ON THE COMPUTATION OF WIND FROM PRESSURE DATA


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ABSTRACT

From a statistical analysis of geostrophic- and gradient-wind computations for all stations reporting winds on two 700-mb charts, it is shown that there is a variability of about 25 per cent in the computations by different individuals, and that the computed values differed by about 35 per cent from the observed wind speed on the average. The percentual deviation was smaller for strong winds. The gradient wind computed using approximate trajectory curvature was only slightly better than the geostrophic, and using contour curvature it was worse. The geostrophic wind thus appears to be the best approximation which can be computed from the pressure field alone.

Theoretical expressions for the deviations of the computed from the observed speeds and velocities are derived. These show that the gradient speed is always greater than the true speed. The gradient speed is shown to be a better approximation than the geostrophic for most cases, but for some cases of curved cross-isobaric flow the geostrophic speed is closer to the true speed. The vector deviation of the gradient wind is, of course, larger than the scalar for cross-isobaric flow.

1. Introduction

During the academic year 1946–1947 the members of the graduate meteorological laboratory course at U.C.L.A. carried out several investigations of the relations between the fields of pressure and wind. In this report some of the more important findings are presented.

For the forecasting of winds in the free air the customary procedure is first to prepare a forecast of the pressure field, and then, using certain generally accepted relationships, to derive the wind from the forecast distribution of pressure. The extent to which computations using these relationships—the geostrophic and gradient-wind formulas—give the true wind speed from the pressure distribution, was one of the subjects studied. As will be shown in the following, the verification is rather poor, and is worse when direction is considered as well as speed. Regarding the geostrophic wind, similar results have been found by Houghton and Austin [1].

According to the usual definition, the geostrophic wind is directed along the isobars on a constant-level chart or along the contour lines on a constant-pressure chart with magnitude \( v_{gr} \) given by

\[
v_{gr} = \frac{b}{f} = \frac{\alpha \frac{\partial p}{\partial z}}{f \frac{\partial n}{\partial z}} = \frac{g \frac{\partial z}{\partial n}}{f \frac{\partial n}{\partial z}},
\]

where \( b \) is the horizontal pressure-gradient force, \( \alpha \) is the specific volume, \( f = 2\Omega \) sin \( \phi \) is the Coriolis parameter, \( \Omega \) angular velocity of the earth, \( p \) pressure, \( z \) height, \( \phi \) latitude and \( n \) is measured normal to the isobars on a constant-level chart or normal to the contour lines on a constant-pressure chart. The gradient wind is directed along the isobars or contours with magnitude \( v_{gr} \) given by the solution of the equation

\[
v_{gr}^2 / f \tau + v_{gr} = v_{gr},
\]

where \( r \) is the radius of horizontal curvature of the trajectory of the air parcel in question. We can compute \( r \) from the formula

\[
\frac{1}{r} = \frac{1}{r_s} + \frac{1}{v \frac{\partial \psi}{\partial t}},
\]

where \( \psi \) is the azimuth of the wind measured counterclockwise, and \( r_s \) the radius of curvature of the streamline. The local rate of turning of the wind, \( \partial \psi / \partial t \), is computed from successive wind reports. This term will be discussed further below.

In practice, data are not available for the evaluation of \( \partial \psi / \partial t \), and the gradient wind is computed using \( r_s \) in place of \( r \), with the further assumption that the streamlines coincide with the isobars or contours. We shall designate by \( v'_{gr} \) the gradient wind computed using contour-line curvature instead of \( r_s \), to distinguish it from \( v_{gr} \) as first defined.

2. The computations

Nine teams of experienced meteorologists each computed \( v_{gr}, v_{gr}, v'_{gr} \) for every station reporting observed winds on each of two 700-mb maps (those for 0300Z on 10 and 11 October 1946), approximately 135 cases in all.2 The teams consisted of pairs of men with an

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1 U.C.L.A., Department of Meteorology, Papers in Meteorology, No. 6; revision of an earlier paper presented 18 June 1947 at the San Diego Meeting of the American Meteorological Society.

2 In computing \( v_{gr} \) it was assumed that the curvature of the streamlines could be approximated by the curvature of the contours; equation (3) was then used to determine \( r_s \).
teams performed their computations, including the analysis of their maps, independently of each other.

The maps used were chosen because there was a large number of pilot-balloons observations reaching the 700-mb isobaric surface. The consequence is that the situation was a relatively "flat" one, with no strong winds. Other than that, it was a random selection. The analysis was carried out with emphasis on fitting the data into a three-dimensional picture, with due consideration of the surface and 850-mb charts. During the analysis the men did not know that the maps would be used for checking the wind relationships, and made the usual allowance for the winds in locating the contours.

It was considered that by having several teams carry out the complete analysis and computations, errors of individual analysis, measurement, and computation would be averaged out. Furthermore, the variation among the teams would give a measure of the reliability of an individual forecaster’s computation.

The two maps used were designated map 0 and map 1 for convenience in entering the data on punch cards. For map 0 each team computed the wind speeds from its own measurements, and averages and standard deviations of the nine values of each computed wind at each station were obtained. The standard deviations for all computed winds were averaged, and the results, expressed in knots and in per cent of the average wind, are given in table 1. For map 1 averages and variances of the measurements of contour spacing and contour curvature at each station were computed. These averages were used to compute the "average" geostrophic and gradient winds. The standard deviations of these winds were computed by differentiating the wind equations and substituting the variances for the differentials in the resulting expressions. They, too, are presented in table 1. The average of all observed wind speeds was 16.4 knots for map 0, and 19.7 knots for map 1.

On both maps the computations were carried out on a nomogram, using successive approximations in the case of the gradient wind. In the method used for map 1, errors which arose from reading values off the nomogram were not averaged out.

It is seen from table 1 that to the extent that the computations approximated a normal distribution, only two-thirds of the computations by the various teams came within about 25 per cent of the average, and the chances are that in one case out of twenty the individual computation deviated as much as 50 per cent from the average. Separation by regions showed that the standard deviations were larger where the analysis was more difficult; this was not the case, however, in coastal regions where uncertainties might be expected because of lack of data. This variability among individual analysts places a practical limitation on the reliability of wind computations, independent of any theoretical considerations.

3. The deviations of computed from observed winds

Table 2 shows the mean deviations (average of differences regardless of sign) of computed from observed wind speeds. On the average, for each map these differences were about one-third of the observed wind speed for rawins and pibals separately, and for each of the three computed winds; if the observed speeds are correct, the computations give rather poor approximations of them.

<table>
<thead>
<tr>
<th></th>
<th>( v_{\text{exp}} )</th>
<th>( v_{\text{gr}} )</th>
<th>( v_{\text{kr}} )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>knots per cent</td>
<td>knots per cent</td>
<td>knots per cent</td>
</tr>
<tr>
<td>Map 0</td>
<td>5.9 36</td>
<td>6.1 37</td>
<td>5.1 31</td>
</tr>
<tr>
<td>Map 1</td>
<td>6.6 34</td>
<td>6.5 33</td>
<td>6.6 33</td>
</tr>
<tr>
<td>All pibals</td>
<td>6.2 35</td>
<td>6.4 35</td>
<td>5.5 31</td>
</tr>
<tr>
<td>All rawins</td>
<td>6.5 35</td>
<td>6.2 32</td>
<td>6.8 38</td>
</tr>
<tr>
<td>All obs.</td>
<td>6.3 35</td>
<td>6.3 34</td>
<td>5.8 32</td>
</tr>
</tbody>
</table>

The accuracy of the observations can only be estimated. Richardson, Proctor and Smith [2] give 1.0 to 1.4 m s\(^{-1}\) for the standard deviation of single-theodolite pibal observations, from double-theodolite observations. If the average wind in their measurements was the same as in the cases studied here, this would be 10 to 15 per cent of the average speed. Studies of the accuracy of rawin observations conducted by the Army Air Forces\(^4\) at Orlando, Florida indicate that the errors of the wind measurements with the radio direction-finding equipment in use, model SCR 658, are about the same percentage. Since errors in observation appear to account for at most 15 per cent of the deviations, it seems clear that at least an equal amount is due to errors in the computations, either in the degree of approximation of the equations, or in their applications.

To compare the relative validity of the three computations \((v_{\text{exp}}, v_{\text{gr}}, v_{\text{kr}})\) it is necessary to limit the mean deviations to cases in which all three can be carried out. Table 3 gives these values.

\(^4\) Radio-wind and pilot-balloons observations.

Table 3. Mean deviations of computed from observed wind speeds (cases for which both $v'_{\text{gr}}$ and $v_{\text{gr}}$ could be computed).

<table>
<thead>
<tr>
<th>Map 0</th>
<th>$v_{\text{gr}}$ (knots)</th>
<th>$v'_{\text{gr}}$ (knots)</th>
<th>Number of cases</th>
</tr>
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<tbody>
<tr>
<td>6.0</td>
<td>6.5</td>
<td>5.1</td>
<td>31</td>
</tr>
<tr>
<td>6.8</td>
<td>7.0</td>
<td>6.6</td>
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<td>6.1</td>
<td>6.6</td>
<td>5.5</td>
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</tr>
<tr>
<td>7.8</td>
<td>7.4</td>
<td>7.1</td>
<td>39</td>
</tr>
<tr>
<td>6.4</td>
<td>6.7</td>
<td>5.8</td>
<td>32</td>
</tr>
<tr>
<td>6.0</td>
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<td>6.7</td>
<td>5.8</td>
<td>32</td>
</tr>
</tbody>
</table>

While the deviation of $v_{\text{gr}}$ appears to be somewhat smaller than the others, this difference cannot be regarded as significant in view of the deviations among individual analysts. The fact that $v_{\text{gr}}$ seems a slightly better approximation than $v'_{\text{gr}}$ is in accord with the idea stated by various authors, that if $1/r_{s}$ is not corrected to $1/r$, it is better to use the geostrophic rather than the gradient-wind equation. Even if the slightly greater accuracy resulting were significant, it could not be taken advantage of in practice, since computing $v_{\text{gr}}$ involves a knowledge of the turning of the wind; hence, the geostrophic-wind equation appears to be the best approximation possible.

As will be shown later, the gradient-wind equation (for $v_{\text{gr}}$), is theoretically a closer approximation to the true wind speed than the geostrophic, except in cases of anticyclonic curvature with significant cross-isobaric flow. It may seem surprising, therefore, that it does not yield substantially better results than the geostrophic-wind equation. The reason for this appears to lie in inaccuracies in evaluating the rate of turning of the wind, $\phi/\delta t$, which is taken into account when computing the curvature of the trajectories from the curvature of the isobaric contours, and in the use of the latter curvature in place of the true streamline curvature.

The rate of turning of the contours at a fixed point might be expected to be a fair approximation of $\phi/\delta t$. However, the comparison of these two rates of turning for about 50 cases showed that this approximation is not a good one. The turning of the trajectories was obtained from the change of observed winds in the period from six hours before map time to six hours after map time, and the turning of the isobars was estimated from the change in the period from twelve hours before map time to twelve hours after map time for the same stations. The average difference was about 50 per cent of the average magnitude of $\phi/\delta t$. A similarly large difference was found when the two estimates were computed for the same time interval.

The observed turning of the winds was used throughout this study, since it was felt that it was the most reliable possible estimate of $\phi/\delta t$. However, even this value was subject to considerable uncertainty, because of the necessity of using 12-hour changes as representing instantaneous time derivatives. This uncertainty is probably largely responsible for the fact that $v_{\text{gr}}$ is so little better an estimate of the wind than $v_{\text{gr}}$.

The importance of the term $r^{-1}\phi/\delta t$ is emphasized by the fact that in 31 out of 42 cases in which it was of opposite sign to $1/r_{s}$, it was larger in magnitude and thus determined the sign of the trajectory curvature. From this consideration alone it is clear that $v'_{\text{gr}}$ cannot be as good an approximation as $v_{\text{gr}}$. Since $\phi/\delta t$ cannot be computed without a knowledge of the winds, it follows that the gradient-wind equation cannot be used in practice to get a better evaluation of the wind from the pressure field than is given by the geostrophic-wind equation.

4. Variation of deviations with speed, curvature, and latitude

The variation of the deviations of the computed winds with observed speed is shown in fig. 1. For actual winds below 10 knots all three approximations are readily seen to be poor. For greater speeds there appears to be no systematic variation, so that the percentage deviation decreases with speed. We may also note from this figure that $v_{\text{gr}}$, $v_{\text{gr}}$, and $v'_{\text{gr}}$ are good as approximations in that order, not only for the average of all observations but consistently throughout the moderate speed range.

The question naturally arises as to the possibility of obtaining greater accuracy in the calculation of the winds by choosing points where the curvature of the contours is small. To test this the deviations were tabulated as a function of curvature. No significant correlation was found in the absolute values of the deviations. However, it was found that while all three computed winds were predominantly greater than the observed in cases with anticyclonic or zero contour curvature, they were more nearly evenly distributed.
between positive and negative for cyclonic curvature of the contours.

To see whether the computations in cases with large turning of wind showed larger deviations from the observed wind than those with little turning, the results were plotted against $\partial \psi / \partial t$. No significant correlation was indicated between any of the computed deviations and $\partial \psi / \partial t$. This suggests that errors in determination of $\partial \psi / \partial t$ and in the computations obscured the expected variation.

Another question which can be examined is the validity of the generally accepted restriction of application of the geostrophic-wind formula to higher latitudes. It would seem that if the formula fails completely to give an approximation of the wind at very low latitudes, it should give increasingly large deviations as one proceeds from high latitudes to moderately low ones. To test this the data were grouped and averaged by latitudes. Fig. 2 presents the average deviation as a function of latitude for all the data on the two maps. While the group centered about 24°N had a somewhat larger average deviation than those immediately to the north, the one centered on 47.5°N had a larger deviation. The vertical lines drawn through the points show the 95 per cent confidence interval for these averages; that is, the probability is only 5 per cent that the true mean is outside the interval centered about the average of the cases used. It is seen that a line intersecting all these lines could be drawn such that no variation with latitude would be indicated. It appears that down to about 20°N the observed deviation of the geostrophic wind is due to factors other than the decrease in latitude.

5. Theoretical expression for deviations of computed speed

It has been mentioned that the computed wind speeds exceeded the observed in most cases. This was true for the gradient-wind computations as well as the geostrophic. This is consistent with the theoretical result, demonstrated below, that the gradient wind is always equal to or greater than the real wind, and that the geostrophic speed is greater than the real speed for all cyclonic curvatures of the real trajectory, and for small anticyclonic curvatures in case of cross-isobaric flow. Petterssen [3] showed this theoretical result for small cross-isobaric angles. It may be seen generally for curved flow from the following considerations.

When the equations of horizontal motion are expressed in natural coordinates [4] the normal component is

$$ v^2 / r + f \nu = b_n, \tag{4} $$

where $b_n$ is the component of the pressure-gradient force normal to the direction of motion. If the angle between the velocity and the isobaric contours is $i$,

$$ b_n = b \cos i, \tag{5} $$

where $b$ is the total pressure-gradient force, acting normal to the contours. From equation (1), $b = f v_{gs}$, so that equation (4) may be written

$$ v(1 + \nu / fr) = v_{gs} \cos i. \tag{6} $$

It is very convenient to introduce the nondimensional curvature parameter $x = \nu / fr$, which is one-half the ratio of the angular velocity of the parcel to the angular velocity of the earth's rotation about the vertical. With this substitution,

$$ v = (v_{gs} \cos i)/(1 + x). \tag{7} $$

With this notation we may rewrite equation (2) for the gradient wind

$$ v_{gr} = v_{gs} / (1 + xy), \tag{8} $$

where $y = v_{gs} / v$. Dividing equation (8) by equation (7) and solving for $y$, we obtain

$$ y = \frac{1}{2x} \left[ \frac{1 + 4x(1 + x)}{\cos i} \right]^{1/2} - 1. \tag{9} $$

The positive square root is chosen to restrict ourselves to the normal baric case [4, pp. 191, 202] in which anticyclonic flow is around high pressure. The ratio $y$ is defined for all values of $x$ for which the gradient wind is defined. The limiting value $x_{gs}$ beyond which $v_{gs}$ and $y$ are not defined is obtained by setting the expression under the radical equal to zero. This minimum value is

$$ x_{gs} = -\frac{1}{4}(1 - \sqrt{1 - \cos i}) $$

for which the ratio of the gradient to the real wind is

$$ y_{gs} = (1 + \sqrt{1 - \cos i}) / \cos i, $$

which is greater than unity except when $i = 0$. By taking the derivative of equation (9) with respect to $x$,
it can be shown that \( y \) has no maximum or minimum but decreases asymptotically to \( (\cos i)^{-1} \) as \( x \) approaches \( +\infty \). This limit is also greater than unity for \( i \neq 0 \).

Thus for all values of \( x \) for which it is defined, the gradient wind speed is equal to or greater than the true wind speed.

The fractional error in the gradient wind is

\[
\frac{(v_{\theta r} - v)}{v} = y - 1.
\]

The variation of this quantity with curvature \( x \) is shown for various cross-isobaric angles \( i \) by solid lines in fig. 3. For a given nonzero cross-isobaric angle the actual wind speed is seen to be always less than the gradient wind. The departure is greatest for maximum anticyclonic curvature, and the departure decreases with increasing cyclonic curvature.

Superimposed on the curves for the variation of \( (v_{\theta r} - v)/v \) in fig. 3 are the analogous curves (in dashed lines) for the variation of \( (v_{\theta g} - v)/v \). From equation (7),

\[
\frac{v_{\theta g} - v}{v} = \frac{v_{\theta g}}{v} - 1 = \frac{1 + x}{\cos i} - 1.
\]

In the figure these curves have been reflected for negative errors to facilitate comparison of the absolute values of the errors for the geostrophic and gradient computations where they are of opposite sign. The following relationships should be noted in the figure:

(a) For certain combinations of cross-isobaric angle \( i \) and anticyclonic curvature \( (x < 0) \), \( (v_{\theta g} - v)/v = 0 \), i.e., the geostrophic-wind equation gives the true wind speed.

(b) For \(|i| > 25^\circ\) and \( x < 0 \), the geostrophic-wind equation gives a better approximation than the gradient-wind equation.

(c) For \(|i| < 25^\circ\) and \( x < 0 \), and sufficiently small numerically, the geostrophic wind gives a better approximation than the gradient wind.

![Fig. 3. Theoretical variation of deviation of geostrophic- and gradient-wind speeds as a function of curvature for various cross-isobaric angles.](image)

![Fig. 4. Theoretical variation of magnitude of vector deviation of gradient wind as a function of curvature.](image)

For most combinations with cyclic curvature and for large anticyclonic curvature with small cross-isobaric angle, the gradient wind is theoretically a better approximation to the wind speed. But as enumerated above, even theoretically the geostrophic is better in some cases.

It should be noted that the real wind does not have a critical curvature. However, if the curvature parameter \( x \) becomes more anticyclonic than the value \( x_a = -\frac{1}{2} \), the flow becomes abnormal. For this value of \( x \), \( v = 2v_{\theta g} \cos i \), and \( r = -4v_{\theta g}f^{-1} \cos i \). Thus any anticyclonic radius of curvature, however small numerically, can exist with normal flow, but as \( r \) approaches zero from the anticyclonic side, \( i \) must approach 90°. While the gradient wind speed can be computed only for sufficiently small anticyclonic curvature, there is no limit to the anticyclonic curvature of real trajectories.

6. The vector deviation of the gradient wind

The introduction of the angle of deviation of the wind from the isobars focuses attention on the vector nature of the wind. Obviously, unless the wind is always directed along the isobars, the average magnitude of the vector difference between the real and the gradient wind will be greater than the average difference in speeds, and larger the larger the angle. In fact, if we write \( Q = |v_{\theta r} - v|/v \), then it can be shown that \( Q = (y^2 - 2y \cos i + 1)^{1/2} \), where \( y = v_{\theta g}/v \) as before. It is readily seen that for \( i \neq 0 \), \( Q > y - 1 \) and the magnitude of the vector error is greater than the magnitude of the difference in speeds.

Fig. 4 shows the variation of \( Q \) with \( x \). Again for each value of \( i \) the error has a maximum magnitude at the critical anticyclonic curvature and decreases asymptotically to a minimum for infinite cyclonic curvature. For moderate anticyclonic and all cyclonic curvature the fractional error is seen to be practically constant for each value of \( i \) up to 30°. At \( x = 0 \),
\[ Q = \tan i, \text{ and this value would apply as the fractional error for } |i| < 30^\circ \text{ and all curvatures except strong anticyclonic ones.} \]

Measurements of the cross-isobaric angle \( i \) on three maps (other than those used in the earlier study) gave an average value (disregarding the sign) of about 20\(^\circ\). For this angle the vector error in the gradient wind would be about 36 per cent for most curvatures, and 51 per cent for the maximum anticyclonic curvature \( x_s \) for which the gradient wind is not imaginary. This value (36 per cent) is approximately that which was obtained for the average deviation of the speed. The vector deviation of the gradient wind obviously must be greater than this, but was not computed. However, the geostrophic vector deviation was evaluated for six maps at 300, 500 and 700 mb on two days, and found to be 59 per cent. Since the geostrophic-speed deviation was not materially different from the gradient, and the geostrophic and gradient-wind directions are by definition the same, it seems plausible that the average vector deviation of the gradient wind would also be greater than 50 per cent.

7. Summary of conclusions

1. Individual variation in analysis and measurement leads to uncertainties of the order of 25 per cent in the computation of wind speed from the pressure distribution, but the percentual error decreases with increasing speed.

2. The computed wind speeds deviate by about 35 per cent from the observed wind speeds. Errors in observed speeds may account for about 1.5 per cent out of this 35 per cent.

3. The gradient wind speed computed using trajectory curvature is not significantly more accurate than the geostrophic wind speed. Using curvature of isobaric contours it is less accurate. Since trajectory curvature cannot be found without observed winds, the geostrophic wind speed is the best approximation possible in practice.

4. There is no significant variation with latitude in the accuracy of the geostrophic wind down to 20\(^\circ\)N.

5. Theoretical considerations that the gradient wind speed is always greater than the actual wind are corroborated by the observations.

6. The magnitude of the vector deviation of the computed gradient wind is about 50 per cent of the wind speed.

Acknowledgments.—It is with regret that acknowledgment cannot be made of the individual contributions by each member of the class who participated in the project. Special mention should be made of Maj. R. G. Suggs and Capt. D. H. Russell, who performed extensive computations during the second semester of operations.

REFERENCES


