NOTES AND CORRESPONDENCE

Normal-Mode Rossby Waves Observed in the Wavenumber 1–5 Geopotential Fields of the Stratosphere and Troposphere

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ABSTRACT

Daily global geopotential height fields have been analyzed for zonally propagating, planetary scale structures with periods in the range of 4 to 30 days. The geopotentials extend from the 850- to 2-mb levels and 85°N to 85°S latitudes, and describe four Northern Hemisphere winters. The analyses indicate that variations in geopotential are dominated by westward moving waves that resemble normal mode Rossby waves. Structures are determined which correspond to the first symmetric modes of zonal wavenumbers 1–5, the second antisymmetric modes of wavenumbers 1–3, the second symmetric modes of wavenumbers 1–3, and perhaps the third antisymmetric mode of wavenumber 1.

Mode structures are found to agree in general with the predictions of theory and numerical models. Only the first symmetric modes are found to maintain their symmetry throughout the troposphere and lower stratosphere. Other modes are found to resemble their predicted Hough function structures in the troposphere but, to varying degree, not in the stratosphere. The observations support the importance of thermal damping and critical lines as causes of this departure. Evidence is found that supports the increasing importance of lower tropospheric damping with increasing mode wavenumber and meridional index.

1. Introduction

Normal-mode Rossby waves are planetary-scale westward moving features that represent the resonant states of the atmosphere. The basic characteristics of these modes may be mathematically obtained from the shallow water, or tidal, equations on a rotating sphere (Longuet-Higgins 1968). A variety of observational studies have provided ample evidence for the existence of normal modes. Eliasen and Machenhauer (1965, 1969) described 500 mb geopotentials in terms of spherical harmonics, and noted that the harmonic coefficients indicated westward moving variance in frequency bands known to correspond to normal modes. One of the modes observed by Eliasen and Machenhauer has also been observed by Madden and Julian (1972) and Madden and Stokes (1975) in long series of sea level pressures. Both studies revealed pressure variations which were recognized as originating with the wavenumber 1 first symmetric normal mode. Tropospheric geopotential fields have also yielded a variety of normal modes when described using Hough functions (Ahlquist 1982, 1985; and Lindzen et al. 1984).

Normal modes have also been observed in the middle atmosphere. Satellite radiances representing stratospheric temperatures were used by Rodgers (1975), Venne and Stanford (1982), Hirota and Hirooka (1984), Venne (1985) and Hirooka and Hirota (1985) to describe a number of modes. Meteor radar winds analyzed by Salby and Roper (1980) have also revealed signals with periods suggestive of several normal modes.

Normal modes have been modeled using a number of methods, with results that begin to delineate the way in which the modes interact with complex zonal wind and temperature fields. A quasi-geostrophic model has been used by Schoeberl and Clark (1980) to examine the resonant response of the atmosphere to periodic forcing. Primitive equation formulations have been applied by Salby (1981a, b) and Grotch and Rose (1985) with the intent of determining normal mode behavior in a realistic atmosphere. Other models have been used to study the normal mode response to realistic tropospheric winds (Kasahara 1980; Ahlquist 1982) and to examine possible sources of mode excitation (Salby and Garcia 1987; Garcia and Salby 1987; Farrell 1988).

Despite the number of studies made of normal modes, the four-dimensional structure of the modes remains poorly determined. This deficiency has remained in part because of the large amount of data which must be analyzed to refine the usually weak signals of the normal modes. Global, multiyear datasets are required to isolate normal modes from other physical sources of variance, observation error, and each other. Another contributing factor has been the use of predetermined basis functions in the analysis of real global data. Hough functions [as applied, for example, by Ahlquist (1982, 1985) and Lindzen et al. (1984)]
have been used to determine the amplitude and behavior of normal modes. The resemblance of normal modes to their theoretical Hough function structures has been demonstrated only for the tropospheric portion of a few modes, however. Theoretical evidence such as that presented by Salby (1981b) and Daley and Williamson (1985) suggests that the normal mode structures may diverge from Hough mode appearance in the stratosphere.

Normal modes have been suggested to be important to the understanding and performance of numerical general circulation and weather prediction models. Hamilton (1987) has used the appearance of normal modes in a general circulation model as a diagnostic of model performance. Daley et al. (1981) have pointed out that normal modes may have inappropriately large amplitudes in numerical weather prediction models. Normal modes may be excited through model initiation with data which are not compatible with the model physics (Daley et al. 1981). One effect of these strong modes may be to produce incorrect modulations of the quasi-stationary waves which, in turn, may produce errors in forecast fields. It would, therefore, be useful to have more accurate representations of observed normal modes in order to better judge and improve the performance of models.

This paper presents the results of an observational study of normal mode structure and behavior, which is based on an empirical analysis method. The analysis method uses no assumptions concerning the meridional structure of the modes and so is free to represent them as they appear in a time-mean sense. This freedom allows more direct comparison with results of models that are similarly unconstrained. The structures of 12 modes are presented and compared to prior observations and numerical model results.

2. Data

The data used in this study are global geopotential heights derived from NOAA/NMC analyses for the four winters 1978/79 through 1981/82. The data originated as hemispheric 65 by 65 grids at 18 pressure levels (1000, 850, 700, 500, 400, 300, 250, 200, 150, 100, 70, 50, 30, 10, 5, 2, 1 and 0.4 mb), and are basically the same data described by Geller et al. (1983) and extended to southern latitudes. The data at the levels above 100 mb are derived from satellite radiances measurements. Because the satellite data are not synoptic, some distortion of transient wave features is assumed to have occurred during data assimilation. The daily averaging of satellite-derived geopotentials around 1200 UTC in a Cressman-type analysis may be expected to result in a reduction of traveling wave amplitude. The degree of this reduction is expected to be small for the range of wave periods examined here, and the results to be described later suggest that this is indeed the case.

The data have been interpolated to a latitude-longitude grid that extends from 85°S to 85°N and then Fourier decomposed in the zonal direction. A 16-point Bessel central difference formula was used to perform the interpolation at all latitudes except between 5°N and 5°S, where a 4-point linear interpolation was used. The 1000, 1 and 0.4 mb data, although available, were not included in this study because of questions concerning their quality. Likewise, the data poleward of 65°S at the 850- and 700-mb levels were not used because of their proximity to the Antarctic land mass. Data from days when only satellite-derived geopotentials were available were not used in order to minimize inconsistency between the troposphere and stratosphere. When data were missing from a given level, the data at levels above it were rejected. Infrequent missing data were restored through linear time-interpolation of the zonal Fourier coefficients.

Each winter studied here was defined to be 120 days long, although not all winters had the same starting date because of data availability. The 1978/79 and 1979/80 winters began on 1 November and ended on 28 February, while the winters of 1980/81 and 1981/82 began on 16 October and 9 September, respectively.

3. Analysis

The goal of the analysis performed here is to provide the space and time structure of coherent geopotential variations within specific frequency bands in which normal mode variance is expected to be concentrated. The analysis therefore proceeds through several steps: calculation of a frequency- and space-weighted complex correlation matrix, complex principal component analysis, and (optional) rotation of the principal components (PCs). The analysis methodology is similar to that used by Denbo and Allen (1984). After filtering to remove variations with period longer than 40 days, the sine and cosine zonal Fourier coefficients are combined to form a complex variable $z_{kj}$ given by $z_{kj} = c_{kj} - i s_{kj}$, where $c_{kj}$ and $s_{kj}$ are the cosine and sine coefficients at position $k$ (in the height–latitude cross section) and time $j \Delta t$, respectively, $i = (-1)^{1/2}$, and $\Delta t = 1$ day. The time-fourier transform of $z_{kj}$ is given by

$$ z_{kj} = \sum_{n=-59}^{60} Z_{kn} \exp(i \omega_n j \Delta t), $$

$$ Z_{kj} = \frac{1}{120} \sum_{j=1}^{120} z_{kn} \exp(-i \omega_n j \Delta t), $$

where $\omega_n = \pi n / 60$.

It can be shown that the elements of the covariance matrix $V$ for positions $k$ and $m$ are given by

$$ V_{km} = \frac{1}{120} \sum_{j=1}^{120} z_{kj}^* z_{mj} = \sum_{n=-59}^{60} Z_{kn}^* Z_{mn}, $$

where the asterisk denotes complex conjugation. The
elements of the $Z$ matrix are weighted in this study to isolate the frequency bands of interest and to account for the different atmospheric mass represented at each position. The frequency weights describe a bell-shaped passband (with response functions shown in Fig. 1), while the spatial weights are largest in the tropical troposphere and decrease with height and latitude. The weight covariance matrix $C$ is given by

$$C_{km} = \sum_{n=1}^{N} w_{kmm} Z_n^* Z_{mn},$$

with the weight at frequency $n$ and positions $k$ and $m$ given by $w_{kmm}; N$ is the number of frequencies considered. The covariance matrix is converted to a correlation matrix $R$ through division by the diagonal elements. $R$ is a complex hermitian matrix and therefore has a real-valued eigenvalue matrix $\Lambda$ and a spatial structure matrix $A$ that satisfy the equation $RA = \Lambda A$. EISPACK eigenvalue/eigenvector computer routines (Smith et al. 1976) are used to calculate the $A$ and $A$ matrices.

The structures $A$ are rotated in some cases to aid in the physical interpretation of the results. The merits of rotation have been discussed by Walsh and Richman (1981) and Horel (1984). In brief, rotation enhances the ability of the eigenvectors to portray structures that are small compared to the spatial extent of the data and to split into separate PCs two or more physically distinct features within the analyzed dataset. A complex varimax rotation developed by Horel (1984) is used here. This rotation produces eigenvectors (spatial structures) which are not necessarily orthogonal, and PCs (time series or spectral estimates) which are orthogonal.

Normally only a small subset of the determined PCs are rotated, corresponding to the largest eigenvalues that are judged significant. In this study the normal modes dominated the atmospheric variance to the extent that they were always represented by the first or second PC. The number of PCs to rotate was determined subjectively from the eigenvalues using the breaking-slope criterion. No rotation was performed when the first eigenvalue was clearly significant (using

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**Fig. 1.** The power spectra of the observed normal modes determined from four NH winters. The modes are indicated in $(s, \pi - s)$ notation with wavenumber increasing from left to right and meridional index increasing from top to bottom. The dotted line indicates the effective filter envelope, the solid line indicates westward moving power, and the dashed line indicates eastward moving power. The period of the power is given in days, and the power is in arbitrary units. Note that the shapes of the spectral peaks are determined largely by the filter envelope.
FIG. 2. The observed spatial structure of the (1, 1) normal mode, in terms of signal to noise ratio ("normalized amplitude", top), zonal phase in eastward degrees (middle), and geopotential amplitude in meters (bottom). Signal to noise ratio is contoured in increments of 0.1 with no stippling for the 0.0 to 0.2 range, lightest stippling for the 0.2 to 0.4 and heaviest stippling for the 0.8 to 1.0 range. The central period of the filter used to restrict variance is given at the top of the figure.

The breaking-slope method) or when all but the first eigenvalue were degenerate (using the test of North et al. 1982). Use was not made of the selection methodology developed by O'Lenic and Livezey (1988).

The rotation of PCs begins with the construction of the rotated structure matrix $B = AT$, where $T$ is a transformation matrix obtained from the rotation algorithm. The unrotated PCs, $f$, are related to the rotated PCs, $g$, by $f = Tg$. Now $f$ is given by $Z = A f$, which can be written

$$f = A^{-1}Z = (A'^{-1}A)A'^{-1}Z,$$

where the prime indicates the transpose form of the matrix. Since

$$\Lambda^{-1} = (A'^{-1}A)^{-1},$$

$f$ can be written as

$$f = A^{-1}A'^{-1}Z$$

$$g = T'f = T'\Lambda^{-1}A'^{-1}Z$$

which is the complex form of the equation given by Harman (1967) for the rotation of real PCs. Column vectors of the matrix $B$ contain the spatial structures.

FIG. 3. As in Fig. 1 but for the (2, 1) mode.
row vectors of $g$ contain the PCs, or time-spectral estimates of the spatial structures. Since $A$ is made up of four winters of data, individual $g$ matrices for each of the winters can be computed. These can then be inverse Fourier transformed to give time series of each PC. These structures and their time development will be examined in the next section.

4. Results

a. First symmetric modes

The first symmetric modes of waves 1 through 5 were analyzed using passbands centered at periods of 5.5, 4.0, 4.3, 5.5 and 7.1 days. The frequency response functions are indicated by the dotted lines in Fig. 1. [Normal modes are often referred to by their zonal wavenumber $s$ and meridional index $n$ as $(s, n − s)$, making the set of first symmetric modes $(1, 1), (2, 1), (3, 1), (4, 1)$ and $(5, 1).] These periods were selected on the basis of the model predictions of Salby (1981b) and Kasahara (1980), and also the observational studies by Ahlquist (1985), Venne (1985), and Hirota and Hirooka (1984), or were defined empirically by maximizing the coherence of westward variance in the analyses.

![Fig. 4. As in Fig. 1 but for the (3, 1) mode.](image)

![Fig. 5. As in Fig. 1 but for the (4, 1) mode.](image)

1) THE (1, 1) MODE, OR 5-DAY WAVE

This often-observed mode appears as the first PC of geopotential filtered to retain variance near 5.5-day period. It is a clearly westward moving feature (Fig. 1) and has a spatial structure (Fig. 2) that is in phase throughout the middle latitudes and tropics of the troposphere. The signal-to-noise ratio, or “normalized amplitude”, is defined as the ratio of mode amplitude at a given location to the standard deviation of the filtered geopotential there. The $(1, 1)$ signal to noise ratio is greatest in the tropics, and the wave geopotential amplitude grows rather steadily with height. The zonal phase is quite uniform in latitude except in the high southern latitudes, and the wave tilts slightly westward with height above 50 mb. The mode is observed to explain 36% of the atmospheric variance in the filter passband and 39% of the variance in the troposphere (Table 1).

The observed structure is in fair agreement with the $(1, 1)$ mode modeled by Salby (1981b). Salby’s mode shows a somewhat greater symmetry between the hemispheres. The magnitude of the vertical phase tilt seen here is close to that determined by Salby, being
In fact, the quasi-geostrophic potential vorticity gradient calculated by Karoly (1982) from climatological data shows a region of negative values near the Antarctic coast during summer.

Experiments with the GFDL spectral general circulation model by Hayashi and Golder (1983) have indicated that the (1, 1) mode appearance at Southern Hemisphere summer high latitudes that changes with the inclusion of mountains in the model. With mountains included, an amplitude peak was found near 80°S at 515 mb that was roughly 130 degrees eastward in phase from the rest of the mode, similar to that observed here. This suggests that the presence of Antarctica may be producing changes in the zonal wind field which cause the mode to become locally propagating meridionally.

2) The (2, 1), (3, 1) and (4, 1) Modes

The (2, 1) structure indicated by the first PC (Fig. 3) is characterized by tropospheric amplitude peaks near 35 degrees latitude that are about 20 degrees out approximately 60 degrees between the 850 and 2 mb levels. Ahlquist (1985) has observed the region of strongest coherence to be centered in the tropics, and his estimate of the December–February mean 500 mb amplitude (5 m) compares well with the amplitudes seen here at midlatitudes. Hirota and Hirooka (1984) have observed the (1, 1) mode as being much more prominent during the equinoxes; however, the vertical phase tilts they observe between 100 and 1 mb are consistent with the solstice structure found here.

It is interesting to note the deviation from simple normal mode structure in the high latitudes of the Southern Hemisphere. The mode has a maximum amplitude there of about 6 meters and is out of phase with the lower latitudes by about 120 degrees. The eastward change in phase with increasing southern latitude could be interpreted as indicating an equatorward energy flux. This would seem to be contrary to the idea that the gravest normal modes are excited by low-latitude disturbances. However, Salby (1982) has suggested that there is a region of negative vorticity gradient near 70°S during the Southern Hemisphere summer, which would serve to reverse the sense of wave energy flux.
The (3, 1) and (4, 1) modes (Figs. 4 and 5) are represented by the first PCs and are observed to retain simple symmetric structures up to the 2-mb level. The modes explain 32% and 26% of the passband variance, comparable with the (2, 1) mode. Both modes are indicated to have negligible phase tilts with height. The amplitudes of these modes observed by Ahlquist (1985) agree well with the present study. Both studies show the (3, 1) and (4, 1) 500 mb amplitudes to be about 4 and 5 m, respectively. In the case of the (4, 1) mode, Ahlquist determines the centers of tropospheric coherence to be near 25 degrees latitude. Here they are observed at 15 degrees, and the amplitude maxima occur between 30 and 40 degrees latitude in both hemispheres.

3) THE (5, 1) MODE AND THE SOUTHERN HEMISPHERE SUMMER WAVE 5

Analysis of the (5, 1) mode is complicated by the presence of a pronounced wavenumber 5 feature in the middle latitudes of the Southern Hemisphere. This wave, as described by Salby (1982), is eastward traveling with a period between 7 and 12 days and a zonal

![Diagram](image-url)
after rotation of the first three PCs. As in the case of the wavenumber 5 summer wave, the rotation has been performed to separate the summer wave from the normal mode. This separation is facilitated by the waves’ opposing directions of propagation. The (5, 1) mode appears to have relatively large signal only in the tropics, and its amplitude maxima are located near 50°S, 30°S and 30°N. The latter two maxima are in phase, whereas the former is out of phase by about 100 degrees. The direction of the phase shift indicates energy transport from the 30°S to 50°S region and, as it is difficult to reasonably invoke the argument for negative vorticity gradient here, must be viewed at this time as evidence of the failure of the analysis to adequately distinguish between the summer wave and the (5, 1) mode.

b. Second antisymmetric modes (1, 2), (2, 2) and (3, 2)

The three antisymmetric modes observed here are not as well defined as the symmetric modes. Unlike most of the (s, 1) modes, the (1, 2) and (2, 2) modes

scale between wavenumbers 4 and 6. An analysis was performed to use its PC-represented structure as a diagnostic of the analysis methodology through comparison with Salby’s results. Rotation of the first five PCs was performed because the broad filter used to isolate the wave allowed the inclusion of (5, 1) signal. The structure of this wave, represented by the first PC (Fig. 6), is found to be very much like that determined by Salby (1982). The horizontal and vertical phase variations are almost exact in their match, as is the location of the maximum amplitude near 48°S and 300 mb. The amplitude of the wave is about two-thirds of that calculated by Salby, and may be smaller because of the combined effects of different data sets, filters, and the noise rejection properties of PC analysis. The percentage of explained passband variance in the Southern Hemisphere is 40%. It is important to note that the weak amplitude maximum near 50°N, probably a result of random correlation, is less than 10% of the true amplitude in the Southern Hemisphere. This may indicate that the production of spurious structures in the troposphere by the analysis method is unlikely.

The (5, 1) mode structure (Fig. 7) is given by PC 2
are represented by the second PCs and rotation is required to properly define their structures. It is also at the longer periods of the (s, 2) modes that spectral red noise becomes a complicating factor in the modes’ interpretation. This is evidenced by a tendency for the power spectrum of each component (Fig. 1) to be shifted slightly towards the longer-period end of each passband. However, analyses of the data at the longer periods do not retain the normal mode structures, suggesting that the passbands shown are properly chosen. As in the case of the symmetric modes, the passbands were chosen on the basis of the results of several model and observational studies. The relative weakness of the modes is evident in the percentage of variance that each explains (Table 1), particularly in the cases of (2, 2) and (3, 2).

Whereas the (s, 1) modes all displayed amplitude symmetry about the equator, the (1, 2) mode has larger amplitudes in the winter hemisphere stratosphere (Fig. 8). This may be due to the asymmetry of the stratospheric wind field and the smaller phase speed of the mode, and will be discussed later. Salby (1984a) has suggested that another factor may be forcing by mid-latitude winter disturbances which the mode begins to resemble (by virtue of its smaller phase speed and more localized latitudinal structure). This effect will become more pronounced for the rest of the modes to be described.

The hemispheric asymmetry has been observed by Hirooka and Hirota (1985) at the 1-mb level. They determined that the amplitudes at 1 mb were about 150 and 50 m in the NH and Southern Hemisphere (SH), respectively, during November–February (from their Fig. 4). The phase differences they calculated, 190 degrees between 50°N and 50°S and 30 degrees between 100 and 1 mb are similar to the differences found here. The growth of amplitude with height is about the same for both studies.

The (1, 2) mode calculated by Salby (1981b) is more symmetric and has a much greater phase tilt with height. Salby indicates that the SH mode structure is modified by a critical line in the summer easterlies.
Although the wave here is not directed equatorward with height as Salby calculates, strong evanescence is seen above 10 mb. Salby also suggests that there be an amplitude minimum near 20 mb and a second maximum at about 7 mb. In this study the second maxima are observed near the 20-mb level and minima occur at about 60 mb.

Both Salby (1981b) and Daley and Williamson (1985) suggest that the 180-degree phase change which produces the mode antisymmetry occurs some 15 to 20 degrees of latitude into the SH. The phase shift line is observed here to be much closer to the equator.

The (2, 2) and (3, 2) modes have structures (Figs. 9 and 10) that are similar to that of the (1, 2) mode with the major difference being an increasing confinement to the troposphere of wave amplitude with increasing wavenumber. Accompanying this is a greater degree of latitudinal symmetry, as might be anticipated for modes trapped in the troposphere. Both modes have tropospheric amplitude maxima that are shifted about 10 degrees northward of those of Ahlquist (1985), an effect which may be due to the consideration here of only winter data.

c. Second symmetric modes

1) THE (1, 3) MODE (16-DAY WAVE)

The (1, 3) mode is observed to have a central period near 17 days and tropospheric structure very much like its anticipated Hough function (Fig. 11). Tropospheric amplitude peaks are observed near 50°S, 25°S, 25°N and 70°N, with the subtropical peaks being about 180 degrees out of phase with the peaks at higher latitudes. The amplitude minimum at the equator is consistent with the Hough structure and significantly nonzero (as in the case of the first symmetric modes). However, the amplitude in the winter hemisphere is several times larger than that of the summer hemisphere, and the difference grows rapidly with height. Salby (1981b) and Daley and Williamson (1985) have suggested that this is caused by both transmission characteristics of the mean wind field and the similarity of the wave's spatial and temporal structure to that of its supposed midlatitude forcing.

Perhaps the most intriguing aspect of the (1, 3) mode is its attributed behavior during January 1979 when a strong amplification of wavenumber 1 amplitude oc-
Fig. 15. As in Fig. 12, but for the NH winter of 1979–1980.

curred. Madden and Labitzke (1981) have suggested that this amplification was actually the exceptionally strong growth of the (1, 3) mode. Their evidence involves the meridional structure of the observed Northern Hemisphere wavenumber 1 geopotential field and the westward motion (corresponding to a period of about 16 days) of the wave crest. Their interpretation of this event suggests that the interference between the (1, 3) mode and the large quasi-stationary wavenumber 1 is of at least occasional importance to the general circulation (Madden 1983).

The time series of (1, 3) amplitude during this time (Fig. 12) does not support a significant role for the global mode in the January 1979 wave amplification event. The mode is not observed to undergo strong amplification during that time and reaches a maximum amplitude of less than 200 m near 10 mb, far less than the 800 m suggested by Madden and Labitzke (1981). Several factors which may serve to reduce the mode amplitude in the current study must be cited. The mode structure is an average based on four winters of data and may not be representative of the 1979 winter structure. In a similar manner, if the mode's propagation is significantly different in character from that retained by the band pass filter, amplitude reduction will occur. Finally, the 1979 event was largely confined to the NH and so may not have had a strong resemblance to the (1, 3) mode structure determined here. Straus et al. (1987) have suggested that the projection of such a NH disturbance onto global basis functions can result in a substantial reduction in (1, 3) amplitude from values obtained at single latitudes (as in Madden and Labitzke 1981). Straus et al. also suggest that the January 1979 event is a disturbance with latitudinal location and extent such that its Rossby frequency is similar to that of the (1, 3) mode. Rather than being projected onto a number of basis functions, in this study disturbance variance such as that suggested by Straus et al. may be concentrated in a PC other than the one that describes the normal mode. This could result in a PC of small amplitude (due in part to averaging over the four winters) that would be difficult to discern from background noise. That no PC exhibiting the NH confinement of the above disturbance is observed here should not be taken as evidence against the existence of such a disturbance. Rather, it is an
indication of the limitations of using extended datasets to describe relatively short-lived phenomena.

2) THE (2, 3) AND (3, 3) MODES

The (2, 3) and (3, 3) modes are also found to have central periods near 17 days and their structure (Figs. 13, 14) is quite like that of the (1, 3) mode. The (3, 3) mode differs in that its winter hemisphere amplitude does not grow with height above the tropopause.

Lindzen et al. (1984) have noted that the (1, 3) and (3, 3) modes seem to be correlated, particularly during the winter of 1978–1979. The (1, 3), (2, 3) and (3, 3) modes are observed here to amplify strongly during January 1980 (Fig. 15). However, this appears to be the only mutual amplification event during the four winters analyzed here, as other winters show what appears to be a random relationship between the two modes.

d. Third antisymmetric mode (1, 4)

The (1, 4) mode identification is marginal because of its proximity to the (1, 3) mode and the red noise portion of the spectrum. A rotation was performed that resulted in the first PC resembling the (1, 3) mode and the second PC giving the (1, 4) structure (Fig. 16). The characteristic Hough function structure is evident only below the 200 mb level, where amplitude peaks are located near 70 and 40 degrees of latitude in both hemispheres. The latitudinal structure at the 2 mb level is quite different and has two strong maxima that are about 40 degrees out of phase.

e. Vertical amplitude variations

1) SURFACE DISSIPATION

Salby (1980) points out that dissipation due to viscous and thermal diffusion in the boundary layer becomes increasingly important as the wavenumber and meridional index increases. The presence of this dissipation may be inferred from the profiles of wave amplitude (Fig. 17). All 12 modes decrease in amplitude as the surface is approached. Inspection of the amplitudes between the 400- and 850-mb levels shows that the rapidity of this decrease strengthens as wavenumber and meridional index increase. This is particularly evident for the modes (1, 1) through (5, 1) (increasing wavenumber) and modes (1, 1) through (1, 4) (increasing meridional index). A measure of the rate of decrease can be obtained by fitting the mode amplitude between 850 and 400 mb to the function \( (p_0/p)^c \). In the absence of boundary layer dissipation \( c \) should be equal to 0.286, the ratio of the gas constant to specific heat at constant pressure. Dissipation near the surface may be assumed to be present when \( c \) exceeds 0.286 and to increase with increasing \( c \). The NH and SH averaged values of \( c \) for the (1, 1) through (5, 1) modes are 0.23, 0.46, 0.48, 0.84 and 0.91, respectively. The averaged values of \( c \) for the (1, 1) through (1, 4) modes are 0.23, 0.39, 0.72 and 1.00, respectively. The averaged values of \( c \) for the (2, 2), (2, 3), (3, 2) and (3, 3) modes are 0.74, 0.74, 0.92 and 0.78, respectively, and so less clearly indicate the role of mode scale in dissipation.

2) STRATOSPHERIC WIND EFFECTS

Salby (1981a) has examined the influence of the wind field on normal modes, and suggests that the wave index of refraction (as defined by Dickinson 1968) and the wave intrinsic frequency are useful indicators of wave propagation. The intrinsic frequency is the angular wind speed relative to a frame of reference in which the traveling wave appears motionless. Salby demonstrates that when the intrinsic frequency is small, it determines the sign of the index of refraction. A positive index of refraction indicates enhanced vertical wave propagation and rapidly growing wave amplitude, while a negative index means that no propagation is allowed and the wave undergoes evanescent decay. Critical lines are found where the intrinsic frequency
goes to zero and the index of refraction goes to plus or minus infinity. In this study critical lines in the four-year mean NH winter winds are observed only in the SH at levels above about 50 mb.

The effect of the summer hemisphere stratospheric easterlies seems to be manifested most extensively for the \( (s, 1) \) modes. Their latitudinal symmetry of amplitude (Fig. 17) is maintained despite the asymmetry of the zonal wind field. Wave amplitude above the 100-mb level is observed to decrease with height more rapidly as the mode phase speed decreases. The change in vertical profile is fairly continuous, ranging from nearly Lamb modelike for \( (1, 1) \) to essentially having no stratospheric presence for \( (5, 1) \).

The \( (s, 3) \) sequence of modes does not display the amplitude uniformity of the \( (s, 1) \) modes, suggesting that the meridional scale of the modes is related to their degree of interhemispheric coupling. This is supported by the observation that increasing the meridional structure does not seem to have the same consequences as increasing the zonal structure. The NH portions of the \( (1, n - 1) \) modes are little changed between \( (1, 1) \) and \( (1, 4) \), whereas the SH amplitudes change from Lamb modelike to constant to chaotic in height (Fig. 17).

5. Summary

A global dataset, representing geopotential heights in the troposphere and stratosphere during the NH winter, has been analyzed for spatial patterns of coherence having periods corresponding to normal modes. Twelve normal modelike structures have been identified, corresponding to the first symmetric modes of wavenumbers 1–5, the second symmetric and antisymmetric modes of wavenumbers 1–3, and the third antisymmetric modes of wavenumber 1. In the case of each mode, the PC representing that mode had the characteristic of westward propagation and was the largest or second largest component in terms of explained variance. Analyses were also performed to detect the modes \((1, 0)\) through \((5, 0)\), \((2, 4)\), \((3, 4)\), \((4, 2)\) through \((4, 4)\), \((5, 2)\) through \((5, 4)\), and \((6, 1)\).

![Fig. 17. Geopotential amplitude as a function of log pressure for the observed normal modes. The solid line is the NH amplitude, while the dashed line is the SH amplitude. The dotted line indicates the amplitude profile of a Lamb mode. Pressure is given in mb and geopotential is given in meters. The profiles are from latitudes at which the tropospheric portion of the mode has maximal amplitude.](image-url)
The failure of these analyses to detect appreciable westward moving variance may be attributed in part to limitations of the data, particularly in the case of the short-period modes (1, 0) through (5, 0). The other absent modes perhaps were too small in amplitude to be clearly resolved from the red noise background or signals of other atmospheric medium-scale waves.

The tropospheric structures of the observed modes generally resembled the expected Hough functions. However, some appreciable asymmetries were found, particularly in the antisymmetric modes. The (1, 1) mode also showed an unexpected degree of structure in the SH, where the effects of the Antarctic continent may influence the mode's horizontal structure.

Mode structures in the stratosphere were somewhat similar to those in the troposphere, with some modifications which may be attributable to thermal damping and mean zonal wind effects. The former was observed as a westward phase tilt with height seen in several of the modes [most clearly in the (1, 1) mode]. Zonal wind effects were seen in the vertical profiles of mode amplitudes, where some evidence was found to support the hypotheses that critical lines trap mode energy in the troposphere and that the modes with large positive intrinsic frequencies resemble Lamb modes throughout the stratosphere.

Evidence for the dependence of lower tropospheric dissipation on zonal wavenumber and meridional index was also found. Mode amplitudes were observed to decrease more rapidly as the surface was approached for modes with higher wavenumbers and larger values of $n - s$.

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