Evidence of Deterministic Chaos in the Pulse of Storm Rainfall

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ABSTRACT

Rainfall data obtained by a highly sensitive raingage have been analyzed for the presence of strange attractors. Analysis of three storms that occurred in Cambridge, Massachusetts revealed, for each storm, the presence of a low-dimensional strange attractor with correlation dimension that was less than 4. The datasets consist of the discrete time series of the interarrival times of one-hundredth of a millimeter rainfall amounts. In all cases, the number of data points in the datasets was at least 3300, which makes the evidence of determinism in storm rainfall strong.

1. Introduction

Several experimental studies recently documented in the literature show evidence of determinism in climate and weather related variables. There is evidence regarding the existence of strange attractors in finite-amplitude convection (Lorenz 1963), local-climate data (Grassberger 1986), wind data (Tsonis and Elsner 1989), and atmospheric-pressure data (Henderson and Wells 1988). For an excellent review of chaos and strange attractors, the reader is referred to Grebogi et al. (1987).

In this paper we document further studies regarding the existence of deterministic chaos in nature. In particular, we show strong evidence that supports the existence of a low-dimensional strange attractor in storm-rainfall observations. Such a finding, if followed by studies that would characterize the distribution of points on the attractor, has significant implications for the reliable long-term prediction of rainfall and, consequently, for the design of large hydraulic structures that are at risk due to excessive rainfall rates.

2. Rainfall data

It is difficult to analyze rainfall for the existence of strange attractors because intense rain usually lasts a short time and yields a short time series when observed with conventional on-site sensors. For statistical stability of the estimates of the correlation dimension of the attractor (if one is indeed present), there should exist several thousand data points in the time series under study for dimensions less than 4 (Ramsey and Yuan 1987). Such an opportunity was afforded by the existence of a limited number of prototype datasets that were obtained by a highly sensitive raingage designed and operated for a short period at the Massachusetts Institute of Technology (MIT) in Cambridge, Massachusetts. The datasets were kindly made available by Professor Earl Williams of the Department of Atmospheric Sciences at MIT. The raingage design was based on a tipping bucket and was capable of sending a signal every time it collected a depth of one-hundredth of a millimeter of rain. The raingage response time could be as short as one-eighth of a second. The datasets consisted of the times the raingage signalled the collection of an amount of rain equal to one-hundredth of a millimeter.

Three storms were studied. The storm dates and times are shown in Table 1 together with the total number of data points in the time series of each storm. Figures 1a through 1c present the time series data for the three storms studied. The vertical axis represents the time between raingage signals (each corresponding to an accumulation of 0.01 mm of rainfall) and the horizontal axis represents the count number of the signal. Low values of the ordinate variable indicate frequent signals and, therefore, periods of heavier rain. The air temperature was above 10°C during the storms that produced heavy rainfall rates. The average rainfall rate for the 9 hours of the first storm was computed to be 4.35 mm h^{-1}. The average rainfall rate for the 13

TABLE 1. Storm dates and times (LST).

Storm number	Initial date and time (LST)	Ending date and time (LST)	Data points
1	25 Oct 1980; 1200	25 Oct 1980; 2059	4000
2	4 Jun 1982; 1900	5 Jun 1982; 0759	3991
3	2 Jun 1982; 0500	2 Jun 1982; 0859	3316

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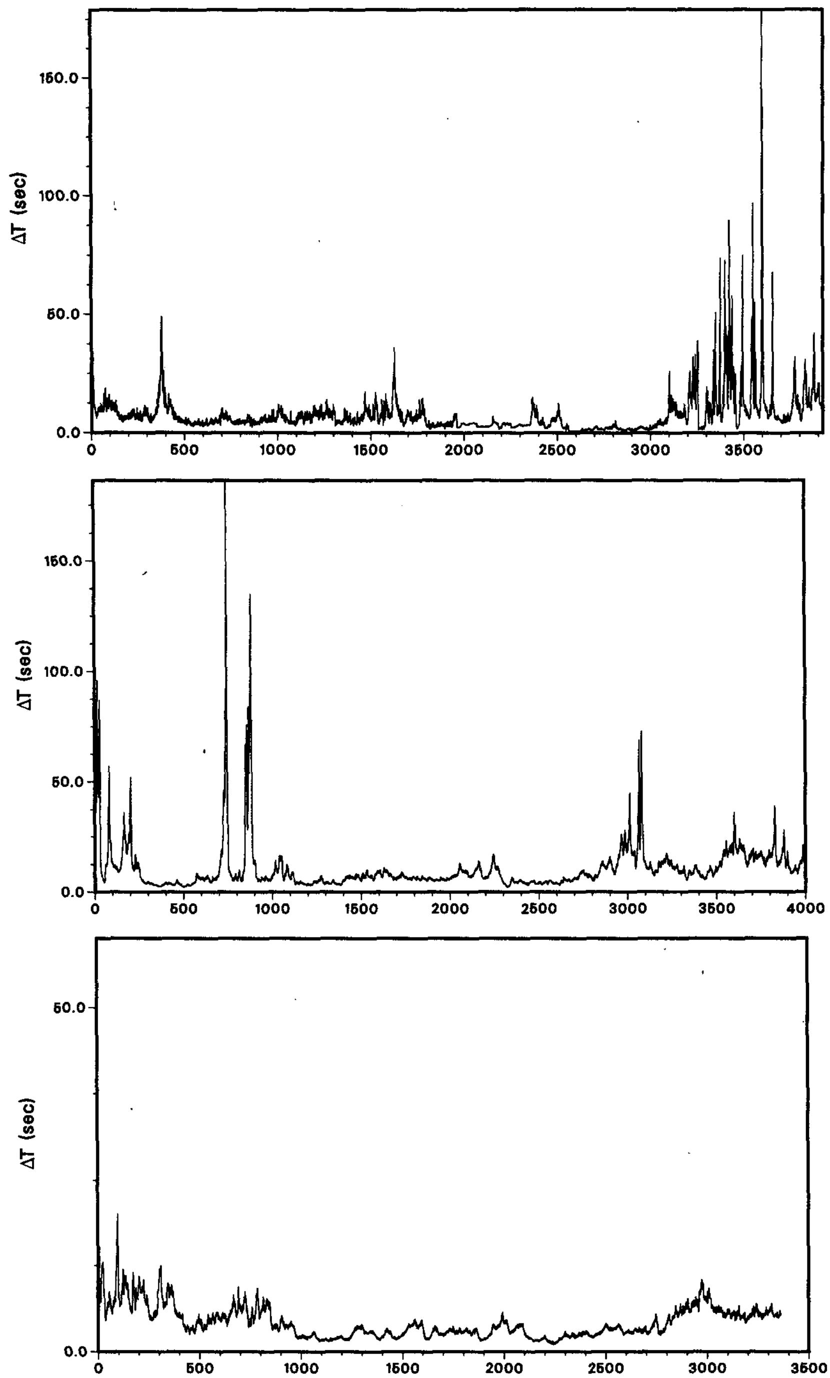

FIG. 1. Time series of the time Δt (in seconds) between successive raingage signals, each corresponding to the collection of 0.01 mm of rain. The abscissa is for the count number of the signals. (a) Storm 1, (b) storm 2, (c) storm 3. The storm dates and times are shown in Table 1.

hours of the second storm was 3.07 mm h^{-1}, while the analogous figure for the 4 hours of the third storm was 8.4 mm h^{-1}. Data from the first storm of Table 1 have been used previously in a related study of chaotic dynamics in rainfall (Rodriguez-Iturbe et al. 1989), after the raw data had been converted by a linear transformation to rain depths over 15 seconds. Such a transformation yielded about 1990 data points in the time series for rainfall depths.

The normalized spectral densities of the three storms are shown in Figs. 2a through 2c. The spectral estimates were normalized by the total variance so that the area under the curves is equal to 1. The estimates were computed using the International Mathematical and Statistical Library (IMSL) software package routines. The Priestley–Bartlett method was used with a spectral window of count 30. Those estimates can be compared to the normalized spectral estimates of 5000 data points of computer-generated Gaussian white noise that are shown in Fig. 2d. It can be seen that the spectra reveal significant variance over a wide range of frequencies (in this case dimensionless), and one is tempted to conclude that the storm-rainfall time series is a sample of a purely random process (e.g., compare Figs. 2a and 2d). The spectrum alone, however, is not a sufficient indicator of randomness in a time series. Time series that possess low-dimensional strange attractors and are generated by deterministic processes exhibit the behavior seen in Figs. 2a through 2c. Thomson and Stewart (1986), and Burge et al. (1984) present classical examples. A more accurate determination of the nature of the time series can be made by computing the correlation dimension v and plotting it against the embedding dimension p. The correlation dimension is a measure of the density of points on the attractor. Next, the method followed for the determination of v is described. The reader is referred to specialized literature for a more detailed treatment (Froehling et al. 1981; Farmer et al. 1983; Grassberger and Procaccia 1983).

3. Determination of the correlation dimension

At first, the initial embedding dimension is set to p. Then, points are located on the p-dimensional space based on the coordinates: $X(t)$, $X(t+\tau)$, . . . , $X(t + (p-1)\tau)$, where $X(t)$ represents the data point of the time series corresponding to count number t and τ is a delay count. The total number of points that can be located in the p-dimensional space for the given time series is called the phase-space portrait of the series. A function $C(r)$ is formed by counting the number of pairs with Eucledian distance in p-dimensional space less than r, and normalizing by the square of the total number of points in the phase portrait. Both cases of including zero-distance points and not including such points were considered. Due to the sufficiently large number of points on the phase portraits, the resultant correlation dimension was not significantly different for those two cases.

Plotting the function $C(r)$ against r on a log–log plot yields the correlation dimension v as the slope of the straight-line segment of the plotted line. The correlation dimension v and the embedding dimension p yield a point in a v versus p plot. Then, the dimension of the embedding space, p, is increased to p' and a new function $C'(r)$ and a new correlation dimension v' are computed. The dyad (v', p') forms a second point in the v versus p plot, etc. If a low-dimensional attractor is present in the time series under study, as p is in-

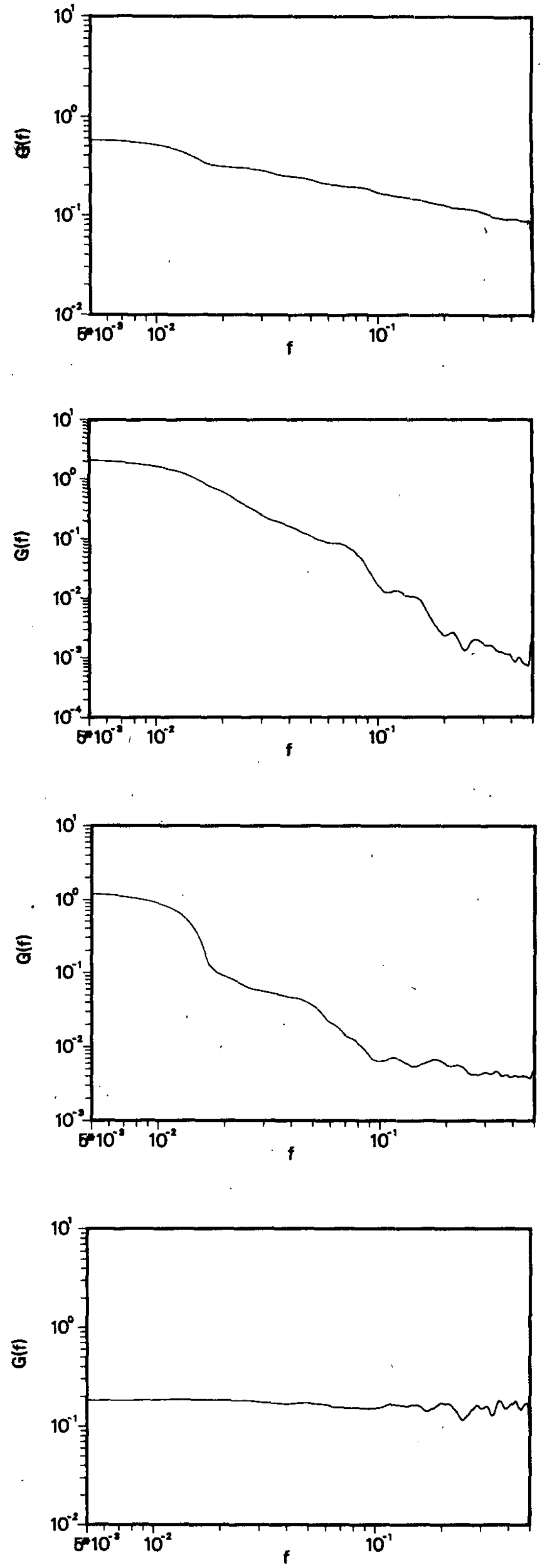

FIG. 2. Spectral estimates of the observed time series normalized by the total variance in counts per cycle as a function of frequency in cycles per count. (a) Storm 1, (b) storm 2, (c) storm 3, (d) Gaussian white noise.

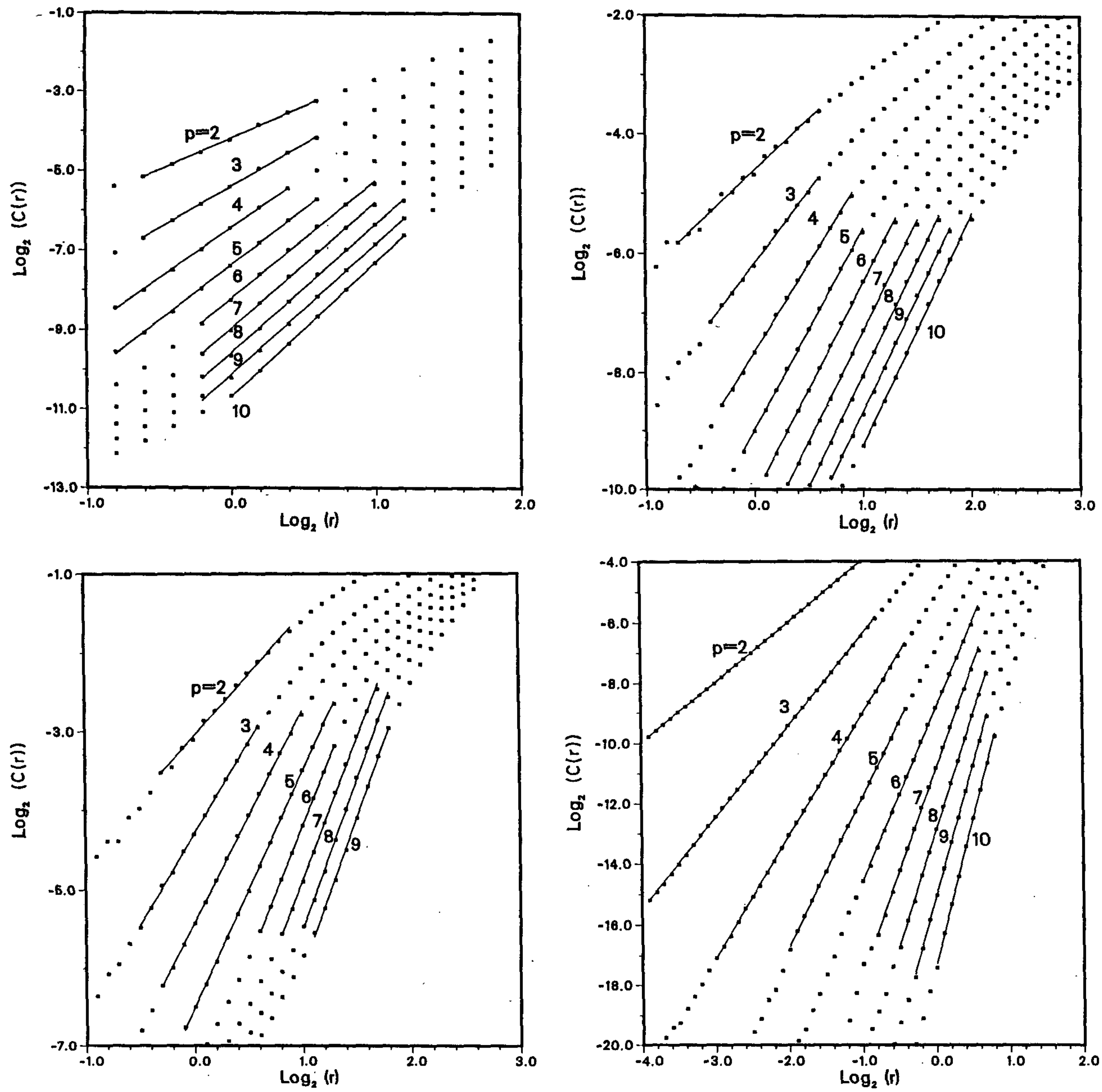

FIG. 3. Logarithm of the function $C(r)$ in base 2 as a function of the logarithm of r in base 2. The curves are parameterized by the corresponding values of the embedding dimension p. (a) Storm 1, $\tau = 4$; (b) storm 2, $\tau = 16$; (c) storm 3, $\tau = 134$; (d) Gaussian white noise, $\tau = 4$.

creased beyond the dimension of the attractor, the correlation dimension would stop increasing and saturation of the values of v would occur in the v versus p plot. On the other hand, if the time series is a sample of a purely random process, saturation is not expected to occur for low values of v.

For this study, the delay count τ was determined by computing the correlation function of the time series and selecting the value of the lag count τ for which the correlation function took the value 0.5. Determining the correct value of the delay count τ usually involves trial and error and no strict rules are known to apply. Several values of τ were tried and the value for which the correlation function of each time series assumed the value of 0.5 gave as good results as any. Such a choice is proposed in Schuster (1988).

Figures 3a through 3c, corresponding to storms 1 through 3, show the log–log plots of the family of functions $C(r)$ versus r together with the linear segment on each line used for the determination of v. Figure 3d shows analogous results for the time series of Gaussian white noise mentioned previously. Figures 4a through

4d are the v versus p plots, with the last figure corresponding to the generated white noise and the former three corresponding to storms 1 through 3. The difference between the former three figures and the last figure is remarkable. In particular, compare the results of Fig. 4a with those in Fig. 4d that correspond to series with similar spectral properties. The saturation value of v obtained for the storm data was in the range of 3.3 to 3.8, indicating a low-dimensional strange attractor in the 0.01-mm pulses of rainfall for all storms examined. It is noted that for the delay counts τ selected and for p up to 10, the phase-space portraits of the time series corresponding to the storm data included at least 3700 points each for all the storms except storm 3. For the latter storm and for $p = 10$ it included 1956 points. Those facts generate more confidence in the results obtained for the former two storms for large p. For all storms, however, the evidence is strong that, for small p ($p < 5$), the values of v saturate to a value less than 4.

Recently, Osborne and Provenzale (1989) proved that colored noises with a spectrum that shows a power-law decay exhibit saturation of v with increasing embedding dimension. In light of their proof and to ascertain that the rainfall data analyzed was indeed a sample of deterministic chaos rather than a sample of colored noise, it was necessary to examine the spectra of all the storms (Figs. 2a, 2b, and 2c). It is apparent that while one can fit a power-law to the spectrum of Fig. 2a and perhaps to the spectrum of Fig. 2b, it is impossible to do so for the spectrum of Fig. 2c over the whole range of frequencies. Furthermore, the exponent of the power law fitted to the spectrum in Fig. 2a was less than 1, which indicates a saturation value of v greater than 10 for colored noise (Osborne and Provenzale 1989). The saturation value obtained for this case (see Fig. 4a) is sufficiently smaller than 10, which makes the evidence of deterministic chaos in the pulse of storm rainfall strong.

The effects of additive random noise on the rainfall

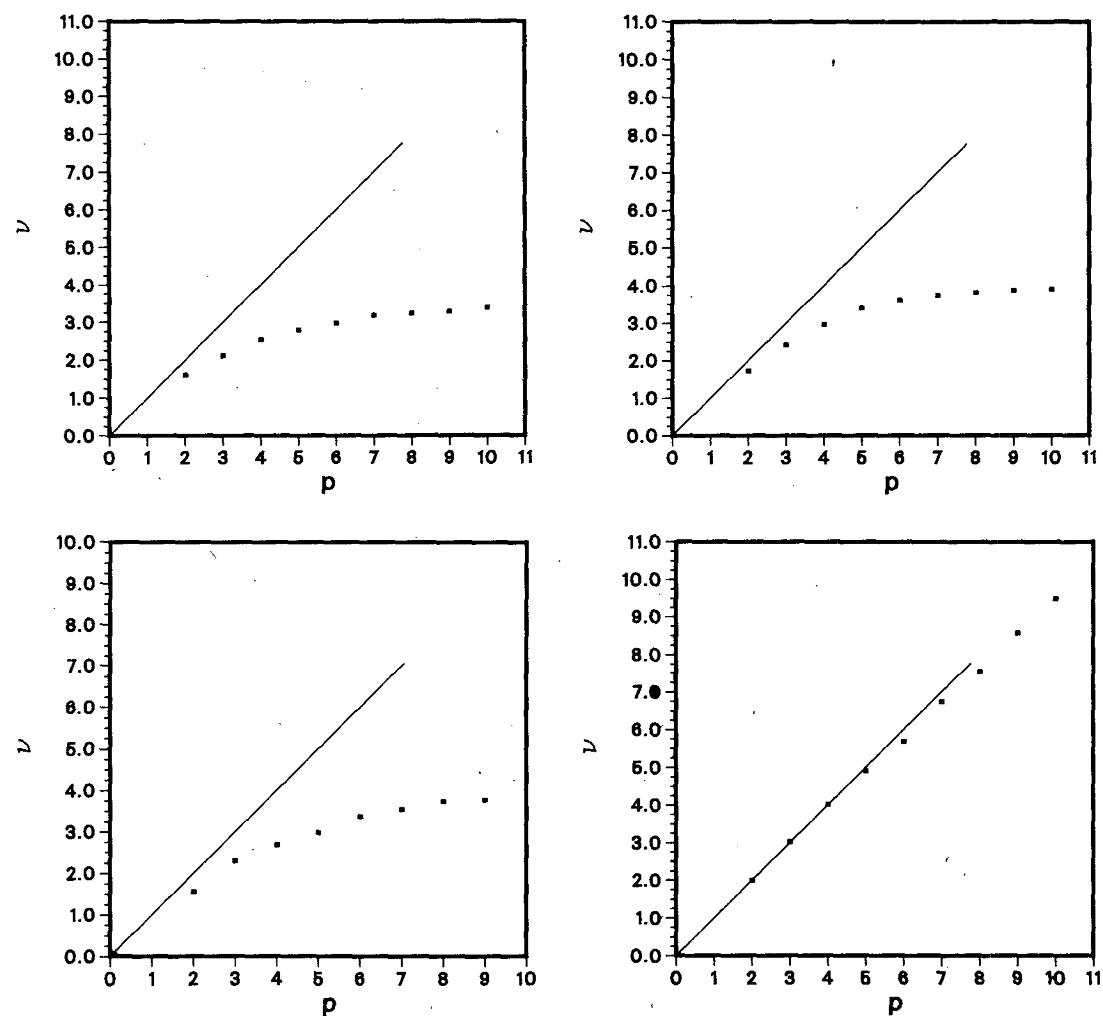

FIG. 4. Correlation dimension v versus embedding dimension p for each time series. (a) Storm 1, $\tau = 4$; (b) storm 2, $\tau = 16$; (c) storm 3, $\tau = 134$; (d) Gaussian white noise, $\tau = 4$.

data examined are now exemplified for storm 1 and for two values of the noise standard deviation. A standard deviation equal to one percent of the standard deviation of the rain data, and a standard deviation equal to 10 percent of the standard deviation of the rain data are considered. The v versus p plots for the two cases are shown in Figs. 5a and 5b, respectively. Comparison of those two figures with Fig. 4a shows that the saturation value of the correlation dimension is sensitive to the strength of the additive random noise and that it increases with increasing noise strength. This implies that additive random instrument-errors would tend to artificially increase the correlation dimension computed for the strange attractor and, in that sense, the results should be considered as being conservative.

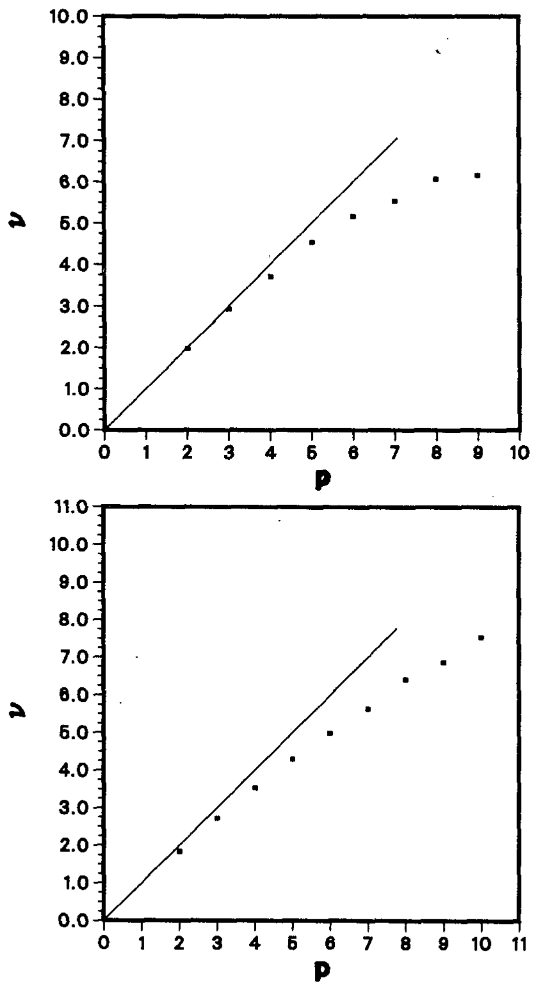

FIG. 5. Correlation dimension v versus embedding dimension p for storm 1. (a) Case of additive random noise of standard deviation equal to 1 percent of the rain-data standard deviation; (b) case of additive random noise of standard deviation equal to 10 percent of the rain-data standard deviation.

4. Prospect

The evidence shown here regarding the deterministic nature of storm-rainfall pulses (as sampled by a prototype raingage) offers hope for more reliable long-term rainfall predictions and presents an opportunity for expanded research in this area of hydrometeorology. Necessary prerequisites toward the former goal appear to be the confirmation of the low-dimensional attractor in the time series of other storms from other locations and climates, and the identification of the important parameters of the distribution of points on the attractor. The key to a successful research effort in this area appears to be the availability of very high temporal resolution samples of storm rainfall.

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