Rotating Stratified Flow over a Mountain Ridge as an Initial Value Problem

PETER R. BANNON

Department of Meteorology, The Pennsylvania State University, University Park, Pennsylvania

(Manuscript received 27 November 1989, in final form 15 August 1990)

ABSTRACT

Solutions for steady inviscid quasi- and semi-geostrophic flow over a mountain ridge on the f-plane are degenerate in the sense that an arbitrary constant mountain-parallel flow can be added to the solution. It is shown that consideration of the problem as an initial value one removes this degeneracy. The quasi-geostrophic results presented here for a semi-infinite atmosphere vary for different initial conditions according to whether the flow is Boussinesq, anelastic, or deep. We enumerate conditions for which a mountain drag and an upstream influence exists.

1. Introduction

A mountain ridge subjected to a normally incident wind on a uniformly rotating reference frame generates a net deflection of the inviscid flow to the right (looking downstream). An expression for this turning is

$$\Delta v(z) = -\frac{U(z)\mathrm{LL}}{\mathrm{MF}}, \qquad (1.1)$$

where $\Delta v = v(\text{downstream}) - v(\text{upstream})$ is the net change in the meridional (i.e., mountain parallel) wind and $U(z)$ is the incident zonal (i.e., normal) wind profile. Here LL is the lift per unit ridge length,

$$\mathrm{LL} = \int_M \rho(z) f U(z) dx dz, \qquad (1.2)$$

where the integral is over the mountain cross section, $\rho(z)$ is the static density field, f is the Coriolis parameter, and MF is the momentum flux,

$$\mathrm{MF} = \int_0^{z_T} \rho(z) U^2(z) dz, \qquad (1.3)$$

where z_T denotes the depth of the atmosphere. The validity of (1.1) has been proved for finite-amplitude topography in homogeneous flow (Batchelor 1967; p. 574) and for stratified, semi-geostrophic, barotropic (Jacobs 1964) and baroclinic (Merkine 1975; Bannon and Zehnder 1989) flow with a rigid lid (z_T finite). Confirmation of (1.1) for non-Boussinesq effects appear in Smith (1979) for linearized, quasi-geostrophic, and in Bannon and Chu (1988) for finite-amplitude, semi-geostrophic topography in a semi-infinite atmosphere (z_T infinite).

A related issue is whether the turning occurs symmetrically (half upstream and half downstream of the ridge crest) or asymmetrically. The symmetric case implies an upstream influence of the mountain:

$$v(\text{upstream}) = -\frac{\Delta v}{2}, \qquad (1.4)$$

but no mountain drag, DL, per unit ridge length,

$$\mathrm{DL} = \int_M \rho(z) f v(x, z) dx dz. \qquad (1.5)$$

We note that the drag force (1.5) acts in the upstream direction while the lift (1.2) is directed to the left (looking downstream). For asymmetric flow, we can let $v(\text{upstream})$ vanish, thereby suppressing an upstream influence, but then the drag DL is nonzero, and we violate D'Alembert's paradox. This duality prompted Blumen (1988) to question the uniqueness of rotating inviscid flow over a ridge on the f-plane.

The purpose of the present paper is to remove the degeneracy in the ridge solutions by considering the problem as an initial-value one, as originally suggested in Bannon and Chu (1988). It is shown that the two solutions (symmetric and asymmetric) are physically distinct and reflect differences in the initial conditions.

Section 2 presents the quasi-geostrophic formulation of the initial value problem and its general solution. We employ the deep (or modified) theory of White (1977) in the analysis since this theory most accurately describes the dynamics of the larger scale flow, an attribute of crucial importance to the far-field behavior of the solution. Section 3 compares the results for Boussinesq, anelastic, and deep flows.

Figure 1 describes the initial conditions used in this

Corresponding author address: Dr. Peter Bannon, Dept. of Meteorology, Pennsylvania State University, University Park, PA 16802.

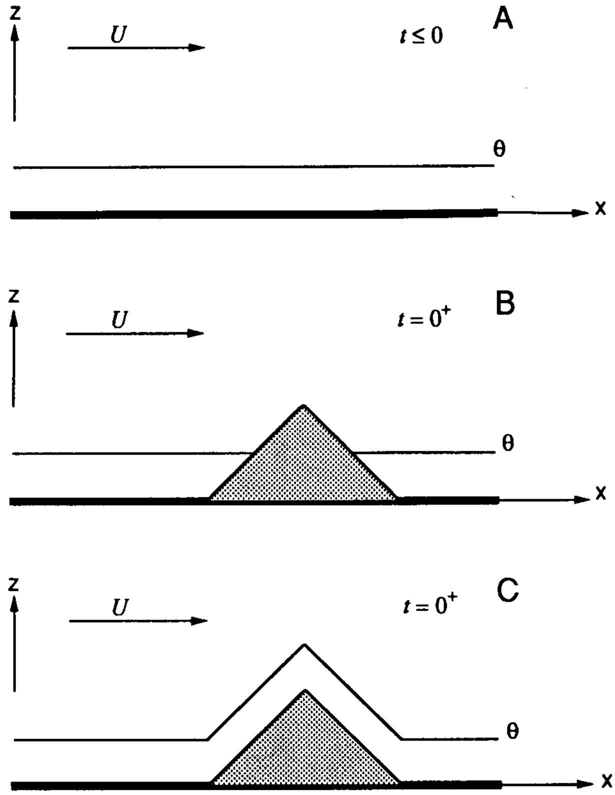

FIG. 1. Schematic illustration of the initial value problem. A uniform zonal wind blows over flat terrain for $t \leq 0$, (a). At $t = 0$, a mountain is impulsively inserted, (b), or uplifted (c). In (b) the initially flat isentropes are undisturbed (case I). In (c) the isentropes are raised as the lower boundary remains an isentropic surface (case II).

study. For case I, the mountain is impulsively inserted into a uniform zonal flow.[1] As a consequence the mountain surface is not zonally isentropic initially. In case II, however, the mountain is impulsively grown from below and the lower surface remains zonally isentropic.

2. The model

a. Formulation

The potential vorticity equation for deep, adiabatic, quasi-geostrophic flow on an f plane is

$$\frac{d}{dt} q \equiv \frac{\partial q}{\partial t} + J(\psi, q) = 0, \quad (2.1)$$

where q is the potential vorticity,

$$q = \nabla^2 \psi + \frac{f^2}{N^2}\left(\frac{\partial^2 \psi}{\partial z^2} - \frac{1}{H}\frac{\partial \psi}{\partial z}\right), \quad (2.2)$$

and ψ is the geostrophic streamfunction. We take the basic-state resting atmosphere to be uniformly stratified with constant buoyancy frequency N and constant density scale height H. Here ∇^2 is the horizontal Laplacian and J the Jacobian for a Cartesian coordinate system.

The presence of orography is incorporated into the problem through the kinematic lower boundary condition on the vertical motion field, using the heat equation in the form

$$\frac{d}{dt}\left[f\left(\frac{\partial}{\partial z} - \sigma\right)\psi + N^2 h\right] = 0, \quad \text{at} \quad z = 0. \quad (2.3)$$

Here $h = h(x, t)$ is the mountain profile, and $\sigma \equiv N^2/g \equiv \theta_s^{-1} d\theta_s/dz$ is the static stability where $\theta_s(z)$ is the static contribution to the potential temperature field.

As expressed by (2.1)–(2.3), the problem is formulated for deep flow. We obtain the standard anelastic theory (e.g., Pedlosky 1987) by setting σ to zero. If, in addition, we let H go to infinity, we obtain the Boussinesq equation. Henceforth, we refer to these theories as deep, anelastic, and Boussinesq, respectively.

We treat the flow as an initial value problem and take

$$\psi = -Uy + \phi(x, z, t), \quad (2.4)$$

where U is the constant zonal wind and ϕ describes the streamfunction response to the imposition of the orography at $t = 0$. Assuming uniform potential vorticity, ϕ satisfies

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{f^2}{N^2}\left(\frac{\partial^2 \phi}{\partial z^2} - \frac{1}{H}\frac{\partial \phi}{\partial z}\right) = 0, \quad (2.5)$$

subject to the lower boundary condition;

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left[f\frac{\partial \phi}{\partial z} + N^2 h\right] = f\sigma\frac{\partial \phi}{\partial t}, \quad \text{at} \quad z = 0, \quad (2.6)$$

and a boundedness condition at $z = \infty$. Those readers not interested in the details of the mathematical analysis may now proceed directly to section 3.

b. General solution

Standard transform techniques provide a closed form solution to the problem (2.5) with (2.6). The inverse Fourier transform in the horizontal for ϕ,

$$\phi(x, z, t) = \mathrm{Re}\,\frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{\phi}(k, t) e^{ikx} e^{-\gamma z} dk, \quad (2.7)$$

where

$$\gamma = \left(\frac{N^2 k^2}{f^2} + \frac{1}{4H^2}\right)^{1/2} - \frac{1}{2H}, \quad (2.8)$$

satisfies (2.5) and the upper boundary condition.

[1] This case is equivalent, for the Boussinesq and anelastic cases only, to the situation of a resting atmosphere surrounding the mountain that is impulsively accelerated to the east.

Introduction of (2.7) and a Laplace transform in time

$$\hat{\phi}(k, s) = \int_0^\infty \hat{\phi}(k, t)e^{st}ds, \qquad (2.9)$$

into (2.6) yields

$$\hat{\phi}(k, s) = \frac{N^2[(s + ikU)\hat{h}(k, s) - \hat{h}_0(k)] - g\dfrac{\delta\hat{\theta}_0(k)}{\theta_s}}{f(\gamma + \sigma)(s + ikc)}, \qquad (2.10)$$

where $\hat{h}_0(k) = \hat{h}(k, t = 0)$ and $\delta\hat{\theta}_0(k) = \delta\hat{\theta}(k, t = 0)$ are the Fourier transforms of the initial topographic and potential temperature anomalies, respectively. Here

$$c(k) = \frac{U\gamma}{\gamma + \sigma}. \qquad (2.11)$$

Taking the inverse Laplace transform of (2.10) along the Bromwich contour and then an inverse Fourier transform formally closes the problem.

c. Transform solution to specific initial conditions

For the specific initial conditions discussed earlier and displayed in Fig. 1, the mountain profile is

$$h(x, t) = h(x)H(t), \qquad (2.12)$$

where $H(t)$ is the Heaviside step function. Then

$$\hat{h}_0(k) = \hat{h}(k), \qquad (2.13)$$

and

$$\hat{h}(k, s) = \frac{1}{s}\hat{h}(k), \qquad (2.14)$$

where $\hat{h}(k)$ is the Fourier transform of $h(x)$. Henceforth, we assume that $h(x)$ is symmetric in x (e.g., a Gaussian or witch-of-Agnesi profile) and, thus, that $\hat{h}(k)$ is symmetric in k. Equations (2.13) and (2.14) hold for both cases I and II.

The two cases differ, however, in their treatment of the initial potential temperature anomaly. In case I the isentropes are initially undisturbed. Thus,

$$\delta\hat{\theta}_0(k) = 0, \quad \text{for case I.} \qquad (2.15)$$

In case II the surface isentropes are displaced upward, resulting in adiabatic cooling of the form

$$g\frac{\delta\hat{\theta}_0(k)}{\theta_0} = -N^2\hat{h}(k), \quad \text{for case II.} \qquad (2.16)$$

Substitution of (2.13)–(2.16) into (2.10) yields

$$\hat{\phi}(k, s) = \frac{N^2}{f}\frac{\hat{h}(k)}{(\gamma + \sigma)}\left[\frac{ikU}{s(s + ikc)}\right], \quad \text{(case I)}, \qquad (2.17)$$

and

$$\hat{\phi}(k, s) = \frac{N^2}{f}\frac{\hat{h}(k)}{(\gamma + \sigma)}\left[\frac{s + ikU}{s(s + ikc)}\right], \quad \text{(case II)}. \qquad (2.18)$$

The associated inverse Laplace transforms are straightforward. The result is

$$\hat{\phi}(k, t) = \frac{N^2}{f}\frac{\hat{h}(k)}{\gamma}\left[1 - b(k)e^{-ikct}\right], \qquad (2.19)$$

where

$$b(k) = 1, \quad \text{case I,} \qquad (2.20a)$$

and

$$b(k) = \frac{\sigma}{\gamma + \sigma}, \quad \text{case II.} \qquad (2.20b)$$

Evaluation of the inverse Fourier transform, (2.7), for (2.19) is nontrivial and depends on whether the flow is Boussinesq, anelastic, or deep. Results are discussed in the following section.

3. Results

The qualitative features of the flow solutions depend critically on the form of the zonal phase speed, $c(k)$, and the vertical decay rate, $\gamma(k)$, as well as the value of the static stability σ. Figure 2 is a plot of $c(k)$ as given by (2.11). If the flow is either Boussinesq or anelastic,

$$c(k) = U, \qquad (3.1)$$

and the system is nondispersive. For deep flow ($\sigma \neq 0$), the waves are dispersive with $c(k) \leqslant U$. Bannon (1989) discusses the physics of the wave propagation in the deep theory. In particular, for small k, the phase speed has the form

$$c(k) \sim U\left(\frac{N^2H}{\sigma f^2}\right)k^2, \quad \text{as } k \to 0. \qquad (3.2)$$

Figure 3 is a plot of $\gamma(k)$ as given by (2.8). For Boussinesq flow,

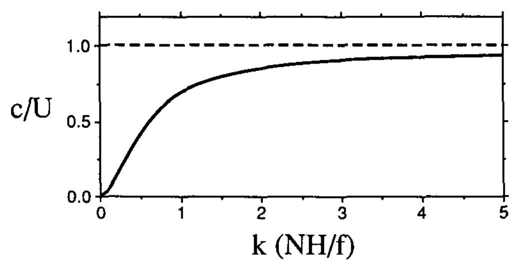

FIG. 2. Zonal phase speed, c, as a function of wavenumber, k, for deep flow (solid line) and for Boussinesq and anelastic flows (dashed line).

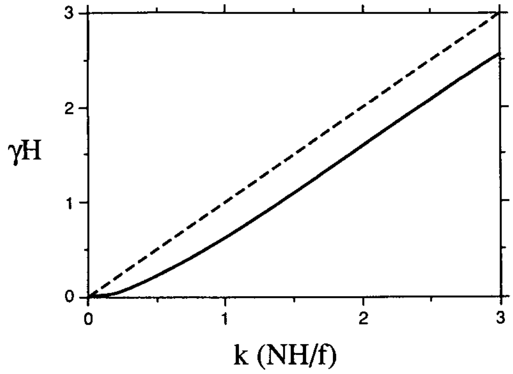

FIG 3. Vertical decay rate, γ, as a function of wavenumber, k, for deep and anelastic flows (solid line) and for Boussinesq flow (dashed line).

$$\gamma = \left(\frac{N^2k^2}{f^2}\right)^{1/2} \tag{3.3}$$

and the vertical decay rate increases linearly with $|k|$. If the flow is either anelastic or deep, γ asymptotes to (3.3) for large k while γ is quadratic in k for small, k:

$$\gamma(k) = \left(\frac{N^2H}{f^2}\right)k^2, \quad \text{as } k \to 0. \tag{3.4}$$

As discussed in the Introduction, the degeneracy of the solution appears in the meridional wind field. We, therefore, focus our discussion on the v-field and its associated permanent turning (1.1) and drag, (1.5). The solution for v may be written as

$$\begin{aligned} v &= v_S(x, z) + v_T(x, z, t) \\ &= \text{Re}\left(\frac{i}{2\pi}\right)\left(\frac{N^2}{f}\right)\int_{-\infty}^{+\infty} k\frac{\hat{h}(k)}{\gamma} \\ &\qquad \times [e^{ikx} - b(k)e^{ik(x-ct)}]e^{-\gamma z}dk, \end{aligned} \tag{3.5}$$

where v_S is the steady and v_T the transient component of (3.5), associated with the first and second terms in the square brackets, respectively.

Figure 4 shows the steady v-field at the surface as a function of distance x. For definiteness, we have evaluated (3.5) numerically for a Gaussian mountain

$$h(x) = h_0e^{-x^2/a^2}, \tag{3.6a}$$

with transform

$$\hat{h}(k) = h_0a\sqrt{\pi}e^{-k^2a^2/4}. \tag{3.6b}$$

Consistent with the discussion in the Introduction, the Boussinesq solution exhibits no permanent turning. Since v_S is independent of σ, the non-Boussinesq (i.e., anelastic and deep) flows both display a permanent turning given by

$$\Delta v_S = -\frac{f\hat{h}(0)}{H}, \tag{3.7a}$$

or, nondimensionally, by

$$\frac{\Delta v_S}{Nh_0} = -\lambda\sqrt{\pi}, \tag{3.7b}$$

where $\lambda = fa/NH$. Note that the non-Boussinesq cases display an upstream influence with v(upstream) $= -\frac{1}{2}\Delta v_S$. Here v_S(upstream) $= 0.886Nh_0$ for the Gaussian mountain with $\lambda = 1$. Mathematically the presence of the turning reflects the presence of a simple pole in the integrand for v_S at $k = 0$ associated with (3.4). We note that both curves in Fig. 4 can also be obtained by convolution from the Green's function solution of steady flow over an isolated mountain (see Bannon 1986) for an infinite mountain ridge.

The analysis of the transient behavior is straightforward for the nondispersive ($\sigma = 0$) cases where (3.1) holds. The function $b(k)$ in the integrand (3.5) mathematically reflects the differences in the initial conditions (see Fig. 1). If the mountain is impulsively uplifted from below (case II), then $b(k) = 0$ [see (2.20b) with $\sigma = 0$] and no quasi-geostrophic transients are excited. In this case the lower boundary is always an isentropic surface, and the total solution is the steady solution. It should be noted that an upstream influence is physically accomplished by the propagation of inertia–gravity waves (e.g., Pierrehumbert and Wyman 1985) whose effect is parameterized in quasi-geostrophic theory by an instantaneous geostrophic adjustment.

For case II, however, the mountain is impulsively inserted into the stream, creating a warm thermal anomaly on the lower boundary. Since $b(k) = 1$, this warm anomaly is equal in structure but opposite in sign to the steady cold-core mountain anticyclone. This cyclonic starting vortex is advected downstream nondispersively at the speed $c = U$. Graphically, the total solution for v consists of the steady curve of Fig. 4 minus the steady curve centered at $x = Ut$. At $t = 0$ the v-field vanishes. As $t \to \infty$, $v \to v_S$ near the mountain for the Boussinesq flow and there is no permanent turning or drag. For the anelastic flow, however, the

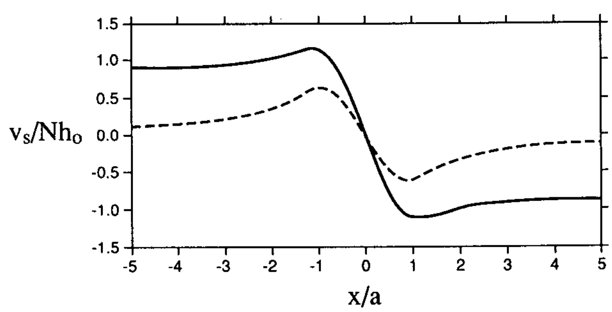

FIG. 4. Steady component of the mountain-parallel wind, v_S, as a function of the distance, x, for deep and anelastic flows (solid line) with $\lambda = fa/NH = 1$ and for Boussinesq flow (dashed line) with $\lambda = 0$.

asymptotic v-field is asymmetric about $x = 0$; there is no upstream influence, there is a local turning to the right over the mountain and a mountain drag but no net permanent turning (Fig. 5).

We now examine the results for deep flow. Here the phase speed c is dispersive. Thus, an initial anomaly will be differentially advected downstream with the shorter waves propagating faster. The appendix analyzes the asymptotic ($t \to \infty$) behavior of the flow. The result,

$$v_T(x, z, t) \sim \frac{f\hat{h}(0)}{H}\left[\frac{1}{\tau}\int_0^x \mathrm{Ai}\left(\frac{-x}{\tau}\right)dx - \frac{1}{6}\right], \quad (3.8)$$

is independent of initial conditions I or II. Here Ai is the Airy function and (A4) defines τ. A plot of (3.8) (see Fig. 6) displays a leftward turning of the flow fixed over the mountain of magnitude given by (A5) that is equal and opposite to (3.7). Moreover, v(upstream) $= -\frac{1}{2}\Delta v_T = \frac{1}{2}\Delta v_S$. Thus, the total v-field has no upstream influence: v(upstream) $= 0$ and there is no net turning: $\Delta v = 0$.

We next consider the drag associated with these cases. The quasi-geostrophic form of (1.5) is

$$\mathrm{DL}(t) = -\rho_0 f \int_{-\infty}^{+\infty} v(x, z = 0, t)h(x)dx, \quad (3.9)$$

where ρ_0 is the reference surface value of the density. Substitution of the v-field of (3.5) into (3.9) yields

$$\mathrm{DL}(t) = \mathrm{DL}_S + \mathrm{DL}_T(t), \quad (3.10)$$

where, by symmetry, the steady component of the drag vanishes

$$\mathrm{DL}_S = 0, \quad (3.11)$$

and

$$\mathrm{DL}_T(t) = \rho_0 N^2 \,\mathrm{Re}\left(\frac{i}{2\pi}\right)\int_{-\infty}^{+\infty} |\hat{h}(k)|^2 b(k)\left(\frac{k}{\gamma}\right)e^{-ikct}dk. \quad (3.12)$$

Application of the method of stationary phase to (3.12) for Boussinesq flow yields

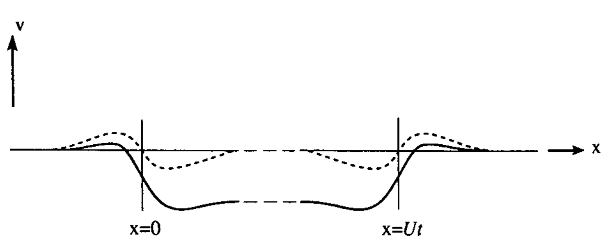

FIG. 5. Schematic illustration of the total mountain parallel wind, v, as a function of distance, x, for anelastic (solid line) and Boussinesq flow (dashed line).

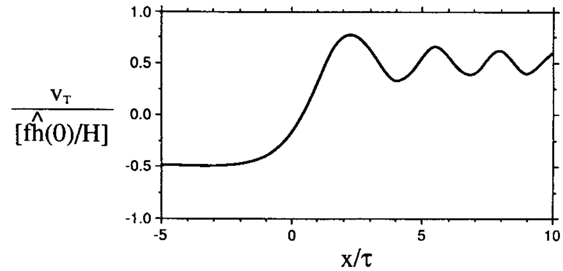

FIG. 6. Asymptotic behavior of the transient component of the mountain-parallel wind, v_T, as a function of distance, x, for deep flow.

$$\mathrm{DL}_T(t) \sim \frac{\rho_0 N f \hat{h}(0)^2 b(0)}{2Ut}, \quad (3.13)$$

and a drag exists for case I ($b = 1$) as the shed vortex propagates downstream. Ultimately, the drag vanishes as $t \to 0$. Evaluation of (3.12) for the non-Boussinesq flows using contour integration yields,

$$\mathrm{DL}_T(t) = C\left[\frac{\rho_0 f^2 \hat{h}(0)^2 b(0)}{2H}\right], \quad (3.14)$$

and the drag is constant for $t \to \infty$. For anelastic flow, a drag exists only in case I. For deep flow, there is a drag of the same amplitude for both initial conditions. Here the constant C in (3.14) equals 1 ($^1/_3$) for the anelastic (deep) flow and the anelastic model overestimates the drag by a factor of 3. It must be emphasized that the presence of a constant drag does not invalidate d'Alembert's paradox since the flow is time dependent and the drag is associated with the shed vortex.

4. Conclusions

We have examined quasi-geostrophic flow over a mountain ridge as an initial-value problem in a vertically semi-infinite atmosphere. Two distinct initial conditions have been addressed: case I, where the mountain is impulsively inserted, and case II, where the mountain is impulsively uplifted from below. Table 1 presents a summary of the salient qualitative features of the results for Boussinesq, anelastic, and deep flows. The steady solutions for non-Boussinesq flow predict a permanent turning and an asymmetric mountain-parallel wind field with an upstream influence. These results are mathematically consistent with the steady solution over an isolated three-dimensional ridge in the limit as the ridge becomes infinitely long. In contrast, the solutions of the initial value problem predict an upstream influence only for the uplifted mountain in anelastic flow. *Thus, the asymptotic behavior of the initial value problem can differ qualitatively from the steady-state solutions.* Since the deep theory more accurately describes the dispersive dynamics of the largest waves, its predictions should be the most applicable to

TABLE 1. Summary of results.

Flow type	Phase speed $c(k)$	Permanent turning of steady solution Δv_S	Shed vortex $b(k)$	Drag DL($t \to \infty$)	Upstream influence $v(x = -\infty)$
Boussinesq	Nondispersive	No	Yes (no)	No (no)	No (no)
Anelastic	Nondispersive	Yes	Yes (no)	Yes (no)	No (yes)
Deep	Dispersive	Yes	Yes (yes)	Yes (yes)	No (no)

Note: The double entries in the last three columns describe the result for the inserted mountain (case I) and, in parentheses, the uplifted mountain (case II).

the atmosphere. It should also be noted that the qualitative results for the Boussinesq flow *with a rigid lid* correspond to those for anelastic flow of Table 1.

The initial conditions used here appear to be the most general ones with uniform potential vorticity: the inserted mountain contains a surface anomaly of potential temperature while the uplifted mountain is isentropic. Superposition would yield the intermediate cases. Furthermore, the inserted mountain case may also be viewed as the sudden switching-on of the mean flow U. This relation holds rigorously only for Boussinesq and anelastic flows. For deep flow an impulse of diabatic warming is required which invalidates the uniform potential vorticity assumption. It should also be noted that a shed vortex can be avoided for deep flow if the mountain is uplifted isothermally. Then

$$g\frac{\delta\hat{\theta}_0}{\theta_0}(k) = -N^2\left(\frac{\gamma+\sigma}{\gamma}\right)\hat{h}(k), \qquad (4.1)$$

and $b(k) = 0$. In this case, only the steady solution remains and there is an upstream influence, and a permanent turning but no drag. Such a situation is physically doubtful, however,

This study addresses inviscid flow on the f-plane. Further research should include frictional processes and the β-effect.

Acknowledgments. I benefited from discussions with Professor Robert Wells on asymptotic analysis in general and the evaluation of (A8) in particular. This material is based upon work supported by the National Science Foundation under Grant ATM-8813315 and ATM-9017043.

APPENDIX

Asymptotic Analysis for Deep Flow

The expression for the transient component of the relative vorticity, ζ, may be written, using the assumed symmetry of $\hat{h}(k)$, as

$$\zeta_T = \text{Re}\,\frac{1}{\pi}\frac{N^2}{f}\int_0^\infty F(k)e^{i\psi t}dk, \qquad (A1)$$

where $\psi = kc - k(x/t)$ and

$$F(k) = \hat{h}(k)b(k)e^{-\gamma z}(k^2/\gamma). \qquad (A2)$$

The asymptotic behavior as $t \to \infty$ [with (x/t) held fixed] of an integral of the form (A1) is,

$$\zeta_T \sim \frac{f\hat{h}(0)b(0)}{H\tau}\,\text{Ai}\left(-\frac{x}{\tau}\right), \qquad (A3)$$

where Ai is the Airy function and

$$\tau = \left[\frac{3HN^2Ut}{\sigma f^2}\right]^{1/3}. \qquad (A4)$$

In deriving (A3), we have followed Lighthill (1978, section 4.11) but incorporated a factor of ½ since the critical point is an endpoint of the integrand (Bleistein and Handelsman 1986, p. 222).

The associated permanent turning is

$$\Delta v_T = \int_{-\infty}^{+\infty}\zeta_T dx \sim \frac{f\hat{h}(0)b(0)}{H}, \qquad (A5)$$

since

$$\frac{1}{\tau}\int_{-\infty}^{+\infty}\text{Ai}\left(-\frac{x}{\tau}\right)dx = 1. \qquad (A6)$$

We can obtain an asymptotic expression for v_T from a definite integral of ζ:

$$v_T(x, z, t) = v_T(x = 0, z, t) + \int_0^x \zeta_T(x', z, t)dx', \qquad (A7)$$

where

$$v_T(x = 0, z, t) = -\frac{1}{\pi}\frac{N^2}{f}\int_0^\infty F(k)\frac{\sin kct}{k}dk. \qquad (A8)$$

From the concepts utilized in the method of stationary phase, the value of v_T at the origin, (A8), will have a dominant asymptotic contribution as $t \to \infty$ for $kc \approx 0$. We, therefore, expand $F(k)$ in a Taylor series about $k = 0$ and substitute (3.2) for c. Evaluation of the lowest order term yields the result

$$v_T(x = 0, z, t) \sim -\left(\frac{1}{6}\right)f\frac{\hat{h}(0)b(0)}{H}. \qquad (A9)$$

Combining (A3) and (A9) into (A7) yields the final result (3.8) for the asymptotic mountain-parallel wind for deep flow.

It is important to note that, from (2.20),

$$b(0) = 1, \tag{A10}$$

for both initial conditions when $\sigma \neq 0$. Thus, the deep response is asymptotically the same for these two scenarios.

REFERENCES

Bannon, P. R., 1986: Deep and shallow quasi-geostrophic flow over mountains. *Tellus,* **38A,** 162–169.

——, 1989: On deep quasi-geostrophic theory. *J. Atmos. Sci.,* **45,** 3457–3463.

——, and P. C. Chu, 1988: Anelastic semigeostrophic flow over a mountain ridge. *J. Atmos. Sci.,* **45,** 1020–1029.

——, and J. A. Zehnder, 1989: Baruclinic flow over a mountain ridge. *J. Atmos. Sci.,* **46,** 703–714.

Batchelor, G. K., 1967: *An Introduction to Fluid Dynamics.* Cambridge, 615 pp.

Bleistein, N., and R. A. Handelsman, 1986: *Asymptotic Expansion of Integrals.* Dover, 425 pp.

Blumen, W., 1988: Stratified, rotating flow over orography: The rigid-lid condition and the far-field circulation. *J. Atmos. Sci.,* **45,** 1417–1422.

Jacobs, S. J., 1964: On stratified flow over bottom topography. *J. Mar. Res.,* **22,** 223–235.

Merkine, L. O., 1975: Steady finite-amplitude baroclinic flow over topography in a rotating stratified atmosphere. *J. Atmos. Sci.,* **32,** 1881–1893.

Pedlosky, J., 1987: *Geophysical Fluid Dynamics,* second edition. Springer-Verlag, 710 pp.

Pierrehumbert, R. T., and B. Wyman, 1985: Upstream influence of mesoscale mountains. *J. Atmos. Sci.,* **42,** 977–1003.

Smith, R. B., 1979: Some aspects of the quasi-geostrophic flow over mountains. *J. Atmos. Sci.,* **36,** 2385–2393.

White, A. A., 1977: Modified quasi-geostrophic equations using geometric height as vertical coordinate. *Quart. J. Roy. Meteor. Soc.,* **103,** 383–396.