The Structure of the Ageostrophic Wind Field in Baroclinic Waves

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ABSTRACT

A high-pass filtered time series of the 500-mb height at a grid point in the Pacific storm track is regressed against the ageostrophic wind field at all grid points on the NMC grids for nine winters in order to determine the characteristic relationship between the ageostrophic wind and the geopotential distribution in baroclinic waves. The resulting ageostrophic wind pattern does not consist simply of a zonally oriented divergent circulation as sometimes pictured in theoretical models. Rather, it is dominated by the zonal component along the axis of the baroclinic waves at the 250-mb level, and the meridional component along the poleward and equatorward flanks of the waves at the 850-mb level.

Diagnosis of the disturbance fields indicates that the finite meridional width of the baroclinic jet and its associated wave disturbances is the key factor in determining the distribution of the ageostrophic wind.

It is shown that the ageostrophic flow based on a variable Coriolis parameter should not be used alone to estimate the divergent circulation since the divergence of the geostrophic wind is of the same order of magnitude and partly compensates the divergence of the ageostrophic wind at upper levels and augments it at lower levels.

1. Introduction

Lim and Wallace (1991, hereafter designated LW) used linear regression analysis to study the observed three-dimensional structure of baroclinic waves in the Northern Hemisphere winter. Simultaneous and lagged regressions of time series of various meteorological parameters were computed and the results were displayed as regression maps on isobaric surfaces. Such maps are composed of the linear regression coefficients between unfiltered time series of a given field at every point on a data grid [the National Meteorological Center (NMC) octagonal grid in LW] and a high pass filtered reference time series at a selected base grid point.

The linear regression technique is described in detail in LW. Here it is only necessary to observe that since the reference time series (which is generally a high-pass filtered 500-mb height series at a point in the center of a major storm track) is normalized with respect to its standard deviation, the regression coefficients carry the units of the regressed variables. They may be viewed as mapping the amplitude of the regressed variable, y_i , for a baroclinic wave in which the amplitude of the reference variable, x_i , is given by the standard deviation of its time series. Thus, regression coefficients provide a more direct indication of the structure of baroclinic waves in vectorial fields such as the ageostrophic wind

field than is given by the corresponding correlation coefficients.

A one-point regression map may be regarded as a composite field that has been constructed from the addition of all the maps in the sample, where each map is weighted by the magnitude of the corresponding value in the reference time series. In regression analysis all the high frequency fluctuations are taken into account and preference is not given to a few pronounced events as in composite studies.

Lim and Wallace found that in most respects the baroclinic wave structures deduced by their regressions were consistent with those based on theoretical and other observational studies. The three-dimensional structure of the ageostrophic wind, however, differed in an important way from the pattern anticipated: the ageostrophic wind at the 850-mb level was dominated by the *meridional* component rather than the zonal component as in the upper troposphere.

This paper explores the nature of the ageostrophic flow in some detail, and elucidates the dynamical processes that lead to the dominance of the zonal component at 300 mb and the meridional component at 850 mb. The focus is on the distribution of ageostrophic winds relative to the corresponding disturbances in the geopotential height field. It is shown that the observed structure of the ageostrophic circulation is a consequence of the finite meridional scale of the waves and the meridional shear of the mean zonal flow in which the baroclinic waves are imbedded. The relationship of this ageostrophic circulation to the vertical motion field is examined for two alternative definitions of the ageostrophic wind, which are based on departures of the actual horizontal wind from geostrophic winds de-

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FIG. 2. The structure of a baroclinic wave based on the Eady model: (a) the waves in the geopotential height and temperature fields at the upper level, (b) streamfunction for the ageostrophic flow in the longitude–height plane, (c) contours of the (solid) meridional component of wind and (dashed) potential temperature, and (d) waves in the geopotential height and temperature fields at the lower level. From Fig. 13.4 of Gill (1982).

at every grid point on the NMC octagonal grid and high-pass filtered, normalized 500-mb height at the base grid point in the central Pacific for the nine winters used in the study of LW. These regression coefficients define the VF ageostrophic wind vector at each grid point related specifically to baroclinic waves at the base grid point. The ageostrophic wind regressions for the 300-mb and 850-mb levels are shown in the top and bottom panels of Fig. 4, respectively. In these maps a few contours from the geopotential height regressions at the respective levels (see LW) have been transcribed in order to show the distribution of the winds relative to the corresponding disturbances in the geopotential height field. The solid (dashed) loop denotes a positive (negative) contour, so an area enclosed by the solid loop indicates the ridge of the corresponding geopotential height waves and the dashed loop the trough.

In Fig. 4 the maximum ageostrophic wind speed at the 300-mb level is on the order of $3\text{--}4\text{ m s}^{-1}$, which corresponds to approximately half of the disturbance geostrophic wind speed associated with synoptic scale motions (but only one-tenth of the observed total wind speed) at that level. At the 850-mb level it is slightly less than 1 m s^{-1} , which corresponds to roughly one-fifth of the high-frequency component of the wind in

the lower troposphere. The ageostrophic wind vectors are oriented zonally at 300 mb and meridionally at 850 mb. The observed distribution of the ageostrophic winds implies horizontal convergence to the east of the ridges at the 300-mb level and to the east of the troughs at the 850-mb level.

The ageostrophic flow at the 700-mb level, which corresponds to the approximate steering level for baroclinic waves, is displayed in the middle panel of Fig. 4. It appears to be a hybrid of the 300-mb and 850-mb patterns and is very weak. (Note the differing scales for the wind arrows as shown at bottom right on each panel in the figure.) The zonal component of the ageostrophic wind field in a longitude–height cross section through a baroclinic wave is shown in Fig. 5. It is obvious from this cross section that the u -component of the ageostrophic wind cannot combine with the vertical motion to form a closed circulation cell as implied in the schematic diagrams shown in Figs. 1 and 2.

Considering the sparsity of observing stations over the oceans and the influence of the data assimilation procedures used in the NMC model, it is reasonable to question whether the ageostrophic wind fields shown in Fig. 4 are robust. As a test of robustness one-point regression maps were computed for the VF ageostrophic wind field based on a grid point over the central United States, which has a high density of upper-air observations. With some minor exceptions the main features of the results (not shown) are similar to those on the maps shown in Fig. 4. Regression maps were also computed for individual winter seasons. For all

FIG. 3. The ageostrophic wind field at the earth's surface on day 5.5 during the evolution of a square Eady wave in an integration of the Boussinesq, hydrostatic, frictionless, and adiabatic equations with the quasi-geostrophic momentum approximation, after Hoskins (1976). The longest arrow is equivalent to 7 m s^{-1} . The solid and dashed contours are for the geopotential height and potential temperature fields, respectively. Contour interval is 50 m for the height field and about 5 K for the temperature field.

winters from 1980/81 onward the results were found to be quite reproducible. (For further discussion see LW.)

4. Theoretical interpretation

In this section the VF ageostrophic winds at the 300-mb and 850-mb levels are diagnosed by assuming that the transient eddies can be approximated as linear, steady waves. The horizontal equation of motion may be written as

$$\frac{D\mathbf{V}}{Dt} + f\mathbf{k} \times \mathbf{V} = -\nabla\Phi \quad (1)$$

where $\mathbf{V}$ is the horizontal wind, $\mathbf{k}$ the vertical unit vector, f the Coriolis parameter, Φ the geopotential, and ∇ the horizontal gradient operator on a surface of constant pressure p . The coordinate conventions are the same as commonly used in meteorological studies. Making use of the VF geostrophic wind, the horizontal momentum equation (1) may be written as

FIG. 4. Maps of (a) 300-mb, (b) 700-mb, and (c) 850-mb ageostrophic wind (V_{a1}) regression, based on high-pass filtered, normalized 500-mb height at the base gridpoint marked with a solid circle. The solid and dashed loops are ± 10 -m contours, respectively, transcribed from the corresponding geopotential height regressions for the same reference time series. The scale for the wind vectors is shown at the lower right of each map.

FIG. 5. Longitude-height section of the VF zonal ageostrophic wind component assembled from one point regression maps at ten standard pressure levels. See LW for details.

$$\mathbf{V}_{a1} \equiv \mathbf{V} - \mathbf{V}_{g1} = \frac{\mathbf{k}}{f} \times \frac{D\mathbf{V}}{Dt} \quad (2)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla + \omega \frac{\partial}{\partial p}.$$

In the above expressions, $\mathbf{V}_{a1}$ refers to the ageostrophic component of wind, $\mathbf{V}_{g1}$ to the geostrophic wind, $\mathbf{V}$ to the observed wind, and ω to the individual pressure change Dp/Dt .

a. The quasi-geostrophic momentum approximation¹

According to Hesselberg (1915) and Jeffreys (1919) [see the review by Phillips (1990)], the horizontal wind in the acceleration and advective terms in (2) can be replaced by the VF geostrophic wind. Hesselberg, Jeffreys, and many later authors simply ignored the vertical advection term in (2). Charney (1948) showed by scaling arguments that to a first approximation the vertical advection and horizontal advection by the ageostrophic wind component may both be ignored. The resulting approximate form of (2) is, except for the use of variable f , identical to the standard quasi-geostrophic approximation in which the ageostrophic wind field is determined solely from the fields of geopotential height and geopotential height tendency.

$$\mathbf{V}_{a1} = \frac{\mathbf{k}}{f} \times \frac{D_{g1}\mathbf{V}_{g1}}{Dt} \quad (3)$$

where

$$\frac{D_{g1}}{Dt} = \frac{\partial}{\partial t} + \mathbf{V}_{g1} \cdot \nabla.$$

¹ The actual horizontal velocity is replaced by the VF geostrophic wind in the total derivative term in the horizontal momentum equation. This approximation must be distinguished from the quasi-geostrophic theory and the geostrophic momentum approximation of Hoskins (1976).

Equation (3) can be rewritten in a form that explicitly shows the contributions of the local change and the horizontal advection of the geostrophic wind to forcing of the VF ageostrophic wind:

$$\mathbf{V}_{a1} = \frac{\mathbf{k}}{f} \times \frac{\partial \mathbf{V}_{g1}}{\partial t} + \frac{\mathbf{k}}{f} \times (\mathbf{V}_{g1} \cdot \nabla) \mathbf{V}_{g1}. \quad (4)$$

The first term on the right-hand side of (4) is often called the isallobaric wind. It is always directed down the gradient of pressure tendency. The second term was referred to by Eliassen (1950) as the sum of the curvature and confluence effects. The terms isallobaric wind and horizontal advective wind are used to designate contributions to the ageostrophic wind from the first and second terms on the right-hand side of (4), respectively, throughout this study.

b. A schematic model of the ageostrophic circulation

The observed distribution of the VF ageostrophic wind examined in the previous section can be understood by considering an idealization that retains the most important aspect of the patterns, namely, the relative orientation of the ageostrophic winds with respect to the corresponding disturbances in the geopotential height field at the same level. A simplified view of the relation between the ageostrophic winds and the corresponding geopotential height disturbances is given in Fig. 6.

The fact that the 850-mb convergence and the 300-mb divergence centers are located nearly 1/4 cycle to the east of the troughs can be understood by considering the vorticity budget. At 300 mb the vorticity centers are moving eastward slower than the advecting zonal flow, while at 850 mb they are moving faster than the zonal flow. Thus, at the 300-mb level there must be divergence (convergence) to the east of the troughs (ridges) to partly cancel the vorticity advection. At 850 mb the opposite situation applies; there must be convergence (divergence) to the east of the troughs (ridges) in order to reinforce the vorticity advection (see Holton 1979, pp. 225–227). Of course, there are other configurations of the ageostrophic wind field (such as that shown in Fig. 2) that are consistent with this pattern of divergence and convergence. Thus, the distribution of the ageostrophic flow cannot be diagnosed in terms of the vorticity budget alone.

For waves with similar meridional and zonal scales, as is observed, the above arguments imply a pattern of irrotational velocity components as shown by the broad solid arrows in Fig. 7. The observed ageostrophic flow shown schematically in Fig. 6 differs dramatically from the irrotational flow. In order to understand the reason for the dominance of the zonal component at 300 mb and the meridional component at 850 mb, it is useful to consider a pattern of linear, steady progressive waves moving at zonal phase speed c . The waves are embedded in a mean zonal jet $\bar{u}(y, z)$, with the maximum

FIG. 6. Summary of the ageostrophic wind distribution with respect to the corresponding disturbances in the geopotential height field. Upper and lower panels are for the 300-mb and 850-mb levels, respectively. Solid and dotted loops denote highs and lows in the geopotential field.

disturbance geopotential fluctuations occurring along the axis of the jet.

Neglecting vertical advection and assuming linearized steady wave disturbances we can write the total differential operator in (3) as

$$\frac{D_{g1}}{Dt} \approx (\bar{u} - c) \frac{\partial}{\partial x}.$$

So that the components of the VF ageostrophic wind are given by

$$u'_{a1} \approx -f^{-1}(\bar{u} - c) \frac{\partial v'_{g1}}{\partial x} \quad (5a)$$

$$v'_{a1} \approx +f^{-1} \left[(\bar{u} - c) \frac{\partial u'_{g1}}{\partial x} + v'_{g1} \frac{\partial \bar{u}}{\partial y} \right] \quad (5b)$$

where the primes denote wavelike perturbations about the mean zonal flow.

The portion of the ageostrophic wind forcing proportional to c is the isallobaric wind forcing, while the portion proportional to $\bar{u}$ is the advective forcing. Since the isallobaric forcing is everywhere directed down the local gradient of pressure tendency, it will force a meridional component of the ageostrophic wind on the northern and southern flanks of the waves, and a zonal component along the central axis of the waves as indicated by the solid arrows in Fig. 8. The advective forcing tends to compensate the isallobaric forcing, but the degree of compensation depends on the strength of the mean zonal wind at each location, as shown by the open arrows in Fig. 8.

FIG. 7. Separation of the ageostrophic wind pattern of Fig. 6 into irrotational and nondivergent parts. Broad arrows show the irrotational part. Thin lines show the streamfunction for the nondivergent part. Note the cancellation and reinforcement of the two parts of the ageostrophic flow at the two levels. See text for discussion.

The relative contributions of the two forcings can be understood by considering the two levels separately:

(i) *300-mb level.* At the center of the storm track the air parcels are moving eastward faster than the wave pattern ($\bar{u} \gg c$, and $|v'_{g1}| \gg |u'_{g1}|$). Thus, advective forcing is much larger than isallobaric forcing and a large u'_{a1} is required to balance the advection of v'_{g1} by the mean zonal wind. On the flanks of the waves air parcels are nearly stationary relative to the wave pattern ($\bar{u} \approx c$, and $|v'_{g1}| \ll |u'_{g1}|$). Thus, the isallobaric and advective forcings tend to balance and the resulting u'_{a1} and v'_{a1} are both small.

(ii) *850-mb level.* At the center of the storm track, $\bar{u} \approx c$, (air parcels are nearly stationary relative to the waves), so that again the advective and isallobaric forcings tend to balance, and u'_{a1} and v'_{a1} are both small. On the flanks of the waves air parcels move westward relative to the wave pattern ($\bar{u} \ll c$, and $|v'_{g1}|$ is small). Thus, isallobaric forcing dominates, $v'_{a1} \approx -c \partial u'_{g1} / \partial x$ and $u'_{a1} \sim 0$.

From a Lagrangian point of view the above arguments indicate simply that where the mean wind and wave speeds are nearly equal parcels remain near the same phase of the waves, and hence the geostrophic wind following the parcel motion is nearly constant so that the ageostrophic wind is very small. On the other hand, where the parcels move rapidly through the waves (either westward or eastward relative to the wave speed) there must be large changes in the geostrophic wind

following the motion, which implies that strong ageostrophic flows are induced.

Therefore, it can be concluded that the jetlike structure of the mean wind, with its resulting meridional confinement of the waves, is the major factor that determines the shape of the ageostrophic flow.

An alternative explanation for the structure of the observed ageostrophic wind pattern, suggested by B. J. Hoskins (personal communication), follows from dividing the ageostrophic flow into nondivergent and irrotational parts. For a wave with similar meridional and zonal scales the irrotational component has the structure depicted in Fig. 7. The nondivergent component can be estimated by taking $\partial(5b)/\partial x - \partial(5a)/\partial y$ to obtain an expression for the vorticity of the ageostrophic wind:

$$\zeta'_a = \frac{2}{f} \frac{\partial v'_{g1}}{\partial x} \frac{\partial \bar{u}}{\partial y}.$$

The pattern of the streamfunction for ζ'_{a1} is shown in Fig. 7. Clearly the wind pattern associated with this streamfunction tends to cancel the meridional component of the irrotational ageostrophic flow at 300 mb and the zonal component of the irrotational ageostrophic flow at 850 mb while reinforcing the zonal component at 300 mb and the meridional component at 850 mb, respectively. For waves in which the meridional scales of the jet and the wave are comparable

FIG. 8. A simple model for interpreting the VF ageostrophic wind pattern. The open arrows show the contribution to V_{a1} from the advective forcing; the solid arrows show the contribution by the isallobaric forcing. Upper panel: 300 mb, lower panel: 850 mb.

to the zonal scale of the wave the nondivergent and irrotational parts of the ageostrophic flow are of the same magnitude so that the above described cancellation is possible.

c. A linearized model

A more quantitative evaluation of the dynamical processes that contribute to the observed structure of the ageostrophic wind regression maps can be obtained by dividing the time series of the field variables into an idealized zonally uniform basic state and time-dependent components, with the latter denoting the high frequency wavelike disturbances characteristic of baroclinic waves:

$$\Phi = \bar{\Phi} + \Phi', \quad u_{g1} = \bar{u} + u'_{g1}, \quad v_{g1} = v'_{g1}.$$

As before, the subscript "g1" refers to the VF geostrophic component of the wind, the overbar denotes the basic state flow and the primed quantities denote wavelike perturbations. It is assumed that the basic state varies slowly in x compared to the disturbance variables so that x -differentiated basic state variables may be neglected compared to the corresponding x -differentiated disturbance variables. Thus, the wave disturbance propagates on a zonal flow that locally behaves as though it were independent of longitude.

If the dependent variables are separated by employing the above scheme, Eq. (3) or (4) may be written in component form without the nonlinear terms as

$$u'_{a1} = -\frac{1}{f^2} \left(\frac{\partial}{\partial t} + \bar{u}_{g1} \frac{\partial}{\partial x} \right) \left(\frac{\partial \Phi'}{\partial x} \right) \quad (6a)$$

$$v'_{a1} = -\frac{1}{f^2} \left[\left(\frac{\partial}{\partial t} + \bar{u}_{g1} \frac{\partial}{\partial x} \right) \left(\frac{\partial \Phi'}{\partial y} \right) - f v'_{g1} \frac{\partial \bar{u}_{g1}}{\partial y} \right]. \quad (6b)$$

FIG. 9. Meridional profiles of the time-mean zonal geostrophic wind averaged over longitudes 140°E through the date line to 140°W; the solid curve for 850 mb and the dashed curve for 300 mb.

FIG. 10. The VF ageostrophic wind fields computed by using the assumed disturbance field of geopotential and the zonal component of the geostrophic wind over the Pacific. The upper panel is for 300 mb and the lower panel for 850 mb. The longest arrows in upper and lower panels are equivalent to 4.5 m s⁻¹ and 0.5 m s⁻¹, respectively.

In order to obtain an analytic expression for the ageostrophic wind field, we define the following basic state and disturbance fields for a given level, taking into account what we believe to be the essential aspects of the basic state profile of the geostrophic zonal wind presented in Fig. 9, and the geopotential height regression maps in Fig. 4. The prescribed distributions for the parameters of interest are

$$\bar{u}_{g1}(y) = U_0 \cos y$$

for the time-mean meridional profile and

$$\Phi'(x, y, t) = \Phi_0 \cos y \sin k(x - ct)$$

for the disturbances. Here U_0 is the maximum basic state geostrophic zonal wind, Φ_0 the amplitude of the disturbances in the geopotential field, c the eastward phase speed of the disturbances, and k and l are the wavenumbers in the x and y directions. The origin of the y axis is placed at 40°N, and it is assumed that the mean zonal wind and disturbances have the same meridional scale. The latitude of the maximum wind is closer to 36°N in Fig. 9, but this small shift in the position of the maximum wind should not significantly alter the distribution of the computed ageostrophic winds. The above expressions are inserted into (6) to yield

TABLE 1. The parameter values for the calculation of the ageostrophic wind fields shown in Fig. 10. Φ_0/g the amplitude of the disturbances, U_0 the maximum zonal wind speed over the Pacific, and c the phase speed of the disturbances.

Parameter (unit)	Level	
	300 mb	850 mb
Φ_0/g (m)	78	37
U_0 (m s ⁻¹)	36	11
c (m s ⁻¹)	13	11

$$u'_{a1} = \frac{\Phi_0 k^2}{f^2} (U_0 \cos ly - c) \cos ly \sin k(x - ct) \quad (7a)$$

$$v'_{a1} = -\frac{\Phi_0 c k l}{f^2} \sin ly \cos k(x - ct). \quad (7b)$$

The u -component of the ageostrophic wind obtained from (7a) is proportional to the difference between the basic-state zonal wind and the phase propagation speed of the disturbances, i.e., $(U_0 \cos ly - c)$, whereas the meridional component of the ageostrophic wind in the first term of (7b) is proportional to phase speed alone. Taking into account the shapes of the cosine and sine functions, maximum values of u'_{a1} will be found along $y = 0$ and the maximum values of v'_{a1} will be found at $y = \pm 2000$ km.

The ageostrophic wind field computed based on these equations is shown in Fig. 10.² The parameter values used are given in Table 1. The amplitude Φ_0 is based on the largest coefficient in the geopotential height regression map for the level in question, multiplied by g . The quasi-geostrophic momentum approximation seems to be capable of reproducing the main features of the ageostrophic wind field observed in the regression maps as summarized in Fig. 6.

The ageostrophic wind field was also computed from (6) based on the observed mean zonal wind fields and the regression maps of geopotential height and 24-hour geopotential height tendency for the base grid point. The results are shown in Fig. 11, which may be compared with the observed results in Fig. 4. The overall features are similar in shape on the corresponding panels of the two figures, but the meridional component of the computed ageostrophic wind field at 300 mb is somewhat stronger than the observed. The computed winds at 850 mb shown in the lower map of Fig. 11 are smaller by a factor of two than the observed winds shown on the bottom panel of Fig. 4. Presumably, this reflects the inaccuracies of the linearization at 850 mb where the mean flow is weak.

² In plotting the figure the f^2 factor is taken constant, which is not the same as using the CF geostrophic wind [see (9) below].

The contributions of the isallobaric and advective terms in (5) to the ageostrophic wind field are shown in Figs. 12 and 13 for the 300-mb and 850-mb levels, respectively. At 300 mb the forcing of the ageostrophic wind by horizontal advection is much larger than the forcing by the isallobaric wind. At 850 mb the forcing by horizontal advection is comparable in amplitude with the isallobaric component. The isallobaric wind opposes the wind due to the horizontal advection throughout the domain analyzed at the 300-mb level and in the regions along the axis of the baroclinic wave train at the 850-mb level.

5. The ageostrophic wind field defined on the basis of constant f

In this section the field of the CF ageostrophic wind in baroclinic waves is derived and compared to the VF ageostrophic wind field examined in the previous section. The CF ageostrophic wind is computed by subtracting the value of the CF geostrophic wind at every grid point on a $(5^\circ \times 5^\circ)$ latitude/longitude grid from the wind fields analyzed by NMC.

Blackburn (1985) emphasized differences between the V_{a0} and V_{a1} fields in his analysis of the ageostrophic wind field in a free, nondivergent planetary-scale Rossby mode superimposed upon a basic state consisting of a constant zonal wind. His V_{a0} field was dominated by the corrections to the geostrophic wind required away from his reference latitude, which proved

FIG. 11. The computed VF ageostrophic wind fields based on the mean fields of geopotential height and temperature and the regression maps of geopotential height and vertical velocity. The upper map is for 300 mb and the lower map for 850 mb. The velocity scale is shown at the lower right of each map.

FIG. 12. Regression maps showing the relative contributions of various components to the total VF ageostrophic wind at 300 mb in the quasi-geostrophic momentum approximation; the upper panel is for the isallobaric component and the lower panel is for the component due to horizontal advection. The scale for the wind vectors is at the lower left.

to be much larger than V_{a1} . The results given here are somewhat different from Blackburn's because the reference latitude is centered on the storm track (whereas his was centered on a node in the wave geopotential field) and because this study takes into account the strong meridional dependence of the observed basic state flow.

The 300- and 850-mb CF ageostrophic wind fields for the Pacific sector are shown in Fig. 14 together with a few contours transcribed from the corresponding geopotential height regression maps at the respective levels. One important difference between the fields of V'_{a1} and V'_{a0} is the larger meridional component in the latter field. At the 300-mb level (upper panel) the maximum zonal wind along the wave axis is comparable with that of V'_{a1} (panel a of Fig. 4), while the v component is much larger than in V'_{a1} . Inspection of Fig. 14 indicates that the 300-mb ageostrophic wind has a substantial nondivergent component in this case associated with vortices in the ageostrophic wind centered on the poleward and equatorward flanks of the waves. At the 850-mb level the CF ageostrophic wind is qualitatively similar in shape to its VF counterpart but it is, on average, two or three times as strong. The reason for this difference will be discussed below.

Making use of the beta-plane approximation, $f = f_0 + \beta y$, and using CF geostrophy, (3) can be expressed as

$$\mathbf{V}_{a0} = \frac{\mathbf{k}}{f_0} \times \frac{D_{g0}}{Dt} \mathbf{V}_{g0} - \frac{\beta y}{f_0} \mathbf{V}_{g0}, \quad (8)$$

where

$$\frac{D_{g0}}{Dt} = \frac{\partial}{\partial t} + \mathbf{V}_{g0} \cdot \nabla.$$

The important advantage of (8) compared to (3) is that the divergence of the $\mathbf{V}_{a0}$ field is the same as that of the observed wind field since the divergence of the CF geostrophic wind vanishes. Thus, the secondary vertical circulation is determined solely by the distribution of $\mathbf{V}_{a0}$.

Separating the variables into basic state and baroclinic wave components as in section 4b, ignoring the x derivatives of the basic state flow, and linearizing, it is readily shown that

$$u'_{a0} = -\frac{1}{f_0} \left(\frac{\partial v'_{g0}}{\partial t} + \bar{u}_{g0} \frac{\partial v'_{g0}}{\partial x} + \beta y u'_{g0} \right) \quad (9a)$$

$$v'_{a0} = -\frac{1}{f_0} \left(\frac{\partial u'_{g0}}{\partial t} + \bar{u}_{g0} \frac{\partial u'_{g0}}{\partial x} + v'_{g0} \frac{\partial \bar{u}_{g0}}{\partial y} - \beta y v'_{g0} \right), \quad (9b)$$

where the ageostrophic winds in (9a,b) represent the components specifically related to the wave part of the motions with periods shorter than about 7 days.

The maps in Figs. 15 and 16 show the fields of total CF ageostrophic wind and the various factors that con-

FIG. 13. As in Fig. 12, but for the 850-mb level.

FIG. 14. The CF ageostrophic wind fields of baroclinic waves in the midlatitude beta plane approximation ($\mathbf{V}_{a0} = \mathbf{V}_{\text{obs}} - \mathbf{V}_{g0}$) at the 300-mb level (upper) and the 850-mb level (lower). The solid and dashed loops represent ± 20 m (± 10 m) contours transcribed from the corresponding regressions of geopotential height. The scale arrows for the wind vectors are shown at the lower left.

tribute to them at the 300- and 850-mb levels, respectively. The computed total ageostrophic wind ($\mathbf{V}'_{a0}$) fields shown in the upper panels of these figures are similar to the observed $\mathbf{V}'_{a0}$ shown in Fig. 14. At 300 mb, advection by the mean flow dominates in the forcing of $\mathbf{V}'_{a0}$. At 850 mb the contributions of the isalobaric and advective terms to the zonal component of the ageostrophic wind cancel. The meridional components of all three forcing terms are in the same sense, and the β term cannot be neglected.

6. Relation between the ageostrophic wind and the divergence

The fact that the CF geostrophic wind is nondivergent makes it easy to relate the corresponding ageostrophic wind field to the divergence of the observed wind field. The first-order relationship between the CF and VF ageostrophic wind fields may be derived from the two different definitions of the geostrophic wind using the β -plane approximation $f = f_0 + \beta y$, where y is the meridional distance from the reference latitude. The relationship is

$$\mathbf{V}'_{a0} = \mathbf{V}'_{a1} + (\mathbf{V}'_{g1} - \mathbf{V}'_{g0}) \approx \mathbf{V}'_{a1} - \frac{\beta y}{f_0} \mathbf{V}'_{g0}.$$

By applying the horizontal divergence operator to both sides of this equation, we obtain an explicit relationship between the divergence fields associated with the VF and CF ageostrophic wind fields.

$$\nabla \cdot \mathbf{V}'_{\text{obs}} = \nabla \cdot \mathbf{V}'_{a0} \approx \nabla \cdot \mathbf{V}'_{a1} - \frac{\beta}{f_0} v'_{g0} \approx \nabla \cdot \mathbf{V}'_{a1} - \frac{\beta}{f_0} v'_{g1}. \quad (10)$$

The subscript "obs" denotes the winds analyzed at

FIG. 15. Computed 300-mb ageostrophic winds ($\mathbf{V}_{a0}$) on the midlatitude beta plane: (a) the total, (b) the isalobaric component, (c) the component due to advection by the mean flow, and (d) the beta part. The solid and dashed loops are ± 20 -m contours of the corresponding 300-mb geopotential height fields. The scale for the wind vectors is shown in the lower left corner.

FIG. 16. As in Fig. 15, but for the 850-mb level and with ± 10 m contour loops.

NMC. The first equality reflects the nondivergence of the CF geostrophic wind.

If the horizontal divergence is computed for the 300-mb and 850-mb levels using only the VF ageostrophic winds, the upper-level horizontal convergence (divergence) is much larger than the corresponding divergence (convergence) in the lower troposphere. The differences between the absolute magnitudes of the divergence fields at the two levels must be due to the horizontal divergence of the geostrophic wind. Thus, both geostrophic and ageostrophic divergence must be considered in computing the secondary circulation for the VF geostrophic wind.

In order to demonstrate this fact the divergence of the VF ageostrophic and geostrophic wind components

at the 300-mb and 850-mb levels is computed. At the 300-mb level (Fig. 17) the two patterns are almost exactly out of phase, whereas at 850 mb (Fig. 18) they are in phase. At both levels the divergence of the VF geostrophic wind is similar in magnitude to the divergence of the VF ageostrophic wind. The combined divergence shown in the bottom panels in the two figures is the same as that of the actual wind field.

7. Conclusion

The observed ageostrophic wind distribution in baroclinic waves in the lower troposphere can be interpreted by considering the isallobaric wind associated with the phase propagation of the waves, the component owing to horizontal advection of the waves by the background flow, and the advection of the background quantities by the perturbation wind field. Although

FIG. 17. The 300-mb divergence fields associated with the VF ageostrophic component (upper), the geostrophic wind field (middle), and the sum of the two fields (bottom). Negative contours dashed and zero contours omitted. Contour interval: $1.0 \times 10^{-5} \text{ s}^{-1}$.

temperature advection by the ageostrophic wind amounts to only about one-half degree per day, it does play a role in tightening the meridional temperature gradient to the south and weakening the gradient to the northwest of surface lows as is evident in Fig. 19. The temperature advection is somewhat stronger in the midlatitude β -plane approximation (not shown).

In the upper troposphere the divergence of the VF geostrophic and ageostrophic fields are of opposite sign, with the ageostrophic divergence field dominating. Thus, consistent with Blackburn's (1985) argument based on his analysis of a free, nondivergent planetary-scale Rossby mode, we found that for observed baroclinic waves the divergence of the VF ageostrophic wind field is considerably larger than the divergence of the CF ageostrophic wind field.

In the lower troposphere, below the steering level for baroclinic waves, the divergence of the VF ageostrophic wind field is much smaller than at the upper levels, but it is reinforced by the divergence field associated with the VF geostrophic wind field, which is

FIG. 18. As in Fig. 17, but for the 850-mb level.

FIG. 19. The advection of time-mean temperature by the VF ageostrophic wind field at the 850-mb level. Contour interval: $2.0 \times 10^{-6} \text{ } ^\circ\text{C s}^{-1}$. The arrows denote the VF ageostrophic wind field. The longest arrow is equivalent to about 0.5 m s^{-1} .

of roughly comparable amplitude. Consistent with this result, our CF ageostrophic wind field resembles the corresponding VF ageostrophic wind field but is almost twice as strong.

The structure of the ageostrophic wind field in baroclinic waves at the 850-mb level suggests that the ageostrophic flow in baroclinic waves does not comprise a complete circulation cell in the longitude–height plane along the axis of the wave train. The lower branch of the secondary flow in the waves is transverse to the axis of the wave train rather than parallel to it. The pattern is reproducible in subsets of the data and in the regressions for various base grid points and is not sensitive to the choice of the framework for computing the ageostrophic wind, i.e., constant f or variable f . The dynamical consistency of this pattern has been confirmed by a simple numerical calculation based on the quasi-geostrophic momentum approximation. The crucial factors contributing to the observed structure of the ageostrophic wind field are that the waves have meridional scales comparable to their zonal scales and that advection by the zonal flow is not uniform across the meridional extent of the waves due to the meridional dependence of the mean zonal flow. The waves are centered on an upper-level jet that has a meridional scale similar to that of the waves themselves. Thus, at the 300-mb level the advecting flow moves much faster than the waves at the jet core, but at nearly the same speed as the waves on their flanks. At 850 mb, on the other hand, the waves move at nearly the speed of the mean flow in the core, but much faster than the mean flow on their flanks. This leads to different balances in the isallobaric and advective forcing of the ageostrophic wind at the two levels so that zonal component dominates at the upper level and the meridional component dominates at the lower level.

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