomes sufficiently large (~1), two-dimensional low level flow upstream will be blocked and will not pass over the obstacle (Baines and Hoinka 1985).

For planetary scales, we first consider the quasigeostrophic vorticity equation

\[
\frac{d}{dt}(\zeta + f) = f_0 \frac{\partial \omega}{\partial p}
\]

where \( f \) is the Coriolis parameter and \( \zeta \) the vorticity. For orographic forcing \( \omega_b \sim -\rho_0 g w_b \) and \( \Delta p \sim \rho g H \),

![Fig. 3. The 400 mb ω' field for the standard eastern orography. The contour interval is 1 \times 10^{-4} \text{ mb s}^{-1}.
](image-url)
so that the linearized form of (4), appropriate for orographic forcing, becomes

$$\hat{u} \frac{\partial \psi'}{\partial x} + \hat{v} \beta \sim \frac{f w_b}{H}$$

where $\beta = df/dy$. For flow over the mountain, (2) is used to replace $w_b$. We assume that the north–south scale $L_y$ is half the meridional scale of the mountain complex while the east–west scale $L_x$ is that of the planetary waves. Consequently, $L_x \gg L_y$ and the main balance is

$$\psi' \beta \sim \frac{\int \hat{u} \frac{\partial h}{H}}{\hat{u} \beta \frac{\partial h}{H}}$$

so that

$$\psi'_{\text{over}} \sim \frac{\int \hat{u} \frac{h}{\beta}}{H}$$

For flow around the mountain (3b) applies, and the dominant effect is for cancellation between $w_x$ and $w_E$ and thus $u'_x \sim -\hat{u}_y$, as must occur on a pressure surface inside the mountain in the $\sigma$-coordinate models.
Thus flow will tend to go over for \( h < h_c \) and around for \( h > h_c \). Choosing typical values for midlatitudes and \( H \sim 7 \text{ km} \), then \( h_c \sim L_y \times 1.1 \times 10^{-2} \), which for \( L_y = 900 \text{ km} \) gives \( h_c \sim 1 \text{ km} \). For the Himalayan complex at 35°N and \( L_y \sim 1000 \text{ km} \) then \( h_c \sim 1.5 \text{ km} \).

For synoptic scales, with \( L_x \sim L_y = L \) the first term in (5) becomes more important and the resulting scaling produces \( h_c = \text{Ro}H \) where Ro is the Rossby number. Since a typical value of \( \text{Ro} \sim 0.15 \), then \( h_c \sim 1 \text{ km} \). This scaling argument captures the essence of that by Phillips (1963) in which he noted that cancellation between \( w_y \) and \( w_x \) in (3c) is essential if the large-scale flow is to remain quasigeostrophic.

Equation (10) is independent of \( \bar{u} \) but has intuitive appeal by incorporating the meridional scale of the mountain. As the meridional scale increases, so the critical height also must increase before the flow will tend to go around rather than over. For the Rockies, the mean height is \( \sim 1 \text{ km} \) when resolved with planetary waves 1–4, and Chen and Trenberth (1988a,b) found that the flow mostly tended to go over rather than around, presumably because of the large meridional extent of the north–south oriented Rockies, i.e., much larger \( L_y \sim 1500 \text{ km} \) and thus \( h_c \sim 1500 \text{ m} \). For the Himalayas, however, \( h \sim 3710 \text{ m} \) so that \( h \gg h_c \) and the flow should prefer to go around. For the Greenland plateau, \( L_y \sim 850 \text{ km} \) but the higher latitude effects give \( h \sim 340 \text{ m} \), so that the flow should also tend to go around rather than over. The results of these scaling arguments are consistent with the simulations of Chen and Trenberth (1988a).

In this paper the height of the Himalayas is systematically varied while keeping the configuration fixed in order to more accurately document the applicability of these scaling arguments.

3. The model

The planetary wave model to be used was described in detail in Chen and Trenberth (1988a). It is formulated in \( p \) coordinates using the linear balance set of equations with spherical geometry but is symmetric about the equator. The model is spectrally truncated using spherical harmonics to four zonal waves and to mode 15 meridionally. There are 11 levels in the vertical at which the vorticity equation is applied with the top level at 5 mb. The model is linearized about a realistic basic state taken from mean January observed conditions. All parameters, such as damping coefficients and static stability, are the same as given in Chen and Trenberth (1988a).

This model is forced only by the orography from the eastern hemisphere, which is dominated by the Himalayan complex, truncated at four zonal waves and 15 meridional modes, see Fig. 1. The maximum height of the orography is 3706 m and, owing to the spectral

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1 The atmosphere will respond with a minimum perturbation in order to produce the smallest perturbation kinetic energy.
truncation, there are relatively small orographic waves in the rest of the domain of order a few hundred meters. This configuration is kept constant in all experiments. In Fig. 1 and in most figures, the outer border is at 20°N since the perturbations are all small farther south (in Fig. 1 the maximum not shown is ~67 m at 10°N, 90°E).

The LBC is applied using the wind at 850 mb which is at ~1500 m at 35°N. This appears to be the most appropriate level overall and is about 40% of the height of the highest peak when resolved with only planetary-scale waves. It would be most realistic to change the level at which the LBC is applied as the mountain elevation is changed, but this would make the results more difficult to interpret. We have performed experiments in which this level is altered while keeping the orography fixed. The results indicate only small changes in the pattern of the planetary wave response, as expected from (10) in which \( h \) is independent of \( \bar{u} \), but the amplitude tends to increase or decrease according to the value of \( \bar{u} \) at that level. The specified \( \bar{u} \) at 35°N changes linearly from 4.41 m s\(^{-1}\) at 900 mb to 9.8 m s\(^{-1}\) at 700 mb.

Thus, in all experiments the LBC has been applied at 850 mb. Therefore if linear theory is to be valid, there should be a linear relationship between the planetary wave response and the mountain height.

One concern over a model such as ours is that the so-called nonlinear terms are included in the LBC (1) but not in the model dynamics. The most important nonlinear terms not included are those arising from perturbation heat fluxes. When the thermodynamic equation is applied at the lower boundary

\[
v \cdot \nabla \Phi_p = -\sigma \omega_k = \sigma \rho_0 g v \cdot \nabla h
\]

where \( v = k \times \nabla \psi \), and \( \Phi_p = -RT/p \).

Therefore

\[
v \cdot \nabla (\Phi_p - \sigma \rho_0 gh) = 0
\]

(11)

at the lower boundary and thus \( \Phi_p - \sigma \rho_0 gh \) is conserved along streamlines. When (11) is expanded it becomes

\[
\bar{u} \frac{\partial}{\partial x} \Phi_p' + \bar{v} \frac{\partial}{\partial y} \Phi_p' + \bar{u} \frac{\partial \Psi_p'}{\partial x} + \bar{v} \frac{\partial \Psi_p'}{\partial y} = \sigma \rho_0 g \left( \bar{u} \frac{\partial h}{\partial x} + \bar{u} \frac{\partial h}{\partial x} + \bar{v} \frac{\partial h}{\partial y} \right).
\]

(12)

200MB \( u_r' + \bar{u} \)

**Fig. 6.** The total westerly wind component at 200 mb resulting from the standard eastern orography, in m s\(^{-1}\).
Tung (1983) notes that when the linearized equations are used and $\bar{u}$ is constant (so that $\partial \Phi_p / \partial y = 0$), only the first terms on each side of (12) are retained so that

$$
\bar{u} \frac{\partial}{\partial x} (\Phi_p - \sigma \rho_0 gh) = 0.
$$

(13)

Thus the constraint is satisfied. However, as noted earlier, the omitted terms on the right hand side (RHS) are not second order and are important for finite orography so that (13) becomes a very inaccurate approximation to (12).

In contrast, in our model, all of the RHS of (12) is retained but the third and fourth terms on the LHS are neglected, so that the constraint is violated. Nevertheless, for small orography, all the nonlinear terms are small so that the extra terms do not cause a problem. For large orography there is strong cancellation on the RHS (e.g. see Fig. 2) and, for very large orography, the RHS $\to 0$. The perturbations remain modest and the second order perturbation terms on the LHS are small (about a factor of 5 less than the individual terms on the RHS, Chen and Trenberth 1988a). Consequently,

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**Fig. 7.** The 900 mb $Z'$ resulting from eastern orography multiplied by factors $F = 0.25, 0.5, 1.0$ and 2.0.

The contour interval is 5, 10, 20 and 20 gpm.
although the constraint is not retained exactly, the overall result is much more accurate than the linear approximation. We therefore believe that the model is a significant improvement over those with a linear LBC formulation although there is scope for further improvements, perhaps using iterative techniques. All of these aspects will be documented and reexamined in detail using the results of the experiments in this paper.

4. Results

a. Standard Himalayas

The response of the model to the standard Himalayas is described first in some detail. Figure 2 shows $\omega_{z}$, $\omega_{e}$ and the total, $\omega_{t}$, corresponding to the LBC w components in (3c). Only $\omega_{z}$ is the externally imposed orographic forcing. As a consequence of the internal dynamics in the model, the flow has adjusted to set up eddy components that give rise to the $\omega_{e}$ component which is seen to be roughly opposite to $\omega_{z}$. The strong cancellation, which arises from (7), results in an $\omega_{e}$ that is therefore much smaller than either component, and is indicative of the marked tendency for the total flow to go around rather than over the mountain complex (see also Fig. 11, presented later). Similar results were found when all orography was included (Chen and Trenberth 1988a).

The vertical structure of the $\omega$ field is quite coherent. Figure 3 shows $\omega$ at 400 mb and reveals three centers near 30°N, 65°E (down), 45°N, 70°E (up), and over China (down) which are linked closely to the corresponding surface centers in $\omega_{s}$ in Fig. 2. Further centers (e.g. southeast of Japan) are present downstream. The nature of the $\omega$ field and its link to $\omega_{e}$ emphasizes they are not to be regarded as forcing the wave response but are part of the response. The $\omega$ is that field, acting through the static stability term in the thermodynamic equation and the divergence term in the vorticity equation, that maintains the thermal wind balance which would otherwise be upset by thermal and vorticity advections. Only $\omega_{z}$ can be regarded as external forcing in this model.

The tropospheric planetary wave response is illustrated in Fig. 4 for the perturbation geopotential heights at 1000, 500, 200 and 50 mb. Figure 5 shows the corresponding cross sections along 25°, 45° and 60°N. Near the surface, maximum response is in the vicinity of the orography with the Siberian high located near its observed location with the observed low center to the south. Maximum values occur farther downstream with height. At 200 mb it is clear that the main wave train propagates equatorward as well as downstream and upward. The extra high center near the British Isles at 200 mb probably stems from the fact that the orography includes a representation of all the eastern hemisphere including Europe and Africa, not just the Himalayas, and combined with the spectral truncation in the orography (Fig. 1), this produced a secondary mountain 552 m high just south of Spain. Note that propagation into the stratosphere occurs only at high latitudes so that at 50 mb the waves have significant amplitude only north of 45°N. This was also found in Chen and Trenberth (1988b).

The eastern hemisphere orographic forcing has resulted in surprisingly realistic features in the flow at 200 mb, that although weak are observed. These are manifested in the zonal mean wind field as jets at 30°N in the vicinity of 30°–60°E and over East Asia, as shown in Fig. 6.

b. Variable Himalayas

Six experiments have been carried out in which the only change is that the heights of all the mountains in Fig. 1 are multiplied by the factors $F = 0.1, 0.25, 0.5, 0.75, 1.0$ and $2.0$. The heights of the highest Himalayan peak therefore, as resolved with four zonal waves and 15 meridional waves, are 371, 927, 1853, 2780, 3706 and 7412 m, respectively. The only external forcing of the model is felt through the $\omega_{e}$ term, see Fig. 2, which will similarly change linearly by the factor $F$. However,

![Fig. 9. The 900 mb Z' for the main high and low centers as a function of mountain height factor.](image-url)
based upon the arguments in section 2, a linear response should be found only for factors up to \( F = 0.25 \), and beyond that, nonlinearities should become more pronounced in the wave response as the flow increasingly goes around the mountain complex. As will be seen, this is indeed the case. Accordingly, we have chosen to mainly present results from the cases with \( F = 0.25, 0.5, 1.0 \) and 2.0.

Figure 7 shows several 900 mb \( Z' \) fields and Figs. 8 and 9 summarize the results for 900 mb \( Z' \) for all cases. Note the changing contour interval in Fig. 7. There are two primary features in the geopotential height response in Fig. 7, a high (H) and a low (L), which for \( F = 0.25 \) are located on the upslope and downslope, respectively. But the location and intensity of the high and low change as \( F \) is increased. Figure 8 shows the locus and intensity of these centers and the magnitude alone is graphed in Fig. 9. The latter indicates a fairly linear response for \( F \leq 0.5 \) but with the amplitude of the response leveling off as \( F \) is increased beyond \( F \sim 1 \). However, it is important to note that not only is the magnitude of the response nonlinear, but also that

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**Fig. 10.** The 200 mb \( \psi' \) fields for \( F = 0.25, 0.5, 1.0 \) and 2.0. Note that the outer boundary is at the equator. The contour interval is 12.5, 25, 50 and 50 m² s⁻¹.
there are shifts in the location of the centers, see Fig. 8. The high moves from the upslope position for $F = 0.25$ to a location north of the Himalayas for higher mountains, into a position similar to that observed for the Siberian High. Doubling the current mountain height ($F = 2.0$) actually changes the response very little.

At 200 mb we have chosen to present the perturbation streamfunction $\psi'$ on charts that extend to the equator in order that the low latitude perturbations may be better distinguished. At $F = 0.25$ a distinctive wavetrain is present with centers indicating propagation downstream and equatorwards. But the two dominant centers are east of the main orographic feature. In wavenumber domain, for $F = 0.25$, maximum amplitudes for $Z'$ occur at $45^\circ N$ for wave 1 (17 m) and $30^\circ$–$35^\circ N$ for waves 2, 3 and 4 (all 8 to 11 m amplitude). As the factor $F$ is increased, the main change at 200

FIG. 11. The total 850 mb geopotential height in gpm for the cases $F = 0.25$ and $F = 1.0$. For the latter, the 1 km orography is stippled.

FIG. 12. The $\omega_E$ and $\omega_b$ for the case where $F = 0.25$. The contour interval is $0.2 \times 10^{-4}$ mb s$^{-1}$.
mb is a poleward shift in the response by 10°-20° latitude and a more global coverage, with significant perturbations both upstream and downstream (Fig. 10). For \( F > 0.5 \) and for the maximum amplitude in \( Z' \), wave 1 is centered at 55°N, wave 2 moves to \( \sim 60°N \), wave 3 is at \( \sim 50°N \) and wave 4 is centered at 45°N. Thus for \( F = 1.0 \), the respective maximum amplitudes near 200 mb are 57, 30, 53 and 23 m for waves 1-4. Relative to expectations based upon linear theory the biggest change is in wave 2 which is much weaker and farther north. On the other hand, wave 3 has become much more prominent.

In order to help account for these changes, we examine what is happening in the LBC in more detail. Figure 11 shows the total flow at 850 mb, which is the level of the flow that enters into the LBC, for \( F = 0.25 \) and \( F = 1.0 \). The 1 km height contour of the orography is stippled on the latter. Clearly, for \( F = 0.25 \) the perturbation is small and the flow is going over the mountain. In contrast, for \( F = 1.0 \), the perturbation is large and the flow has a much greater tendency to go around the high orography.

In order to examine quantitatively the extent to which the flow is going around, Fig. 12 shows \( \omega_E \) and \( \omega_z \) [see Eq. (3)] for \( F = 0.25 \). This is to be contrasted with the corresponding parameters for \( F = 1.0 \) in Fig. 2. For \( F = 0.25 \), \( \omega_z \) is simply 0.25 times that in Fig. 2. Thus Fig. 12 reveals that \( \omega_E \ll \omega_z \) so that \( \omega_b \sim \omega_z \) for this case. In contrast, \( \omega_E \sim -\omega_z \) in Fig. 2 and \( \omega_b \) is a fairly small residual for \( F = 1.0 \).

There are two main centers of vertical motion, up (U) and down (D), of similar magnitude, for both \( \omega_E \) and \( \omega_z \). Figure 13 shows the locus and intensities of the upward and downward motions from \( \omega_E \) as a function of \( F \). Note the significant movement of these centers of vertical motion as \( F \) changes. For small \( F, \omega_E \) grows in magnitude as the square of \( F \), as expected, but for \( F \sim 1 \) the increase in \( \omega_E \) becomes more nearly linear with \( F \), following that in \( \omega_z \).

To see this relation more clearly, we have averaged the magnitudes of the two main centers of vertical motion, and plotted the ratio \( \omega_E/\omega_z \) in Fig. 14. Also shown is the magnitude of the maximum net upward motion center \( -\omega_b \). As expected the ratio \( \omega_E/\omega_z \) in Fig. 14 is small when \( F \) is small. It reaches a value of 0.5 when \( F \sim 0.37 \) and is close to unity for \( F \approx 1.0 \). As \( F \) increases beyond 1, the cancellation between \( \omega_E \) and \( \omega_z \) becomes so great that the maximum \( \omega_z \) perturbation actually decreases although the mountain is higher! It seems reasonable to consider linear theory applicable for \( F \leq 0.25 \). For \( F < 0.37 \) the flow is preferentially going over the orography. However, for \( F > 0.37 \) the component around the orography is at least comparable in size and for \( F \approx 1.0 \) the "over" component becomes relatively small.

c. Nonlinear terms

Two nonlinear terms of interest are those associated with the zonal mean poleward momentum and heat fluxes by the waves. These fluxes together make up the Eliassen–Palm (E–P) flux of wave activity (Chen and Trenberth 1988a,b), and, through the divergence of the E–P flux, give the wave driving of the zonal mean flow. Once again, the changes in the fluxes as the height of the orography is increased are quite different than would be expected from linear theory. To illustrate this we present \( \nabla^T \) in Fig. 15 for the two cases \( F = 0.25 \) and \( F = 1.0 \). Note that for \( F < 0.5 \) the maximum poleward heat flux (or vertical wave flux) is near 35°N, at the latitude of the maximum orography. But for \( F > 0.5 \) the maximum shifts to \( \sim 55°N \), close to where the maximum is observed for the Northern Hemisphere stationary wave poleward heat flux. The \( u \nabla^T \) for \( F = 1.0 \) (not shown) also has a similar pattern to that observed but it is also much weaker with a maximum.

![Fig. 14. The ratios (left scale) of maximum values of \( \omega_E/\omega_z \) and \( \nabla^T/\omega_z \) at 850 mb as a function of the mountain height ratio \( F \). Also given is \( -\omega_b \) at the main upward motion center (scale at right) in 10^{-4} \text{ mb s}^{-1}.](image-url)
of 6 m² s⁻². The E-P flux confirms the upward and equatorward flow of wave activity south of 60°N.

As noted in section 3, one concern with a model such as ours is the importance of the linearization and the size of the neglected nonlinear terms. Consequently, in Fig. 16 we present \( -\mathbf{\nabla} \cdot \mathbf{v} \cdot \nabla T' \) at 850 mb, which is the only nonlinear term affecting the simulated planetary waves of any significance. This quantity is proportional to the sum of the third and fourth terms in Eq. (12) and if multiplied by \( R/\alpha \pi \approx 2 \times 10^{-4} \) the units become mb s⁻¹ so that they can be directly compared with the \( \omega \) terms. Perhaps not unexpectedly by now, as \( F \) increases, the pattern of \( -\mathbf{\nabla} \cdot \mathbf{v} \cdot \nabla T' \) changes substantially (Fig. 16). Since this is a nonlinear term, it should be expected from linear theory that its amplitude would increase as \( F^2 \) and that it should become highly significant. This is not the case at all. The ratio of the mean magnitude of the two main centers of \( -\mathbf{\nabla} \cdot \mathbf{v} \cdot \nabla T' \), appropriately scaled, to those of \( \omega \), is shown in Fig. 14. Instead of growing linearly with \( F \) as expected, the ratio reaches a maximum for \( F \approx 0.6 \), and for no \( F \) does the ratio reach 0.2 so that the neglected nonlinear terms in all cases are indeed relatively small compared with the external forcing.

On the other hand, the \( \mathbf{v} \cdot \nabla T' \) term is not so small relative to \( \omega \), for large \( F \). The ratio is \( \approx 10\% \) for \( F = 0.25 \) but increases to \( \approx 50\% \) for \( F = 1 \). Since it would be possible to emulate the results of our model by substituting an externally imposed \( \omega \), in place of our nonlinear LBC, the above result might be interpreted to mean that the nonlinear heat flux terms would profoundly affect the solution for large \( F \), in which case they should not be neglected. However, this is not true for our model since only \( \omega \) is externally imposed. Instead, it would be possible to include the \( \mathbf{v} \cdot \nabla T' \) term as an extra “external forcing” term added onto the \( \omega \) term and solve for the wave solutions again. Such a process can be iterated and, since the new term is relatively small (Fig. 14), the new solution should not change much from what we already have and the process would be expected to converge.

It is important to recognize that, in our model, the external orographic forcing arises through a basic state westerly flow impinging on the orography and this sets up the planetary waves. The fact that the net orographic forcing \( \langle \omega \rangle \) differs greatly from that imposed \( \langle \omega \rangle \) is a consequence of the planetary wave response and it is therefore not correct to interpret \( \omega \) as an externally imposed forcing for the planetary waves. It can only be regarded as the orographic forcing if the whole fluid (i.e. the solution) is specified as a “basic state” but then it is no longer valid to add other effects, such as the perturbation heat flux or diabatic heating, to the simulation since the “basic state” would change.

We conclude that the evidence in Figs. 14 and 16 supports both the scaling arguments in section 2 and the discussion in section 3 concerning the importance of the nonlinear terms. The latter helps provide an a posteriori justification for the model. Although we have shown that the \( \mathbf{v} \cdot \nabla T' \) term is relatively small, it would have a modest impact so that the quantitative results given above would no longer be exact.

5. Discussion and conclusions

In this paper we have shown how the model flow changes in response to increases in the height of the Himalayan mountain complex. The results shown in the previous section clearly demonstrate the importance of the additional “eddy” terms in the LBC, and
amplitudes of the forced planetary waves increase at a rate much less than expected from linear theory. Moreover, instead of the maximum response occurring at the latitude of the mountain (35°N) it is shifted poleward to ~55°N consistent with the observed location of the wintertime stationary waves in the Northern Hemisphere.

In analyzing the model results we have computed the largest nonlinear terms neglected in the model and shown a posteriori that they are indeed relatively small. This occurs because of the cancellation between \( \omega_e \) and \( \omega_b \) in the LBC, as discussed in sections 2 and 3, so that \( \omega_b \) is but a small residual. It provides justification for using the model for the purpose of obtaining qualitatively correct simulations of the planetary wave response to Northern Hemisphere stationary forcing. Nevertheless, it is desirable to include the heat flux terms into the model if possible. In the future, we plan to compute the nonlinear terms and use them as additional forcings for the model, which will then need to be iterated to produce a total solution.

The guidelines developed in section 2 for the critical mountain height beyond which the nonlinear aspects of the solution become important appear to be quite reasonable. The departures from the linear solution can be neglected for \( F \leq 0.25 \) but not for \( F \geq 0.5 \). For our model, the flow around the mountain becomes dominant for \( F > 0.37 \) or when then planetary-scale mountain height becomes ~1370 m. In view of the limitations in the model, in practice we expect this value to be in the range 1 to 1.5 km. However, the appropriate formula for the critical height \( h_c \) for this to occur, as given by (10), is a function of latitude and the meridional scale of the orography as resolved on the planetary scale. Thus it is primarily the Himalayas and the Greenland Plateau in the Northern Hemisphere and Antarctica in the Southern Hemisphere where the preferred tendency for the flow to go around rather than over should be manifested for present day conditions. The effects are more equivocal for the Rockies. But such effects should have been considerable over North America and even Europe during the last glacial maximum when ice sheets reaching up to 3 km thick were present.

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REFERENCES


