NOTES AND CORRESPONDENCE

Effects of Turbulence on the Growth of a Cloud Drop Spectrum

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ABSTRACT

The collection equation is solved using a probabilistic collection kernel that includes the effects of overlapping turbulent eddies. The numerical results show that turbulence contributes to the collection process, and as the turbulence increases, so does the broadening of the drop spectrum. But even for very intense turbulence, the droplet growth rate remains fairly slow.

1. Introduction

Some degree of turbulence is always present in clouds, yet it is not clear to what extent the turbulent air fluctuations influence the microphysical processes of precipitation formation. Of particular interest is the role of turbulence on the collision coalescence process in young developing cumulus clouds (Pruppacher and Klett 1978). Studies by East and Marshall (1954) and Saffman and Turner (1956) suggested that turbulence enhances droplet growth. This was further supported by Jonas and Goldsmith (1972) and Woods et al. (1972), who made experiments indicating that shear flow increases the collection rate above its value in still air. Because it is extremely difficult to take accurate observations of drop collisions in convective clouds, Panchev (1971) suggested numerical modeling as the best approach to investigate the problem. The effects of turbulence on the inertial impaction collisions between drops have been modeled by de Almeida (1976, 1979a,b). He argued that weak turbulence could cause a significant broadening in a maritime drop spectrum, as a result of the fact that drops of different sizes respond with different inertia to the fluctuating turbulent velocity field.

In addition to the shear and inertia effects, turbulence is likely to enhance drop collisions also through the mechanism of overlapping of eddies. Initially, cloud drops can be contained in two spatially separated eddies. Later, it might happen that these eddies overlap, causing collisions and coalescence of some of the drops. This mechanism is of interest for the initial stages of raindrop formation as it operates in drops with equal or different sizes. Recently, Reuter et al. (1988) formulated a model using stochastic differential equations to estimate the probabilistic collection kernel including some effects of overlapping turbulent eddies. The model derivation is based on the assumption that the turbulent air fluctuations can be approximated using a white noise spectrum with a constant variance related to a turbulent diffusion coefficient. This simple formulation makes it possible to derive a set of stochastic ordinary differential equations that can be associated with a Focker–Planck equation. However, the white noise formulation also raises concern about its validity in representing realistically the motion of drops in a turbulent field (Cooper and Baumgardner 1989). The assumption that drops react instantaneously to the fluctuations in the ambient flow is not consistent with observations indicating that drops need time to adjust to the flow. Also, the assumption of a constant diffusion coefficient neglects correlations between motions of droplets that result from their proximity. Despite these weaknesses, the stochastic model is of interest, as it allows us to make an estimate of the turbulence effect, even for the case when the two drops have equal sizes. Previous model formulations failed to address this situation.

The work reported in Reuter et al. deals only with individual pairs of drops. To estimate the overall effect of turbulence on the evolution of a cloud droplet spectrum, the collection equation must be solved using a kernel that includes the effects of turbulence. The purpose of this note is to document results of the collection equation using the turbulent kernel of Reuter et al.

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2. Collection equation with turbulent collection kernel

The spectrum of cloud drop sizes can be described using a number density function $N(m, t)$, which is defined such that $N(m, t)dm$ is the average number of drops per unit volume in the mass interval $(m, m + dm)$ at time t. Assuming that the drop spectrum changes only due to coalescence, its evolution can be described using the collection equation

$$\frac{\partial N(m, t)}{\partial t} = \frac{1}{2}\int_0^m N(m-M, t)N(M, t)K(M, m-M)dM - N(m, t)\int_0^\infty N(M, t)K(M, m)dM \quad (1)$$

where $K(m, M)$ denotes the collection kernel, defined as the probability that a drop of mass M collects a drop of mass m in unit time in unit volume. Gillespie (1975a,b) and Valioulis and List (1984) showed that (1) is indeed a good approximation to the fully stochastic collection process for typical cloud conditions. Numerical schemes for solving (1) were developed by Bleck (1970), Berry and Reinhardt (1974a), Gelbard and Seinfeld (1978), Brown (1985), Tzivion et al. (1987), Eyre et al. (1988), and others. We adopted the spline-collocation scheme of Eyre et al. because of its computational efficiency. As the scheme is described in detail in their above reference, only the major features are summarized here. The scheme uses cubic splines to yield a discrete equation with very good convergence properties. An adaptive mesh grading is used to assure that the nodal points are always in their optimal positions as the spectrum evolves. The optimization is done by equally distributing the nodal points with respect to the arc lengths of the approximate solution function. A collocation method is used to obtain a system of first-order differential equations for the expansion coefficients. This system is solved using an explicit multistep method. The scheme conserves liquid water content and yields results which agree well with analytic solutions for simple collection kernels (Scott 1968). In this study, 20 nodal points are used. Tests have shown that the addition of more nodal points does not change the results. Optimal mesh grading is done every 5 min. The nondimensional cutoff parameter q_c has the value 0.95. The free mass parameter ζ is equal to $(7 \times m_0)$, where m_0 is the mass of the drop with mean drop radius r_0 at initial time.

The collection kernel is computed using the stochastic model of Reuter et al. (1988). Its basic assumptions have been dealt with critically by Cooper and Baumgardner (1989) and Reuter (1989). As these references contain a detailed description of the physical model and the numerical methods, only the major features are mentioned here. The model is based on stochastic differential equations that describe the relative motion of a pair of drops falling in turbulent air. The forcing contains a deterministic term representing the effect of gravity and stochastic terms representing the turbulence effects. The turbulent air fluctuations are modeled by white noise with its variance related to the diffusion coefficient σ. Theoretical considerations suggest that σ should increase with increasing intensity of the turbulence and also with decreasing separation distance, but the exact relationship is not known for the viscous range (Panchev 1971). Because of this uncertainty and also to simplify the calculations, it is assumed that σ is a constant. For the scales of drop interactions, the maximum value of σ should not exceed about 25×10^{-6} m^2 s^{-1} (Reuter et al. 1988). The stochastic equations of relative drop motion can be related to a Fokker–Planck equation that governs the evolution of the probability density function for drop collisions. Integrating the probability density function gives an estimate of the turbulent collection kernel (Reuter 1989).

In the stochastic model, two functions have to be specified—the terminal fall velocity $u(r)$ of a drop with radius r, and the collection efficiency $E(r, R)$ of drops with radii r and R. To keep the computer coding simple, it is convenient to use analytic formulas. For $u(r)$, we adopt the curve of Berry and Pranger (1974) that provides an accurate fit to empirical drop fall velocities (e.g., Beard and Pruppacher 1969). Because we are interested in the evolution of drop spectra during their early stages when drops have radii less than 30 μm, a unit coalescence efficiency is assumed. Thus the collection efficiencies are equal to the collision efficiencies. As to the choice of collision efficiencies, de Almeida (1976, 1979a) computed the values for cloud droplets in turbulent air. However, his representation of the turbulent energy spectrum is considered doubtful for the size scales of droplets, and the inertial resistance of drops to turbulent accelerations has not been modeled correctly (Grover and Pruppacher 1985). Because of this we did not use de Almeida's values, but adopted the collision efficiencies of Davis and Sartor (1967), as approximated by the analytical formula of Scott and Chen (1970). The Davis–Sartor efficiencies were derived assuming a Stokes flow regime without making any corrections for the slip-flow effect or Oseen flow. This may account for the fact that the Davis–Sartor collision efficiencies are smaller than those computed later by Klett and Davis (1973), which used the complete equation governing the flow around the two drops.

3. Results

The initial condition for the collection equation is given by a drop size spectrum centered at radius r_0 with liquid water content L. Specifically the initial spectrum is assumed to be a Gamma distribution of the form

$$N(m, 0) = 4\,Lmm_0^{-3} \exp(-2m/m_0) \qquad (2)$$

where m and m_0 are the masses of drops with radii r and r_0. Equation (2) fits closely many observed droplet spectra (Scott 1968).

In the presentation of our results, we will use the mass density function $\tilde{N}(r, t)$ instead of the number density function $N(m, t)$. The two functions are related by the expression $\tilde{N}(r, t)dr = mN(m, t)dm$. The advantage in using $\tilde{N}$ is that, in comparison to N, $\tilde{N}$ emphasizes more the larger drops in the spectrum. Also, $\tilde{N}(r, t)dr$ gives directly the contribution of the drops with radii between r and $r + dr$ to the liquid water content L, which is the conserved quantity.

Figure 1 shows an example of the evolution of a drop spectrum in strongly turbulent air with $\sigma = 25 \times 10^{-6}$ m^2 s^{-1}. Initially the drop spectrum is specified by $r_0 = 10$ μm and $L = 1$ g m^{-3}. As time evolves, the spectrum becomes broader, but the growth remains fairly slow. Our results are similar to those obtained by Berry and Reinhardt (1974b) who neglected effects of turbulence.

The relation between droplet growth and turbulence intensity is examined by computing the evolution of the spectrum for different values of σ. All cases start with the same initial spectrum centered at 10 μm with a liquid water content of $L = 1$ g m^{-3}. The density function $\tilde{N}$ (expressed in percentage of the total liquid water content L) at 50 min is plotted against radius for three different values of σ in Fig. 2. The droplet mass spectrum for $\sigma = 10^{-6}$ m^2 s^{-1} is broader than that in still air, where $\sigma = 0$ m^2 s^{-1}. And in the case of extremely intense turbulence ($\sigma = 25 \times 10^{-6}$ m^2 s^{-1}), the spectrum is the broadest. Calculations for intermediate values of σ give further support that the broadening of the droplet spectrum increases monotonically with increasing intensity of turbulence.

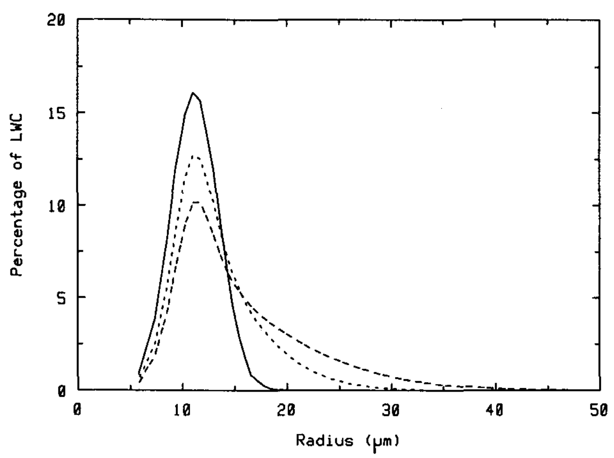

FIG. 1. Evolution of drop mass density distribution $\tilde{N}(r, t)$ (in percentage of liquid water content) for intense turbulent air with $\sigma = 25 \times 10^{-6}$ m^2 s^{-1}. The curves are plotted at initial time (solid), after 30 min (dotted) and after 50 min (dashed).

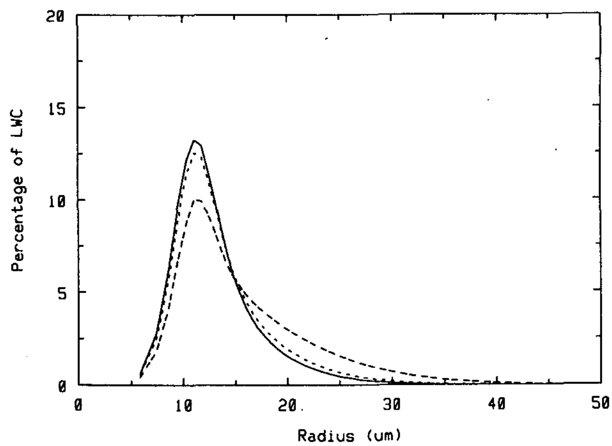

FIG. 2. Comparison of drop mass density distributions $\tilde{N}(r, t)$ (in percentage of liquid water content) after 50 min for $\sigma = 0$, (solid), 10^{-6} (dotted) and 25×10^{-6} m^2 s^{-1} (dashed).

Few experiments were made with different mean drop sizes. For the case r_0 equal to 5 μm, the spectrum remains almost unchanged in time even in the presence of strong turbulence. The collision efficiencies and the cross sections for these small drops are so small that the rate of collection is insignificant. For the case r_0 equal to 15 μm, the spectrum is found to grow faster. Again the broadening of the droplet spectrum can be broadened by overlapping of turbulence, but the contribution of turbulence still remains very small.

4. Discussion

In this note, we examined the effect of overlapping turbulent eddies on the collisional growth of cloud droplets. Our results suggest that a broadening of the spectrum can occur when the turbulence intensity is very strong. However, the effect seems to be weak and does not add significantly to the process of gravitational collection. As in all modeling studies, the conclusions are limited to the validity of the basic model assumptions. Cooper and Baumgardner (1989) expressed concern that the assumptions used in deriving our probabilistic collection kernel would have a net effect of overestimating the effects of overlapping eddies. Against the background of this criticism, our results may be thought of as an upper bound for the contribution of overlapping eddies to droplet collection. However, one must stress that our study is also limited by the assumption of droplet collision efficiencies valid in nonturbulent air. The results presented would have to be reexamined when reliable data on collision efficiency in turbulent air become available.

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