Hail Growth Hysteresis

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ABSTRACT

The transition between wet and dry growth for graupel and hail is examined, and new figures are presented illustrating the critical water contents necessary for transitions into or out of the wet-growth regime. These figures are extended to smaller sizes and lower bulk densities than considered in previous studies. In addition, the possibility of hysteresis in the transitions is examined.

1. Introduction

As a hailstone or graupel particle grows by riming, the latent heat released by the freezing of accreted supercooled cloud droplets locally raises the surface temperature of the collecting particle. In regions of high supercooled water content, the overall rate of latent heat release may exceed the rate at which the collecting particle is able to transfer the heat back to the environment by forced convection and evaporation. In this case, the surface temperature of the particle will tend to rise until it reaches 0°C. At this point, the subsequent rate of freezing of the accreted water will be controlled by the net rate at which the particle transfers heat to the environment. Water collected in excess of this rate will either be retained as a liquid coating, soak into a low-density ice core, be incorporated into a layer of "spongy" ice, or be shed. This is usually termed the wet-growth regime (Schumann 1938; Ludlam 1950, 1958). By extension, dry growth is the term used to describe the normal rime growth processes that do not generate latent heat rapidly enough to overwhelm the natural rates of heat transfer.

Recently, Lesins and List (1986) have examined in detail the development of spongy ice on model hailstones using a pressure-controlled icing wind tunnel. Their observations suggest that the transition from a *dry* regime to one that produces spongy ice deposits and possible shedding of accreted water is not a simple transition, but rather an involved process that can be broken down into a number of separate stages or distinct growth environments (i.e., dry regime, moist regime, spongy regime, spongy-shedding regime, soaked-shedding regime, and dry-shedding regime). In addition, they note that conditions can vary considerably over a hailstone surface, with dry growth observed in one area while another area might be experiencing spongy growth. Fearing that the traditional usage of the simple terms, wet and dry, to describe the overall mode of growth might be misleading, they recommend that these terms not be used. In spite of their concerns, however, there are many reasons for examining the average growth conditions for a hailstone as a whole. In this context, the "classical" partitioning of the growth modes into wet or dry should not be misleading since the wet-growth regime can be envisioned as including the various spongy or shedding regimes.

In this article, we examine the critical liquid water contents required for the transition between wet-growth and dry-growth regimes, defined in the traditional sense, with particular reference to how small an ice particle can be and still accrete cloud droplets rapidly enough to become *wet*. In addition, we examine Johnson's (1987) suggestion that the boundary between wet and dry growth should exhibit a significant degree of hysteresis. That is, that the initial transition from a dry-growth regime to a wet-growth regime (the onset of wet growth) will often require higher water contents than the reverse transition from wet to dry.

2. Discussion

For any ice particle growing by riming in a cloud of supercooled liquid droplets, there exists a critical maximum liquid water content, W_c, for which the rate of latent heat released by the freezing of the supercooled droplets is exactly balanced by the natural processes of heat transfer back to the environment. Quantitatively, this critical water content can be expressed (e.g., Pruppacher and Klett 1978) as:

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$$W_c = \frac{8[k_a(T_0 - T_e)f_h + L_e D_v(\rho_0 - \rho_e)f_v]}{DVE_c[L_m - c_w(T_0 - T_e)]}, \quad (1)$$

where T_0 is the surface temperature of the hail or graupel as it enters the wet-growth regime (0°C), T_e is the temperature of the environment, ρ_0 is the water vapor density at 0°C, ρ_e is the water vapor density in the surrounding environment, k_a is the thermal conductivity of air, D_v is the diffusivity of water vapor in air, L_e is the latent heat of vaporization, L_m is the latent heat of fusion, c_w is the heat capacity of liquid water, f_h and f_v are the effective ventilation coefficients for heat and water vapor, respectively, D and V are the diameter and terminal velocity of the graupel or hailstone, and E_c is an appropriately defined mean collection efficiency.

The numerical evaluation of the terms in (1) will involve a set of equations similar to that employed in microphysical particle-growth models (e.g., Rasmussen and Heymsfield 1987a). As such, the specific formulation used involves a consistent set of equations for evaluating particle growth rates and heat transfer over a wide range of environmental conditions. This permits extension of the results to smaller particle sizes than have usually been reported. The most commonly used portrayal of the critical water content, for example, is that presented by Macklin and Bailey (1968). Their results, however, apply only to hailstones that are 2 cm in diameter and larger.

Terminal velocities can be calculated using the now familiar Best number–Reynolds number approach. By appropriate selection of the input parameters, this approach permits estimation of terminal velocities for particles of any specified bulk density. Figure 1 shows the underlying dependence of the drag coefficient, C_D, on Reynolds number. For smooth spheres (solid line) the relation is based on the summary of observational and theoretical results by Clift et al. (1978). The curve for rough-surfaced particles closely follows the relation used by Rasmussen and Heymsfield (1987a) for graupel and hail. For simplicity, we have restricted these curves (and all our subsequent analyses as well) to subcritical flow regimes.[1] The primary difference between these two curves is the increased drag due to the surface roughness of the graupel or dry hail. For small particles (in this case meaning graupel with Reynolds numbers in the vicinity of 100 to 1000) roughness can add significantly to the drag (e.g., Selberg and Nicholls 1968; Heymsfield 1978). For larger particles, however, the skin-friction component of the drag becomes increasingly small and variations in the small-scale surface roughness have less and less of an effect (e.g., Macklin and Ludlam 1961; Young and Browning 1967; List et al. 1969; Clift et al. 1978). This is reflected

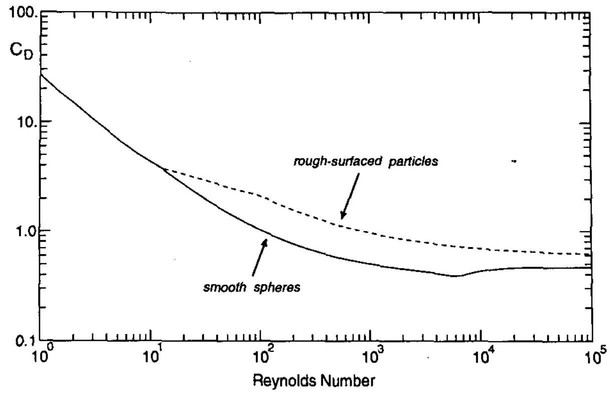

FIG. 1. Drag coefficients, C_D, as a function of Reynolds number for smooth spheres (solid line) and rough-surfaced particles (dashed line).

in Fig. 1, where the drag coefficients for smooth spheres and rough-surfaced particles begin to converge at high Reynolds numbers.

The effect of the surface roughness on heat transfer is less well known, with different sources often giving somewhat contradictory results. In part, this confusion represents differing definitions of roughness. Schuepp and List (1969), for example, reported a significant (up to a twofold) increase in the heat transfer for rough hailstone models over the Reynolds number range of 3×10^3 to 1×10^5, using models covering a wide range of surface roughness. As roughness elements become increasingly large, however, they become indistinguishable from more complex features, such as the lobe structure often observed in large hailstones (Browning 1965, 1966). For this study, we have assumed that rough-surfaced spheres (graupel and "dry" hail) will have heat transfer characteristics that are slightly enhanced over those of smooth spheres (10% larger), except for large hail. For these largest particles, we will follow the observations of Bailey and Macklin (1968), who found dramatic increases in heat transfer coefficients compared to smooth spheres. In part, this increase reflects the presence of lobe structure in the large hailstones they were investigating. This increase in the heat transfer for Reynolds numbers in excess of 10^5 has been found to be essential for the growth of very large hail (see Rasmussen and Heymsfield 1987b). As expressed in (1), the specific terms describing the net rate of heat transfer have been combined into the terms representing the effective ventilation coefficients for heat and water vapor, f_h and f_v. Figure 2 illustrates the variation of f_h with Reynolds number (assuming $T = -10$°C, $P = 450$ mb). The corresponding ventilation coefficient for water vapor is virtually identical. In both cases, the numerical formulation follows Bailey and Macklin (1968), incorporating the specific temperatures and pressures required for each calculation.

To the extent that rough-surfaced particles have larger drag coefficients than smooth particles of the

[1] At the critical point (roughly Re = 4×10^5 for smooth spheres) the drag coefficient, C_D, decreases abruptly as the whole of the boundary layer surrounding the particle becomes fully turbulent.

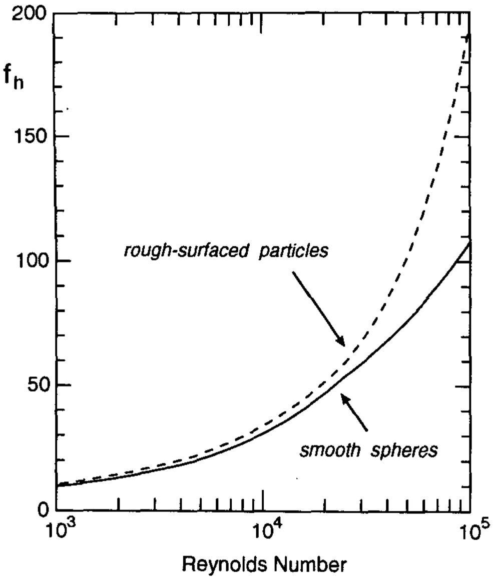

FIG. 2. Effective ventilation coefficient for heat, f_h, as a function of Reynolds number for smooth spheres (solid line) and rough-surfaced particles (dashed line). This example is based on an ambient temperature and pressure of −10°C and 450 mb.

same size and mass, they will fall slower and accrete fewer cloud droplets. If at the same time, these rough-surfaced particles are at least as efficient at heat transfer, then the critical water content for rough-surfaced particles should be larger than the critical water content for an otherwise similar particle with a smooth surface. This was the rationale for Johnson's (1987) suggestion that there might be a significant degree of hysteresis involved in transitions between wet-growth and dry-growth regimes. An implicit assumption in this suggestion is that the surface properties of a graupel or hailstone will differ in various growth regimes, and that these surface properties will in turn modify the particle's drag, heat, and mass transfer coefficients. Wet growth, with its spongy ice and surface coatings of liquid water, should modify the properties of particles in the wet-growth regime in the direction of smooth spheres. In this sense, the critical water contents for smooth spheres can be considered a lower limit for the transition point between wet and dry growth. Even under wet growth, however, most particles will seldom become truly "smooth." The experiments of Rasmussen et al. (1984), for example, have shown that in free fall, melting particles larger than about 9-mm diameter will start to shed their water coating. Beyond 1.5- to 2.0-cm diameter, most of the surface water will be shed. While spongy ice growth may be able to smooth some of the large- or moderate-scale surface structures, water coatings during wet growth will generally be quite thin and will only be able to smooth the smaller-scale roughness elements (see Lozowski 1991; List 1991).

Although hailstones are often assumed to sweep out all the cloud droplets in their path, the actual collection efficiencies for small water drops can be quite a bit less than unity. This can make a noticeable difference in the critical water contents (Macklin and Bailey 1968). As would be expected, the collection efficiencies decrease with decreasing cloud droplet diameters. Somewhat surprisingly, however, the collection efficiency decreases with *increasing* hailstone diameter as well (e.g., Macklin 1978). In our calculations, we have estimated collision efficiencies for large graupel and hailstones using the approach introduced by Beard and Grover (1974) and Hall (1980), and used by Johnson (1987) and Rasmussen and Heymsfield (1987a,b). Ultimately, this approach is based on the work of Langmuir and Blodgett (1946) and produces collection efficiencies similar to those measured by Macklin and Bailey (1968).

To the extent that prior studies have considered nonunity collection efficiencies for the sweep out of supercooled cloud droplets, the calculations have generally been based on uniform or monodisperse droplet distributions. In this study we assume that the cloud droplets can be represented by a gamma, or Pearson Type III, distribution as discussed by Berry and Reinhardt (1974). In its most general form, this is a three-parameter distribution. Two of the three parameters, however, can be defined directly in terms of the overall number concentration of activated cloud droplets, and the total liquid water content. The third parameter, ν, specifies the relative variance of the distribution. For $\nu = 2$ the relative variance in radius (relative dispersion) is 0.2. This value is typical of a wide variety of natural cloud droplet distributions, and will be used in the subsequent calculations.

Figures 3–6 illustrate the results of evaluating (1) for a variety of initial conditions. In each case, the computational procedure is to make a preliminary estimate of the critical liquid water content using (1), with an assumption of unity collection efficiencies. From this point an iterative procedure is followed in which the total droplet concentration (fixed for a given initial condition) and the currently estimated critical water content are used to define a cloud droplet distribution. The calculated distributions range from 1 to 100 microns in diameter in one-micron steps. Appropriate collection efficiencies are calculated for each of the 100 categories of cloud droplets, and a mass-weighted mean collection efficiency is determined for the distribution. The preliminary estimate of the critical water content is then adjusted using this mean collision efficiency. At this point the process is repeated until successive estimates of the critical water content differ by less than 0.05 g m^{-3}.

For each set of initial conditions, four separate curves are drawn. The light-gray curve represents the critical water content relation for smooth spheres (bulk density = 0.91 g cm^{-3}) having a surface temperature of 0°C. This is the classic starting point for estimates of this

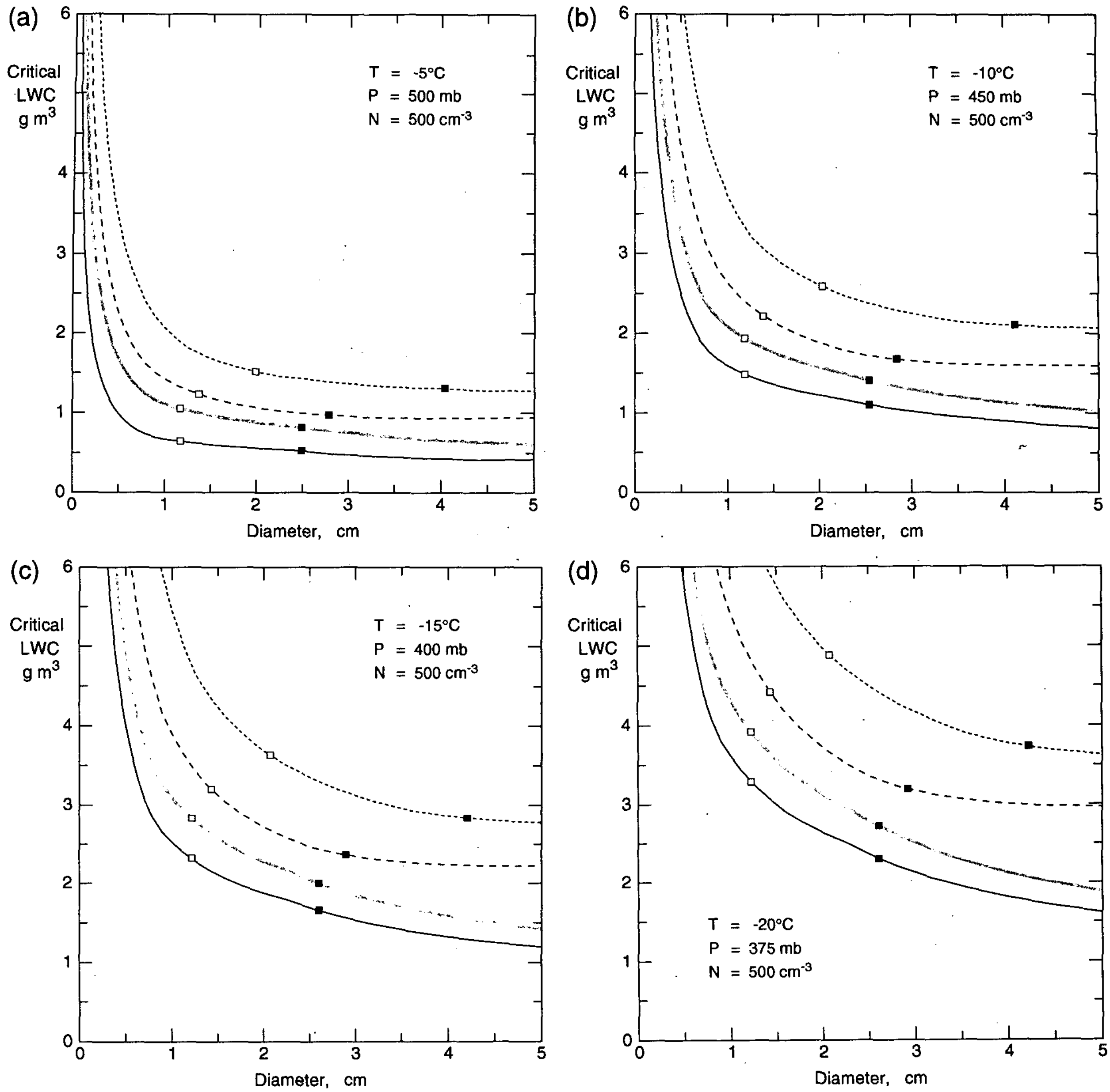

FIG. 3. Critical liquid water contents (g m^{-3}) for wet growth as a function of the diameter of the collecting particle. The light-gray line is appropriate for high-density, smooth spheres (bulk density 0.91 g cm^{-3}, surface temperature 0°C). The dashed line (immediately above the gray line) corresponds to rough-surfaced spherical particles with bulk densities of 0.91 g cm^{-3}. The dotted line indicates the critical water contents for rough-surfaced particles with significantly lower bulk densities (0.3 g cm^{-3}). Like the gray line, the bottom-most solid black line is based on high-density (0.91 g cm^{-3}) smooth spheres, but with a surface temperature of −2°C. The open squares indicate the position on the curves corresponding to a Reynolds number of 1×10^4, while the filled in squares indicate Reynolds numbers of 3×10^4.

sort. The dashed line represents the critical water contents for rough-surfaced high-density (0.91 g cm^{-3}) ice particles. The dotted line represents critical water contents corresponding to rough-surfaced low-density (0.3 g cm^{-3}) ice particles. Recently, List (1990) has argued that the surface of the water coat on a hailstone undergoing wet growth *must* be supercooled by several degrees, and cannot stay at precisely 0°C. The final curve (solid black line) repeats calculation for high-density (0.91 g cm^{-3}) smooth spheres, but with the surface temperature reduced to −2°C. In effect, this last curve just shows the sensitivity of the calculated critical water contents to the surface temperature of the growing particle. In order to orient these curves in terms of the Reynolds numbers of the collecting particles (cf. Figs. 1 and 2), open squares have been drawn at Reynolds numbers of 1×10^4, and solid squares at 3×10^4.

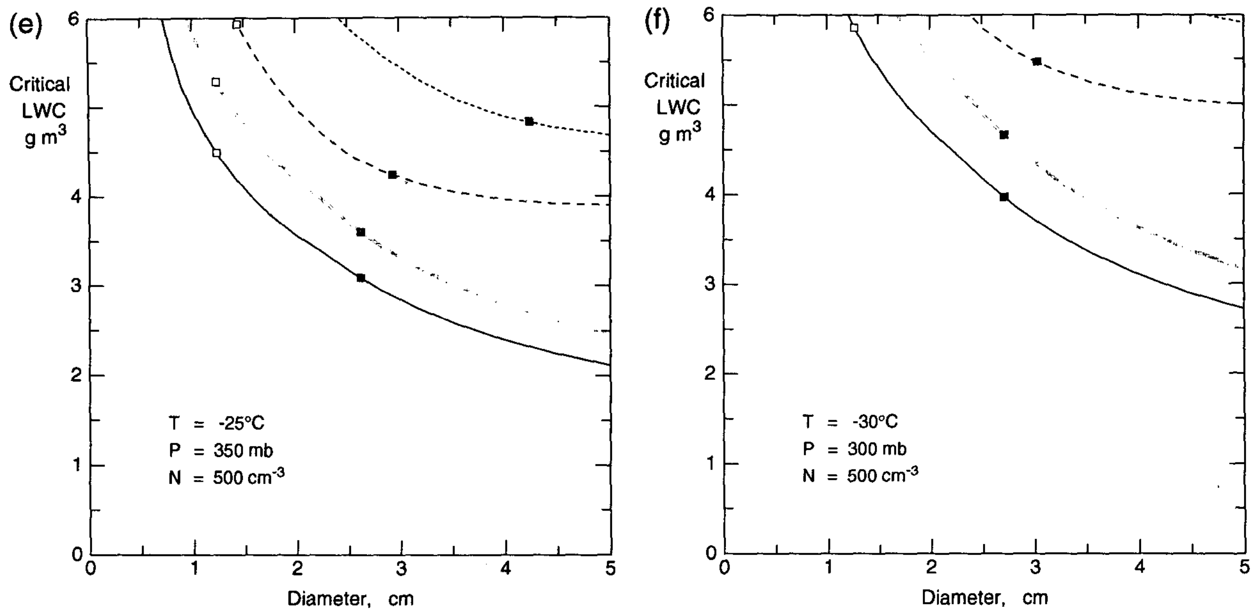

FIG. 3. (*Continued*)

Figure 3 presents results from a number of calculations, illustrating the variation in W_C as a function of the size of the collecting ice particle for six different sets of environmental conditions (air temperatures and pressures ranging from $-5°$ to $-30°$C and 500 to 300 mb). For now, the overall cloud droplet concentration has been maintained at 500 cm^{-3}, although the distribution itself is not fixed. If these illustrations were viewed merely as a set of sensitivity studies, their interpretation would be straightforward. Trying to understand the curves in light of possible hysteresis effects and particle evolution is a bit more complex. The best starting point for interpreting these curves are the calculations corresponding to high-density, rough-surfaced particles (dashed lines). These curves correspond to the "standard" estimates of critical water contents presented in most other studies. The companion curves for smooth spheres (gray lines) are everywhere below the curves for rough-surfaced particles. The critical water contents for these high bulk-density smooth spheres represent a lower limit on the water contents necessary to *sustain* wet growth once it has started. Prior to the onset of wet growth, however, rimed ice particles are not going to act like smooth spheres, and will not enter into wet growth until the water contents exceed the higher critical water contents associated with rough-surfaced particles. This is the essence of the hysteresis, since transitions between wet growth and dry growth would require different water contents, depending on the direction of the transition.

In reality, however, hailstones larger than about one centimeter in diameter will seldom behave like completely smooth spheres. Particles of this size will typically shed much of their excess water coating and will often have large-scale roughness elements (lobe structures) that dominate their drag as well as heat and mass transfer. During melting studies, for example, Bailey and Macklin (1968) found that large lobed hailstones had heat transfer coefficients that were essentially the same irrespective of whether their surfaces were wet or dry. In this case, hysteresis effects would be minor at best, so long as the surface temperature of the wet hailstone stays very close to 0°C. If, as List (1990) suggests, *growing* hailstones in the wet-growth regime develop surface coatings of water that are supercooled by several degrees, then the critical water contents would be shifted downward by several tenths of a gram per cubic meter. In effect, this would push the envelope for wet growth downward and reinstate the possibility of hysteresis.

The remaining curve, the dotted line corresponding to low bulk density particles, is always considerably above the other curves. This is to be expected, since the low bulk density particles will tend to fall slower than higher-density particles of the same size and accrete cloud droplets at a slower rate. Consideration of low-density particles extends the hysteresis in the opposite direction, raising the water contents that are needed before wet growth can begin. For example, at $-15°$C (see Fig. 3c), a 2-cm diameter particle with a bulk density of 0.3 g m^{-3} would require a liquid water content of 3.7 g m^{-3} or more to initiate wet growth. Once into the wet-growth regime, water will rapidly penetrate into the low-density ice structure and increase the particle's overall bulk density. A high-density particle of the same diameter will accrete cloud droplets more rapidly, and can maintain wet growth in water contents as low as 2.7 g m^{-3} (dashed curve). In the unlikely event that the surface of this particle were smoothed enough to act as a completely smooth sphere, it could maintain its wet growth in water contents as low as 2.3 g m^{-3}, even if it maintained its surface temperature at or near 0°C (gray curve). If the surface temperature of the particle were to drop to −2°C, then the particle could still maintain itself in the wet-growth regime in water contents of about 2.3 g m^{-3}, even if it maintains a rough surface (i.e., dashed curve minus 0.4°).

If the ambient air temperature is relatively near 0°C, it is not difficult to achieve wet growth. Figure 4 presents an expanded look at the small-particle end of two of the plots (−5° and −10°C) shown in Fig. 3. In both cases, the x axis has been expanded by a factor of ten, with the size range being limited to particles of five millimeters in diameter or less. At −5°C, graupel that are only a few millimeters in diameter are potential candidates for wet growth. For particles in this size range, however, the critical water contents increase rapidly with decreasing temperature. For example, the 3-mm diameter ice particle (rough surface, bulk density = 0.91 g cm^{-3}) that required water contents of only 3.4 g m^{-3} to initiate wet growth at −5°C will now have to have water contents in excess of 7 g m^{-3} to grow wet at −10°C. For temperatures of −10°C or warmer, a similar 1-cm diameter particle will need a water content of only 2.6 g m^{-3}, or less, to initiate wet growth. Such small particles are of special interest in the current discussion, since they are generally too small to shed the liquid water coats that will form on their surface during either melting or wet growth. With a substantial water coat, such particles could well behave very much like classically smooth spheres, if only over a limited size range.

Figures 5 and 6 extend the calculations of Fig. 3 by considering different cloud droplet concentrations. In particular, Fig. 5 repeats the calculations presented in Fig. 3c, for droplet concentrations of 200 cm^{-3} and 800 cm^{-3}. Higher droplet concentrations result in a small but systematic shift in the critical water content curves toward higher values. For a given water content, of course, higher droplet concentrations mean smaller cloud droplets and an associated reduction in the mean collection efficiency term. Lower droplet concentrations push the curves in the other direction, resulting in systematically lower curves for the critical water contents. Similarly, a reduction to a droplet concentration of 200 cm^{-3} at −25°C, as shown in Fig. 6, reduces by a bit (0.2 to 0.5 g m^{-3}) the water contents necessary to initiate wet growth or to maintain it once it is initiated.

3. Conclusions

The primary purpose of this study is to take a fresh look at the transition between the wet-growth and dry-growth regimes. In the region between 0° and −10°C, relatively small ice particles (diameters ranging from several millimeters up to one centimeter) may experience water contents that are large enough to initiate wet growth. For the most part, however, wet growth is primarily the domain of particles of centimeter diameters and above. In general, the critical liquid water contents necessary to initiate wet growth are lower for

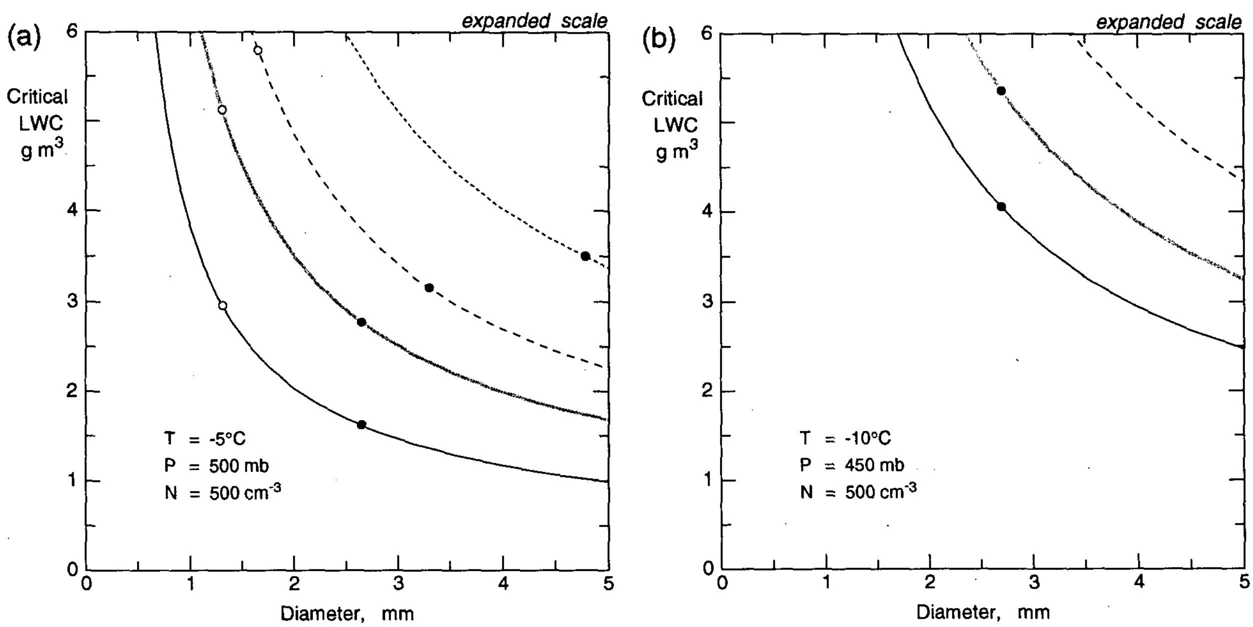

FIG. 4. An expanded view of two panels from Fig. 3, illustrating the critical water contents for small (0.5-cm diameter and below) ice particles. The open circles indicate the position on the curves corresponding to a Reynolds number of 3 × 10^2, while the solid circles indicate Reynolds numbers of 1 × 10^3.

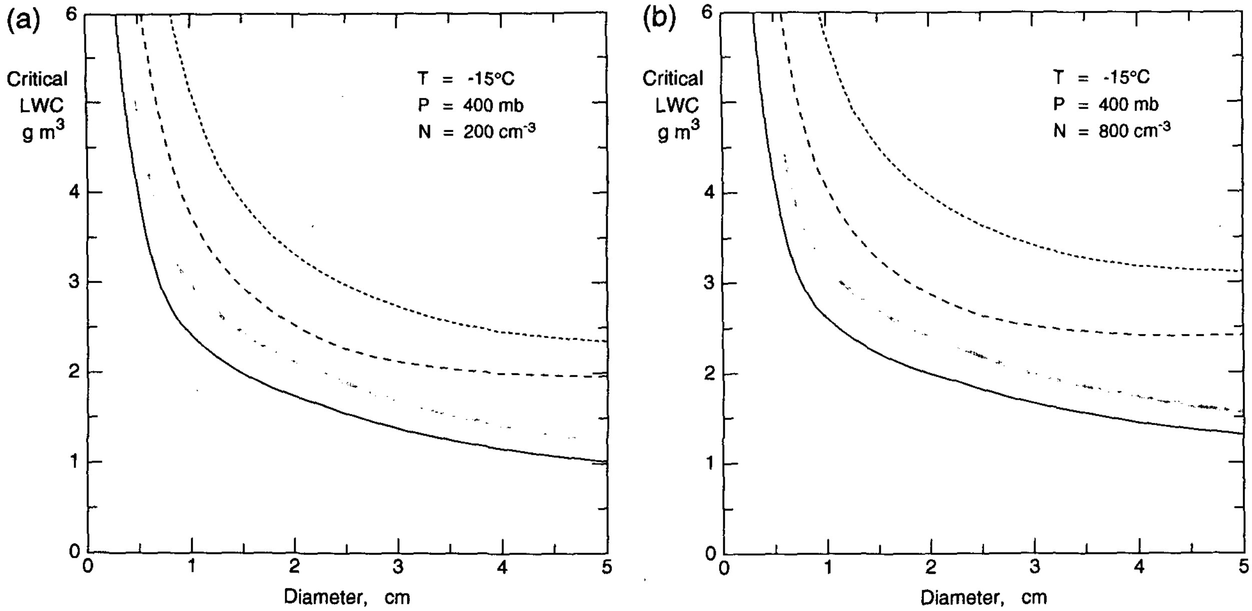

FIG. 5. Critical water contents at −15°C and 400 mb, assuming cloud droplet concentrations of 200 cm^{-3} and 800 cm^{-3} (see caption for Fig. 3).

large collecting particles, higher bulk densities, lower cloud droplet concentrations, and warmer temperatures. Although the critical water content curves presented in this paper are not always directly comparable with earlier results, the overall results and general magnitude of the critical water contents agree quite well with earlier studies such as List (1960), Macklin and Bailey (1968), and Rasmussen (1987b).

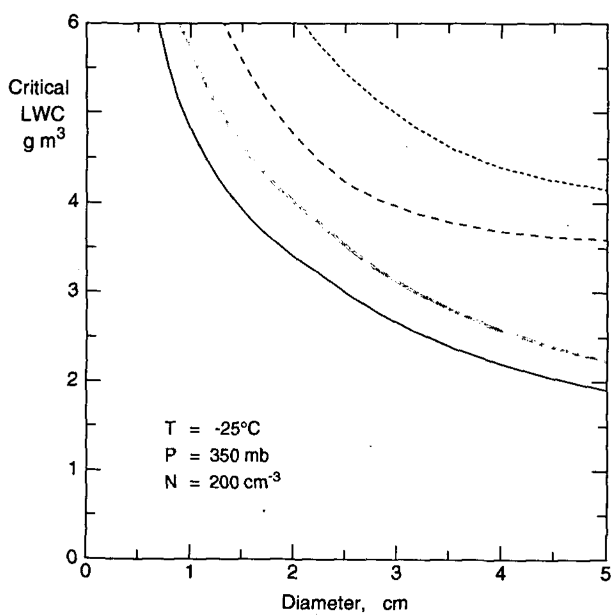

FIG. 6. Critical water contents at −25°C and 350 mb, assuming a cloud droplet concentration of 200 cm^{-3} (see caption for Fig. 3).

The current analysis supports the conclusion that transitions from dry growth to wet growth may often require higher water contents than transitions from wet growth to dry growth. That means that wet growth may be relatively hard to initiate, but once it begins it will tend to continue, even in the face of environmental conditions that would not have been adequate to initiate the wet growth in the first place. Following the transition from dry growth to wet growth, there would have to be a major reduction in the ambient supercooled water content to force the growth back to dry conditions. Similarly, the reverse transition from wet to dry would require a sizeable increase in ambient water contents to force the growth back to the wet-growth regime. As originally envisioned by Johnson (1987), this hysteresis would primarily be the result of differences in the drag coefficients and heat transfer coefficients for smooth and rough particles. This interpretation seems to be limited to collecting particles smaller than one centimeter in diameter. For larger particles, the extension of the analysis to include lower bulk density particles (requiring higher water contents to initiate wet growth) and particles having a surface water coat that is supercooled to several degrees below 0°C (reducing the water contents required to *continue* wet growth) seems to be potentially more important.

Interestingly, the hail growth model of Rasmussen and Heymsfield (1987a) implicitly included most of the hysteresis effects discussed in the current paper. In particular, the model included separate algorithms for the drag coefficients and heat transfer coefficients for wet and dry hailstones. Furthermore, the model maintained an explicit calculation of the bulk density of the hailstone (or graupel), allowing absorption of liquid water into the low-density ice structure without affecting the surface properties of the particle. Most importantly, the model included an explicit formulation for the shedding of liquid water, whether the result of wet growth or melting. The major feature not included in the model was the possibility that the equilibrium surface temperature of a particle undergoing wet growth might be significantly less than 0°C.

In all cases, of course, the present analysis is based on idealized curves that are in some degree intended to represent *average* conditions. Individual particles, however, vary greatly. This means that there will be a considerable amount of variability that will be superposed on the current results, including variability between different portions of a single particle. Hysteresis effects add another kind of variability, one that is dependent not so much on the instaneous physical properties of a given particle as on its prior growth history.

The current analysis also shows that large low-density ice particles require significantly higher water contents to begin wet growth than do higher-density particles of the same size. Once wet growth starts for such a low-density particle, it should rapidly increase in bulk density as liquid water soaks into the low-density rime structure. Such a particle would be relatively stable in the wet-growth regime, and should grow rapidly. This is precisely what would be expected to happen in the two-stage hail-growth process discussed by Pflaum (1980) and Prodi et al. (1986). One interesting aspect of such a process is the possibility that a particle might enter the wet-growth regime at a relatively warm ambient temperature, where the water contents required to initiate wet growth are comparatively modest, and then continue growing wet as it is carried aloft to colder and colder temperatures. Such a particle would never have to experience the high water contents necessary to initiate wet growth directly in a cold environment.

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