A Theory of Cyclogenesis Forced by Diabatic Heating. 
Part II: A Semigeostrophic Approach 

YUH-LANG LIN 

Department of Marine, Earth, and Atmospheric Sciences, North Carolina State University, Raleigh, North Carolina 

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ABSTRACT 

A linear quasi-geostrophic theory of coastal cyclogenesis proposed by Lin has been extended by a semigeostrophic model. The response of an east–west backsheared, quasi-geostrophic baroclinic flow over an isolated heat source is a low pressure near the heating center and a weaker high pressure downstream, as found in part I of the theory. With the inclusion of nonlinear geostrophic advection, the low is weakened slightly and becomes asymmetric, while the high remains about the same strength. With the inclusion of nonlinear ageostrophic advection, the low is strengthened significantly by the warm air advection and becomes more asymmetric. 

With the addition of uniform friction, the cyclogenesis process is weakened. However, the cyclone is strengthened slightly by differential friction. It appears that the primary source of the cyclonic vorticity of the coastal cyclone is the hydrostatic response of the diabatic heating modified by the baroclinicity. With a mountain ridge included, there exists an inverted pressure ridge (trough) over (downstream of) the mountain. A damping effect is evidenced by a pool of cold air located upslope of the mountain ridge, which is formed by the cold air advection and upslope adiabatic cooling. The inverted pressure ridge between the mountain and the heat source is strengthened by the surface heating and is shifted farther upstream of the mountain. 

When the semigeostrophic model is applied to East Coast cyclogenesis with a northeastly surface wind, a cyclone develops near the western boundary of the Gulf Stream. The low is skewed with a larger gradient located to the southeast corner. The mountain-induced high pressure is relatively weak since the surface wind is almost parallel to the mountain ridge. The cold-air damming is negligible and the Appalachians play a minor role in this type of coastal cyclogenesis. With an easterly surface wind, the cold-air damming is more pronounced. The inverted ridge or the damming area is shifted further upstream and more widespread in the region between the southern part of the Appalachians and the Gulf Stream. A confluent–divergent couplet is produced to the northwest and southwest of the heat source. The results are consistent with observations. 

1. Introduction 

Observational studies suggest that there are two major mechanisms responsible for cyclogenesis over the U.S. East Coast. The first may be called the boundary-layer control of cyclogenesis (e.g., Bosart 1981, 1988). It is proposed that the cyclonically curved coastline under a northeastly flow is favorable for the growth of cyclonic vorticity in response to differential diabatic heating and differential friction between a relatively warm ocean and colder landmasses. The second mechanism may be called the upper-level jet streak/trough control of the cyclogenesis (e.g., Uccellini et al. 1984; Uccellini and Kocin 1987). It is proposed that the circulation patterns associated with jet streaks establish an environment within which low-level processes can further contribute to cyclogenesis. The transverse ageostrophic components associated with jet streaks aloft combine with the longitudinal components as- 

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heat source (Lin 1987; Lin and Li 1988). When applied to East Coast cyclogenesis, a cyclone develops near the western boundary of the Gulf Stream. The cyclone is mainly produced hydrostatically by the less dense air above and modified by the baroclinic effects. Two regions of weaker high pressure form to the southeast and northwest of the low. A confluent–diluent couplet, which represents a localized speed maximum, forms to the downwind side of the cyclone. The inverted trough–ridge couplet is more pronounced with an easterly surface wind. The genesis region and the flow pattern associated with the cyclone, as predicted by the theory, are consistent with observations. The low-level sensible heating has played a key role in the formation of the cyclone, which is consistent with the findings of Sutcliffe (1947) and Petterssen et al. (1962).

The study of L89 is based on a quasi-geostrophic approximation and analogous to the theory of lee cyclogenesis proposed by Smith (1984, 1986). Even though the quasi-geostrophic approximation has been used to simulate some weather phenomena rather successfully, its validity is still not clear. One major weakness of the quasi-geostrophic theory is that the ageostrophic motions are predicted by the geostrophic wind advection. Because of the neglect of the ageostrophic advection, the convergence and cyclonic relative vorticity are underestimated, while the divergence and anticyclonic vorticity are overestimated (Holm 1975). To include the nonlinear ageostrophic advection, Honskis and Bretherton (1972) and Honskis (1975) have proposed the geostrophic coordinate transformation. It is found that the inclusion of ageostrophic advection gives the strengthening of the lows and weakening of the highs, the occlusion process, and the formation of a warm frontal region (Holm 1976). Using a semi-geostrophic model, Chen and Smith (1987, denoted as CS hereafter) have shown that the nonlinear ageostrophic advection effect plays an important role in modifying the location and shape of the cyclone and the associated frontal collapse. The process of lee cyclogenesis proposed by Smith (1984, 1986) has also been investigated numerically by Lin and Perkey (1989) using a primitive-equation numerical model. It is found that the nonlinear effect plays an important role in the process of flow splitting upstream and in forming the front in the vicinity of the lee cyclone. Thus, we anticipate that the nonlinear ageostrophic advection may play a similar role in the thermally induced cyclogenesis.

Based on a power series expansion in the Rossby number, Buzzi and Tibaldi (1977) have investigated the frictional effects on rotating, stratified, and barotropic flow over an isolated topography. It is found that the disturbance associated with orographic forcing becomes asymmetric in the front–rear direction, generating a long tail of positive relative vorticity to the lee side of the mountain. A similar result has been found in a study of barotropic flow over an isolated warm region (Lin 1989a) with an Ekman friction included. In addition, Lin found that the positive relative vorticity and low pressure in the inviscid layer induced by the diabatic heating are weakened because the fluid sucked out of the Ekman layer in the center of the vortex must flow outward in the inviscid layer (Pedlosky 1982). In studying the Charney (1947) and Eady (1949) problems, Farrell (1985) has shown that the inclusion of an Ekman friction can stabilize the flow. The impact of the Ekman friction has also been investigated by Lin and Pierrehumbert (1988) and Valdes and Hoskins (1988) in baroclinic flow. Lin and Perkey (1989) has shown that the lee cyclone is weakened when the frictional effects are included. In a study of New England coastal frontogenesis, Bosart (1975) suggests that the differential friction plays the role of the synoptic scale geostrophic deformation in packing the isotherms together. However, Ballentine (1980) has shown that the differential friction is not a primary source of mesoscale convergence during coastal frontogenesis in his numerical simulations. Therefore, it is interesting to study the effects of differential friction in the proposed process of coastal cyclogenesis.

It is well documented that East Coast cyclogenesis is often preceded by a wedge of cold air and high pressure on the eastern slope of the Appalachians (e.g., Richwein 1980; Forbes et al. 1987; Bell and Bosart 1988). This phenomenon is associated with a synoptic high pressure located in New York or New England and is known as "cold-air damming." The cold air is formed by the cold advection and upslope flow, which is then trapped by the mountain. The number of damming events is greatest in March, with a secondary maximum in December, and a minimum in July (Bell and Bosart 1988). Bell and Bosart also suggest that the damming events are likely to be stronger than average in December, since that month represents a period in which the land–water temperature contrast is the highest. This indicates that the cold-air damming may be related to differential heating across the coastal line. In fact, it has been found that an inverted ridge forms downwind of the diabatic heating for an easterly surface wind in Part I of the theory (L89). Since East Coast cyclones often form and develop near the baroclinic zone associated with the cold-air damming and the coastal front, it is essential to understand the combined effects and the relative importance of orography and diabatic heating in the cyclogenesis process.

The purpose of this paper is to extend the quasi-geostrophic theory of coastal cyclogenesis proposed in Part I (L89) using a geostrophic momentum approximation and to investigate the effects of nonlinear geostrophic and ageostrophic advections, differential friction, and orography. In section 2, we describe the model and the numerical methods for solving the problem. Results of linear quasi-geostrophic, nonlinear quasi-geostrophic and nonlinear semigeostrophic inviscid flow are presented and compared in section 3. Both
effects of nonlinear geostrophic and ageostrophic advectons are investigated in this section. The frictional effects are investigated in section 4 with the addition of Ekman friction. Effects of differential friction between land and water on cyclogenesis will also be investigated. In section 5, the orographic effects associated with flow over a diabatic heating region will be studied. The theory will then be applied to the cyclogenesis along the East Coast in section 6. Both surface sensible and orography of the Appalachian mountain are included and idealized by an elongated heat source and an elongated bell-shaped mountain, respectively. Concluding remarks can be found in section 7.

2. The model

The nonlinear quasi-geostrophic potential vorticity equation and the thermodynamic equation applied at the surface for an inviscid Boussinesq fluid on an f-plane can be written

\[
\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \left( \nabla^2 p + \nabla^2 p_{zz} \right) = \left( \frac{g \rho_0 f^2}{c_p T_0 N^2} \right) q,
\]

where \((u, v, w)\) is the wind velocity, \(p\) the pressure, \(\theta\) the potential temperature, and \(q\) represents the diabatic heating rate per unit mass. Other symbols are defined in Part I of the theory (L89).

Following a procedure similar to that of CS, Eq. (2) may be written as

\[
p'_{xz} + \left( U_x + \frac{1}{\rho_0 f} p_x' \right) p'_{xx} + \left( U_y + \frac{1}{\rho_0 f} p_x' \right) p'_{xy} - U_z p_x' - V_z p_y' + \rho_0 N^2 w = \left( \frac{g \rho_0}{c_p T_0} \right) q'
\]

\[\text{at } z = 0\] (3)

where \(p'\) is the perturbation pressure and \((U_x, V_y)\) is the basic wind at the surface. In deriving Eq. (3), the hydrostatic equation, geostrophic wind and thermal wind relations,

\[
\theta' = (\theta_0 / g \rho_0) p_x','
\]

\[
\nu_x = (1 / \rho_0 f) p_x', \quad V_x = (1 / \rho_0 f) \theta_x,'
\]

\[
U_x = (g / \theta_0) \Theta_x, \quad V_x = (g / \theta_0) \Theta_x,
\]

have been applied. With the assumption of shallow heating as in Part I (L89), the quasi-geostrophic potential vorticity equation in the interior may be reduced to

\[
\nabla^2 p' + \left( f / N^2 \right) p'_{zz} = 0, \]

because of the absence of interior forcing. The system of Eqs. (3) and (7) is closed if the vertical velocity and the diabatic heating at the lower boundary are known. The development is relevant to the Eady problem (Eady 1949) and to the problem of lee cyclogenesis (CS) if the diabatic heating is excluded.

The vertical velocity at the lower boundary, with Ekman friction and mountain included, may be written

\[
w = \left( U_x + u_x \right) \frac{\partial}{\partial x} + \left( V_y + v_y \right) \frac{\partial}{\partial y} h(x, y) + \sqrt{\nu / f, \xi_x} \quad \text{at } z = 0.
\]

where \(h(x, y)\) represents the mountain shape and \(\xi_x\) the perturbation geostrophic vorticity. In the above equation, we have implemented a simple Ekman layer boundary condition. The Ekman number corresponds to \(\nu / f H_0^2\) where \(H_0\) is a height scale. Use of the geostrophic wind relationship [Eq. (5)], the above equation becomes

\[
w = \left( U_x + u_x \right) \frac{\partial}{\partial x} + \left( V_y + v_y \right) \frac{\partial}{\partial y} h(x, y) + \sqrt{\nu / f, \nabla^2 p'} \quad \text{at } z = 0.
\]

We then substitute Eq. (9) into Eq. (3) for the lower boundary condition.

To solve the system of Eqs. (3) and (7), the leapfrog and the second-order center-difference schemes are applied to the time and space derivatives, respectively. The finite difference form of Eq. (3) can be written (with primes dropped)

\[
p_{x}^{n+1} = p_{x}^{n-1} - 2\Delta t \left[ U_x - \frac{1}{\rho_0 f} \left( \frac{P_{i+1,j} - P_{i-1,j}}{2\Delta y} \right) \right]
\]

\[
\times \left[ P_{i+1,j} - P_{i-1,j} \right] + \left[ V_y + \frac{1}{\rho_0 f} \left( \frac{P_{i+1,j} - P_{i-1,j}}{2\Delta x} \right) \right]
\]

\[
\times \left[ P_{i+1,j} - P_{i+1,j} \right] - \frac{U_x P_{i+1,j} - U_x P_{i-1,j}}{2\Delta y} - \frac{V_y P_{i+1,j} - V_y P_{i-1,j}}{2\Delta x} + \frac{1}{\rho_0 N^2} \left[ U_x + u_x \right]
\]

\[
\times \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta x} + \left( V_y + v_y \right) \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta y} + \frac{1}{\rho_0 N^2} \left[ U_x + u_x \right]
\]

\[
\times \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta x} + \left( V_y + v_y \right) \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta y} + \frac{1}{\rho_0 N^2} \left[ U_x + u_x \right]
\]

\[
\times \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta x} + \left( V_y + v_y \right) \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta y} + \frac{1}{\rho_0 N^2} \left[ U_x + u_x \right]
\]

\[
\text{at } z = 0.
\]
where the superscript $n$ denotes the time step and the subscripts $i$ and $j$ denote the grid numbers in $x$ and $y$ directions, respectively. Making the double Fourier transform in $x(-k)$ and $y(-l)$ of Eq. (7) for the interior flow gives

$$
\hat{p}_{zz} - \left( \frac{N^2}{f} \right) \hat{p} = 0
$$

(11)

where $k = (k^2 + l^2)^{1/2}$ is the horizontal wavenumber. The general solution of Eq. (11) can be written

$$
\hat{p}(k, l, z, t) = \hat{p}_s(k, l, 0, t) e^{-N|z|/2f}.
$$

(12)

where $p_s$ denotes the surface perturbation pressure. Notice that the bounded upper boundary condition has been applied at $z = \infty$ to the above solution. Equation (12) implies that

$$
\hat{p}_s = \frac{f}{N|k|} \hat{p}_z \quad \text{at} \quad z = 0.
$$

(13)

The algorithm for solving Eqs. (10) and (13) is described as follows. With the surface values of $p_{z}^{n-1}$, $p_{z}^{n}$, $p_{z}^{n+1}$ and $p_{z}$ known, $p_{z}^{n+1}$ can be obtained from Eq. (10). The surface perturbation pressure $\hat{p}_z$ in the Fourier space can then be solved by making Fourier transform of $p_{z}^{n+1}$ by FFT and substituting into Eq. (13). To obtain the $p_z$ in the physical space, we apply the FFT to $\hat{p}_z$. Throughout this paper, we have used the following numerical parameters: $\Delta t = 10$ min, $\Delta x = \Delta y = 60$ km. The total grid numbers are 64 in both $x$ and $y$ directions. As mentioned earlier, a bounded upper boundary condition is applied at $z = \infty$. A periodic lateral boundary condition is assumed implicitly by the use of a FFT algorithm.

Similar to L89, the perturbation potential temperature ($\theta$), relative geostrophic vorticity ($\xi_g$), and perturbation geostrophic velocity ($u_g, v_g$) are calculated from the following relationship

$$
\hat{\theta} = \left( \frac{\theta_0}{g \rho_0} \right) \left( \frac{N x}{f} \right) \hat{p},
$$

(14)

$$
\xi_g = -\left( \frac{\sigma^2}{f \rho_0} \right) \hat{p},
$$

(15)

$$
\hat{u}_g = -\left( \frac{i l}{f \rho_0} \right) \hat{p},
$$

(16)

$$
\hat{v}_g = \left( \frac{i k}{f \rho_0} \right) \hat{p},
$$

(17)

in the Fourier space and then transformed back to the physical space by FFT. The basic pressure field is calculated from the geostrophic wind balance

$$
U = (-1/f \rho_0) P_y,
$$

(18)

$$
V = (1/f \rho_0) P_x,
$$

(19)

which are integrated and combined to give

$$
P(x, y, 0) = f \rho_0 (V_S x - U_S y) + P(x_0, y_0, 0)
$$

at $z = 0.$

(20)

The reference pressure $P(x_0, y_0, 0)$ is chosen arbitrarily to be 1005 mb at the reference point $(x_0, y_0, 0) = (0, 0, 0)$. The basic potential temperature field is calculated from the thermal wind relations,

$$
\Theta_y = -f \theta_0 / g U_z,
$$

(21)

$$
\Theta_x = (f \theta_0 / g) V_z,
$$

(22)

which are integrated and combined to give

$$
\Theta_S(x, y, 0) = (f \theta_0 / g) (V_S x - U_S y) + \Theta_S(x_0, y_0, 0)
$$

at $z = 0.$

(23)

Similar to the basic pressure field, the reference potential temperature $\Theta_S(x_0, y_0, 0)$ is chosen arbitrarily to be 293 K at the reference point $(x_0, y_0, 0) = (0, 0, 0)$. Notice that we have assumed a constant vertical wind shear throughout this study as well as in L89.

Using the geostrophic momentum approximation (Eliassen 1962; Hoskins 1975), the nonlinear ageostrophic advection of the geostrophic wind can be included in the model. The advantage of the geostrophic momentum approximation is that the calculation is very accurate when the curvature vorticity is small. To include the ageostrophic advection effects, we have applied the geostrophic coordinate transform (Hoskins and Bretherton 1972; Hoskins 1975)

$$
X = x + v_g / f
$$

$$
Y = y - u_g / f
$$

$$
Z = z
$$

$$
T = t
$$

$$
\Pi = p + (\rho_0 / 2) (u_g^2 + v_g^2)
$$

(24)

where $(X, Y, Z, T)$ are the coordinates in the geostrophic space. With the above coordinate transformation, the following relationship can be verified.

---

FIG. 1. Surface fields after 24 h of baroclinic flows over a bell-shaped heat source with circular contours as described by Eq. (40) with $Q_0 = 0.24 J kg^{-1} s^{-1}$ and $a_x = a_y = 150$ km. The basic wind is assumed to be $U(z) = (-10 + 0.005z) m s^{-1}$ and $V(z) = 0 m s^{-1}$, which blows from the east at the surface and reverses its direction above $z = 2 km$. Other parameters are assumed to be $f = 10^{-4} s^{-1}, N = 10^{-2} s^{-1}, T_0 = 260 K,$ and $\rho_0 = 1 kg m^{-3}$. Three cases of linear quasi-geostrophic (LQG), nonlinear quasi-geostrophic (NQG), and nonlinear semigeostrophic (NSG) flows are presented. The surface perturbation pressure and the perturbation potential temperature fields are shown on the left (a, b, c) and right (d, e, f) panels, respectively. Dashed lines represent negative values. The contour of heating rate of 0.04 J kg^{-1} s^{-1} is denoted by bold dash lines in (a) and (d).
FIG. 2. As in Fig. 1 except for the surface total pressure (a, b, c) and potential temperature (d, e, f) fields.
$J = 1 \left[ 1 - \frac{1}{\rho_0 f^2} (\Pi_{XX} + \Pi_{YY}) \\
+ \frac{1}{\rho_0 f^4} (\Pi_{XX} \Pi_{YY} - \Pi_{XY}^2) \right] = \frac{\zeta}{f}, \quad (25)$

$$
\begin{pmatrix}
fv_g, -fu_g, \frac{g}{P_0} \theta \\
\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}
\end{pmatrix}
= \frac{1}{\rho_0} \begin{pmatrix}
\frac{\partial \Pi}{\partial x}, \frac{\partial \Pi}{\partial y}, \frac{\partial \Pi}{\partial z}
\end{pmatrix}, \quad (26)
$$

where $J$ is the Jacobian of the transformation and $\zeta$ is the vertical component of the absolute vorticity. The conservation of potential vorticity may be written

$$
\left( \frac{\partial}{\partial T} + u_\theta \frac{\partial}{\partial X} + v_\theta \frac{\partial}{\partial Y} + w \frac{\partial}{\partial Z} \right) q_s = 0
$$

where $q_s$ is the potential vorticity

$$
q_s = (\zeta f \rho_0) \Pi_{ZZ}. \quad (28)
$$

The thermodynamic equation at the surface then becomes

$$
\left( \frac{\partial}{\partial T} + u_\theta \frac{\partial}{\partial X} + v_\theta \frac{\partial}{\partial Y} \right) \theta + \frac{N^2 \theta_0}{g} w = \left( \frac{\theta_0}{c_p T_0} \right) q. \quad (29)
$$

Following Bannon (1984), $\partial \theta / \partial Z$ is replaced by a constant $\partial \theta / \partial Z$. In addition, $\partial \theta / \partial Z$ is approximated by $N^2 \theta_0 / g$ (Appendix A). In the interior of the fluid, the governing equation for the pressure $\Pi$ follows from Eqs. (25), (26), (28), and the constraint of uniform potential vorticity, i.e. $q_s = \zeta N^2 / f$,

$$
\frac{1}{\rho_0 f^2} (\Pi_{XX} + \Pi_{YY}) + \frac{1}{\rho_0 N^2} \Pi_{ZZ} \\
- \frac{1}{\rho_0 f^4} (\Pi_{XX} \Pi_{YY} - \Pi_{XY}^2) = 1. \quad (30)
$$

The pressure can be separated into a basic part and a disturbance part,

$$
\Pi(T, X, Y, Z) = \Pi(Z) + \Pi'(T, X, Y, Z) \quad (31)
$$

where

$$
\frac{1}{\rho_0} \frac{\partial \Pi}{\partial Z} = \frac{g \theta}{\theta_0}. \quad (32)
$$

The basic equation for the disturbance pressure field is then obtained by substituting Eq. (31) into Eq. (30),

FIG. 3. Geostrophic vorticity ($\zeta_g$) for Case LQG (a) and Case NQG (b), and geostrophic kinetic energy (c) for Case NQG of Figs. 1 and 2.
\[
\left( \Pi'_x + \Pi'_y + \frac{f^2}{N^2} \Pi'_z \right)
- \frac{1}{\rho_0 f^2} (\Pi''_{xx} \Pi'_y - \Pi''_{xy}) = 0. \tag{33}
\]

As shown in Blumen and Gross (1987), the quadratic terms in the above equation are relatively small under the same constraint required by the geostrophic momentum approximation. Therefore, we will use the following approximate form in this study:

\[
\nabla_h^2 \Pi' + \frac{f^2}{N^2} \Pi'' = 0. \tag{34}
\]

Thus, the potential vorticity equation in the geostrophic space has the same form as the quasi-geostrophic counterpart. Notice that the nonlinearity is retained through the coordinate transformation (24).

At the lower boundary, the semigeostrophic thermodynamic equation may be written

\[
\Pi_{ZT} + \left( U_S - \frac{1}{\rho_0 f} \Pi'_Y \right) \Pi_{ZX} + \left( V_S + \frac{1}{\rho_0 f} \Pi'_X \right) \Pi'_{ZY}
- U_Z \Pi'_X - V_Z \Pi'_Y + \rho_0 N^2 w = \left( \frac{g \rho_0}{\gamma_p} T_0 \right) q'
\]

at \( z = 0 \). \tag{35}

The vertical velocity, with mountain and Ekman friction included, may be written

\[
w(T, X, Y) = \left[ \frac{\partial}{\partial T} + (U_S + u_e) \frac{\partial}{\partial X} + \left( V_S + v_e \right) \frac{\partial}{\partial Y} \right] h(T, X, Y) + \sqrt{\nu/2f} \xi
\]

where \( h(T, X, Y) \) represents the mountain shape in the geostrophic space, which is time-dependent, and \( \xi \) the disturbance vorticity. The mountain shape \( h(T, X, Y) \) and the heating function \( q(T, X, Y) \) in the geostrophic space are obtained by substituting \( x = X - v_e/ f \) and \( y = Y + u_e/ f \) into \( h(x, y) \) and \( q(x, y) \) in the physical space [e.g., Eqs. (40) and (42)]. The forcing functions then become time dependent because \( u_e \) and \( v_e \) vary with time. The lower boundary condition is applied at \( Z = 0 \) instead of at the mountain surface. The error has been shown by Blumen and Gross (1986) to be small. Besides, we have neglected the time derivative in Eq. (36) as did in Hayes et al. (1987). However, the boundary condition contains the ageostrophic advection is still retained through the coordinate transformation. The vertical component of the absolute vorticity may be obtained from Eq. (25),

\[
\xi = \int \left[ 1 - \frac{1}{\rho_0 f^2} (\Pi''_{xx} + \Pi''_{yy})
+ \frac{1}{\rho_0 f^2} \Pi''_{xy} \right] \] \tag{37}

Substituting (31) into (37) and linearizing it gives

\[
\xi = \xi' = \left( 1/\rho_0 f \right) \nabla_h^2 \Pi'. \tag{38}
\]

This gives a form similar to that of Blumen (1980). The neglected ageostrophic component of \( u \) has been shown by Blumen (1979) to be \( O(\text{Ro}^2) < 0.1 \) compared to \( v_e \). Therefore, Eq. (36) becomes

\[
w(T, X, Y) = \left[ (U_S + u_e) \frac{\partial}{\partial X} + (V_S + v_e) \frac{\partial}{\partial Y} \right] \] \times h(T, X, Y) + \sqrt{\nu/2f} \xi
\]

at \( Z = 0 \) \tag{39}
FIG. 5. As in Figs. 1 and 2 except with the addition of uniform Ekman friction (Cases LQG-F, NQG-F, and NSG-F). Only the surface perturbation pressure (a, b, c) and surface total potential temperature (d, e, f) are shown. The viscosity $\nu$ is assumed to be 4.5 m$^2$ s$^{-1}$ over the whole domain.
The above equation can then be substituted into Eq. (35) to give the lower boundary condition. Notice that the lower boundary condition and the vertical velocity at $Z = 0$ (Eqs. (35) and (39)) in the geostrophic space have the same forms as their quasi-geostrophic counterparts [Eqs. (3) and (9)] in the physical space. The semigeostrophic system, Eqs. (34) and (35), can then be solved numerically in the geostrophic space in the same way as the nonlinear quasi-geostrophic system. The solution in the physical space can be obtained by the inverse geostrophic coordinate transform.

3. The inviscid flow

Three cases, linear quasi-geostrophic (LQG), nonlinear quasi-geostrophic (NQG), and nonlinear semigeostrophic (NSG) flow, have been performed and compared in this section. In the LQG case, the nonlinear geostrophic advection terms of Eq. (3) are neglected, while they are included in the NQG case. The inclusion of ageostrophic advection in the NSG case is made by solving Eqs. (34)–(36) with the geostrophic coordinate transformation. In all cases studied in this paper, the model is initialized with a zero perturbation.

Figure 1 shows the surface perturbation pressure and perturbation potential temperature fields after 24 h of baroclinic flows over a bell-shaped heat source with circular contours. The diabatic heating function is prescribed by

$$q(x, y) = \frac{Q_0}{[x^2/a_x^2 + y^2/a_y^2 + 1]^{3/2}}$$  \hspace{1cm} (40)

where $a_x$ and $a_y$ are the horizontal scales of the heat source in $x$ and $y$ directions, respectively. The maximum heating rate ($Q_0$) and the horizontal scale ($a_x$) in this case of the heat source are assumed to be 0.24 J kg$^{-1}$s$^{-1}$ and 150 km, respectively. The basic wind is assumed to be $U(z) = (-10 + 0.005z)$ m s$^{-1}$ and $V(z) = 0$ m s$^{-1}$, which blows from east at the surface and reverses its direction at $z = 2$ km. Hoskins (1975) places a loose upper limit on the Rossby number at $Ro = 0.5$ for use in semigeostrophic theory. For flow over mountains, Blumen and Gross (1988) suggest that $Ro < 0.3$ and $h_0/D < 0.5$ for the geostrophic momentum approximation to be valid, where $h_0$ and $D$ ($= 2f a_{mx}/N$) are the mountain height and the deformation depth, respectively. The Rossby number in the present case is about 0.33, which is estimated by $U_S/2f a_x$ with $U_S = 10$ m s$^{-1}$, $f = 0.0001$ s$^{-1}$, and $a_x = 150$ km. Notice that we have used the whole width ($2a_x$) for the horizontal scale of the bell-shaped heat source, instead of the half-width ($a_x$). In later sections, we will use $h_0 = 1$ km and $a_{mx} = 100$ km [e.g., Eq. (42)], which gives a ratio of $h_0/D = 0.5$. Therefore, the geostrophic momentum approximation may still be appropriate for describing the flow in the present study.

The response of a linear quasi-geostrophic flow to the low-level diabatic heating is a region of low pressure with a minimum of about 6.0 mb forms in the vicinity of the heat source and a region of weaker high pressure with a maximum of 1.2 mb downstream of the heat source after 24 h (Fig. 1a). The low–high couplet is a manifestation of the thermally forced baroclinic wave with zero phase velocity due to the presence of the basic wind reversal (L89). Associated with the low and high pressures are two relatively compact regions of warm and cold air, respectively. The maximum (minimum) perturbation temperature is about 9.1°C ($-2.1{°}$C). The relative vorticity field (Fig. 3a) indicates that the low (high) is associated with the positive (negative) vorticity. The low is shallow because the $e$-folding depth of the low, i.e., $f L_a/2\pi N$, is about 1.5 km for $L_a \sim 1000$ km. The perturbation fields for case LQG is
similar to that in L89 (Case 1, C-1) except that the low (high) is slightly stronger (weaker). The difference may be caused by the finite difference method applied to Eq. (3) in the present study, which was not applied in L89 case. By comparing with Fig. 5 of L89, it appears that the present numerical scheme is more accurate due to a less periodic behavior on lateral boundaries. However, the basic features are very similar.

Both total pressure and potential temperature fields (Fig. 2a and 2d) indicate that the fluid parcel experiences a cyclonic circulation near the heating center and an anticyclonic circulation downstream. The total pressures and potential temperatures are calculated by summing the basic and perturbation fields. There exists a jet, which is associated with a larger pressure gradient zone, located to the northwest of the concentrated heating region (Fig. 2a). As also shown in L89, there exists a confluent-diffluent couplet in the vicinity of the heat source (Fig. 2a). Confluent-diffluent bands have been observed to the west of the coastal front during GALE IOP#2 (Riedman 1990), which may be associated with the differential heating in a low-level onshore flow according to the present theory. The total pressure field indicates that an inverted trough, rather than a closed circulation, is produced.

With the inclusion of nonlinear geostrophic advection (Case QNG), the low is weakened slightly from −6.0 to −5.6 mb, while the high stays about the same strength (Fig. 1b). The high is shifted slightly to the south. This shift in the location of the high center is mainly associated with the strong nonlinear geostrophic wind to the northwest of the heating region (Fig. 2b). The asymmetric pattern is also evident in the perturbation potential temperature field (Fig. 2e) and the geostrophic vorticity field (Fig. 3b). An asymmetric pattern in surface perturbation fields has also been found in corresponding lee cyclogenesis simulations (CS; Lin and Perkey 1989). The total fields of pressure and potential temperature (Fig. 2b and 2e) indicate that the jet is turning further to the left of the basic wind as compared with the linear case (Fig. 2a and 2d). This southwestward turning of the jet is responsible for the slightly weakening of the low due to stronger cold-air advection to the west of the heat source. The time evolution of the minimum pressure field indicates that the low pressure develops more rapidly in the first 10 h and then slows down afterwards for both LQG and NQG cases (Fig. 4a). Even though there is a noticeable difference in the potential temperature fields, the maximum thermal gradient stays about the same as the LQG case (Fig. 4b) at a value of 5.4 K/100 km after 24 h. The maximum thermal gradient is calculated numerically by the following formula

$$|\nabla \theta|_{\text{max}} = \max_{i,j} \left[ \frac{(\theta_{i+1,j} - \theta_{i-1,j})^2}{2\Delta x} + \left( \frac{\theta_{i,j+1} - \theta_{i,j-1}}{2\Delta y} \right)^2 \right]^{1/2}$$

(41)

With the nonlinear ageostrophic advection (Case NSG) included, the low is strengthened significantly to about −9.1 mb and the high is strengthened slightly to 1.4 mb (Fig. 1c). The strengthening of the low is consistent with that predicted by the semigeostrophic theory (Hoskins 1975). However, the strengthening of the high is just opposite to what one might expect. This may be explained by the periodic boundary condition implicitly assumed by the FFT algorithm. For example, the weak low pressure region located near the western boundary in Fig. 1c behaves like a repeated disturbance from the low pressure upstream. The low has developed into a cyclone at 24 h as can be seen from the closed isobars (Fig. 2c). A loosely defined term "cyclone" is used here to indicate that there exists a pronounced closed circulation in the flow pattern. The asymmetric patterns of the perturbation pressure and potential temperature are much more pronounced than case NQG, which is related to the geostrophic kinetic energy (Fig. 3c). The jet turns farther to the south and has a higher speed as can be seen from the more compact isobars compared with case NQG (Fig. 2c). The low is strengthened by the warm advection to the southeast of the heat source. This is also evidenced by the local temperature maximum to the southeast of the low (Figs. 1f and 2f). The role of nonlinear ageostrophic advection becomes more prominent in regions of cyclonic geostrophic vorticity (Fig. 3b), since this is a local measure of the Rossby number. This explains why the circulation is stronger in the NSG case than the quasi-geostrophic cases. There is no frontal collapse generated in this case, which has been found in a corresponding lee cyclogenesis simulation (CS). The absence of frontal collapse may be due to the phase difference of the cyclones produced by the diabatic heating and the orography as depicted in Eq. (13) of L89. Notice that the low-level air parcels are allowed to pass over the heat source in the present case, while they are forced to go up or around the mountain in Chen and Smith's case (CS). In this way, the air parcel turns cyclonically around the heat source, while it turns anticyclonically around the mountain. The maximum potential temperature perturbation at 24 h is increased significantly from the linear quasi-geostrophic case. This result is different from the corresponding lee cyclogenesis problem (e.g., CS) in which the low is weakened for the nonlinear semigeostrophic flow. Thus, the nonlinear ageostrophic advection tends to enhance the thermally induced cyclogenesis. The minimum pressure increases almost linearly in the first 24 h (Fig. 4a) and then slows down at later times (not shown). The maximum thermal gradient increases very rapidly to about 20 K/100 km at 24 h.

4. Frictional effects

Figure 5 shows responses of linear quasi-geostrophic, nonlinear quasi-geostrophic, and nonlinear semigeostrophic flow over an isolated heat source with the ad-
dition of Ekman friction after 24 h (LQG-F, NQG-F, NSG-F). The viscosity coefficient ($\nu$) is 4.5 m$^2$ s$^{-1}$, which is distributed uniformly over the whole domain. The basic wind is the same as previous cases (LQG, NQG, and NSG), which blows from the east with a speed of $-10$ m s$^{-1}$ at the surface and reverses its direction at $z = 2$ km. Friction tends to suppress the development of the disturbance for all cases. The minimum perturbation pressure is about $-4.5$ mb after 24 h, which is reduced by 25% for case LQG, 20% for case NQG, and 45% for case NSG. The asymmetric patterns of perturbation fields for nonlinear frictional cases (NQG-F, NSG-F) are much less pronounced than the corresponding inviscid cases. The thermal gradients in the total potential temperature fields (Figs. 5d,e,f) are weaker than the inviscid cases. Weaker disturbance is resulted from the weaker cyclonic vorticity produced by the addition of friction. Figure 6a indicates that the cyclonic vorticity maximum is reduced from $2.4 \times 10^{-4}$ s$^{-1}$ for case NQG to be $1.9 \times 10^{-4}$ s$^{-1}$ for case NQG-F. Even though a pronounced asymmetric pattern is shown in the geostrophic vorticity field, it is too weak to influence the perturbation pressure and potential temperature fields. The value of geostrophic kinetic energy, $\rho_0 (u^2 + v^2)/2$, is significantly reduced to be 112 Pa for case NQG-F (Fig. 6b) from 270 Pa for case NQG (Fig. 5c).

The weakening of the vorticity in the presence of Ekman friction can be explained by the spindown process (e.g., see Pedlosky 1982). That is, the fluid forced out by the cyclonic vortex from the Ekman layer must flow out in the inviscid layer, from the vortex center to its rim. This outward mass flux will produce vortex compression and reduce the inward pressure-gradient force. A similar result has been found in a linear quasi-geostrophic barotropic flow over an isolated warm region (Lin 1989a) and in a corresponding numerical simulation of lee cyclogenesis (Lin and Perkey 1989). This result is also consistent with a number of theoretical studies (e.g., Farrell 1985; Lin and Pierrehumbert 1988; Valdes and Hoskins 1988) in which the baroclinic flow is stabilized with the inclusion of an Ekman friction.

Differential friction between land and water has been shown to play a key role in coastal frontogenesis (Bosart 1975; Danard and Ellenton 1980). To study the impact of differential friction on the East Coast cyclogenesis, we have performed a simulation of semigeostrophic flow over a region with idealized diabatic heating, as in previous cases, and differential friction. The basic flow and heating function are kept the same as for case NSG-F, except that the viscosity coefficient of the Ekman friction is set to 0 to the east of the dash-dot line (Fig. 7). This dash-dot line is intended to represent the coast line. The flow response is very similar to the inviscid case (Case NSG, Figs. 1, 2, and 3). The minimum perturbation pressure decreases slightly from $-9.1$ to $-9.8$ mb, while the maximum potential temperature stays about the same. The maximum geostrophic vorticity for case NQG increases slightly from $2.4 \times 10^{-4}$ (Fig. 3b) to $2.8 \times 10^{-4}$ s$^{-1}$ (Fig. 7). Over the land (to the left of the dash-dot line in Fig. 7), the perturbations are much weaker than the inviscid case since the flow is slowed down over the land due to friction. Thus, the differential friction between the water and land tends to enhance the cyclone to be slightly stronger than the inviscid case. However, as found in Part I, the primary source of the cyclonic vorticity of the coastal cyclone is the hydrostatic response of the diabatic heating modified by the baroclinicity.

5. Orographic effects

Figure 8 shows the response of a nonlinear semigeostrophic flow over an isolated heat source and a mountain after 24 h (Case NSG-M). The mountain shape is given by

$$h(x, y) = \frac{h_0}{\left(\frac{(x-x_m)^2}{a_{mx}^2} + \frac{(y-y_m)^2}{a_{my}^2} + 1\right)^{3/2}} \quad (42)$$

The parameters used in the above equation are: $h_0 = 1$ km, $x_m = -900$ km, $y_m = 0$ km, $a_{mx} = 100$ km, and $a_{my} = 1500$ km. Thus the bell-shaped mountain is elongated in the $y$ direction, which is intended to represent a mountain ridge. The center line of the mountain is located at 900 km to the west of the heating center ($x = -900$ km). The corresponding Rossby number ($U_s/(2fa_0)$) is about 0.5. Since the Ekman friction is not included in this case, the results can be compared directly with case NSG (Figs. 1–3). As also adopted in a lee cyclogenesis simulation (Lin and Perkey 1989), the model is initialized with a zero perturbation.

In the vicinity of the heat source, the low strengthens slightly from $-9.1$ to $-9.8$ mb and the perturbation potential temperature increases from 23 to 26 K. A region of high pressure (Fig. 8a) forms to the upstream of the center line of the mountain ridge, while a low pressure forms to the lee of the mountain. This result is consistent with the theory of Smith (1984). Near the mesoscale mountain ridge, the flow becomes subgeostrophic with a left-turning in the Northern Hemisphere, as shown in the study of barotropic flow over mesoscale mountains (Smith 1982). Regions of high and low pressures near the mountain are elongated.
in the north–south direction, and coincide with the shape of orographic forcing. For the surface easterly flow, the inverted ridge is very weak, almost to the point of nonexistence (Fig. 8c). Away from the heat source, the isobars dip southward very little. The damming effect is evidenced by the pool of cold air upstream of the mountain ridge (Fig. 8d). It appears that the cold air along the eastern slope of the mountain ridge is mainly formed by the cold advection and the adiabatic cooling associated with the upslope wind. Near the heating center, the geostrophic vorticity field (Fig. 8e) is almost identical to that of case NSG (Fig. 3b). Upstream of the mountain ridge, there exists a region of negative vorticity which is associated with the mountain-induced high pressure. On the lee side, there exists a wider region of cyclonic vorticity.

The damming effect will be more pronounced for a smaller Froude-number flow over an elongated heat source. Lin (1989b) shows that the inverted ridge is much more pronounced for a flow over an elongated heat source than a circular one. The Froude number, defined as $F_r = U_0/L$, is about 1 in this case, which is relatively larger than observed values of 0.3 to 0.4 (Forbes et al. 1988; Bell and Bosart 1988). This helps explain the relatively weaker damming effect. In the real case, the Froude number may become smaller because the basic flow speed ($U_0$) is often reduced by the boundary layer effect. Notice that the rotational Froude number can be defined as $F_r = L/Nh_0 = U_0/L$, where $L$ is the horizontal scale of the mountain in the basic flow direction. For semigeostrophic theory to be valid, Blumen and Gross (1986) have shown that the rotational Froude number must be greater than 1.7. In this case, the horizontal scale of the mountain is about 200 km ($2a_x$) and the rotational Froude number is about 2.0. Thus the semigeostrophic theory may still be appropriate for describing the flow. It is interesting to mention that the rotational Froude number is no longer a function of $U_0$, but depends on the reciprocal of the mountain slope ($L/h_0$). Thus, a smaller rotational Froude number is expected for a steeper mountain, which would have a stronger damming effect. In this study, we have assumed a shallow mountain in accordance with the lower boundary condition, Eq. (8).

6. Application to East Coast cyclogenesis

Figure 9a depicts the cyclogenesis frequency and the geography along the east coast of the United States. The total number of cyclone formation events in a 63 month period from January 1978 to March 1983 indicates that a belt of high frequency is located along the western boundary of the Gulf Stream, with the maximum near the South Carolina coast (Smith 1986). The strong clustering along the coast is evidently due to the highly complicated geography. To the east of the coastline, there exists the cool shelf water adjacent to the warm Gulf Stream (denoted by a doubled dash line). To the west of the coastline, there exists the piedmont adjacent to the Appalachians which stretches from northeastern Alabama to Maine. To simplify the problem, the mountain is idealized as an elongated bell-shaped mountain, while the diabatic heating associated with the temperature contrast between land and water is idealized as an elongated bell-shaped heat source as used in L89. The sensible heat flux near the Carolina coast for 0000 UTC 25 January 1986, just before the cyclogenesis event of GALE IOP#2, is shown in Fig. 9b (Riordan 1990). Identical to L89, the surface sensible heating rate and function are idealized from this sensible heat flux distribution.

The diabatic heating function, Eq. (40), is rotated clockwise by 45 degrees to represent this surface sensible heating,

$$q(x, y) = \frac{Q_0}{[(x - y)^2/2a_x^2 + (x + y)^2/2a_y^2 + 1]^{3/2}}. \tag{43}$$

This represents the horizontal pattern of the sensible heat flux observed at 0000 UTC 25 January 1986, just prior to the cyclogenesis of GALE IOP#2 (Fig. 10 of L89; Riordan, 1989). The same horizontal scales, $a_x = 75$ km and $a_y = 300$ km, as in L89 have been used. The maximum heating rate, 0.24 J kg$^{-1}$ s$^{-1}$, is roughly estimated from observations of East Coast cyclogenesis.

The Appalachians is idealized as an elongated bell-shaped mountain which is oriented in the northeast–southwest direction and represented by the following equation,

$$h(x, y) = h_0 \frac{1}{[\{(x - x_{m0}) - (y - y_{m0})]\}^2/2a_{nx}^2 + \{(x - x_{m0}) + (y - y_{m0})\}^2/2a_{ny}^2 + 1]^{3/2}}. \tag{44}$$

The parameters used in the above equation are: $h_0 = 1$ km, $x_{m0} = 0$ km, $y_{m0} = 900$ km, $a_{nx} = 100$ km, and $a_{ny} = 1500$ km. The basic wind is assumed to be $U(z) = (-8.7 + 0.00415z)$ m s$^{-1}$ and $V(z) = (-5.0 + 0.00347z)$ m s$^{-1}$. At the surface, the basic wind blows from 60° with a speed of 10 m s$^{-1}$, as also used in L89. At 5.5 km ($\sim 500$ mb), it blows from 225° (southwest) with a speed of 20 m s$^{-1}$. The thermal wind is almost parallel to the coastline. The hodograph of the basic wind vectors at surface and 5.5 km is sketched in the upper left corner of Fig. 10a. Since the addition of surface friction does not make significant changes from the inviscid case, we have excluded the Ekman friction in the following simulations.

FIG. 8. As in Fig. 7 except with no friction and with a mountain ridge. The height contour of 250 m is denoted by bold dash-double dot lines (•••••••). The mountain ridge has a bell shape in east–west direction. The mountain height and half-width are 1 and 100 km, respectively. Results may be compared with Case NSG (Figs. 1, 2 and 3).
Fig. 9. (a) Total number of cyclone formation events in a 63 month period from January 1978 to March 1983 in each $2^\circ \times 2^\circ$ rectangle (Adapted from Smith 1986). The Appalachians and the averaged western boundary of the Gulf Stream are denoted by $\wedge\wedge\wedge$ (enclosed by dotted lines) and $\cdots$, respectively. (b) Surface analysis for 0000 UTC January 1986 (GALE IOP#2) for the sensible heat fluxes (in W m$^{-2}$). The dashed lines in (b) represent the estimated values of sensible heat flux where only sparse data are available (Modified from Riordan 1990).
Figure 10 shows the surface fields of perturbation pressure, perturbation potential temperature, total pressure, total potential temperature, geostrophic vorticity, and geostrophic kinetic energy for a semigeostrophic flow over the prescribed heat source (43) and mountain (44) after 24 h (Case EC1). A surface low with an amplitude of $-9.2$ mb (Figs. 10a and 11a) forms near the center of diabatic heating. The low can be called a cyclone since a closed circulation is shown in the total pressure field (Fig. 10c). Again, a loosely-defined "cyclone" is used here to indicate that there exists a pronounced closed circulation in the flow pattern. This indicates that the genesis region of the cyclone is near the western boundary of the Gulf Stream. The result is similar to the quasi-geostrophic case (L89). The low is stronger and skewed with a larger gradient located to the southeast corner, which is due to the nonlinear ageostrophic advection of warm air. Two regions of high pressure develop in the vicinity of the low, with the stronger (weaker) one located to the west (south) (Fig. 10a). The high pressure located to the west of the cyclone is enhanced by the mountain-induced high as compared with a case without mountain (Case EC2, Fig. 12a). The absolute magnitude of the mountain-induced high pressure is also weaker than the heat-induced low as found in Case NSG-M (Fig. 8a). The mountain-induced high pressure becomes much weaker for the present case since the surface wind is almost parallel to the mountain ridge. In addition, since the mountain is closer to the diabatic heat source (636 km), the high is overcome by the heat-induced low except near the southern part of the mountain. This explains the weak cold-air damming in the present case. The low pressure produced by the mountain is evident on the lee side.

Associated with the cyclone is a relatively compact region of warm air and positive vorticity (Figs. 10b and 10c). Even though the mountain-induced high is overcome by the heat-induced low in the northeast part of the elongated mountain, a pool of slightly cold air over the mountain is still evident (Fig. 10d). This cold air is produced by the diabatic cooling as the air parcel climbs up the mountain ridge and the anticyclonic turning because the flow becomes subgeostrophic upstream of the mountain. However, the cold air located to the southern part is mainly formed by the cold air advection by the ageostrophic wind as can be seen from the anticyclonic turning (Fig. 10c). The geostrophic vorticity field shows a region of negative vorticity in the vicinity of the mountain, while a region of positive vorticity forms on the lee side. This is consistent with the results of corresponding lee cyclogenesis simulations (Smith 1984, 1986). The maximum thermal gradient reaches a value of 22.4 K/100 km at 24 h (Fig. 11b). The fluctuations (oscillations) present in the curve may be due to the shift of locations of maximum thermal gradient with time.

To isolate the heating effect, a case excluding the mountain (Case EC2) is performed and shown in Fig. 12. Compared with the previous case (Case EC1), the flow field in the vicinity of the heating, the time evolutions of minimum pressure and the maximum thermal gradient are identical (Fig. 11). Therefore, we may conclude that the Appalachians play a minor role in the cyclogenesis process with a basic surface flow nearly parallel to the orientational axis of the mountain ridge. Figure 13 shows the flow response at 24 h of a case (Case EC3) similar to EC1 but with an easterly wind at the surface. The basic state flow is given by $\mathbf{u}(z) = (-10 + 0.005z) \text{ m s}^{-1}$ and $\mathbf{v}(z) = 0.004z \text{ m s}^{-1}$. This situation may be realized as the synoptic high to the north of the genesis region moves further eastward to the ocean. The basic characteristics of the flow response near the heat source are similar to EC1. Compared with the quasi-geostrophic case (L89), the low is strengthened by the ageostrophic wind advection and reaches a value of $-7$ mb. However, the low is weaker than that of Case EC1 due to less heat received by the easterly wind at the surface. This is because the residence time of air parcels over the heating region is shorter compared with that of EC1. In the vicinity of the mountain ridge, the flow response is much stronger than the case with a northeasterly surface wind (Case EC1). The inverted ridge is stronger in this case because the basic wind has a larger impinge angle with the mountain. A pool of cold air is dammed to the upslope of the mountain, with the maximum located between the southern part of the mountain and the heat source (Figs. 13c and 13d). It is interesting to point out that the inverted ridge or the damming area is shifted further upstream and is more widespread in the region between the southern part of the mountain and the heat source. This result is consistent with observations (Forbes et al., 1987; Bell and Bosart, 1988). One example of cold-air damming is given in Fig. 14, which shows the pressure distribution at 600 m surface at 1200 UTC 13 January 1980 (Forbes et al. 1987). Notice that the low on the lee side may not present in the real atmosphere since the basic flow structure behind the mountain usually is different from that upstream. The flow response also shows that a confluent-diffusive couplet forms to the northwest and southwest of the heating, as also found in L89. This confluent-diffusive couplet may be related to the coastal frontalogenesis as observed (e.g., Riordan 1990). The cyclone and the thermal gradient are weakened in an easterly flow due to a reduction in the heat received as shown in Fig. 11.

7. Concluding remarks

A linear quasi-geostrophic theory of cyclogenesis forced by surface diabatic heating proposed by Lin (1989b) has been extended by a semigeostrophic model. The nonlinear ageostrophic advection is included by applying the geostrophic coordinate transform. The model solves the semigeostrophic system numerically by a finite difference method and a Fast Fourier Transform algorithm. The model is then used
Fig. 10. A developing cyclone produced by a semigeostrophic baroclinic current over a diabatic heating region and a semiridge after 24 h (Case EC1). The diabatic heating function and the mountain shape are given by Eqs. (43) and (44), respectively. The parameters used are \( Q_0 = 0.24 \text{ J kg}^{-1} \text{s}^{-1} \), \( \alpha_x = 75 \text{ km} \), \( \alpha_y = 300 \text{ km} \), \( \eta_0 = 0 \text{ km} \), \( \eta_0 = 1000 \text{ m} \), \( a_{x_T} = 100 \text{ km} \), \( a_{y_T} = 1500 \text{ km} \), \( x_{	ext{max}} = 0 \text{ km} \), \( y_{	ext{max}} = 900 \text{ km} \), and \( P(0,0,0) = 1015 \text{ mb} \). The hodograph is shown in (a), where \( U_S \) and \( U_T \) denote the basic winds at surface and 5.5 km, respectively. The contours of heating rate of 0.04 J kg\(^{-1}\) s\(^{-1}\) and terrain height of 250 m are denoted by bold dashed line and dash-dotted line, respectively. Six fields are shown: (a) perturbation pressure, (b) perturbation thermal gradient, (c) total pressure, (d) total potential temperature, (e) geostrophic vorticity, and (f) geostrophic kinetic energy.

Fig. 11. Time evolutions of minimum pressure (a) and maximum thermal gradient for Cases EC1 (Fig. 10), EC2 (Fig. 12), and EC3 (Fig. 13).

to study the effects of nonlinear geostrophic and ageostrophic advectons, differential friction, and orography. Finally, the model is applied to East Coast cyclogenesis.

The response of an east–west backsheared, quasi-geostrophic baroclinic flow over an isolated heat source is a low pressure near the heating center and a weaker high pressure downstream, as found in Part I of the theory. The low–high couplet is a manifestation of the thermally forced baroclinic wave with zero phase velocity due to the presence of the basic wind reversal. Associated with the low and high pressures are two relatively compact regions of warm and cold air, respectively. There exists an easterly jet located to the north of the low.

With the inclusion of nonlinear geostrophic advection of geostrophic wind, the low is weakened slightly, while the high remains about the same strength. Perturbation fields of pressure and potential temperature become asymmetric. This asymmetric pattern is associated with the southwestward turning of the jet which is also responsible for the slightly weakening of the low due to the cold-air advection to the west of the heat source. With the inclusion of nonlinear ageostrophic advection, the low is strengthened significantly by the warm air advection to the southeast of the heat source. Thus, we conclude that the nonlinear ageostrophic advection tends to enhance the cyclogenesis process forced by diabatic heating.

With the addition of uniform friction, the cyclonic circulation is weakened. This phenomenon is explained by the spindown process. However, differential friction between the water and land tends to enhance the cyclone to be slightly stronger than the inviscid case. It appears that the primary source of the cyclonic vorticity of the coastal cyclone is the hydrostatic response of the diabatic heating modified by the baroclinicity. With a mountain ridge included, there exists an inverted pressure ridge (trough) over (downstream of) the mountain. The result is consistent with corresponding lee cyclogenesis simulations. Damming effect is evidenced by the pool of cold air located upslope of the mountain ridge. A cold air along the eastern slope of the mountain ridge is formed by the combined effects of cold air advection and upslope adiabatic cooling. The inverted pressure ridge between the mountain and the heat source is strengthened by the surface heating and is shifted farther upstream of the mountain.

When the semigeostrophic model is applied to East Coast cyclogenesis with a northeasterly surface wind, a cyclone develops near the center of the low-level sensible heat source. That is, the western boundary of the Gulf Stream. The low associated with the cyclone is stronger than the corresponding quasi-geostrophic case, but is skewed with a larger gradient located to the southeast corner. The mountain-induced high pressure is relatively weak since the surface wind is almost parallel to the mountain ridge. The cold-air damming is
Fig. 12. As in Fig. 10 except with no mountain (Case EC2).
Fig. 13. As in Fig. 10 except with surface easterly wind (Case EC3).
negligible and the Appalachians play a minor role in this type of coastal cyclogenesis in which a surface flow is nearly parallel to the orientational axis of the mountain. With an easterly surface wind, a pool of cold air is dammed to the up-slope of the mountain. The inverted ridge or the damming area is shifted farther upstream and more widespread in the region between the southern part of the Appalachians and the Gulf Stream. A confluent–diffluent couplet forms to the northwest and southwest of the heat source, which may be related to the coastal frontogenesis. The result is consistent with observations.

The present theory of coastal cyclogenesis can be extended by a fully nonlinear primitive-equation model. This will allow us to include the moisture effects and advection effects on the ageostrophic wind, which may play important roles as the cycloic vorticity becomes more intense and the horizontal scale of the cyclonic circulation becomes smaller. The coupling of Ekman friction and low-level diabatic heating can be improved by using a more realistic boundary layer parameterization. A steeper mountain can also be incorporated into the numerical model, which may produce a more realistic and stronger cold-air damming along the windward slope. The relationship between the confluent–diffluent zones and the coastal frontogenesis can also be studied in more detail by using a finer resolution in a numerical model.

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APPENDIX A

Approximation of $\partial \theta / \partial Z$ of Eq. (29)

According to Hoskins (1975), $J \partial \theta / \partial Z$ may be written as the following:

$$J \frac{\partial \theta}{\partial Z} = \left[ \frac{1}{\rho_0 f^2} p_{xz} \left( 1 + \frac{1}{\rho_0 f^2} p_{yy} \right) - \frac{1}{\rho_0 f^4} p_{yx} p_{x} \right] \frac{\partial \theta}{\partial x} - \left[ \frac{1}{\rho_0 f^2} p_{yz} \left( 1 + \frac{1}{\rho_0 f^2} p_{xx} \right) - \frac{1}{\rho_0 f^4} p_{xz} p_{y} \right] \frac{\partial \theta}{\partial y} + J \frac{\partial \theta}{\partial z}$$

(A1)

where $J$ is the Jacobian of the geostrophic coordinate transform, which equals $\xi' f$. After making linearization and neglecting higher order terms, we have

$$\frac{\xi}{f} \frac{\partial \theta}{\partial Z} = - \frac{1}{\rho_0 f^2} p_{x} \partial_x \theta_x - \frac{1}{\rho_0 f^2} p_{y} \partial_y \theta_y + \frac{\xi}{f} \frac{\partial \theta}{\partial z}$$

(A2)

Applying Eqs. (18), (21), and (22) to the above equation yields

$$\frac{\xi}{f} \frac{\partial \theta}{\partial Z} = - \frac{\theta_0}{g} (V_z^2 + U_z^2) + \frac{\xi}{f} \frac{\partial \theta}{\partial z}.$$  

(A3)

The typical scales for East Coast cyclogenesis are

$\xi = 0.5 f$,  \hspace{1em} $\theta_0 = 300$ K,  \hspace{1em} $g = 9.8$ m s$^{-1}$,

$U_z$, $V_z = 0.005$ s$^{-1}$, and $\partial \theta / \partial z = 0.3$ m$^{-1}$.

Using the above scales, we obtain

$$\frac{\theta_0}{g} V_z^2, \hspace{1em} \frac{\theta_0}{g} U_z^2 \sim 7.5 \times 10^{-4}$ K m$^{-1}$,

$$\frac{\xi}{f} \frac{\partial \theta}{\partial z} \sim 0.15$ K m$^{-1}$.

Thus, to a first approximation, we may neglect the first two terms on the right hand side of Eq. (A3). This gives $\partial \theta / \partial Z = \partial \theta / \partial z$.

APPENDIX B

Acronyms of Cases Simulated

LQG Linear quasi-geostrophic case
NQG Nonlinear quasi-geostrophic case
NSG Nonlinear semigeostrophic case
LQG-F Same as LQG except with uniform friction included
NQG-F Same as NQG except with uniform friction included
NSG-F Same as NSG except with uniform friction included
NSG-DF Same as NSG except with differential friction included
NSG-M Same as NSG except with a mountain ridge included
EC1 (East Coast cyclogenesis case) Nonlinear semigeostrophic flow over a heat source and a mountain with a surface wind blowing from 60°.
EC2 Same as EC1 except with no mountain
EC3 Same as EC1 except with an easterly surface wind.

REFERENCES


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