Turbulent Entrainment and Mixing in Clouds: A New Observational Approach

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ABSTRACT

A statistical analysis of cloud droplet interarrival times, measured using an aircraft-mounted forward-scattering spectrometer probe, is used to deduce spatial scales of inhomogeneity of droplet concentration in cumulus clouds. The analysis often indicates inhomogeneity of the droplet concentration at small scales (about 1 cm) within regions that appear homogeneous when viewed at larger scales. An explanation of the small-scale inhomogeneity is offered, and the larger-scale homogeneity of the droplet concentration is interpreted in terms of the processes of entrainment and mixing.

1. Introduction

Entrainment has been studied using measured thermodynamic variables and mixing diagrams (Paluch 1979; Blyth et al. 1988) and mixing has been studied indirectly by interpreting measured droplet size spectra (Warner 1969; Paluch and Knight 1984). Others have attempted more direct observation of the mixing process. A general aspect in these studies has been to determine whether the droplets (or particles) are randomly located in space. Since this is also fundamental to the approach of the present study, those works are briefly reviewed.

Preining (1983) compared measured high-resolution aerosol particle concentration data and aerosol interparticle spacing data with the ideal distributions expected if the particles are randomly located in space. In this study, high-resolution droplet concentration data are compared with similar ideal distributions. Their analysis was made visually on graph paper.

Austin et al. (1985) compared droplet concentrations, measured at 2-m resolution, with the Poisson distribution. Although less rigorously defined, they used essentially the same statistic to be used in the present study.

Baumgardner (1986) described the collection and analysis of interarrival times of droplets at the forward-scattering spectrometer probe (FSSP). That work provided the impetus to collect, for the present study, similar data in a more complete form by recording the arrival time of each droplet at the FSSP in sequential order.

Paluch and Baumgardner (1989) used the interarrival times of droplets, mentioned above, to estimate a "local droplet concentration." Brenguier (1990) derives a similar quantity called "local drop rate" using a method, based on the effect of coincident droplets, requiring only standard FSSP data. Where the droplet concentration is homogeneous, these derived quantities should indicate the actual droplet concentration. Where the droplet concentration is not homogeneous, they can be shown to indicate a droplet concentration in between the maximum and minimum droplet concentrations, biased toward the maximum. Therefore, if the indicated local droplet concentration is sufficiently greater than the standard FSSP droplet concentration measurement [corrected for coincident and dead time losses; Baumgardner et al. (1985), Brenguier (1989)], such that the difference cannot be explained by sampling statistics or errors, then the region must be inhomogeneous.

The main advantages of the data and method of the present study are 1) much finer spatial resolution; 2) the statistical analysis is more robust, that is, significance levels for rejection of the null hypothesis that the droplets are randomly situated in space are determined and the test's sensitivity (power) demonstrated; and 3) the test provides some information about the scales of the inhomogeneities.

Droplet positions and velocities in three dimensions have been recorded on holograms (Conway et al. 1982; Kozikowska et al. 1984). These data are the most complete, but difficulties with automated analysis limit their usefulness. If sufficient automation is achieved, then the methods being introduced in this study could be adapted for three-dimensional analysis.

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In section 2 the new data are described, and in sections 3 and 4 the analysis method is presented. Observations are presented in section 5, with some possible explanations in section 6. Because the analysis method is new, its description is necessarily lengthy. Readers not interested in the details may prefer to skip part or all of sections 2 through 4c and begin reading again at section 4d.

2. The data

The data consist of each droplet's time of arrival at the FSSP laser beam. The data can be viewed either as a sequence of the time intervals \((y)\) between droplets (interarrival times) or as a time series of droplet concentrations \((x)\) based on the number of droplets detected during each time interval of fixed length \((L)\). Traditionally, the FSSP has been used to measure droplet concentration and size spectrum at a fixed frequency. Now the concentration time series can be viewed at different frequencies by varying the length of the time interval \((L)\) over which the droplets are counted. The clock is a 250-kHz oscillator. Therefore, 4 \(\mu s\) will be used as the smallest unit of time and is called "click." Since the airplane traveled at a fairly constant velocity, typically 80 m \(s^{-1}\), time and distance are interchangeable:

\[
1 \text{ click} = 4 \mu s = 0.32 \text{ mm}. \tag{1}
\]

Because the clock is digital, any interarrival time with value \(y \) clicks, \(y \) an integer greater than 0, corresponds to an actual interarrival time between \(y - 1\) and \(y + 1\) clicks. Any datum \((y)\) with value 0 clicks corresponds to an actual interarrival time between 0 and 1 click. This distinction makes the distribution of interarrival times different than the waiting time distribution. The well-known exponential "waiting time distribution" is what would result if interarrival times of randomly located droplets could be measured with an ideal continuous clock.

There is a finite amount of time (about 2 clicks) after a droplet is detected during which new arrivals are not detected. This will be called the blind time. Thus, the majority of short interarrival times are missed. Also, once each second the computer responsible for receiving and recording the data causes a blind time that is much longer, typically about 5000 clicks, and sometimes (once each six seconds) is even longer. These longer blind times are easily detected and ignored in data from a dense cloud region. Compensation for the short blind times will be discussed after describing the method of analysis.

During part of the flight program, the size of each droplet was recorded along with its arrival time.

3. The Fishing test

The goal of the analysis is to make observations of the turbulent mixing process in clouds. As used in this text, the following statements are, by definition, equivalent. 1) The droplets are randomly located in space with equal chance of being found everywhere. 2) The droplet concentration is homogeneous. If a cloud region is well mixed, then the droplet concentration is homogeneous and the interarrival times \((y)\) follow a certain probability distribution as do measurements of the droplet concentration \((x)\). If the mixing of air parcels with different droplet concentrations is incomplete, then spatial variations in the concentration cause the measured distributions of interarrival times and concentrations to differ from those in the well-mixed case.

A useful method of analysis is one that can detect and characterize inhomogeneity of the droplet concentration by comparing the measured distributions with expected distributions.

We will analyze the data viewed as a time series of concentration measurements. Our method, called the Fishing test, is a statistical test of the hypothesis that the data come from a region homogeneous in droplet concentration. The sensitivity of the test, that is, the probability of rejecting the hypothesis of homogeneity when there are in fact inhomogeneities, depends on both the length scale of the inhomogeneities and the length of interval \((L)\) over which each datum \((x)\) is calculated. This provides information about the scales of the inhomogeneities.

If the droplets are randomly located with equal chance everywhere, then since their diameters are very much smaller than their average separations, the number per length interval \(L\) is a Poisson-distributed random variable. In a given region of length \(NL\), there are \(N\) intervals of length \(L\) and, therefore, \(N\) measurements of \(x\). For the Poisson probability distribution the variance equals the mean, so the Fishing statistic \(F\) is defined as follows:

\[
F(L) = \left[ \frac{1}{N-1} \sum \frac{(x - \bar{x})^2}{\bar{x}} - 1 \right] (\delta(N))^{-1},
\]

where

\[
\bar{x} = N^{-1} \sum x \tag{2}
\]

and \(\delta(N)\) is the standard deviation of the expression inside the brackets, assuming the \(x\) are Poisson distributed, as discussed below. Therefore, under the null hypothesis that the data are Poisson distributed (that is, from a homogeneous cloud region), \(F\) is a statistic with mean of 0 and variance of 1. If \(F\) is large enough, then the null hypothesis is rejected. We use a critical value of 3, which implies 1% or less chance of erroneous rejection.

The distribution of

\[
\frac{1}{N-1} \sum \frac{(x - \bar{x})^2}{\bar{x}} - 1
\]

(3)
depends on \(N\) and the distribution of \(x\). Since the distribution of \(x\), under the null hypothesis, is a one-
parameter distribution, the distribution of (3) should be completely determined by \(N\) and the mean of the
Poisson distribution from which the \(x\) are drawn (\(\mu\)). The variance of (3) follows a simple formula (Cox and
Lewis 1966):

\[
\delta^2(N, \mu) = \frac{2(1 - (\mu N)^{-1})}{N - 1}.
\]  

In our case, \(\bar{x}N\) is typically many thousands and in no case will an analysis be done with \(\bar{x}N\) less than 100.
Therefore, since \(\bar{x}\) is an estimate of \(\mu\), \(\mu N\) will always be much greater than 1 and (4) is approximated well
by \(2/(N - 1)\). Accordingly

\[
\delta(N) = \left(\frac{2}{N - 1}\right)^{1/2}
\]  

is used.

For an inhomogeneous region of cloud, the value of
\(F\) depends on the value of \(L\) and the length scales of the inhomogeneities. If there is a dominant length scale
\((L_o)\) of inhomogeneity, then \(F\) is maximized for \(L\) near \(L_o\). Therefore, to look for preferred scales, \(F\) is calculated and graphed as a function of \(L\).

4. Calibrating the Fishing test

Artificial data were generated to determine a critical value for rejecting the null hypothesis (section 4a), to
model the effects of data error (section 4b), and to demonstrate the sensitivity of the method (section 4c).

a. Under the null hypothesis

The Fishing test is typically applied to regions about 100 meters (=300 000 clicks) long with mean droplet
interarrival times of about 10 clicks. To build approximate distributions of \(F\) under the null hypothesis, 5000
synthetic sequences of interarrival times were generated, each about 300 000 clicks long with a mean interarrival
time of about 10. For each synthetic data sequence, \(F\) was calculated for each of the following groupings of events: \(L = 1, 10, 200, 4000,\) and \(59\ 900\) clicks. The distributions are shown in Figs. 1a–e along with \(M\) and \(V\) (the mean and variance) of each \(F\) distribution and the percentage of values greater than 3.

Only data sequences for which the number of droplet detections \((\bar{x}N)\) is greater than 100 (typically much
greater) and for which \(\bar{x}\) is greater than 0.01 (even for \(L = 1\) click) were analyzed. To cover these low values of \(\mu (\approx \bar{x})\) and \(N\), 5000 sequences of interarrival times were generated, each about 10 000 clicks long with a
mean interarrival time of about 100 clicks; \(\bar{x}N\) is therefore \(\approx 100.\) For each data sequence, \(F\) was calculated
at the following values of \(L: 1, 10, 100, 900,\) and 1900. These distributions are shown in Figs. 1f–j. The odd shape, for the case \(L = 1, \bar{x} = 0.01,\) arises because a relatively small number of events (about 100) go into
a large number of cells (10 000). Thus, the probability of getting more than one in any cell is low, and therefore
the probability of getting some values of \(F\) (including the mean) is very low and high for other values of \(F\).
The narrow high peak at values less than 0 is due to cases in which no cells had more than one event. The other
bumps at values greater than 0 are due to cases in which one or more cells had two or more events.

Despite their different shapes, all the \(F\) distributions have mean \(\approx 0,\) variance \(\approx 1,\) and a low probability
of \(F\) greater than 3. Thus, a value of \(F\) greater than 3 is used as grounds for rejection of the null hypothesis
(homogeneity). Ideally, a value of \(F\) less than \(3\) would also be grounds for rejection of the null hypothesis.
However, the existence and treatment of error in the data (next subsection) tends to reduce \(F\) and can
artificially cause some values of \(F\) less than \(3.\) Therefore, the null hypothesis is rejected only if \(F\) is greater
than 3. Because of the error, the confidence of rejection when \(F\) is greater than 3 is increased. So far and in the
following subsection, for a given value of \(L, F\) refers to a single calculation of Eq. (2). Starting with section
4c, \(F\) will refer to \(F_{ave},\) an average of four calculations of \(F.\) This also increases the confidence of rejection
when \(F_{ave}\) is greater than 3.

b. Error in the data

Because of the blind time after each droplet detection, the majority of small interarrival times \((y = 0, 1, 2\) clicks) are missed, and since the clock doesn't stop, each missed time is added to the next interarrival time. This error is easily modeled in the synthetic data and
its effect is small. In fact, in the following, it will be seen that it reduces further the already low probability
of erroneously rejecting the null hypothesis.

Due to the finite length of the smallest unit of time, the click, the distribution of measured interarrival times
is slightly different from the waiting time distribution, which would result if the clock were continuous. That
is, for each measurement of interarrival time \(y,\) the actual waiting time (WT) is between \(y - 1\) and \(y + 1\)
clicks. Actual waiting times, if they were known, could be binned such that all times from 0 to 1 clicks are in
bin 0 and all times from 1 to 2 clicks are in bin 1, etc. (Fig. 2a). This binned waiting time distribution is different
than the measured interarrival distribution. Specifically, on average one-half of the events that would fall in bin A of the binned waiting time distribution will fall in bin A of the measured-interarrival
distribution and one-half will fall in bin \(A + 1.\) Thus, in going from the binned waiting time distribution to the
measured-interarrival distribution, each bin loses one-half to the next bin and gains one-half from the preceding bin, resulting in a small net increase for that bin. It can be shown that, except for bin 0, which loses
Fig. 1. Distributions of \( F \), when \( x \) is Poisson distributed, for different values of the parameters \( \mu \) (the mean number of droplets in the distance \( L, \bar{x} \approx \mu \)) and \( N \) (the number of observations of \( x \)) spanning the domain of interest in this work. Also shown for each distribution is its mean \( (M) \), variance \( (V) \), and percent of values greater than 3. (a)–(e) The typical case where the number of droplets \( (N\bar{x}) \) is large \( (\approx 30,000) \). (f)–(m) The case where \( N\bar{x} \approx 100 \). All real data sequences analyzed have at least 100 droplet detections.
one-half to bin 1 but gains nothing, the percentage increase is the same for each bin and increases with decreasing mean waiting time. The percentage increase is only 5% for a mean waiting time of 10, which is the smallest typical mean waiting time observed in the real data. Thus, the measured distribution of interarrival times looks like a binned waiting time distribution, except the first bin is only about one-half full. Figure 2b is a computer-generated measured-interarrival time distribution for which homogeneous cloud is simulated and the error is not simulated. Real data (Fig. 2c) looks basically the same, except nearly all the 0's and 1's and about three-fourths of the 2's are missing. If 2 clicks are subtracted from each interarrival time in the real data, the resulting distribution looks more like Fig. 2b. The compensation for the blind time error consists of just that; 2 clicks are subtracted from each measured-interarrival time.
The effects of the blind time, with this compensation, can be assessed in three parts:

1) the effect if all the 0’s and 1’s and the "expected" number of 2's (52.5% if the mean waiting time is 10) are missed, the missed times are not added to the next interarrival time, and 2 is subtracted from each remaining interarrival time;

2) the effect of adding the missed times to the next interarrival times; and

3) the effect of missing more then the expected number of 2’s.

The first part has no effect on the statistics of a data sequence under the null hypothesis, but it shortens the sequence for the following reason. Under the null hypothesis, a sequence of interarrival times is simply a random ordering of numbers from an interarrival distribution, such as Fig. 2b for a mean waiting time of 10. The interarrival distribution is simply a transformed binned waiting time distribution (one-half of each bin is put in the next larger bin). It can be shown that the waiting time distribution is not affected by removing all the times less than b and then subtracting b from the remaining times. The same is true for the interarrival distribution, except all the times in bins 0 to b and the expected (about 50%) number of times in bin b + 1 must be removed and then b + 1 subtracted from each remaining time. Thus, if all the 0’s, 1’s, and randomly the expected number of 2’s are removed from a data sequence (under the null hypothesis) and 2 is subtracted from each remaining time, the sequence is still a random ordering from the same interarrival distribution and thus a data sequence with the same statistics, except shorter.

Empirical quantile-quantile (q-q) plots compare two distributions by sorting (from small to large values) each of them and then plotting one versus the other. If the two distributions are the same, then the points will lie approximately on the line passing through the origin, with slope = 1. Five hundred typical data sequences (150 000 clicks long, with a mean waiting time of 10) were generated without simulating the error, and for each, F was calculated at L = 1, 100, and 9000. Then part 1 of the error was modeled and F recalculated at the same values of L for each new data sequence. Figures 3a-c are q-q plots of the resulting F distributions before and after simulating part one of the error. As expected, they show no effect of part 1 of the error on the distributions of F. Since the amount of data is reduced, part 1 of the error has the effect of reducing the chance of detecting inhomogeneities when they are present.

For real data each missed time is added to the next one (part 2 of the error). This does change the statistics of the data sequence slightly. It reduces the variance compared to the mean, and thus also reduces F, particularly when L is small. Figures 3d-f demonstrate this with q-q plots of F distributions at L = 1, 100, and 9000 calculated on 500 synthetic data sequences before and after parts 1 and 2 of the error are simulated. The effect is noticeable only for the case L = 1.

Finally, part 3 of the error is that more than the expected number of 2’s are missed. This also has the effect of reducing the variance compared to the mean and thus F. In this case the effect can be large but again only for small L. Figure 5a is F(L) for a synthesized data sequence, simulating homogeneous cloud, for which all parts of the error were simulated. The large negative values of F at small L are due to part 3 of the error. Part 3 of the error can be compensated for by inserting 2’s randomly into the data sequence, but this is not done in the majority of cases. Not compensating for this error only decreases the chance of an erroneous rejection of the null hypothesis.

c. Sensitivity to inhomogeneities

The F statistic tests the (null) hypothesis that the data from which it is calculated are Poisson distributed. If F is greater than 3, the null hypothesis is rejected. The data from which F is calculated are measurements of the droplet concentration; that is, the number of droplets detected in a distance L, measured in clicks, where 1 click = 4 μs ≈ 0.32 mm. When inhomogeneities exist at a spatial scale L_o, F is most sensitive to those inhomogeneities when L is close to L_o. Therefore, F is calculated for many values of L and plotted as a function of L. When L is greater than 4 clicks, F is calculated four times, starting first at the beginning and then at L/4, L/2, and 3L/4 clicks into the data sequence. Here F_ave is the average of the four values of F. Under the null hypothesis, the probability of F_ave being greater than 3 is less than the probability of F being greater than 3, resulting in an even greater confidence of rejecting the null hypothesis when F_ave is greater than 3. Throughout the remainder of this manuscript F refers to F_ave.

While the probability of rejecting the null hypothesis when it is true can be quite precisely quantified, the probability of rejecting the null hypothesis when it is false is not so easily quantified. Rather than even attempting such a task, the sensitivity of the test will be demonstrated by example, that is, by using synthetic data that models inhomogeneous cloud.

According to current ideas on turbulent mixing (Tennekes and Lumley 1972; Broadwell and Breidenthal 1982), when turbulence is initially mixing fluids with different droplet concentrations, there should be concentration differences at many scales reflecting the effects of different-sized eddies. Diffusion of fluid properties is slow compared to turbulent transport and therefore can act only at very small scales. Therefore, there should be regions of different sizes within which the droplet concentration is homogeneous but between which the droplet concentration differs (these are referred to as clumps). To demonstrate the sensitivity of
Fig. 3. (a)-(c) $q$-$q$ plots of $F$ distributions before and after simulating part 1 of the error. (d)-(f) $q$-$q$ plots of $F$ distributions before and after simulating parts 1 and 2 of the error. The line passing through the origin with slope $= 1$ is also drawn on each plot.
the Fishing test, simplified models are used. Namely, inhomogeneous data is modeled as consecutive homogeneous clumps, with lengths varying around \( L_o \), within which the mean concentration has one of two possible values (see Fig. 4). Each of the following parameters is varied singly: \( L_o \) (Figs. 5b–f), the difference in concentration between clumps (Figs. 6a–d), the average concentration (Figs. 6g–h), and the length of the sequence (Figs. 6d–g). Except where otherwise stated, the error is simulated and parts 1 and 2 are compensated for, as discussed. Part 3 of the error is not compensated for, except where stated.

For this simple model of inhomogeneous cloud, \( F(L) \) has a maximum when \( L_o \) approximately equals \( L_o/2 \) and the width of the peak is about \( L_m \). Figure 5 shows that the magnitude of the peak (\( F_m \)) increases with \( L_o \). Thus, a peak (with \( F_m \) greater than 3) at \( L_m \) implies that there are inhomogeneities at spatial scales on the order of magnitude of \( L_m \). If \( F \) decreases as \( L \) increases above \( L_m \), then any inhomogeneities at scales much larger than \( L_m \) are weak compared to those at the scale \( L_m \). Otherwise the peak would be at those larger scales. Here weak or strong inhomogeneity implies small- or large-concentration differences between the clumps. Nothing can be inferred about the existence of inhomogeneities on scales much smaller than \( L_m \).

In Fig. 5f there is a small local maximum where \( F \) is always negative. The negative \( F \) values are due to the lack of 2's in the data. This was compensated for by randomly inserting the expected number of 2's into the data sequence, resulting in Fig. 5g. The compensation is not complete because, unlike the homogeneous case, the density of 2's, and therefore the number lost, varies spatially. Figure 5h shows how the Fishing test would respond to the same inhomogeneity as in Figs. 5f and 5g if there were no blind time error at all. The sensitivity of \( F \) to inhomogeneities is greatly reduced by the error at these small scales.

Because the error causes large negative values of \( F \) at small \( L \), in all following \( F(L) \) graphs those values of \( F \) less than \(-3 \) are not plotted. Except where it can be explained by the error, \( F \) is not found to be significantly low (less than \(-3 \)) in data from actual clouds. If it were observed, the physical interpretation would be that the droplets are more evenly spaced than random placement would imply.

Figures 6a–d demonstrate that if all other factors are constant, increasing the concentration difference between clumps increases \( F_m \).

Figures 6d–h demonstrate that increasing the number of events either by increasing the total length of the sequence or by decreasing the mean waiting time also increases \( F_m \).

In Fig. 5 the clumps were a length \( L_o \pm 0.2L_o; F(L) \) may oscillate due to this quasi periodicity. This is most noticeable in Figs. 5d and 5e. In Fig. 6 this is less evident as the clumps were a length \( L_o \pm 0.5L_o \). Real data is probably less periodic. In further tests it was found that \( F_m \) does not depend on the periodicity of the inhomogeneities.

The examples so far provide an intuitive feeling for how the Fishing test responds to inhomogeneities of the type shown in Fig. 4. The response of the Fishing test increases as the concentration difference between clumps increases. When clear air is mixing with cloud, the Fishing test is sensitive to small ratios of clear to cloud air. This was modeled by randomly inserting clear air gaps (length 200 ± 100 clicks or 6.4 ± 3.2 cm) into otherwise homogeneous data sequences. In Fig. 7 the homogeneous data sequence is the typical 100 m with mean waiting time of 10 and the only variation is the number of gaps. Also shown is the ratio of clear to cloud air. The magnitude of the peak is roughly proportional to this ratio and the peaks are somewhat broader than in the previous cases, with \( L_m \) between 0.5\( L_o \) and \( L_o \). This model, although also simple, may resemble some parts of real clouds more closely than the previous model.

The inhomogeneous structure in real clouds is undoubtedly more complex than the above simple models. However, it must be similar and the Fishing test should be able to detect it with similar sensitivity. That is, alternative hypothesis that would not be rejected as inhomogeneous by the Fishing test would have to involve either inhomogeneities at length scales smaller than the resolution of the clock, a structure physically unrealizable in a cloud, or only weak contrasts in droplet concentration. The longer the length scale of inhomogeneity, the weaker the contrast must be in order to be undetected. Thus, when \( F(L) \) is less than 3, the
Fig. 5. (a) Fishing test \( F(L) \) for synthetic data for which homogeneous droplet concentration is modeled. (b)–(f) \( F(L) \) for synthetic inhomogeneous droplet concentration data, after Fig. 4, with varying \( L_0 \). (g) Same as (f), except part 3 of the error is compensated for. (h) Same as (f), except no error is simulated.
Fig. 6. Fishing test $F(L)$ for synthetic data sequences in which inhomogeneous droplet concentration is modeled (see Fig. 4). $L_0 = 3.2$ cm. (a)–(d) The concentration difference between clumps is varied for each plot, while the average concentration is held constant. (d)–(g) The length of the region (NL) is varied. (g)–(h) The average concentration is varied.
region must be nearly homogeneous, especially at the longer length scales. Therefore, the test will be used to make the qualified conclusion that a region is homogeneous within the sensitivity of the test. The qualifier will be occasionally omitted in the following text.

d. Summary of the Fishing test

The Fishing statistic $F$ tests whether the data from which it is calculated (droplet concentrations at variable resolution $L$) are Poisson distributed, which is equivalent to the droplet concentration being spatially homogeneous. If $F$ is greater than 3, the data are not Poisson distributed and the cloud sample is not homogeneous. Since $F$ is more sensitive to large-scale inhomogeneities than to small, it may be useful to think of $F(L)$ as a power spectrum operated upon by a transform that amplifies small wavenumbers compared to large wavenumbers. A well-defined maximum (with $F_m$ greater than 3) implies that there are inhomogeneities at approximately the scale of the maximum ($L_m$) and that inhomogeneities at larger scales must be weaker (have less contrast in droplet concentration).

No conclusions can be drawn about inhomogeneities at smaller scales. Where $F(L)$ is less than 3, homogeneity cannot be concluded. However, it is homogeneous within the sensitivity of the test, which experimental runs with modeled data demonstrate is very sensitive.

5. Observations (field data)

Data were collected using the University of Washington’s Convair C-131A cloud physics research aircraft. Prior to analyzing the new data with the Fishing test, clouds were selected using flight videos, voice notes, and data on liquid water content. Generally, young isolated cumulus cloud turrets were selected; however, they varied from very hard looking, visibly rising cloud turrets with bases intact to turrets that ap-
peared to be descending with dissipating bases. Penetrations through these clouds varied from just nipping the very top or edge to penetrations through the center at several levels down to cloud base. Over 50 cloud penetrations from four different flights over eastern Washington have been analyzed.

Figures 8a–d are typical time series at 50 Hz of droplet concentration (which is equivalent to about 1.6-m resolution measurements) of regions inside clouds specifically chosen to demonstrate the range of possible outcomes of the Fishing test, which are also shown. The spatial variations in concentration responsible for the large F values in Figs. 8a and 8b are visible to the eye. However, the spatial variations in concentration responsible for the large F values in Fig. 8c are not detectable in the figure or in a concentration time series at any frequency. The natural (Poisson process) variation hides the inhomogeneous variations when they occur at such small scales. In the case of Fig. 8d, the sensitivity of the Fishing test was increased by compensating for part 3 of the error. Still, no inhomogeneities were exposed. Thus, the region is homogeneous within the sensitivity of the test. The synthetic data demonstrate that only weak inhomogeneities could go undetected.

Remarkably, regions exist for which the Fishing test indicates homogeneity at all except small scales. Figure 8c is one of many such cases. In such cases, \( L_m \) is between 0.5 and 5 cm. Most of the cloud penetrations studied could be divided into regions 10–100 m in length, within each of which the Fishing test indicates homogeneity either at all scales or at all scales greater than 10 cm. Figure 9 is a typical case. This penetration was through the center of the upper part of a turret but is typical of any level. Figure 9a is a time series of droplet concentration through the entire penetration. The Fishing test applied to the entire penetration or to any two adjacent regions, demarcated by the vertical dashed lines, yields a maximum (with \( F_m \) greater than 3) at a length scale about as long as the sample region itself. The regions marked b–g in Fig. 9a were picked out (by eye) and tested separately. Figures 9b–g are the results of the Fishing test applied to each region. Each of these regions is similar to the region in Fig. 8c in that the Fishing test indicates inhomogeneity at small scales (0.5 to 5 cm) and homogeneity at all larger scales. Regions testing homogeneous at all scales were found, together in the same penetration, with regions like those in Fig. 9 in only 6 penetrations out of 50.

If turbulent motions are actively mixing together parcels with different droplet concentrations, then the variations should exist at many scales. This situation was commonly observed in small parts (often near the edges) of many clouds and throughout a few clouds. However, the clouds overall consisted of more regions that were homogeneous than inhomogeneous (at least at scales greater than 10 cm). Figure 10 is a droplet concentration time series of one of the more extreme cases of an actively mixing cloud. This penetration was near cloud top but similar regions were found at all levels. Throughout most of the penetration, the Fishing test applied to any region that is long enough for statistical significance indicates inhomogeneity at the scale of the region itself. This is consistent with what the eye sees: that there are inhomogeneities at many scales from large to small. The low-concentration regions well inside the cloud are also noteworthy. These regions have reduced droplet concentration but are not stretches of clear air. In a few clouds, one or several small clear air gaps were observed well inside the cloud, but these are rarer than low-concentration regions.

Figure 10b shows a particularly interesting region from Fig. 10a. The Fishing test applied to the higher concentration regions (a, c, e, and g) lumped together indicates the regions have nearly the same droplet concentrations (Fig. 11a). If the concentrations were different, then the Fishing test would indicate inhomogeneity at a scale about as long as the individual regions. The same is true for the low-concentration regions (Fig. 11b). The region shown in Figure 10b therefore looks like the early stage of mixing between two parcels with different droplet concentrations. Figures 12a–b, to be discussed later, are the droplet spectra for each of the combined regions.

6. Discussion

Two aspects of the observations will be discussed: first some speculative explanations for the small-scale inhomogeneity phenomenon and then some implications for large-scale entrainment and mixing that are independent of the explanation of the small-scale inhomogeneities.

a. Small-scale structure

The Fishing test commonly detects inhomogeneities at scales between 0.5 and 5 cm in regions that, within the sensitivity of the test, are homogeneous at all larger scales (e.g., Fig. 9). This is a most unexpected observation and thus warrants some discussion. However, since a definitive explanation has not been found, this discussion must be labeled speculative. The typical dimension of the small-scale inhomogeneities (1 cm) is similar to many of the probe dimensions (e.g., the inlet tube diameter is about 4.5 cm) and is also close to typical mean distances between detected droplets. Therefore, a first suspicion must be that the inhomogeneities are spurious. Careful consideration of the FSSP operation has failed to reveal any cause for this effect. Also, circumstantial evidence that the inhomogeneities are real is provided by the fact that they are not always found and that when they are found, the FSSP appears to be functioning well in all respects. While there is no proof that the inhom-
Fig. 8. Time series, 50-Hz, of droplet concentration (from real data) and Fishing test \( F(L) \) graphs for each region.
Fig. 9. (a) Time series, 50-Hz, of droplet concentration; (b)–(g) $F(L)$ for the corresponding regions in (a).

geneities are real, their reality must be considered and it is assumed in the following discussion.

Since the sensitivity of the Fishing test increases with the length scale of inhomogeneity, the test is indicating the larger end, that is, upper bound, of a range of scales at which inhomogeneities persist after larger-scale structure has been removed. Since the length scale indicated by the Fishing test consistently lies between 0.5
and 5 cm it must be an important length scale of the flow. There are only a few length scales that could be relevant.

1) The Kolmogorov microscale \( \eta \), which corresponds to the smallest-scale eddies, is the length scale at which viscous forces equal inertial forces. At a range of scales around the Kolmogorov microscale, most of the flow’s turbulent kinetic energy is dissipated into molecular energy. Also, it is at the Kolmogorov microscale that the eddies’ Reynolds numbers become of order unity, and thus it is unlikely that eddies at smaller scales will form. It is reasonable that \( \eta \) could be 0.5–5 cm for the clouds in which the observations were made. For stationary turbulent flows where buoyancy effects are negligible, the following proportionality may be derived by dimensional reasoning (see Tennekes and Lumley 1972):

\[
\eta \sim \frac{\nu^{3/4} l^{1/4}}{u^{3/4}},
\]

where \( \nu \) is the fluid viscosity and \( u \) and \( l \) are velocity and length scales at which energy is fed into the turbulent flow. Assuming the constant of proportionality is 1 and choosing a typical cloud-scale velocity and width as 5 m s\(^{-1}\) and 1000 m, one gets \( \eta \) equal to about 0.5 mm. However, such an estimate is crude since the constant of proportionality in (6) could differ considerably from 1. Evidence from many different turbulent flows suggests it is about 10 (Chapman 1979). Furthermore, cloud flow is not stationary and buoyancy effects are not negligible. The spatial variation in buoyancy, which must occur during mixing due to latent heat effects, may increase the turbulent kinetic energy and thereby decrease the size of the smallest eddies. The nonstationarity of the production of turbulent kinetic energy from the mean flow has the opposite effect on the size of the smallest eddies. Since the smallest eddies are the last to form and the first to decay, they will be the most affected by the nonstationarity of the flow.

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**Fig. 10.** Droplet-concentration time series, 50-Hz, of an actively mixing cloud.

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**Fig. 11.** (a) Fishing test \( F(L) \) for the combined regions a, c, e, and g of Fig. 10b; (b) \( F(L) \) for the combined regions b, d, and f of Fig. 10b.
2) The Batchelor microscale ($\eta_q$) is the smallest scale at which inhomogeneities of a quantity ($q$) can exist. Molecular diffusion of that quantity rapidly removes any smaller-scale inhomogeneities. The Batchelor microscale varies with the diffusivity of the quantity. The Batchelor microscale for momentum is the Kolmogorov microscale. The Batchelor microscale for any quantity $q$ with diffusivity $D_q$ may be written

$$\eta_q \sim \left( \frac{D_q}{\nu} \right)^{1/2} \eta. \tag{7}$$

This may be derived as follows: where two fluids initially meet, the interface is infinitely sharp. Diffusion immediately creates a diffusive interface that grows with time. However, it cannot grow indefinitely because the interface is stretched and thinned by turbulent eddies. Thus, the thickness of the diffusive layer (Th) depends on the local strain rate ($s$) as well as the diffusivity ($D_q$):

$$Th \sim \left( \frac{D_q}{s} \right)^{1/2}. \tag{8}$$

The highest strain rate is due to the smallest eddies. The strain rates of Kolmogorov microscale eddies are of order $u^{1/2} \nu^{-1/2} t^{-1/2}$ (see Tennekes and Lumley 1972) from which (7) follows. If the Schmidt number ($\nu/D_q$) is large, then the Batchelor microscale is smaller than the Kolmogorov microscale. The actual mechanism for homogenizing the droplet concentration at the smallest scales (droplet "diffusion") is not classical Brownian diffusion, which is too slow for droplets. Rather it is a randomization due to the different velocities and rates of acceleration of different sized droplets under gravitational, centrifugal, and fluid drag forces. Thus, the droplets' diffusivity depends on the very same small-scale turbulent characteristics that are being investigated.

3) The Taylor microscale corresponds to a diffusive interface that is being strained only by large-scale eddies. The largest eddies have the smallest strain rates, an estimate of which is $s \sim u/l$. Using (8) the Taylor microscale ($\lambda$) can be written

$$\lambda \sim \left( \frac{D_q}{u} \right)^{1/2}. \tag{9}$$

We have two Taylor layers: one for momentum $\lambda_\nu \sim \nu^{1/2} l^{1/2} u^{-1/2}$ and, if the droplets' Schmidt number is large, another smaller one for droplets. A crude estimate of $\lambda_\nu$ in the clouds studied is about 6 cm.

Because the Batchelor scale is by definition the smallest scale at which inhomogeneities in droplet concentration can exist, it cannot be the upper bound detected by the Fishing test. If the droplets' Schmidt number is of order one, then the Kolmogorov microscale is approximately equal to the droplets' Batchelor scale and also cannot be the upper bound. This leaves only the Taylor microscale. The Taylor microscale is the upper-bound length at which diffusion can occur, but it is difficult to understand physically why inhomogeneities at this scale would persist even after the diffusion layer has been sufficiently convoluted by all sizes of eddies such that no larger clumps remain.

If the Schmidt number is large then another possibility exists. In this case the Batchelor microscale is smaller than the Kolmogorov microscale and the latter could be the upper bound detected by the Fishing test. In this scenario, the eddies are very effective at mixing from the large scale to the Kolmogorov microscale because they exist at all of those scales. But mixing from the Kolmogorov microscale to the Batchelor microscale may be slower, thus permitting inhomogeneities to persist on that range of scales for some time after larger-scale inhomogeneities are gone. This scenario seems
the most likely because there is a physical reason for the mixing efficiency to decrease at scales smaller than the smallest eddies.

b. Large-scale picture

Independent of the explanation for the small-scale inhomogeneities sometimes observed, the other observations can be interpreted to make some deductions about large-scale entrainment and mixing.

Because low-concentration regions and occasionally clear air gaps are found well inside a dense cloud, cloud-scale motions must be present and important to the entrainment and mixing process. Note that the region blown up in Fig. 10b is separated from the edge by a higher concentration region. Such observations can only be explained by the effects of large, cloud-scale eddies. A gradient diffusion model of mixing is clearly inadequate.

The fact that regions diluted with environmental air are found well inside clouds but gaps of clear air are rare indicates that the clear air is initially mixed with cloud air before the large-scale motions transport it inwards. Logically this would begin at the cloud edges in regions smaller in scale than the large eddy size, therefore on faster time scales. Since clear air has been occasionally observed inside the clouds, the time scale for initial mixing at the cloud edges, although shorter than that for cloud-scale transport, cannot be very much shorter.

The observations of regions 10–100 m in length that are, within the sensitivity of the Fishing test, homogeneous (at least at scales greater than 10 cm) indicate that regions in that size range have time to homogenize before cloud-scale eddies intermix them or affect them with new entrainment. The time scale for homogenization of such regions cannot be very much shorter than that for cloud-scale eddies or else larger homogeneous regions would be observed.

The observation that the clouds consist of more homogeneous regions than inhomogeneous regions suggests that the large-scale eddy entrainment may be better described as intermittent events than as a continuous process.

The droplet spectra of Fig. 12 have similar shapes and the same mode diameters; only the concentration differs. This phenomenon has been observed and discussed before (Paluch and Knight 1984). If entrainment has caused evaporation of droplets in the low concentration regions, then the spectra are consistent with a mixing process in which some droplets experience total evaporation while others are unaffected. This scenario has been named "inhomogeneous mixing" (Baker and Latham 1979) and occurs when the evaporation time of droplets ($\tau_e$) is considerably smaller than the mixing time scale. The insensitivity of the droplet spectrum modal diameter to dilution, in this case, could be partly due to inhomogeneous mixing and partly due to the insensitivity of the FSSP sizing measurements (Paluch and Baumgardner 1989, section 7). However the modal diameter insensitivity to dilution could also result from the entrained clear air having been already saturated with water vapor.

The time scales in this physical situation are such that no easy visualization can be made. That is, all the relevant time scales are similar in magnitude. A time scale for the largest-scale motions is the characteristic cloud turret size divided by its characteristic velocity, which for the present study is 100–1000 meters divided by several meters per second. Thus, the time scale is on the order of one to several minutes. It has been argued above that mixing between clear and cloud air occurs initially at the cloud edges on time scales shorter but not very much shorter than those for large-scale transport. Likewise, it is argued that the time scale for homogenization of regions 10–100 m in size is shorter but not very much shorter than that for cloud-scale eddies. Finally, cloud droplet evaporation times are also on the order of 1 min. Evidently all the physical processes—initial mixing of clear and cloud air, transport of the mixing regions from the edges to the interior, homogenization of interior regions, and evaporation (with associated latent heat effects) of cloud droplets—occur simultaneously. Figure 13 is an attempt to illustrate the above ideas.

![Fig. 13. Illustration of entrainment and mixing in small cumulus clouds. Key characteristics: initial entrainment and mixing near edges, simultaneous but discrete large-scale entrainment events due to cloud-scale eddies, subsequent homogenization of regions 10–100 m in length.](image-url)
Note that although this picture seems consistent with the clouds observed, it was assumed that all clouds had a similar entrainment and mixing process. That is, it was assumed that the differences between cloud penetrations were simply the differences between snapshots of the same process at different stages and not due to different processes in different clouds. Since the environment through which a cloud rises and from which it entrains varies from day to day and cloud to cloud, so might the entrainment and mixing characteristics. Many more observations must be made along with environmental soundings in order to describe general patterns applicable to most clouds.

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REFERENCES


Chapman, D. R., 1979: Computational aerodynamics development and outlook. AIAA, 17, 1293–1313 (see Fig. 13).


