Slow Instabilities in Tropical Ocean Basin–Global Atmosphere Models

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ABSTRACT

The effect of ocean boundaries on instability in coupled ocean-atmosphere models is determined. Eigenvalues and eigenvectors are calculated for coupled systems featuring an ocean basin bounded zonally by a flat continent. The atmosphere is periodic zonally about the globe. The oceanic and atmospheric dynamics are both represented by linear shallow water equations on the equatorial $\beta$-plane. The calculation involves latitudinal series expansion and longitudinal finite differencing.

Certain of the modes (i.e., eigenvectors) have amplitudes that grow with time, the growth being a direct result of the ocean–atmosphere coupling. Comparison is made to modes previously determined for the theoretically simpler case where the ocean is zonally unbounded. Growing modes in the bounded ocean case correspond in aspects of behavior and structure to particular modes of appropriate wavelength in the unbounded ocean case. The basic mechanism of instability is the same in both cases. Growing modes in the bounded ocean case feature wavelike disturbances that propagate slowly across the ocean basin. The direction of propagation and period of the oscillation are very sensitive to the values prescribed for oceanic thermodynamic coefficients. The period is set by the length of time that the coupled disturbance takes to propagate across the basin, and, for a 15 000 km wide basin, ranges from about a year to many years. Dependence of modal behavior on other parameters is also documented. Instability occurs for a greater range of parameters, and growth rates are larger when the ocean basin is wide (e.g., 15 000 km) than when it is narrow. The zonal width of the continent has little effect on modal behavior. The Kelvin and symmetric low-latitude Rossby components of the oceanic and atmospheric motion fields are of primary importance in modal growth.

1. Introduction

A major stimulus to tropical climate research is provided by the recurrent El Niño and associated atmospheric anomalies (Rassmusson and Carpenter, 1982). El Niño occurs every few years as part of the irregular “El Niño–Southern Oscillation” (ENSO) cycle; the severe economic consequences associated with the particularly strong 1982–83 El Niño are documented in Barber and Chavez (1983) and Canby (1984). It now appears almost certain that the occurrence of the ENSO cycle is a result of ocean–atmosphere coupling (e.g., see Sarachik, 1987, for a recent review). Consequently, there is increasing interest in the development and analysis of coupled ocean–atmosphere models, so as to understand the essential mechanisms of the cycle, and perhaps ultimately to forecast El Niño onset.

One class of models are nonlinear dynamical models of the coupled ocean–atmosphere (e.g., Anderson and McCreary, 1985; Cane and Zebiak, 1985), which are used in attempts to simulate the complete ENSO cycle. Anderson and McCreary (1985) find that their numerical solutions gradually become dominated by periodic disturbances that propagate slowly eastward. The numerical solutions of Cane and Zebiak (1985) display irregular oscillations that have some similarities to the observed ENSO cycle; their disturbances develop and decay approximately in situ in the central and eastern part of their ocean basin. However, mechanisms underlying the oscillations in these models remain somewhat unclear.

Another class of models are linear dynamical models of the coupled ocean–atmosphere (e.g., Rennick, 1983; Philander et al., 1984; Gill, 1985). While linear models cannot hope to simulate aspects of the ENSO cycle as well as a nonlinear model, diagnosis of their behavior is generally more straightforward. The extent to which linear models will be helpful in understanding ENSO is not yet clear. It may be possible to simulate crudely the overall ENSO cycle in terms of a slow, self-excited oscillation in a linear model. However, it seems likely that nonlinearities are very important in some phases of the cycle, e.g., in limiting the growth of anomalous sea-surface temperature (SST) or latent heating during El Niño. Linear dynamics may be more appropriate at some phases of the cycle than others. In particular, linear models might help clarify the mechanism of

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ocean–atmosphere instability suspected of being important in the rapid development of anomalies during the early stages of El Niño (e.g., Bjerknes, 1969; Philander, 1983; Cane, 1986) and possibly at other times in the cycle (Philander, 1985). Small initial perturbations were found to grow rapidly in Philander’s et al. (1984) model, clearly indicating the presence of an instability. Self-excited disturbances also developed in the models of Rennick (1983) and Gill (1985). In other respects, however, the linear model results are in conflict. The slowly growing disturbances found by Rennick and Gill propagate westward and have structure quite different to Philander’s et al. rapidly growing disturbance, which propagates eastward. In all five aforementioned coupled models, the system is initially perturbed and the subsequent motion is computed numerically without external forcing. Thus, motions internal to the coupled system behave very differently from model to model in terms of growth rate, structure, and speed and direction of propagation. The models differ most radically in the form of the SST equation, suggesting that the parameterization of ocean thermodynamics is crucial to model behavior.

In a first step towards a systematic analysis of coupled models, Hirst (1986a) determined free modes for several linear coupled models. The models differed in the form of the SST equation. The most general thermodynamic case had highly parameterized representations of three processes thought to be most important in the development/decay of equatorial SST anomalies during the ENSO cycle; namely, zonal temperature advection, modulation of vertical temperature advection by the depth of the thermocline, and damping of SST anomalies by, for example, anomalous surface heat flux (e.g., Gill, 1983; Zebiak and Cane, 1983; Harrison and Schopf, 1984). Thermodynamic limits of this most general case correspond to, first, the models of Gill (1985) and Rennick (1983), second, the model of Philander et al. (1984), and, third, a model that resembles a linearized version of the Anderson-McCreary (1985) model. In each model, one or more oceanic equatorial waves were shown to be destabilized by ocean–atmosphere coupling. An oceanic wave is destabilized when associated SST perturbations induce (via atmospheric latent heating) wind fields, which reinforced the oceanic wave through atmosphere-to-ocean transfer of kinetic energy by wind stress. In this context, many of the previously mentioned conflicts in results were explained as a consequence of the different ocean thermodynamics assumed by different authors. The study of free modes offered many insights into the behavior of instabilities in linear coupled models and the role of ocean thermodynamics in model behavior.

One major limitation of Hirst’s (1986a) analysis was that the oceans and atmospheres considered had no horizontal boundaries; perturbations were assumed to be zonally periodic. Real world oceans have eastern and western boundaries, as do the oceans in the previously mentioned numerical coupled models. The apparent agreement between Hirst (1986a) and other modelers suggests that results for the unbounded ocean–atmosphere case are relevant to the bounded ocean case, to some degree. However, the extent to which this is so has not been clearly demonstrated; reflection of wave energy at the rigid ocean boundaries may complicate the situation considerably. The present paper continues the systematic development of the theory of coupled ocean–atmosphere models, through an analysis of instabilities in coupled models featuring a zonally bounded ocean. A particular purpose of this paper is to demonstrate the extent to which the free mode results of Hirst (1986a) extend to the more realistic zonally bounded ocean case.

The paper is organized as follows. The model equations are presented in section 2, along with a brief review and additional interpretation of key results from Hirst (1986a). The method for computing modes when the ocean is zonally bounded is outlined in section 3a; it is much more complicated than that for the unbounded ocean–atmosphere and requires that some simplifying assumptions be made. A simple change in the boundary conditions permits the method of section 3a to calculate modes for the unbounded ocean–atmosphere, and a preliminary study (section 3b) tests results obtained thereby against key results from Hirst (1986a). Section 4 presents results for the bounded ocean basin. Two configurations, illustrated in Fig. 1, are analyzed in detail. First, the zonal circulation of the ocean is broken by imposing a barrier of negligible zonal width across the equatorial duct (Fig. 1b). Second, the barrier is broadened to form a continent wider than the ocean (Fig. 1c); this configuration is similar to that used by most other investigators in their numerical calculations. The continent is flat, and the latent heating perturbation is zero over the continent.

2. Coupled model

The coupled system consists of a single baroclinic mode atmosphere and an ocean mixed layer interacting via (i) wind stress and (ii) atmospheric heating parameterized in terms of the SST perturbation field. Both oceanic and atmospheric motions are governed by the linear shallow water equations on the equatorial β-plane. The basic model equations are essentially as in Hirst (1986a); the only differences include adoption of the “longwave approximation” for atmospheric and oceanic motions and the assumption that the atmosphere is in equilibrium with the underlying SST pattern (e.g., Gill, 1985). The above approximations are made for computational expediency, but are appropriate for motions of large zonal extent and low frequency (Cane and Sarachik, 1981; Gill, 1980). Unstable modes for the unbounded ocean–atmosphere are shown in section 3b to be little affected by these assumptions. The significant alteration from the system
of Hirst (1986a) involves imposition of rigid east and west boundaries on the ocean. The ocean basin extends from \( x = 0 \) to \( x = x_E \) (where \( x \) is the zonal coordinate) and the continent, if any, extends from \( x = x_E \) to \( x = x_C \).

### a. The atmosphere

The atmospheric response to a middle tropospheric latent-heat source \( Q(x, y, t) \) is given by

\[
\begin{align*}
\beta y V + \phi_x + A U &= 0 \quad (1a) \\
\beta y U + \phi_y &= 0 \quad (1b) \\
c_0^2 (U_x + V_y) + B \phi &= Q. \quad (1c)
\end{align*}
\]

Independent variables \( (x, y, t) \) refer to eastward and northward displacement \( (y = 0 \) at equator) and time. Dependent variables \( U, V \) and \( \phi \) are the lower tropospheric zonal and meridional wind and geopotential height, respectively (Gill, 1980). Perturbations are subject to Rayleigh friction and Newtonian cooling. Values and meanings for model coefficients are listed in Table 1. The Rayleigh friction coefficient, \( A \), over the continent is set equal to that over the ocean.

The atmospheric period around the globe. Thus the zonal atmospheric boundary conditions are

\[
(U, V, \phi)|_{x=x_C} = (U, V, \phi)|_{x=0}.
\]

### b. Ocean dynamics

The basic ocean model consists of a mixed layer overlying a deep, cold, quasi-motionless layer. An infinitesimally thin thermocline separates the two ocean layers. Motions and temperature are assumed constant with depth within the mixed layer. The dynamical equations for the mixed layer, subject to the longwave approximation, are

\[
\begin{align*}
u_t - \beta y v + \alpha g \Delta \bar{T} h_x + au &= \tau^y/(\rho_0 \bar{h}) \quad (2a) \\
\beta y u + \alpha g \Delta \bar{T} h_y &= 0 \quad (2b) \\
h_t + \bar{h}(u_x + v_y) + bh &= 0. \quad (2c)
\end{align*}
\]

The dependent variables include perturbations in thermocline depth \( \bar{h} \), zonal \( (u) \) and meridional \( (v) \) motion, and zonal \( (\tau^y) \) wind stress; \( \Delta \bar{T} \) is a typical mixed layer—deep ocean temperature difference. The background state features a thermocline at the uniform depth \( \bar{h} \).

The characteristic speed in the ocean model, \( c_o = (\alpha g \bar{h} \Delta \bar{T})^{1/2} \), is generally set at 1.4 m s\(^{-1}\) (following Philander et al., 1984); correspondingly, we take \( \bar{h} = 70 \) m and \( \Delta \bar{T} = 14 \) K. This value of \( c_o \) implies an oceanic Rossby deformation radius \( \lambda_o = (c_o/\beta)^{1/2} = 250 \) km, which is much smaller than that for the atmosphere \( (\lambda_a = (c_a/\beta)^{1/2} = 1170 \) km).

Oceanic boundary conditions at the rigid eastern and western boundaries are as recommended by Cane and Sarachik (1981) for low frequency motions [i.e., of period > 150 days or \( \text{Re}(\sigma) < 0.1(c_o/\beta)^{1/2} \)] in ocean models where the long wave approximation is employed:

\[
u( x = x_E, y, t ) = 0 \quad (3a)
\]

### Table 1. Values for basic parameters in Models I-IV.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( 5.0 \times 10^{-4} \text{ s}^{-1} )</td>
<td>Atmospheric Rayleigh friction coefficient</td>
</tr>
<tr>
<td>( B )</td>
<td>( 5.0 \times 10^{-4} \text{ s}^{-1} )</td>
<td>Atmospheric Newtonian cooling coefficient</td>
</tr>
<tr>
<td>( a )</td>
<td>( 1.16 \times 10^{-7} \text{ s}^{-1} )</td>
<td>Oceanic Rayleigh friction coefficient</td>
</tr>
<tr>
<td>( b )</td>
<td>( 1.16 \times 10^{-7} \text{ s}^{-1} )</td>
<td>Damping coefficient for ( h ) field</td>
</tr>
<tr>
<td>( d )</td>
<td>( 1.16 \times 10^{-7} \text{ s}^{-1} )</td>
<td>Oceanic Newtonian cooling coefficient</td>
</tr>
<tr>
<td>( c_o )</td>
<td>( 1.4 \text{ m s}^{-1} )</td>
<td>Oceanic gravity wave speed parameter</td>
</tr>
<tr>
<td>( c_o' )</td>
<td>( 30.0 \text{ m s}^{-1} )</td>
<td>Atmospheric gravity wave speed parameter</td>
</tr>
<tr>
<td>( K_s )</td>
<td>( 1.6 \times 10^{-7} \text{ s}^{-1} )</td>
<td>Wind stress coupling coefficient</td>
</tr>
<tr>
<td>( K_Q )</td>
<td>( 3.5 \times 10^{-3} \text{ m}^2 \text{ s}^{-3} \text{ K}^{-1} )</td>
<td>Atmospheric heating coupling coefficient</td>
</tr>
<tr>
<td>( T_o ) (II, III)</td>
<td>( -5.0 \times 10^{-7} \text{ K m}^{-1} )</td>
<td>Background zonal SST gradient</td>
</tr>
<tr>
<td>( K_T ) (III, IV)</td>
<td>( 3.5 \times 10^{-8} \text{ K m}^{-1} \text{ s}^{-1} )</td>
<td>Vertical advection thermal forcing coefficient</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 2.2 \times 10^{-11} \text{ m}^2 \text{ s}^{-4} \text{ K}^{-1} )</td>
<td>Meridional gradient of Coriolis parameter</td>
</tr>
<tr>
<td>( g )</td>
<td>( 9.8 \text{ m s}^{-1} )</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>( 2.0 \times 10^{-4} \text{ K}^{-1} )</td>
<td>Thermal expansion coefficient of water</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>( 1.0 \times 10^3 \text{ kg m}^{-3} )</td>
<td>Density of ocean water</td>
</tr>
</tbody>
</table>

* Exceptions are specified in the text or legends.
\[ \int_{-\infty}^{+\infty} u(x = 0, y, t) dy = 0. \]  

(3b)

Here \( x = 0 \) at the western boundary and \( x = x_E \) at the eastern boundary of the ocean. The integral in (9b) parameterizes the important role that short Rossby waves play in mass transport along the western boundary (e.g., Pedlosky, 1969).

c. Ocean thermodynamics

In Hirst (1986a), special attention was paid to the effect of ocean thermodynamics parameterization on modal behavior. To this end, four models were defined, each distinguished by the form of the SST equation. One model (Model III) features the most general SST equation to be considered:

\[ T_t + \bar{T}_x u - K_T h + dT = 0. \]  

(4c)

Here \( T \) is the SST perturbation and \( \bar{T}_x \) is a background zonal temperature gradient. Hirst (1986a) linearized more complete equations for mixed layer thermodynamics to obtain formulae for the coefficients \( K_T \) and \( d \) in terms of background state variables. The \( K_T \) and \( d \) increase rapidly as \( h \) decreases. Representative values of \( K_T \) and \( d \) are given in Table 1; however, modes are computed for a range of \( K_T \) and \( d \).

SST equations for the other three models of Hirst (1986a) are special limits of (4c). Model I features the limit where \( K_T \) and \( d \) are very large, whence (4c) reduces to

\[ T = (K_T d^{-1}) h. \]  

(4a)

This limit represents a local balance; it is the thermal relation used by Philander et al. (1984).

Model II features the thermal advection limit, i.e.,

\[ T_t + \bar{T}_x u + dT = 0. \]  

(4b)

SST perturbations (\( T \)) result solely from advection by the perturbation current. Equation (4b) has the same form as the SST equation of Rennick (1983) and Gill (1985).

Finally, Model IV features local thermal forcing only, i.e.,

\[ T_t - K_T h + dT = 0. \]  

(4d)

Equation (4d) is similar to a linearization of Anderson and McCreary's (1985) thermal equation in their zonally unbounded ocean case.

Most results presented in this paper are determined using the Model IV SST equation (4d). The reason for concentrating on Model IV is that the ocean thermodynamics are easier to interpret that for Model III, while a prognostic ocean thermal equation is retained (unlike for Model I). Key results for Models II and III thermodynamics are also presented. Results for Model I are inferred by taking the limit of Model IV as \( K_T \) and \( d \) are made large.

d. Wind stress and latent heating parameterization

Perturbation wind stress and latent heating are parameterized in terms of atmospheric variables according to the traditional simple assumptions (e.g., Philander et al., 1984):

\[ \tau^z/(\rho_0 h) = K_S U \]  

(5)

\[ Q = K_Q T. \]  

(6)

Representative values for the coupling coefficients \( K_Q \) and \( K_S \) were estimated in Hirst (1986a) and are listed in Table 1. In order to facilitate comparison of results to those of Hirst (1986a) and other authors, all model coefficients are here set constant with longitude.

e. Motions in the uncoupled ocean and atmosphere

Before discussing motions in the coupled ocean–atmosphere system, it is helpful to review free motions in the separate ocean and atmosphere. Matsuno (1966) thoroughly documented the free equatorial modes permitted by the full linear shallow water equations in the absence of forcing (\( Q = \tau^z = \tau^r = 0 \)); the free modes include the eastward propagating Kelvin wave, a series (numbered \( n = 1, 2, 3, \ldots \)) of westward propagating Rossby waves, the mixed Rossby–gravity wave and two series of inertia–gravity waves. Long (short) Rossby waves have westward (eastward) group velocity. The long-wave approximation filters out all inertia–gravity and short Rossby waves, but the Kelvin and long Rossby waves are not significantly affected (Gill, 1980). The phase and group velocities of atmospheric waves are very much greater than those of corresponding oceanic waves, i.e., \( c_o \gg c_a \). In the case \( a = b \neq 0 \) (\( A = B \neq 0 \)), all free waves in the ocean (atmosphere) decay with an \( e \)-folding decay time of \( a^{-1} \) (\( A^{-1} \)). Atmospheric free waves decay at a much faster rate than oceanic free waves, i.e., \( A \gg a \). The case \( a > b \geq 0 \) is discussed by Mofjeld (1981).

The free modes for uncoupled ocean models that feature a prognostic thermal equation (e.g., as in Models II–IV) include a set of thermal modes as well as the dynamical modes discussed here. The oceanic thermal modes of Models II–IV have zero frequency, decay rate equal to \( d \), and a structure that may be written

\[ (u, v, h) = 0; \quad T(x, y, t) = \text{Re} \{ T(y) e^{i k x} \} e^{-\alpha t} \]

where \( T(y) \) is an arbitrary bounded function of \( y \) and \( k \) is an arbitrary wavenumber. The above facts about uncoupled thermal modes will aid interpretation of modal behavior for the coupled Models III and IV (section 2f).

f. Review of previous coupled mode results

Previous determinations of modes for coupled ocean–atmosphere models include those by Lau (1981), Yamagata (1985), Hirst (1985, 1986a) and Rennick.
and Haney (1986). All calculations were for a zonally unbounded ocean–atmosphere and solutions were sought of the form

\[ U(x, y, t), \ldots, T(x, y, t) \]

\[ = \text{Re}\{U(y), \ldots, T(y)e^{ik(x-y)}\} \quad (7) \]

where \( k \) is the prescribed wavenumber, \( \text{Im}\{\sigma\} \) and \( \text{Re}\{\sigma\} \) are the calculated growth rate and frequency, and \( \{U(y), \ldots, T(y)\} \) is the calculated meridional structure. By the use of (7), all \( x \) and \( t \) derivatives are transformed into simple algebraic terms. Lau performed pioneering analytical calculations for a model similar to Model I in the single case where Coriolis accelerations and meridional motions are neglected. Yamagata used a shooting method to calculate some modes for a model of the same form as Model I. Renwick and Haney performed analytical calculations as per Lau (1981) for models similar to I and II.

Hirst (1985, 1986a) used matrix methods to calculate modes for Models I–IV, and found that just one mode in each model had a significant growth rate \( |\text{Im}\{\sigma\}| \sim (50 \text{ days}^{-1}) \) to (200 days) \(^{-1}\) when model coefficients had representative values (Table I). In Model I, ocean–atmosphere coupling destabilizes the oceanic Kelvin wave. In Model II, the coupling dampens the Kelvin wave but destabilizes the westward-propagating \( n = 1 \) Rossby wave. In Model III, which features both local (as in Model I) and advective (as in Model II) ocean thermal processes, the unstable mode features very low phase speed and oceanic structure intermediate between a Kelvin and an \( n = 1 \) Rossby wave. However, at very small values of the coupling coefficients, the behavior and structure of the Model III unstable mode comes to resemble that of an oceanic thermal mode. Finally, the unstable mode in Model IV propagates slowly eastward and has structure somewhat resembling that of the Model III unstable mode; it likewise stems from an oceanic thermal mode.

Coupling was found to most affect motions of low frequency and large zonal wavelength because, first, only at such wavelengths and frequencies can both oceanic perturbations excite a strong atmospheric response and vice versa (see also Hirst, 1986b, chap. 7) and, second, only at low frequencies is there sufficient time for motions to generate significant SST perturbations in models featuring a prognostic SST equation.

g. Energetics

Interpretation of modal behavior is aided by analysis of perturbation energetics. We begin by expanding on the analysis for modes in the unbounded ocean–atmosphere by Yamagata (1985) and Hirst (1986a). Equations for quantities proportional to the atmospheric available potential energy \( (E_a^p) \), the atmospheric perturbation kinetic energy \( (E_a^K) \), the oceanic available potential energy \( (E_o^p) \) as determined from the full dynamical equations (of Hirst 1986a) are

\[ \langle E_a^p \rangle_t + \langle \phi U_x + \phi V_y \rangle + 2B\langle E_a^p \rangle = \langle Q\phi \rangle/c_a^2 \quad (8a) \]

\[ \langle E_a^K \rangle_t + \langle U\phi_x + V\phi_y \rangle + 2A_1\langle E_a^p \rangle = -\langle \tau U + \tau V \rangle/M_a \quad (8b) \]

\[ \langle E_o^K \rangle_t + \alpha g\Delta T\langle uh_x + vh_y \rangle + 2b\langle E_o^p \rangle = 0 \quad (8d) \]

where \( E_a^p = \phi^2/2c_a^2 \), \( E_a^K = (U^2 + V^2)/2 \), \( E_o^K = (u^2 + v^2)/2 \) and \( E_o^p = \alpha g\Delta T h^2/2H \). Angle brackets indicate an area average over a zonal wavelength and from \( y = -\infty \) to \( y = +\infty \). For the derivation of (8b), the atmospheric Rayleigh friction term \( A(U, V) \) was replaced by the terms \( A_1(U, V) + (\tau^*, \tau^*)/M_a \). The term \( (\tau^*, \tau^*)/M_a \) represents surface wind-stress drag, where \( M_a \) is the mass per unit area of the atmospheric layer. The term \( A_1(U, V) \) represents other dissipative processes in the troposphere (e.g., cumulus friction).

The energetics for an unstable mode is summarized in Fig. 2. The ultimate source term is \( \langle Q\phi \rangle/c_a^2 \); this term builds \( E_a^p \) when there is a negative correlation between atmospheric latent heating \( (Q) \) and surface geopotential \( (\phi) \). The \( E_a^p \) is then converted to \( E_a^K \) through the terms \( \langle \phi U_x + \phi V_y \rangle \) in (8a) and \( \langle U\phi_x + V\phi_y \rangle \) in (8b) \( \langle \phi U_x + \phi V_y \rangle = -\langle U\phi_x + V\phi_y \rangle \) when \( V \to 0 \) as \( y \to \pm\infty \), as in Hirst (1986a). If the resulting surface wind stress \( (\tau^*, \tau^*) \) is generally in the same direction as the ocean current \( (u, v) \), then kinetic energy is transferred from the atmosphere to the ocean via wind stress work (Yamagata, 1985). Finally, \( E_a^K \) is converted to \( E_o^p \). The growing mode is thus fed by release of latent heat supplied by the background state. Further, a mode will grow only if \( \langle Q\phi \rangle < 0 \) and \( \langle Uu + Vv \rangle > 0 \).

![Fig. 2. Schematic diagram of energy transfers for an unstable mode in the unbounded ocean case.](image-url)
The energetics of low frequency modes when the ocean is zonally bounded are essentially the same as that shown in Fig. 2. Energy lost/gained at the rigid boundaries is generally small compared to the conversions indicated in Fig. 2.

3. Method for a bounded ocean basin

The presence of east and west oceanic boundaries considerably complicates the problem of finding modes for the coupled system. The reflection of oceanic wave energy at the rigid boundaries implies that solutions will not in general be sinusoidal in \( x \). Hence it is unlikely that the \( x \) derivatives can be simply transformed into algebraic terms, as was possible for the unbounded ocean–atmosphere. Instead, it is necessary to use finite differences (or series) in \( x \) as well as in \( y \). The resulting number of algebraic equations is much larger than for the unbounded ocean–atmosphere. Adoption of the long-wave and equilibrium-atmosphere assumptions greatly reduce the number of equations and hence the size of the matrix which must be solved to obtain the eigenvectors. A method to compute modes for the bounded ocean case is developed in section 3a. The numerical convergence of this method is examined in the Appendix. The new method is tested against results from Hirst (1986a) in section 3b, after boundary conditions consistent with an unbounded ocean-atmosphere are adopted.

a. Development

The method developed here allows the computation of “symmetric” modes [i.e., those featuring \( u, h, T, U \) and \( \phi \) fields symmetric about the equator] for the bounded ocean case. It involves meridional Hermite function expansion of dependent variables and zonal finite differencing. The method could be adapted for computation of antisymmetric modes, but results of Hirst (1986a) indicate that the faster growing modes are symmetric. Antisymmetric modes are not further considered in this paper.

We begin by nondimensionalizing Eqs. (1), (2) and (4) using a length scale of \( \lambda_o = (c_o/\beta)^{1/2} = 2.5 \times 10^3 \text{m} \) and a time scale of \( \mu_o = (c_o h_o)^{1/2} = 1.8 \times 10^3 \text{s} \). The nondimensional dependent variables, here denoted by asterisks, are

\[
(U^* \quad V^* \quad u^* \quad v^*) = \left( U, V, u, v \right)/c_o,
\]

\[
(\phi^* \quad h^* \quad T^*) = \left( \phi/c_o^2, h/h_o, T/T_o \right).
\]

All variables and coefficients in the following are non-dimensional and asterisks are dropped.

Modes are sought of the form

\[
[U(x, y, t), \cdots, T(x, y, t)]
\]

\[
= \text{Re}\{(U(x, y), \cdots, T(x, y))e^{-i\omega t}\}, \quad (9)
\]

(cf. section 2f). On substituting (9) into (1), (2) and (4c), the equations for the coupled system become

\[
AU - yV + \phi_x = 0 \quad (10a)
\]

\[
K_q T + c^2 U_x + c^2 V_y + A\phi = 0 \quad (10b)
\]

\[
yU + \phi_y = 0 \quad (10c)
\]

\[
u u - y v + h_x - K_S U = i\sigma u \quad (10d)
\]

\[
u_x + v_y + ah = i\sigma h \quad (10e)
\]

\[
y v + h_y = 0 \quad (10f)
\]

\[
\dot{T} + K_T h + \Delta T = i\sigma T \quad (10g)
\]

where \( c = c_o/c_o \). Note that derivatives in both \( x \) and \( y \) remain.

The problem now is to determine the eigenvalues \( \sigma \) and eigenvectors \( \{u(x, y), h(x, y), T(x, y)\} \). Hence \( \text{Re}\{\sigma\} \) (henceforth written \( \sigma_h \)) gives the frequency, \( \text{Im}\{\sigma\} \) (henceforth written \( \sigma_i \)) gives the growth rate and \( \{u, h, T\} \) gives the horizontal structure of a mode. The associated fields \( U, V, \phi, v \) may be reconstructed from the eigenvector by use of the diagnostic relations (10a–c) and (10f).

We turn first to the oceanic dynamical equations (10d–f), and introduce the new variables (Gill, 1980)

\[
s = h + u, \quad r = h - u. \quad (11)
\]

Then \( s, r, U \) and \( T \) are expressed as sums of symmetric Hermite functions, e.g.,

\[
s(x, y) = \sum_{n=0, 2, \cdots} s_n(x)\psi_n(y)
\]

and \( v \) as a sum of antisymmetric Hermite functions. Upon using the following properties of Hermite functions

\[
\psi_{n+1} - y\psi_n = -(2n + 2)^{1/2}\psi_{n+1},
\]

\[
\psi_{n+1} + y\psi_n = (2n)^{1/2}\psi_{n-1}
\]

and the fact that Hermite functions are orthogonal, we obtain from (10d–f)

\[
s_{nx} + as_n - K_S U_n - (2n)^{1/2}\nu_n = i\sigma s_n
\]

\[
n = 0, 2, 4, \cdots \quad (12a)
\]

1 Any function bounded on \( -\infty < y < +\infty \) can be expressed as a sum of the Hermite functions

\[
\psi_n(y) = \pi^{-1/4}(2n!)^{-1/2} H_n(y) \exp(-y^2/2), \quad n = 0, 1, 2, \cdots
\]

where \( H_n(y) \) are the Hermite polynomials (e.g., Abramowitz and Stegun, 1964, chap. 19, 22).
-r_{nx} + ar_{n} + K_{s} U_{n} + (2n + 2)^{1/2} v_{n+1} = i \sigma r_{n} \quad n = 2, 4, 6, \ldots \quad (12b)\\
r_{n} = [(n + 2)/(n + 1)]^{1/2} s_{n+2} \quad n = 0, 2, 4, \ldots \quad (12c)

Then, after eliminating the \( t_{n} \) and \( r_{n} \),

\[
\frac{-1}{(2n - 1)} s_{nx} + a s_{n} - \frac{(n - 1)}{(2n - 1)} K_{s} U_{n} + \frac{(n^{2} - n)^{1/2}}{(2n - 1)} \times K_{s} U_{n-2} = i \sigma s_{n}, \quad n = 2, 4, 6, \ldots .
\quad (13b)
\]

Hence the oceanic dynamical equations (10d–f) have been replaced by an infinite set of equations possessing derivatives only with respect to \( x \) and constant coefficients. The equations are coupled through the wind components. The results from Hirst (1986a) suggest that motions associated with the most unstable modes are confined rather closely to the equator, so the set (13) can probably be truncated at some suitably large \( n \). For the results in section 4, Eq. (13) is truncated after the eighth (i.e., \( n = 14 \)) equation. Tests for convergence (see the Appendix) support such a truncation.

The functions \( s_{n}(x) \) are related to the long-wave components of the oceanic motion field; namely, the value of \( s_{0} \) gives the amplitude and relative phase of the oceanic Kelvin component, \( s_{2} \), similarly for the \( n = 1 \) Rossby component, \( s_{4} \) similarly for the \( n = 3 \) Rossby component, etc.

We now turn to the boundary conditions for the ocean basin. On using (11), the orthogonality of Hermite functions and (12c), the eastern boundary condition (3a) yields

\[
s_{n}(x_{E}) = \left[ \left( \frac{n - 1}{n} \right) \cdot \left( \frac{n - 3}{n - 2} \right) \cdots \frac{1}{2} \right]^{1/2} s_{0}(x_{E}).
\quad (14)
\]

Thus, at the eastern boundary, the values of all the \( s_{n} \) for \( n \geq 2 \) are specified by the value of \( s_{0} \). The value of \( s_{0} \) is undetermined, however. This is consistent with an incident Kelvin wave of arbitrary amplitude being reflected as a series of long Rossby waves at the eastern boundary. Next, on using (11), (12c) and the property

\[
\int_{+\infty}^{\infty} \varphi_{n}(y) dy = (n + 1)/n^{1/2} \int_{-\infty}^{+\infty} \varphi_{n-2}(y) dy,
\]

the western boundary condition (3b) yields

\[
s_{0}(x = 0) = \sum_{n = 2, 3, \ldots} \left[ \left( \frac{n - 1}{n} \right) \cdot \left( \frac{n - 3}{n - 2} \right) \cdots \frac{1}{2} \right]^{1/2} (n - 1)^{-1} \cdot s_{n}(0).
\quad (15)
\]

Thus, at the western boundary, the value of \( s_{0} \) is determined as a sum over the values of the \( s_{n} \), \( n > 2 \). This is consistent with a set of incident long Rossby waves being reflected at the western boundary as, ultimately, a Kelvin wave. The Eqs. (13) for the \( s_{n}(x) \) are essentially a set of first-order ordinary differential equations in \( x \) and there are now as many boundary conditions as equations, as required for solution. The equations for the \( s_{n}(x) \) are coupled via the atmosphere and at the boundaries. We now turn briefly to the ocean thermal equation (10g). On using (11), Hermite function orthogonality and (12c), (10g) yields

\[
dT_{n} + [(\bar{T}_{x} - K_{T})/2] \cdot s_{n} = [(\bar{T}_{x} + K_{T})/2] \cdot [(n + 2)/
\]

\[(n + 1)]^{1/2} \cdot s_{n+2} = i \sigma T_{n}, \quad n = 0, 2, 4, \ldots .
\quad (16)
\]

If the dynamical equations (13) are truncated at a certain \( n = N \), then the set of thermal equations (16) are truncated at \( n = N - 2 \), in order to ensure that both the \( s_{n} \) and the \( s_{n+2} \) terms in (16) are represented for each \( T_{n} \). Representation of both terms is important since they generally partially cancel one another.

The oceanic fields are represented at \( J \) discrete points in \( x \), including east and west boundary points. The points have separation \( \Delta x \), and the derivatives in (13) are represented by centered finite differences. For example, at the point \( x_{j} \), (13a) becomes

\[
as_{n}(x_{j}) + [s_{n}(x_{j+1}) - s_{n}(x_{j-1})]/(2\Delta x)
\]

\[= -K_{s} U_{0}(x_{j}) = i \sigma s_{0}(x_{j}).
\]

Uncentered differencing is used at the easternmost (for \( s_{0} \) or westernmost (for \( s_{n} \), \( n \geq 2 \)) step, in order to determine \( s_{n} \) at that boundary.

The \( x \) finite-differencing scheme does not produce spurious growth or decay of the ocean components (\( s_{n} \)) but it does cause higher index components (\( n \geq 6 \)) to propagate too fast (Hirst, 1986b). Such errors in propagation appear unimportant, since the amplitudes of the errant \( s_{n} \) decrease very rapidly away from a forcing region [the \( e \)-folding decay distance for \( s_{0} \) is about 1000 km, or one grid space]. Since forcing on the ocean associated with ocean–atmosphere instability generally has a much broader zonal scale, higher order Rossby modes are important only in the local response of the ocean to the atmosphere. Indeed, results of section 4 are found to be almost unchanged when \( x \) derivatives for \( \{s_{n}, n > 6\} \) in (13) are deleted (i.e., when propagation of the higher \( n \) components is prevented).

On using finite differencing in \( x \) and boundary conditions (14) and (15), (13) and (16) become linear algebraic equations and may be written

\[
M_{3} \delta^{2} + M_{4} U = i \sigma \delta
\]

\[
M_{5} \delta^{2} + M_{6} T = i \sigma T
\]

where \( \delta \) is a vector of length \((J - 1) \times (M)\) which contains the \( s_{n}(x_{j}) \), \( U \) is a vector of length \((J - 1) \times (M)\) which contains the \( U_{0}(x_{j}) \), \( T \) is a vector of length \((J) \times (M - 1)\) which contains the \( T_{n}(x_{j}) \) [where \( M \) is the number of equations remaining from the set (13)]. \( M_{3}, M_{4}, M_{5}, M_{6} \) are sparse banded matrices.
The oceanic equations are now algebraic, but the equations for oceanic motion (17a) include forcing terms involving zonal wind. The remaining task is to express algebraically the zonal winds (U) in terms of SST (T). This task is relatively simple provided length and time scales suitable for the atmosphere are used. First, make the following definitions:

\[ y^* = c^{-1/2}y, \quad U^* = U/c, \quad V^* = V/c, \quad \phi^* = \phi/c^2 \]
\[
S = \phi^* + U^*, \quad R = \phi^* - U^* \tag{18}
\]

and express the variables R, S, T, U*, V* as sums of Hermite functions whose length scales are appropriate to atmospheric motion, e.g.,

\[ S(x, y) = \sum_{m=0,2,\cdots}^\infty S_m(x)\psi_m(y^*). \]

Then an analysis similar to that for the ocean yields [from (10a–c)] the following equations:

\[ R_m = [(m + 2)(m + 1)]^{1/2}S_{m+2} \tag{19} \]
\[ S_{0x} + (Ac^{-1})S_0 = -(KQC^{-3})T_y \tag{20a} \]
\[ S_{mx} - (2m - 1)Ac^{-1}S_m = (m - 1)KQC^{-3}T_m \]
\[ + (m^2 - m)1/2KQC^{-3}T_{m-2}, \quad m = 2, 4, 6, \cdots \tag{20b} \]

The \( S_m(x) \) give the amplitude and phase of the Kelvin \((m = 0)\) and long Rossby \((m \geq 2)\) components of the atmospheric motion fields. Here, the set (20) is truncated after M equations. The set consists of linear ordinary differential equations in \( x \), forced by terms involving the SST field. On using the periodic atmospheric boundary condition \( [S_m(x = 0) = S_m(x = xc)] \), each of the set (20) can be solved to obtain expressions for the \( S_m \) which involve longitudinal integrals of the \( T_m \). In order to compute the \( S_m(x) \) from a \( T \) field given only at grid points \( x = x_j, j = 1, \cdots, J \), the ocean temperature is assumed to vary stepwise with \( x \). Then the aforementioned integrals may be evaluated analytically, leaving an algebraic equation for each \( S_m(x) \) in terms of the \( T_m(x_j) \). The set of such equations may be written

\[ S = M_7 \cdot T^* \tag{21} \]

where \( S \) is a \((J - 1)\ast(M)\) vector containing the \( S_m(x) \), \( T^* \) is a \((J)\ast(M)\) vector containing the \( T_m(x) \) and \( M_7 \) is a block banded matrix. A vector, \( U^* \), containing the wind components \( U_m^*(x) \), can easily be found from & using (19), (18) and the orthogonality of Hermite functions. Hence, from (21),

\[ U^* = M_8 \cdot T^* \tag{22} \]

where \( M_8 \) is a block banded matrix.

The problem now remains to project the oceanic temperature field onto the atmospheric Hermite functions, \( \psi_m(y^*) \). To find \( T_m(x_j) \) in terms of \( \psi_m(x_j) \), equate the respective Hermite function series, truncate after some \( m \) and \( n \) (for chapter 4 results, truncation is after \( m = 14 \) and \( n = 12 \)), multiply through by \( \psi_m(y^*) \), then integrate from \( y = -\infty \) to \( y = +\infty \) and use the orthogonality of the \( \psi_m(y^*) \) to get

\[ T^* = P \cdot T \tag{23} \]

where \( P \) is a dense matrix. Some small-scale meridional structure in \( T \) will be lost upon making the projection (23), because of the \( n \) truncation.

Also, the atmospheric winds must be projected onto the oceanic Hermite functions. A procedure analogous to the derivation of (23) yields the projection equation

\[ U = C^{1/2}P^T \cdot U^* \tag{24} \]

Equations (22), (23) and (24) give

\[ U = M_9 \cdot T \tag{25} \]

where \( M_9 \) is a dense matrix. The zonal wind \((U)\) is thereby expressed in terms of SST \((T)\). The oceanic equations (17) can now be written as a homogeneous set of linear algebraic equations, which may be written in the standard eigenproblem form

\[ M \cdot \xi = i\sigma \xi \tag{26} \]

where \( M \) is a real square matrix with \( J \times (M - 1) \) \((J - 1) \times M \) rows and \( \xi = (\xi^T, T^T)^T \). Equation (26) can be solved for the eigenvalues \( \sigma \) and eigenvectors \( \xi \) by the QR method, e.g., Smith et al., 1976; Atmospheric and oceanic fields for each mode can then be reconstructed from the respective \( \xi \) and \( T \).

The results presented in sections 3b and 4 are generally computed using \( M = 8 \) and \( J = 15 \). By truncating after eight meridional components, oceanic and atmospheric perturbations poleward of about \((y = 5.0 \text{ units})\) and \((y = 23 \text{ units})\), respectively, are neglected. The grid spacing is \( \Delta x = 4.24 \text{ units (1070 km)} \) when \( J = 15 \). The numerical convergence of eigensolutions is discussed in the Appendix.

b. Unbounded ocean–atmosphere

The method derived in section 3a may be used to calculate modes in an unbounded ocean–atmosphere, if the boundary conditions (18) and (19) are replaced by periodic boundary conditions, i.e.,

\[ s_n(x = 0) = s_n(x = xc), \quad n = 0, 2, 4, \cdots \]

This method of computing modes for an unbounded ocean–atmosphere is inefficient when compared to the methods of Hirst (1986a), because of the finite differencing in \( x \). However, computation of modes in an unbounded ocean–atmosphere by the method of section 3a serves as a test prior to other applications.

Modes for an unbounded ocean–atmosphere computed by the method of Hirst (1986a; section 3a) and by the method of section 3a above are compared here. The method of Hirst (1986a) uses the complete linear shallow water equations and explicitly assumes sinu-
sooidal zonal structure; it is henceforth referred to as method A. The method of section 3a, which uses (1), (2) and (4) and determines the zonal structure by finite differencing, is referred to as Method B. Method B cannot be used with confidence to find the modes in the *bounded* ocean case if it is unable to satisfactorily find modes for the unbounded ocean–atmosphere.

Eigensolutions are computed by method B for Model IV when \( x_E = 15000 \text{ km} \) (59.3 units). They are found to include modes possessing a zonal wavelength of \( x_E \) or an integer fraction thereof \( (x_E/2, x_E/3, \text{etc.}) \). Some modes correspond to those reported for Model IV in Hirst (1986a). In particular, there is a series of destabilized modes. Attributes of the three most destabilized modes are listed in Table 2. The most unstable mode has a wavelength of 15 000 km (i.e., \( k = 0.106 \)) and propagates slowly eastward; it corresponds to the Model IV unstable mode of Hirst (1986a) of the same wavelength. Other destabilized modes have wavelengths of 7500 and 5000 km (i.e., 2 and 3 cycles over the interval), and correspond to shorter wave versions of the Model IV unstable mode. These shorter wave modes are shown in Table 2 to have small negative growth rates, i.e., they are decaying slowly. However, they are decaying at a rate much slower than that of oceanic motions in the absence of coupling [whence \( \sigma_t = -a = -d = -2.1 \times 10^{-2} \text{ units} = -1.16 \times 10^{-7} \text{ s}^{-1} \)]. Thus these modes are destabilized by the coupling, but not by enough to overcome entirely the prescribed oceanic damping. Also listed in Table 2 are attributes of the Model IV unstable mode calculated at appropriate wavelengths by method A, of Hirst (1986a). Clearly the two methods here give results in reasonable agreement.

Method B has been extensively tested versus the results of Hirst (1986a) (Hirst, 1986b). Growth/decay rates and frequencies for modes of Models II, III and IV were computed at wavelengths ranging from 2000 to 40 000 km by setting the value of \( x_E \) equal to the desired wavelength and considering only the fundamental modes (i.e., modes displaying one zonal cycle over the interval). Calculated growth rates of the various modes agree closely with those of Hirst (1986a) (to within \( \pm 0.2 \times 10^{-2} \text{ units} \)) at all wavelengths. Frequencies are likewise in close agreement, except that frequencies for shorter \( n = 1 \) Rossby modes (wavelength < 5000 km) are too high, as expected from use of the longwave approximation. Structures for the unstable modes in Models II, III, and IV were computed, and are similar to the corresponding structures shown in Hirst (1986a). Minor discrepancies between the atmospheric divergence fields are linked to the incomplete response of the atmosphere to the \( T \) field in method B, resulting in part from the truncation after eight atmospheric wave components.

Overall, results for an unbounded ocean–atmosphere computed by method B, of section 3a, are very similar to those presented in Hirst (1986a). Thus, the method passes a test necessary prior to its application to a bounded ocean–global atmosphere. Further, the favorable comparison supports the validity of the equilibrium atmosphere and long-wave assumptions common in coupled models (e.g., Gill, 1985; Anderson and McCreary, 1985; Cane and Zebiak, 1985). Finally, the close agreement between the growth rates computed here and those presented previously implies that the instability depends only on the Kelvin and low \( n \) Rossby components of the motion fields. Full motion fields associated with the unstable mode may contain inertia–gravity and high \( n \) Rossby components, but such components are not essential for the growth of the mode.

### Table 2: Major attributes of destabilized modes computed for the unbounded ocean–atmosphere and for the bounded ocean–global atmosphere, using Model IV thermodynamics. The ocean interval or basin is 15 000 km wide zonally. Units are nondimensional; \( \sigma_t \) of \( 1 \times 10^{-2} \) = (220 days)\(^{-1}\); \( \sigma_R \) of \( 1 \times 10^{-2} \) = period of 1300 days. For uncoupled oceanic motion, \( \sigma_f = -2.1 \times 10^{-2} \text{ units} \).

<table>
<thead>
<tr>
<th>Ocean barrier type</th>
<th>Mode</th>
<th>( \sigma_t ) (10(^{-2}) units)</th>
<th>( \sigma_R ) (10(^{-2}) units)</th>
<th>Direction of propagation</th>
<th>Cycles over interval/across basin</th>
</tr>
</thead>
<tbody>
<tr>
<td>None (Via method A)</td>
<td>Unstable</td>
<td>+0.99</td>
<td>2.81</td>
<td>Eastward</td>
<td>1</td>
</tr>
<tr>
<td>None (Via method B)</td>
<td>Unstable</td>
<td>+0.66</td>
<td>1.19</td>
<td>Eastward</td>
<td>3</td>
</tr>
<tr>
<td>Thin barrier</td>
<td>U1</td>
<td>+0.99</td>
<td>2.56</td>
<td>Eastward</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>U2</td>
<td>−0.14</td>
<td>1.84</td>
<td>Eastward</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>U3</td>
<td>−0.69</td>
<td>1.19</td>
<td>Eastward</td>
<td>3</td>
</tr>
<tr>
<td>Wide continent</td>
<td>U1</td>
<td>+0.83</td>
<td>2.40</td>
<td>Eastward</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>U2</td>
<td>+0.10</td>
<td>1.89</td>
<td>Eastward</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>U3</td>
<td>−0.55</td>
<td>1.31</td>
<td>Eastward</td>
<td>3</td>
</tr>
</tbody>
</table>
from $x = 0$ to $x = x_E$, the continent extends from $x = x_E$ to $x = x_C$. The effects of the continent are to (1) break the zonal ocean circulation and (2) permit propagation of atmospheric energy away from ocean and its dissipation over land. The width of the ocean basin is $x_E = 15,000$ km; unless otherwise indicated, this value roughly corresponds to that of the Pacific Ocean. Oceanic boundary conditions (9) apply. The atmosphere is periodic, thus $(U, V, \phi)|_{x=x_E} = (U, V, \phi)|_{x=0}$. Except where otherwise noted, the continental width is set at $22,500$ km, so that the globe has a realistic circumference of $37,500$ km. This situation is referred to as the “wide continent” case, and is illustrated in Fig. 1c. The case where the ocean is bounded by a continent of negligible width (i.e., a barrier) has also been extensively analyzed (Hirst, 1986b). Such a case is illustrated in Fig. 1b; and referred to as “the thin barrier case.” The only effect of the barrier is to break the zonal oceanic circulation; thus the thin barrier case represents an intermediate step between the unbounded case of section 3b and the wide continent case. However, results are very similar to those for the wide continent case, and so are not discussed in detail here. Section 4a presents results obtained when the ocean thermodynamics are that of Model IV $(\bar{T_x} = 0, K_T = 3.5 \times 10^{-9}$ K m$^{-1}$ s$^{-1}$); results for Models II and III are summarized in section 4b.

### a. Model IV

The set of eigensolutions for the wide-continent case is found to include a series of destabilized modes, whose major attributes are listed in Table 2. Modes in the series are henceforth referred to as U1, U2, U3, where the numeral indicates the number of (approximate) cycles that the perturbations display across the ocean basin. Each mode strongly resembles a mode in the series of destabilized modes reported for the unbounded case (section 3b). Thus, the U1, U2 and U3 modes apparently correspond closely to the unstable mode in the unbounded ocean–atmosphere at wavelengths of $15,000$, $7500$ and $5000$ km, respectively. The shorter wave modes (U2 and U3) again have small negative growth rates, indicating slow decay. However, the rate of decay is again very much smaller than that for uncoupled oceanic motion (where $\sigma_f = -a = -2 \times 10^{-2}$ units), hence these modes are destabilized by the coupling. Table 2 also shows the similar results obtained for the thin barrier case.

The dependence of $\sigma$ for the U1 mode on various parameters is illustrated in Figs. 3–5. Figure 3 shows that both the growth rate and frequency increase as the coupling coefficients $K_Q K_S$ are increased. Growth rates and frequencies remain a little less than values for the corresponding mode in the unbounded case. Figure 4 shows how $\sigma$ varies as the ocean thermal coefficients $K_T$ and $d$ are increased while $K_T / d$ remains constant. Both the growth rate and frequency increase as the Model I limit (large $K_T$ and $d$) is approached, but growth rate does not increase by as much as in the unbounded ocean model. Increases in $K_T / d$ beyond that in Fig. 4 do not significantly change $\sigma$; the Model I limit has essentially been reached at $K_T = 70 \times 10^{-9}$ K m$^{-1}$ s$^{-1}$ ($6 \times 10^{-5}$ units).

Figure 5 shows that the growth rate decreases as the ocean basin width is decreased, and the frequency does likewise when $x_E < 12,000$ km, in fashion similar to that observed in the unbounded case when wavelength is reduced. Motions of larger zonal scale are more affected by coupling, primarily because the atmospheric response (in terms of wind speed) strengthens as the zonal scale of the forcing (here by SST) increases (Hirst, 1986b, chap. 7). Thus the effect of coupling is weak when the basin is narrow, and the U1 mode behaves more like the stationary, dissipative thermal mode from which it stems (section 2f). All values in Figs. 3–5 are computed by method B, of section 3. Clearly the qualitative behavior of the fundamental unstable mode is not much altered by the presence or absence of the barrier.

The dependence of $\sigma$ on the atmospheric coefficients $c_a$ and $A$ at various basin widths is indicated in Fig. 6. Larger values for $c_a$ and $A$ reduce the strength of coupling, as evidenced by a reduction in growth rate and frequency of the U1 mode. The dependence of growth rate on $c_a$ and $A$ is very similar to that reported for
Models I and II in the unbounded ocean case (Hirst, 1986a). Smaller values of the atmospheric Rossby deformation radius \( R = (\omega/\beta)^{-1/2} \) bring the frequencies and horizontal structures of the atmospheric free waves closer to those of the oceanic free waves, and thereby the response of the atmosphere to the ocean (and vice versa) is enhanced (Matsuno, 1966).

Figure 7 shows the change in growth rate and frequency for the U1 mode as the zonal width of the continent is increased from zero. Growth rate and frequency decrease slightly at first, but cease to change when the width is increased beyond about 22,500 km (90 units). Atmospheric energy propagating across a continent of width 22,500 km is attenuated to such an extent that it can no longer influence the U1 mode. For example, the magnitude of the atmospheric Kelvin component decreases by a factor of 50 from the western to the eastern edge of such a continent. Atmospheric Rossby components suffer much greater attenuation. Thus the mode is unaffected by a further increase in continental width.

The horizontal structure and time evolution of the U1 mode are displayed in Figs. 8–10. Figure 8 shows a time sequence of fields for \( T(x, y, t) \). The T perturbation fields are wavelike in \( x \); weak positive and negative perturbations appear alternatively in the western basin, then each such disturbance grows in magnitude as it propagates eastward. The speed of propagation is about 0.22 units (0.31 m s\(^{-1}\)) or just over one-fifth of that for the uncoupled oceanic Kelvin wave; this is slightly slower than for the corresponding mode in the unbounded case and is reflected in a slightly reduced frequency (Table 2). There is generally one principal positive \( T \) and one principal negative \( T \) disturbance present at any one time. Away from the coasts, the field appears similar to that for the unbounded case (Fig. 16 of Hirst, 1986a). On reaching the eastern boundary, the \( T \) perturbation spreads poleward along the coast, following [via (4d)] the \( h \) perturbation which spreads meridionally in order to maintain the longwave relation (2b) and the boundary condition (3a).

At a given longitude, each disturbance features larger perturbations than its predecessors, owing to the exponential growth factor \( \exp(\sigma_I t) \).

Relationships between the variables are illustrated in Fig. 9 at a time when \( T \) is positive in the central basin. Away from the barrier, the structure appears much as in the unbounded ocean case (Hirst, 1986a). There is clearly a positive correlation between winds and ocean currents, and a negative correlation between atmospheric heating \( \mathcal{Q} \) and surface atmospheric pressure \( (-p_v \beta) \). Thus the two necessary conditions for modal growth (section 2g) are satisfied.

In Fig. 9, \( T \) and \( h \) perturbations have large negative values all along the eastern boundary, following the "impact" of a negative \( h \) perturbation from the west.

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**Fig. 4.** Behavior of growth rate and frequency as \( K_T \) and \( d \) are increased from the "representative" values while \( K_T/d \) remains constant. Curves are for the fundamental unstable mode in the unbounded ocean–atmosphere (solid), the thin oceanic barrier case (dashed) and the wide continent case (dot-dashed); \( K_T \) is in \( 10^{-4} \) K m\(^{-1}\) s\(^{-1}\) and \( d \) is in 10\(^{-3}\) s\(^{-1}\). Otherwise as in Fig. 3.

**Fig. 5.** Growth rate and frequency as functions of interval or basin width \( (x_e) \) for the fundamental unstable mode in the unbounded ocean–atmosphere (solid), the thin oceanic barrier case (dashed) and the wide continent case (dot-dashed). Dashed vertical line indicates \( x_e \) of 15,000 km. Otherwise as in Fig. 3.
The large zonal temperature contrasts at the barrier produce distorted wind and pressure patterns; in particular, there is a very rapid strengthening of the easterly wind westward from the barrier. An analysis of the terms in (1c) at a point one half a grid space to the west of the barrier indicates that zonal divergence ($U_z$) there is about 25 percent greater than local atmospheric forcing ($K_Q T$), the difference is made up for by meridional convergence ($-V_y$). The large zonal divergence is related to the broad meridional extent of the atmospheric forcing along the eastern boundary, which locally suppresses meridional divergence. Westerly winds are apparent to the east of the ocean basin. This zonal outflow has the characteristics of a Kelvin component generated by the strong atmospheric forcing over the far eastern ocean. The zonal outflow dissipates over the continent and does not significantly affect winds over the western ocean.

It may appear from Figs. 8 and 9 that much of the mass associated with a positive $h$ disturbance spreads poleward and is lost to the tropical ocean when the disturbance impacts the eastern ocean boundary. Such is not the case. A Kelvin wave impacting on the boundary has 80 percent of its mass reflected as long Rossby waves of $n < 14$; only the remaining 20 percent is lost to the ocean model. Meridional mass fluxes are generally small compared to the zonal mass fluxes along the equator.

The long-wave approximation is not expected to be valid for the atmosphere in the immediate vicinity of the barrier. Under the more complete equations of Hirst (1986a), the sudden zonal change in temperature (hence atmospheric heating) across the barrier should produce low-frequency short Rossby components of atmospheric motion, together with long Rossby and Kelvin components. The short Rossby waves, eliminated under the long-wave assumption, have very slow eastward group velocity ($c_R < c_{el}/20$) and consequently a very short damping distance of about 300 km, i.e., less than a grid spacing. Thus any effects of short Rossby components will be confined to the immediate vicinity of their generation; in particular, short Rossby components generated at the boundary will dissipate over the continent and not affect the ocean.

It is sometimes of interest to illustrate the time evolution of the unstable mode without the exponential growth component $\exp(\sigma_I t)$, in part because some features of the modal structure are masked by the exponential growth. Also, $[U(x, y), \ldots, T(x, y)]$ and $\sigma_R$ are insensitive while $\sigma_I$ is very sensitive to the uncoupled oceanic damping rate set by $a$ in (2). The time evolution of equatorial fields with the exponential growth suppressed is illustrated in Fig. 10. The weak disturbances originating at the western boundary still grow as they propagate towards the eastern ocean. The frequency of the oscillation in the far western ocean is
nearly independent of the width of the continent (Fig. 7). Thus the timing of initial disturbances can be set by factors other than forcing from the west; the speed at which disturbances extend eastward from the western boundary and continuity of mass imposed by (2c) appear to be important. The eastward propagating disturbances bear some resemblance to those found by Anderson and McCreary (1985) in their bounded ocean case. Zonal wind fields are especially similar. The frequency of the U1 mode is about twice that of Anderson and McCreary's oscillation; in part a result of the disparate values given to atmospheric coefficients (Hirst, 1986a).

Maximum positive values attained by the perturbations during the cycle illustrated in Fig. 10 are listed in Table 3. All values assume a modest 20 m amplitude in \( h \). Calculated motions are nearly zonal as expected after the use of the long-wave assumption. Under the complete linear shallow water equations, oceanic meridional motion would be strong in the immediate vicinity of the boundaries because of short Rossby and coastal Kelvin waves (the effects of which are parameterized in the present model). Table 3 also lists cor-

![Diagram](image_url)

**Fig. 7.** Growth rate and frequency as a function of continental width \((x_c-x_e)\) for the fundamental unstable mode in Models II, III and IV. The width of the ocean basin is set at 59.3 units (15 000 km). Crosses \((\times)\) indicate values for the unstable mode of wavelength 59.3 units in an unbounded ocean-atmosphere.

![Diagram](image_url)

**Fig. 8.** SST \((T)\) fields at indicated times for the U1 mode in the wide continent case. Contours for \( T \) are at 0, 20\%, 40\%, 60\% and 80\% of the largest \( T \) value at the eastern boundary during 50 \( \leq t \) \( \leq 311 \). Dashed contours indicate negative values. Heavy vertical lines indicate positions of ocean boundaries. Units of \( x, y \) and \( t \) are non-dimensional; one unit of time \((t)\) equals 2.1 days, of \( y \) equals 2.3° latitude.

responding information for the fundamental unstable modes of Models II and III.

The time evolution of equatorial fields with the exponential growth included is shown in Fig. 11 for the U1 mode. Each disturbance grows strongly as it crosses the ocean, and achieves greatest magnitude in the far eastern basin.

The equatorial \( T \) field for the U2 mode is shown as a function of time in Fig. 12. Perturbations feature approximately two cycles across the width of the ocean. Disturbances travel eastward across the ocean at a phase speed of about 0.095 units (0.133 m s\(^{-1}\)), almost iden-
tical to that for the unstable mode of wavelength 7500 km in the unbounded ocean case, and slightly less than one-half that for the U1 mode in the wide continent case.

Results for the thin barrier case are similar to those described above, except that perturbations in the western ocean basin are somewhat larger. It appears that atmospheric motion extending over the barrier from eastern-ocean forcing does assist initial development of disturbances in the western basin.

Clearly, the imposition of an oceanic barrier or continent does not greatly alter the structure of the U1 mode from that in the unbounded ocean case, when the ocean thermodynamics is that of Model IV. The effect of imposing a continent when the ocean features Model II or III thermodynamics is described in the next section.

b. Models II and III

Key results obtained for bounded ocean basins using Model II or Model III thermodynamics are illustrated in Figs. 7 and 13–15. The eigensolution sets are found to include series of destabilized modes as per Model IV. The growth rate and frequency of the fundamental Model II unstable mode (Fig. 7) are decreased and increased respectively by imposition of a thin barrier, but both increase slightly as the barrier is broadened into a wide continent. Figure 13a shows that the mode features disturbances propagating westward across the basin, as expected from Hirst (1986a) and in agreement with Rennick (1983) and Gill (1985). Largest $T$ values for the Model II mode are located in the central ocean, in contrast to those for the Model IV mode located in the far eastern ocean (Fig. 10c). This difference is partly a result of the different ocean thermodynamics; the modes for both models have $u$ fields with maxima near the center of the basin (Table 3), but only in Model II is the $T$ field directly determined by the $u$ field.

The fundamental Model III unstable mode is split into two modes upon imposition of a thin barrier (Fig. 14). Both modes are nonoscillatory and have similar growth rate and structure; perturbations display approximately one cycle across the ocean basin. The modes feature disturbances that do not propagate, apparently locked into the ocean geography. When the barrier is broadened to just one unit width, the modes acquire a nonzero (although very small) frequency and in all aspects become identical (apart from a phase shift in time). The frequency of the resulting single mode
increases as the barrier is widened further, but never approaches that in the unbounded ocean case. The growth rate decreases somewhat as the barrier is widened. Figure 13b shows that the mode features disturbances that propagate eastward across the basin, as expected from Hirst (1986a). The rate of propagation is extremely slow. A disturbance takes about five years to travel from \( x = 10 \) units to the eastern boundary, giving a propagation speed \( (c_{\text{prop}} = 0.06 \text{ units} = 0.1 \text{ m s}^{-1}) \) about 30 percent smaller than that for the corresponding mode in the unbounded case. In any case, the \( e \)-folding growth time is about 90 days, so the disturbances can be considered to be developing essentially in situ. Equatorial fields with exponential growth included can give the appearance of a developing basinwide warm event (Fig. 14).

The equatorial \( T \) patterns for the Model II and III modes (Fig. 13) have broader zonal extent than that for the Model IV mode (Fig. 10c), displaying less than one complete cycle across the basin. This difference between the modes is again partly a result of different ocean thermodynamics. The equatorial \( u \) patterns for all the above modes display less than one cycle across the basin, but the \( h \) patterns are less broad zonally and
TABLE 3. Maximum positive perturbations associated with the fundamental unstable mode in each of Models II–IV, when maximum $h = 20$ m. Values are for the wide continent case. Overall exponential growth is neglected. $t$, $x$, and $y$ indicate the time and location of the maximum, in nondimensional units; $P$ is the “lower tropospheric pressure” perturbation, defined $P = \rho_d \phi$.

<table>
<thead>
<tr>
<th>Perturbation</th>
<th>Model II</th>
<th>Model III</th>
<th>Model IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ (K)</td>
<td>1.39</td>
<td>1.03</td>
<td>0.37</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$u$ (m s$^{-1}$)</td>
<td>0.75</td>
<td>0.46</td>
<td>0.29</td>
</tr>
<tr>
<td>$v$ (m s$^{-1}$)</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$U$ (m s$^{-1}$)</td>
<td>2.25</td>
<td>1.96</td>
<td>1.82</td>
</tr>
<tr>
<td>$V$ (m s$^{-1}$)</td>
<td>0.96</td>
<td>0.78</td>
<td>0.36</td>
</tr>
<tr>
<td>$P$ (mb)</td>
<td>0.54</td>
<td>0.40</td>
<td>0.23</td>
</tr>
</tbody>
</table>

Tend to display approximately one cycle across the basin (e.g., compare Figs. 10a and 10c). The dominant thermal forcing for the Model II and III modes is advection (Hirst, 1986a), consequently $T$ patterns most closely follow $u$ and so are zonally broader than that for Model IV, where $T$ follows $h$. The $u$ and $h$ patterns are further discussed in section 5.

Clearly the values prescribed for coefficients in the SST equation (4c) have a major effect on the behavior of the fundamental unstable mode. The period of oscillation is particularly sensitive to the values of the ocean thermal coefficients, since it is set by the time required for the coupled disturbance to propagate across the basin, and since the phase speed of the coupled disturbance is very dependent on these coefficients (Hirst, 1986a). The variation in period and $e$-folding growth time of the fundamental mode as ocean thermal coefficients $T_x$ and $K_T$ are varied first from Model II to Model III values, then from Model III to Model IV values are shown in Fig. 15. In the wide-continent case, the period ranges from 1.5 years for Model II and IV values to many years for intermediate values. Figure 15 also shows results for the unstable mode of 15 000 km wavelength in the unbounded ocean case. The change in behavior is qualitatively similar to that for the wide-continent case, but the range of coefficient values for which the mode has a very long period is shifted towards the Model II limit. However, the period of the unbounded-case mode is sensitive to wavelength; the period decreases with larger wavelength near the Model II limit and increases rapidly with wavelength in the vicinity of standard Model III values (Hirst, 1986a). Periods for the unbounded-case unstable mode

Fig. 11. $x$–$t$ diagram showing time evolution of equatorial SST ($T$) and zonal wind ($U$) for the U1 mode in the wide continent case. Overall exponential growth is included. Otherwise as in Fig. 10.

Fig. 12. $x$–$t$ diagram showing periodic time evolution of the equatorial SST ($T$) field for the U2 mode in the wide continent case. Overall exponential growth is neglected. Otherwise as for Fig. 10.
Fig. 13. $x$-$t$ diagram showing periodic time evolution of the equatorial $T$ field for the fundamental unstable mode of (a) Model II and (b) Model III, in the wide continent case. Overall exponential growth is neglected. Otherwise as for Fig. 10.

of 18 000 km wavelength better resemble those shown for the wide-continent case in Fig. 15; the period for this unbounded-case mode is greater than 10 years over the range $(T_x = -5.0 \times 10^{-7} \text{ K m}^{-1}, K_T = 2.3 \times 10^{-9} \text{ K m}^{-1} \text{ s}^{-1})$ to $(T_x = -4.8 \times 10^{-7} \text{ K m}^{-1}, K_T = 3.5 \times 10^{-9} \text{ K m}^{-1} \text{ s}^{-1})$. Thus, the period of a wide-continent case mode most closely resembles that of an unbounded-case mode with wavelength moderately greater than the basin width, when advection is the dominant ocean thermal forcing. On the other hand, growth rates for the wide-continent case mode are less than those for the 15 000 km unbounded-case mode.

The $e$-folding growth times indicated in Fig. 15 are at least a factor of 2 less than the period. Thus, over a full cycle, the amplitude of the oscillation will increase by at least a factor of 7, and generally by much more. In reality, nonlinearities in the ocean thermodynamics and in the supply of moisture for atmospheric heating would soon limit the growth of any oscillation. The oscillations in the nonlinear Anderson and McCreary (1985) and Cane and Zebiak (1985) models do not continue to grow with time, although the Anderson and McCreary oscillation otherwise somewhat resembles that of Model IV here. It remains to be seen whether the oscillations determined here for linear models are directly related to the oscillations found in nonlinear coupled models.

Fig. 14. As for Fig. 13b but with overall exponential growth included.

Fig. 15. (a) Period and (b) $e$-folding growth time (both in years) of the fundamental unstable mode at points along the $(T_x, K_T)$ path indicated by the thick solid line in (c). in the unbounded ocean case for wavelength 15 000 km (solid) and wide continent case (dotted). $T_x$ is in $10^{-7}$ K m$^{-1}$, $K_T$ is in $10^{-9}$ K m$^{-1}$ s$^{-1}$. Dotted rectangle in (c) indicates range of realistic $T_x$ and $K_T$ according to analysis of Hirst (1986a).
5. Summary and discussion

This paper investigates the properties of instabilities in coupled ocean–atmosphere models that feature a zonally bounded ocean, through determination of the normal modes for the system. Certain modes have amplitudes that grow with time. The growth of these "unstable" modes is a direct result of ocean–atmosphere coupling; the ocean perturbation is built through atmosphere-to-ocean transfer of kinetic energy via wind stress; the wind stress is itself a response (via atmospheric latent heating) to the sea surface temperature pattern associated with the ocean perturbation. No unstable modes are found when coupling coefficients are small. Comparison is made to modes calculated for coupled models that feature a zonally unbounded ocean (Hirst, 1986a). In general, the qualitative behavior of an unstable mode in the bounded ocean case is similar to that of the unstable mode of appropriate wavelength in the corresponding unbounded ocean case. The zonal width of the continent separating the eastern and western oceanic boundaries is of little importance. If there is a mode unstable over a range of wavelengths in the unbounded case, then the mode will be represented as a series of unstable modes in the bounded ocean case. The first, second and third modes in the series generally have growth rate, frequency, direction of propagation and structure similar to that for the unbounded-case unstable mode of wavelength equal to one, one-half and one-third of the basin width, respectively. Thus instability is likely to be present for the bounded ocean case if and only if there are modes in the corresponding unbounded ocean case that are unstable at wavelengths equal to or less than the basin width. Therefore, a relatively cheap calculation for the unbounded ocean–atmosphere provides a good indication of modal behavior to be expected for the bounded ocean.

Many of the results of Hirst (1986a) thus carry over to the bounded ocean case. In particular, motions most affected by coupling have large zonal scale and long period. Consequently, instabilities are more likely to develop when the ocean basin is wide (e.g., a "Pacific Ocean" of width 15,000 km) than when it is narrow. In addition, the behavior and structure of the unstable modes are very sensitive to the parameterization used to calculate the SST perturbation. In the thermal advection limit (Model II), the most unstable mode features disturbances which propagate from east to west across the basin with a period of about 1.5 years, and corresponds to the destabilized oceanic Rossby mode in the unbounded ocean case. In the local thermal forcing limit (Model IV), the most unstable mode features disturbances that propagate from west to east, again with a period of about 1.5 years. When thermodynamics intermediate between the two limits is used (Model III), the most unstable mode may have a period of many years. The growth rates of the unstable modes are quite sensitive to values of the damping, coupling and ocean thermodynamic coefficients, and the ratio of the Rossby deformation radii $\lambda_d/\lambda_s$. The Kelvin and long low-$n$ Rossby components of the oceanic and atmospheric motion fields are of primary importance in modal growth.

The period of oscillation for the coupled ocean basin–atmosphere is set by the time required for by the coupled disturbance to propagate from one side of the basin to the other. Thus, the period is set by strength of coupling and type of ocean thermodynamics, among other things, and has nothing to do with the length of time that it takes for uncoupled ocean waves to propagate across the ocean basin and back. In Model IV the period of the oscillation can be actually increased by decreasing the width of the basin (Fig. 5).

Numerical models based on the Model II, III or IV equations should mimic the present results when run from arbitrary initial conditions. All eigenmodes computed for (at least) the Model IV wide-continent case are found to be linearly independent of one another; thus any arbitrary initial condition can be expressed as a sum of these eigenmodes. The U1 mode will in general be represented; it will soon grow to dominate the perturbation patterns calculated by a numerical model based on (17) and (20). Present result for Models II, III and IV have been corroborated by experiments with numerical models, as discussed here in the Appendix.

Some features of the unstable modes in the bounded ocean case resemble the anomaly patterns observed during El Niño (Rasmusson and Carpenter, 1982; Arkin et al., 1983). In particular, the largest perturbations of SST and zonal wind lie on the equator, and the strongest westerly winds lie west of the largest positive SST perturbations. However, model SST perturbations are confined too closely to the equator, as discussed in Hirst (1986a). Easterly winds to the east of a developing positive SST perturbation are too strong. Meridional winds to the north and south of the SST perturbation are unrealistically weak. Regarding the individual models, the westward propagation of all perturbations associated with the Model II unstable mode is unrealistic. The tendency in the Model IV unstable mode for a large positive SST perturbation to be associated with a large negative SST perturbation at some other longitude is also unrealistic. Perhaps the most realistic appearing unstable mode is that of Model III, the most general thermodynamic case. This last unstable mode may display positive equatorial SST perturbations extending most of the way across the basin; all perturbation fields propagate very slowly eastward.

Results for all the bounded ocean cases are unrealistic in that the calculated wind perturbations are generally largest near the eastern ocean boundary (e.g., Figs. 13 and 18), whereas observed wind perturbations in the equatorial Pacific are generally largest in the central to western part of the basin. This erroneous
feature results from the assumption that the coefficient relating atmospheric heating to SST perturbation \((K_Q)\) is constant with longitude. In reality, atmospheric latent heating is far more sensitive to underlying SST in the western than in the eastern equatorial Pacific. In the east, climatological cold SST and tropospheric subsidence act to suppress convection; small values of \(K_Q\) are appropriate there. The weaker atmospheric forcing would result in weaker winds over the eastern basin even if SST perturbations remained large there (as in reality). The importance of permitting coefficients to vary zonally is indicated.

A feature of the most unstable mode in each model is the tendency for the thermocline depth perturbation \((h)\) field to display approximately one cycle across the ocean basin. It would appear that a deficit of mass (indicated by a negative \(h\)) in one sector must be at least partly balanced by a surplus of mass (positive \(h\)) elsewhere along the equator, as a result of continuity imposed by the \(h\) equation (2c) combined with the dominance of equatorial zonal mass transport over meridional mass transport characteristic of unstable modes in both the bounded and unbounded ocean cases. (Wind stress directly forces only the zonal mass transport, meridional mass transport arises in response to zonal mass redistribution so as to keep the zonal motion geostrophic). The SST field in Model IV follows the \(h\) field and therefore displays a similar pattern. Otherwise, the SST fields in Models II and III and the oceanic velocity fields are less closely bound by the continuity constraint and may display perturbations of one sign extending much of the way across the ocean basin (e.g., the \(u\) field for the Model IV U1 mode at time \(t = 160\), Fig. 10a).

Although meridional mass fluxes are smaller than zonal equatorial mass fluxes, the former are responsible nevertheless for substantial fluctuations in the zonally integrated oceanic mass field. For the Model IV U1 mode in the wide-continent case, the surplus of warm water mass in areas of positive \(h\) within the zone \(|y| < 1\) can drop to only 45 percent of the deficit in areas of negative \(h\) within that zone (e.g., at time \(t = 100\) in Fig. 10b). Cane and Zebiak (1985) have recently called attention to the “equatorially zonally integrated heat content” (essentially an integration of equatorial \(h\) across the basin) after finding that this quantity reaches a maximum in their model prior to the peak of a model equatorial warming. Figure 16 shows values for \(h\) and SST \((T)\) integrated over \(0 < x < x_e; |y| < 1\), for the fundamental unstable mode of (a) Model IV, (b) Model III and (c) Model II in the wide-continent case. In each instance, the peak in the zonally integrated \(h\) precedes that of \(T\). This is so even for Model II where \(T\) is entirely determined by the zonal current, \(u\). The lead of zonally averaged \(h\) over \(T\) in Model II reflects the large positive \(h\) in the eastern basin that precedes the large positive \(T\) in the central basin (Table 3).

The interpretation offered by Cane and Zebiak (1985) for their results concerning the zonally integrated heat content is that the ocean-atmosphere evolves gradually during non-El Niño years from a stable to an unstable state. A symptom of this evolution is the increase in zonally integrated heat content. El Niño occurs as a result of the instability once the ocean-atmosphere becomes unstable. By facilitating an outflow of warm equatorial water, water along the eastern oceanic boundary, or otherwise, El Niño resets the ocean-atmosphere back to a stable state. It would appear from this interpretation that linear models may be helpful in determining the necessary conditions for, and mechanisms of, the instability that leads to El Niño. The relevant equations would be linearized about the state of the ocean-atmosphere just prior to the onset of El Niño, and the anomalously high zonally averaged heat content would be built into the background state. An implication is that models linearized about climatological states would probably be unable to produce self-excited oscillations. The existence of the overall...
ENSO cycle would be dependent on nonlinear processes.

An alternative interpretation is that models linearized about the climatological state may produce self-excited oscillations (of period several years) and that the ENSO cycle derives from such an oscillation. The oscillation would of course be considerably modified by nonlinear processes. Nevertheless, variations in the zonally averaged heat content would simply occur as part of the (linear) cycle and a maximum would be reached prior to the full development of the thermal anomalies as per Fig. 16. There would be no need to consider flips from “stable” to “unstable” states. The models analyzed here are not linearized about the climatological state. Such a linearization would result in coefficients that vary zonally and with time. The present models, with their constant coefficients, are yet too simple to give clear insight on these fundamental issues.

It is by deliberate choice that only the few apparently most important physical processes are considered in the present models, and then in often crude linear form with constant coefficients. The present work, and Hirst (1986a,b) offer many insights into the dynamics of coupled model instability; this was possible because the models were kept simple. An attempt to include every physical process that might conceivably be of some importance during the development of El Niño would have been premature given the purpose of the work. A basis is now built for understanding instabilities in more complicated and realistic systems.

Some potential areas of future research are clear. In reality, coefficients (esp. \( \bar{T}_x, K_Q, K_T, d \)) are strongly dependent on longitude. Such dependence can easily be incorporated into the scheme developed in section 3, and so a straightforward future study might involve calculating modes for an ocean-basin global atmosphere model featuring zonally varying coefficients. In addition to providing for a more realistic background state, such a model is of theoretical interest in that the ocean thermodynamics changes from the advective limit (Model II) in the central-west Pacific toward the local equilibrium limit (Model I) in the far east Pacific. As a further step, it may be necessary to include more than one active layer in the ocean, since both total and anomalous currents in the equatorial central and western Pacific change markedly with depth, even above the thermocline (Gill, 1982; Firing et al., 1983). It appears that wind forcing does not act as a body force on the entire ocean layer above the thermocline [cf. Eq. (2)], but more accurately on a thin (\(~50 \text{ m deep}\) “Ekman layer” (Zebiak and Cane, 1983). The momentum coupling between this Ekman layer and the deeper water down to the top of the thermocline may be sufficiently weak that two active momentum layers must be considered (e.g., Zebiak and Cane, 1983; Schopf and Cane, 1983; Schopf and Harrison, 1983; Schopf, 1983). The range of theoretical studies have implications for the strategies of diagnostic studies, both empirical and numerical. For example, preference might be given to verifying calculated phase relationships.

An especially timely theoretical study would involve comparison of results from 1) a nonlinear coupled model capable of producing oscillations that have temporal and spatial characteristics resembling the observed ENSO cycle (e.g., Cane and Zebiak, 1985), 2) a coupled model linearized about the seasonally varying climatological state and 3) coupled models linearized about a variety of temporally constant (but zonally varying) background states. Such a study should reveal the role of seasonal variation and nonlinear processes in modifying or inducing the oscillation. It should indicate whether climatological background states are capable of inducing a slow oscillation or whether background states featuring “preexisting” anomalous conditions are required for instability.

In any case, the model results will provide guidelines for the observational search for precursors to the El Niño (cf. Wyrtki, 1985; Cane and Zebiak, 1985). Observational studies may then clearly test the validity of the model results. Positive results should lead to indices of observed variables that are useful in forecasting the potential for El Niño development. Otherwise, a thorough physical understanding of the ENSO cycle may aid development of numerical models capable of forecasting the potential for El Niño. Forced ocean models might forecast El Niño by a few months (Inoue and O’Brien, 1984, 1986), while coupled ocean–atmosphere models might foreshadow El Niño several years in advance (Cane et al., 1986). However, it is not yet certain that an improved physical understanding of ENSO will lead to improved forecasting of El Niño. Moreover, it is not certain that forecasting ability will be put to effective social and economic use. Nevertheless, it seems very important that a phenomenon having a human impact as vast as does the El Niño–Southern Oscillation be thoroughly understood. Only when a thorough understanding of the phenomenon is achieved may the full application of that understanding become clear.

Acknowledgments. Most of this work was performed as part of the author’s Ph.D. project under the supervision of Professor John A. Young, whose useful advice throughout is gratefully acknowledged. Special thanks to Dr. David S. Battisti, for permission to modify his numerical model in order to test the eigenmode calculation. The author would also like to thank Drs. Mark A. Cane and Edward S. Sarachik for stimulating discussions. This work was supported by NSF Grant ATM-144-S482.

APPENDIX

Tests for Numerical Convergence

The method developed in section 3 requires truncations in both \( x \) and \( y \). In the \( x \) direction, the number
of grid points ($J$) is limited, while in the $y$ direction, the number of symmetric Hermite functions ($M$) is limited. Results to be presented here indicate the extent to which calculated modal properties show convergence toward particular values as $J$ and/or $M$ are increased, and suggest the accuracy of properties calculated with the standard $J = 15$ and $M = 8$. Attention is restricted to the fundamental unstable mode in each of the unbounded ocean, thin barrier and wide-continent cases. Throughout, coefficients are fixed at representative values for Model IV (Table 1), and the ocean basin or interval width is 59.3 units.

In the unbounded ocean case, the effects of Hermite function truncation can be isolated from the effects of finite differencing in $x$. It is possible to effectively set $J = \infty$ by a priori requiring that the solutions to (13), (16) and (20) be sinusoidal in $x$; simple algebra then brings these equations into standard eigenvalue form. The convergence of modal properties with increasing $M$ can now be studied in isolation. Figure 17 shows how growth rate and frequency thus calculated depends on $M$. The curves suggest convergence of $\sigma_I$ and $\sigma_R$ towards values around $1.12 \times 10^{-2}$ and $2.92 \times 10^{-2}$ units, respectively; $\sigma_R$ reaches a maximum at $M \approx 8$ and $\sigma_I$ does likewise at $M \approx 24$, both $\sigma_R$ and $\sigma_I$ decline slightly as $M$ is further increased. Errors in the frequency ($\sigma_R$) and in the difference between growth rate and uncoupled damping rate ($a + \sigma_I$) that result from truncation at $M = 8$ are less than 3 percent of the suggested converged values. Figure 17 also shows convergence characteristics of $\sigma_I$ and $\sigma_R$ as obtained from the full equations via the method of Hirst (1986a, section 3.1). The latter sequences appear to be converging towards values slightly different to the suggested converged values obtained from (13), (16) and (20). The two sets of equations are not expected to have identical eigenvalues, since several physical approximations were made in deriving (13), (16) and (20).

The values of $\sigma_I$ and $\sigma_R$ as calculated above for “$J = \infty$” provide a reference by which to determine the effect of finite differencing in $x$ on the modal behavior. Specifically, it is possible to test for the correct convergence of $\sigma_I$ and $\sigma_R$ as the number of grid points ($J$) is increased while $M$ is kept constant. Values of $\sigma_I$ and $\sigma_R$ were computed as $J$ is increased while $M$ is set at 8, and are in fact found to converge towards the corresponding $J = \infty$ values. At $J = 15$, the errors in calculated $\sigma_R$ and ($a + \sigma_I$) due to the truncation in $x$ are less than 1% of the $J = \infty$ values. Thus, at $M = 8$
and $J = 15$, $\sigma_R$ is about 3% higher and $(a + \sigma_I)$ is about 3% lower than the fully converged values suggested by Fig. 17.

The preceding results for the unbounded ocean case are helpful in interpreting the dependence of modal properties on $J$ and $M$ in the thin barrier and wide continent cases. In the latter cases, the effect of Hermite function truncation cannot be isolated, since solutions are not sinusoidal in $x$. Documentation of numerical convergence in the thin barrier and wide continent cases is restricted here to the behavior of $\sigma_I$ and $\sigma_R$ as $J$ and $M$ are increased together. Figure 18 shows $\sigma_I$ and $\sigma_R$, calculated for each of the three cases when $(J, M)$ have the values $(5, 3)$, $(7, 4)$, . . . , $(15, 8)$ and $(17, 9)$. Behavior of $\sigma$ in the unbounded ocean case resembles that shown in Fig. 17; in particular, a maximum in $\sigma_R$ occurs when $M = 8$. Behavior of $\sigma$ in the bounded ocean cases is very similar to that in the unbounded ocean case; this suggests that truncation errors in the bounded ocean cases are similar in sign and magnitude to the small errors indicated in the unbounded ocean case.

Finally, eigenmode results for the wide-continent case are compared to the behavior of a corresponding numerical model. The ocean dynamics submodel is as in Cane and Zebiak (1985), the atmosphere submodel is as per Zebiak (1982) except that $K_0$ is constant (the bulk of the code for the numerical model was developed and supplied by Dr. David S. Battisti). The only differences in the numerical model physics from that of Eqs. (1)–(6) are, first, no atmospheric longwave approximation is made and, second, meridional wind stress ($=K_0V$) is permitted to act on the ocean. The dimensions of the ocean and continent are as for the wide-continent case. Coefficients are as per Table 1, and ocean thermodynamics is either Model II, III or IV. The numerical ocean is initially forced for one time step by an applied wind stress, then the coupled model is permitted to evolve free from any further external forcing. In each situation, an oscillation develops that displays behavior and structure independent of the initial forcing and similar to that of the most unstable eigenmode described in section 4. Growth rates and frequencies inferred from the numerical solutions differ by less than $0.4 \times 10^{-2}$ units from the corresponding eigenmode values. This satisfactory agreement provides further evidence for the correct convergence of the method in section 3a.

REFERENCES


