The Generation of Ultralow-Frequency Variations in a Simple Global Model

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ABSTRACT

Fluctuations in the global angular momentum of a two-layer zonally homogeneous global model of the atmosphere are dominated by very low frequencies, periods longer than 100 days. This variability is confined to the tropics, where it is associated with variations of approximately ± 5 m s^{-1} in the zonal winds. Lagged correlations suggest that the variations in angular momentum are a response to variability in the zonally averaged eddy flux of vorticity in the tropics.

A zonally symmetric model is constructed to determine the response to an impulsive rearrangement of vorticity across the tropics. The response of the angular momentum in the zonally symmetric model, in convolution with the tropical eddy flux of vorticity derived from the full model, provides a successful model of the variations in angular momentum in the full model.

The relevance of the results to James and James' finding ultralow-frequency variability in a five-layer model and to the possibility of ultralow-frequency variability in the atmosphere is discussed.

1. Introduction

Baroclinically unstable waves typically have periods and doubling times of a few days. The strongest variability in the atmosphere, however, is at periods longer than 10 days (Blackmon 1976). Recently, it has been demonstrated that the nonlinear dynamics of a model atmosphere, in which available potential energy is released by baroclinic instability, can produce fluctuations with periods as long as months, years, or even decades (James and James 1989). These fluctuations are denoted ultralow-frequency variability.

In the present work, a model that previously was used to study the dynamics of different types of low-frequency variability (Robinson 1991a,b; Hendon and Hartmann 1985) is used to investigate the dynamics of variability on longer time scales. The model is a dry, global, primitive equation model with two levels and zonally homogeneous boundary conditions; that is, land–sea contrasts and orography are absent. The circulation is driven by a Newtonian relaxation of the temperature toward a zonally symmetric distribution that represents radiative equilibrium for December or January. Dissipation in the boundary layer is simulated by a linear drag, $\tau = 5$ days, on winds in the lower layer of the model. The model is spectral and is run with a rhomboidal 15 truncation.

In the earlier work with this model the horizontal diffusion was Fickian, but now a ∇^8 diffusive operator is used. The diffusion constant is such that the smallest horizontal scales in the model, represented by the coefficients of spherical harmonic Y_{30}^{15}, are dissipated at a rate of 4 day^{-1}. This change reduces the influence of diffusion on all but the smallest scales and requires a reduction in the time step from one to one-half hour.

The results presented below are based on the final 5900 days of a 6000-day integration. The time and zonally averaged zonal winds in the upper (250 mb) and lower (750 mb) layers of the model are shown in Fig. 1.

2. Diagnostic analysis of ultralow-frequency variability

Following James and James, we look for ultralow-frequency behavior in the largest scales of motion in the model atmosphere. Figure 2 shows the last 5000 days of temporal evolution of the Y_0^1 spherical harmonic coefficient of the barotropic component of the vorticity. This quantity, denoted M, represents the globally integrated angular momentum of the model atmosphere. The time series of M is dominated by fluctuations with periods greater than 100 days. This is confirmed by the spectrum, shown in Fig. 3. The spectrum was computed by dividing the final 5120 days of the integration, sampled twice daily, into 10 segments of 512 days (1024 points) each. Each segment was Hanning windowed, and the spectra were computed using a fast-Fourier transform. The spectrum shown is the average of the resulting 10 sample spectra.[1]

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[1] The computations were carried out using the "spectrum" routine of the MATLAB™ software package on a MacIntosh computer.

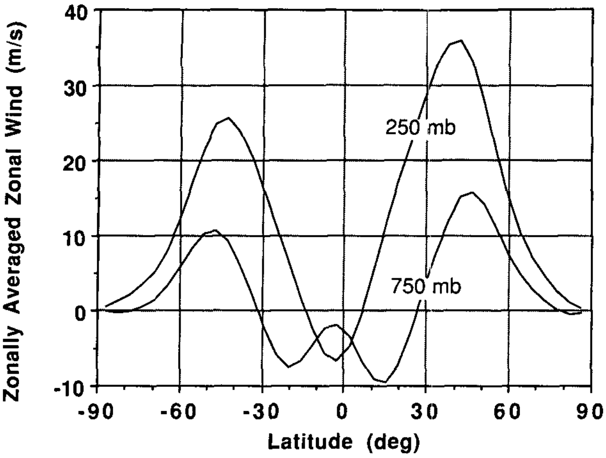

FIG. 1. Time and zonally averaged zonal winds in the upper (250-mb) and lower (750-mb) layer of the two-layer model.

Two questions arise immediately: what is the structure of the changes in the circulation associated with variations in M, and are these changes large enough to be significant? The first question is addressed by Fig. 4, which shows the simultaneous correlations of the zonally averaged zonal winds in both model layers with M. The correlations with upper-layer winds are positive within the tropics and small elsewhere. Near the equator, variations in M explain nearly half of the variance of the zonally averaged zonal wind. Correlations with lower-layer winds are weaker: they are negative near the equator and positive in the Northern Hemisphere tropics.

The strength of these variations is found by separately compositing the zonally averaged zonal winds over those half days on which M takes on values in the highest or lowest tenths of its distribution. High and low M composites differ by about 10 m s^{-1} near the equator, and they differ little from climatology poleward of 30° latitude.

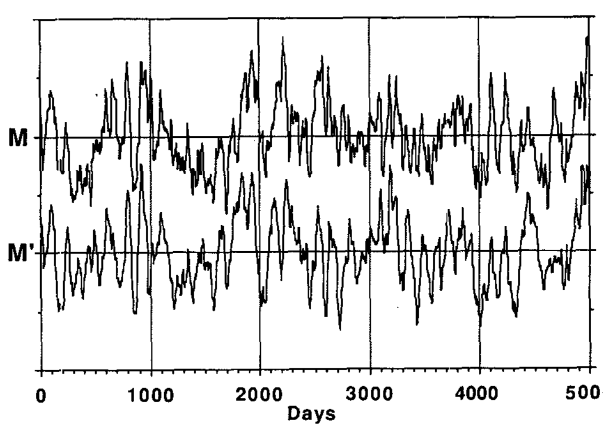

FIG. 2. Time series of M, the global angular momentum, and M', a model of M (see section 3). The means of both time series have been removed, and both series have been normalized to unit variance.

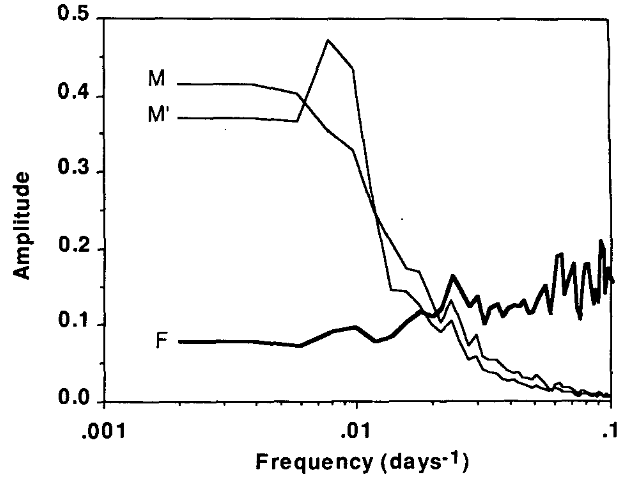

FIG. 3. Spectra of M, the global angular momentum, M', the modeled global angular momentum, and F, the tropical eddy flux of vorticity. The spectra are normalized, and the amplitudes of the spectral coefficients are plotted against the log of frequency.

The very simple nature of our model, together with the special properties of M (the global angular momentum), constrains how the circulation can change in association with fluctuations in M. In the model, M can change only through the action of the drag on the lower-level winds. Specifically,

$$\frac{dM}{dt} = -C_d M_1, \tag{1}$$

where M_1 is the angular momentum of the lower layer and $C_d = 0.2$ day^{-1} is the drag coefficient. Assuming that there are no secular variations in M, a consequence of (1) is that M_1 is uncorrelated with M.

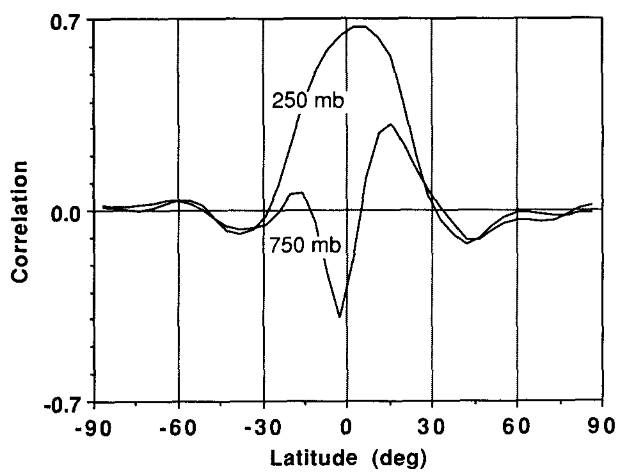

FIG. 4. Correlations of the zonally averaged zonal winds in the model with M.

Correlations of the upper-layer zonally averaged zonal winds with M as a function of latitude and temporal lag are shown in Fig. 5. Consistent with the spectrum of M, its signal in the zonal wind is persistent over tens of days. The signal is confined to the tropics, especially when the winds lead M. At positive lags, the positive correlations spread into the subtropics. The strongest correlations with M (>0.7) are immediately north of the equator, and they lead M by several days; that is, maxima in M are preceded by a weakening of the upper layer's tropical easterlies.

Some insight into the sequence of events that cause M to vary is obtained by correlating various quantities with the time derivative of M, $\dot{M}$. Simultaneous correlations of the zonally averaged zonal winds with $\dot{M}$ are shown in Fig. 6. Correlations with the lower-level winds are primarily negative, particularly in the Northern Hemisphere subtropics; that is, 750-mb easterlies are stronger when M is increasing, as would be expected from (1). In the upper level, increasing M is associated with weakened tropical easterlies and subtropical westerlies. The correlations between the 250-mb winds and $\dot{M}$ suggest that increasing M is associated with anomalously high vorticity in the Northern Hemisphere tropics and anomalously low vorticity in Southern Hemisphere tropics; an inference that is confirmed by the correlations of $\dot{M}$ with the zonally averaged relative vorticity (not shown). This then suggests that increases in M may be preceded by anomalously strong eddy transport of vorticity across the equator.

To check this possibility, the correlation between $\dot{M}$ and the zonally averaged eddy flux of vorticity $\overline{v'\zeta'}$ is calculated as a function of latitude and lag; the results are shown in Fig. 7. While the magnitudes of the correlations are small (this will be explained later), the pattern of correlations shows that increases in M are preceded for several days by stronger than average northward eddy fluxes of vorticity in the tropics, especially in the Northern Hemisphere, together with stronger than average southward vorticity fluxes in the Northern extratropics.

The zonally averaged eddy vorticity flux is the barotropic contribution to the divergence of the Eliassen–Palm flux. Chaotic variations in the generation of eddy activity by baroclinic instability in middle latitudes can produce variations in the flux of wave activity into the tropics and thus lead to variability in the vorticity fluxes within the tropics. There must be some intervening

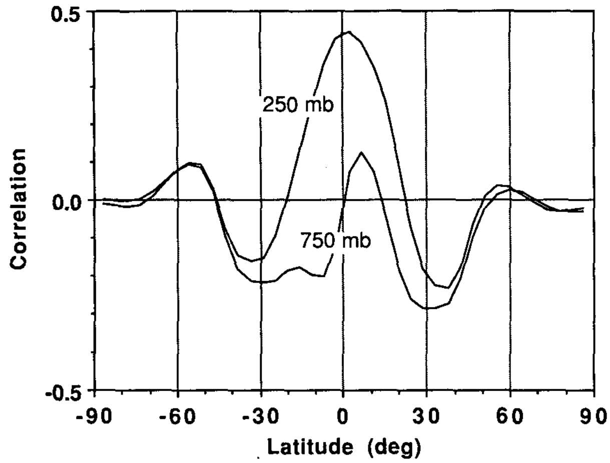

FIG. 6. Correlations between the zonally averaged zonal winds and the time derivative of M, $\dot{M}$. The $\dot{M}$ is estimated from the twice daily record of M.

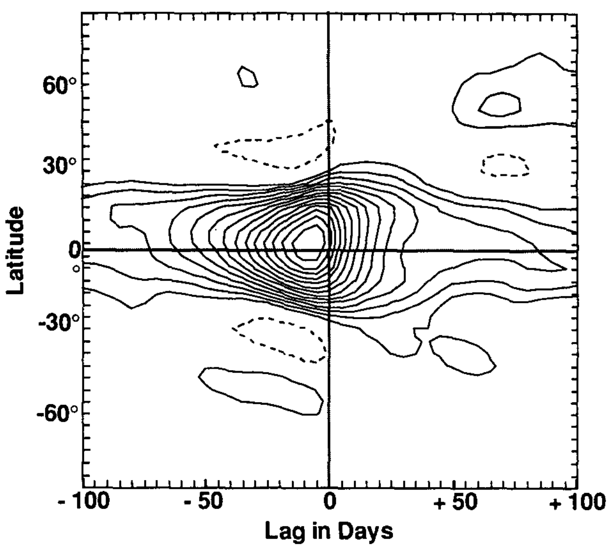

FIG. 5. Correlations, as a function of latitude and lag, between M and the zonally averaged zonal winds in the upper layer. Positive lag means that M leads the zonal wind. The contour interval is 0.05, negative contours are dashed, and the zero contour is absent.

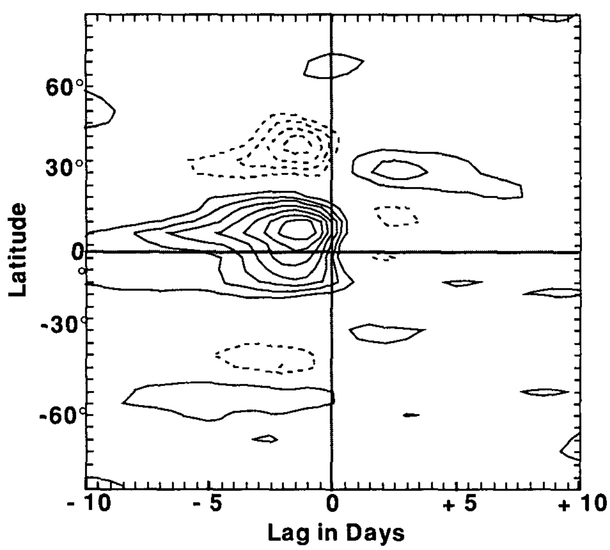

FIG. 7. Correlations, as a function of latitude and lag, between $\dot{M}$ and the zonally averaged northward eddy flux of vorticity in the upper layer $\overline{v'\zeta'}$. The contour interval is 0.025, negative contours are dashed, and the zero contour is absent.

dynamics, however, as horizontal eddy fluxes of vorticity cannot directly modify M.

The zonally averaged vorticity fluxes in the tropics vary on time scales ranging from a few days to hundreds of days. These fluxes are represented by a quantity denoted F, the integral of $\overline{v'\zeta'}$ at 250 mb across the tropics, from 20°S to 20°N. The spectrum of F, the heavy solid curve in Fig. 3, is very different from that of M, in that amplitudes increase slightly with increasing frequency between 0.01 and 0.1 day^{-1}. If the fluctuations in F are responsible for the variations in M, then the intervening dynamics must act as a very effective low-pass filter. The relatively high-frequency variability in F is then irrelevant to its forcing of M, but contributes to the variance in F and so reduces its correlation with M. That this is indeed the case will be demonstrated in the next section.

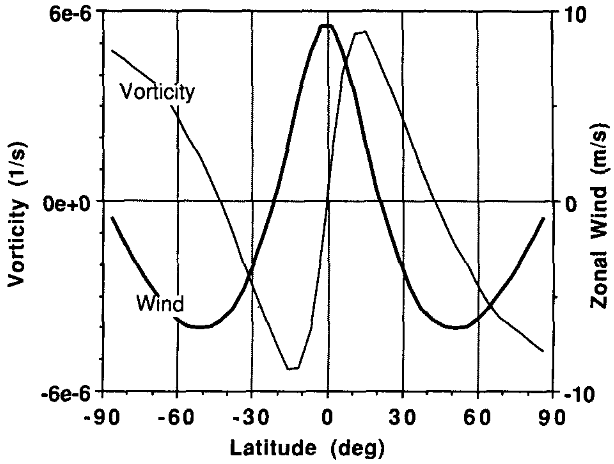

FIG. 8. Initial perturbations to the zonally averaged vorticity and zonal wind in the upper layer of the zonally symmetric model.

3. Spin down of a zonally symmetric model

In order to investigate the dynamics of the response of M to redistribution of tropical vorticity by eddies, we consider how a zonally symmetric version of the two-layer model responds to an impulsive redistribution of its upper-layer vorticity. The zonally symmetric model is identical to the full model, except that all zonally asymmetric components of the circulation are excluded. So that this model can maintain the same time-averaged circulation as the full model, the time and zonally averaged effects of eddies on the vorticity, divergence, and temperature (calculated from the 6000-day run of the full model) are included as forcing terms. With these forcing terms included, the time and zonally averaged zonal winds, temperatures, and meridional circulation of the full model comprise a time-independent state of the zonally averaged model. It turns out to be particularly important that the Hadley circulation of the full model be included in the zonally symmetric model. (The importance of the climatological Hadley circulation in the zonally symmetric spin down of M was pointed out to me by Prof. K. Bowman.)

The zonally averaged model is perturbed at $t = 0$ by a rearrangement of vorticity in the upper layer, as shown in Fig. 8. The global integral of this vorticity perturbation and its projection onto Y_1^0 are identically equal to zero, so that initially M is unperturbed. The initial perturbation in the upper-layer winds is also shown in Fig. 8. There is a westerly jet centered on the equator and easterlies in middle and high latitudes. Figure 9 shows the subsequent evolution of the upper-layer zonal wind anomalies in the zonally symmetric model. The middle- and high-latitude easterlies weaken rapidly, while the equatorial jet splits into separate jets in each hemisphere, which then slowly drift away from the equator and decay.

The evolution of the associated perturbation in M is shown by the thick curve in Fig. 10. There is a rapid increase over the first 25 days followed by a slower decline to small negative values. This is the first of a few strongly damped oscillations with a period of about 150 days. These variations must, according to (1), be driven by variations in the lower-layer winds, which are shown in Fig. 11 for days 1, 10, and 25. In bringing the upper-level winds into thermal wind balance, the meridional secondary circulation generates a pattern of lower-layer winds similar to those in the upper layer. The middle- and high-latitude anomalies in the lower layer are stronger than those in the tropics, because the geostrophic adjustment process is more effective in coupling the upper and lower layers of the model where the Coriolis parameter is greater. Thus, at day 1 the middle-latitude easterly anomalies dominate the tropical westerly anomalies in the lower layer, and drag accelerates the atmosphere. The extratropical easterly anomalies decay more rapidly than the tropical westerly anomalies, so that by day 25 the angular momentum of the lower layer has become positive and M decreases.

The response of the zonally symmetric model to an impulsive redistribution of vorticity shares certain properties with the variations in M in the full model: the persistent anomalies in the zonal flow are confined to the tropics, and the anomaly in M is long-lived, surviving for 70 days after it reaches its peak. It remains to be shown, however, that the zonally symmetric model is reproducing the dynamics that cause M to fluctuate in the full model. It is proposed that the ultralow-frequency variations in M are driven by variability in the zonally averaged eddy flux of vorticity that has been filtered by the zonally symmetric dynamics of the model. If this is the case, then the impulse response of this dynamical filter should resemble the response of the zonally symmetric model to an initial rearrangement of vorticity, the heavy curve in Fig. 10. In other words, if the heavy curve in Fig. 10 is used to

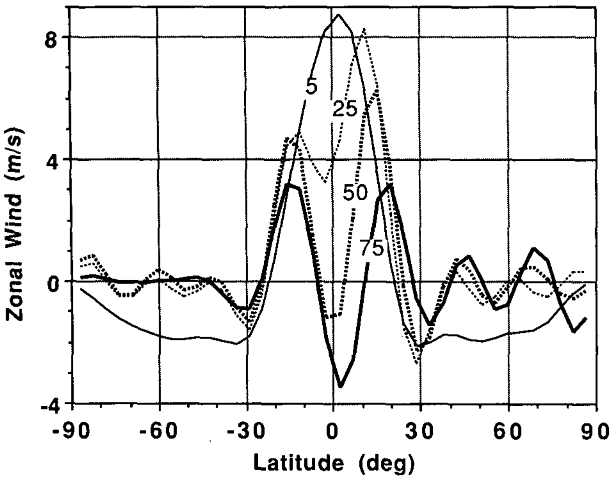

FIG. 9. Zonally averaged zonal-wind anomalies in the upper layer of the zonally symmetric model. The numbers on the curves refer to the model time in days.

define a filter that is then applied to the tropical vorticity flux F, the evolution of M should be reproduced.

To test this, the convolution of F with the impulse response of the zonally symmetric model is computed, and the result, denoted M', is proposed as a model for the variations in M. The impulse response of the zonally symmetric model at day i is denoted R_i, and the value of F on day j is denoted F_j, so that M' on day j is given by,

$$M'_j = \sum_{i=0}^{500} R_i F_{j-i}. \qquad (2)$$

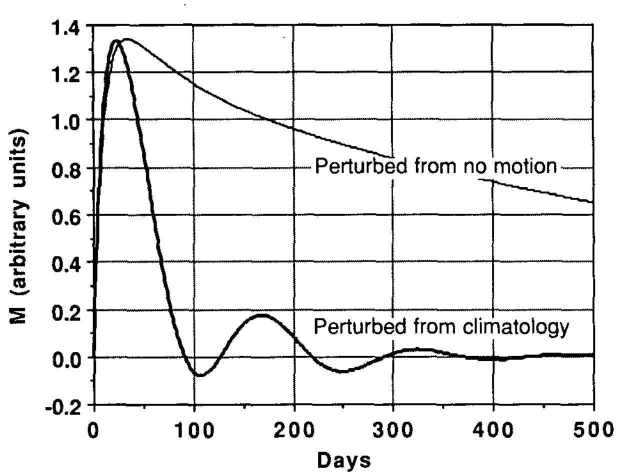

FIG. 10. Evolution of the anomaly in M, in arbitrary units, in the zonally symmetric model. The thick curve refers to the run in which the climatological zonally symmetric circulation of the model is included, and the thin curve is for a run in which the model is perturbed from a state of no motion.

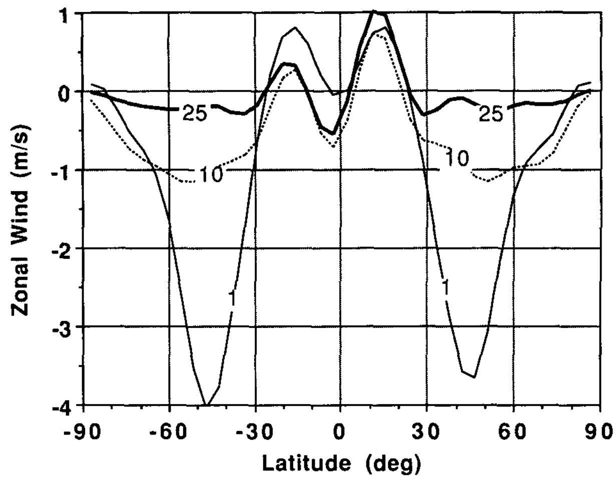

FIG. 11. Lower-layer zonal wind anomalies in the zonally symmetric model, at the indicated days.

Figure 2 shows M and M' for the final 5000 days of the model run. Both are normalized to unit variance. The curves are clearly similar, as is confirmed by their correlation coefficient 0.67. This is good agreement, given that the range of latitudes included in F and the structure of the initial vorticity rearrangement for the zonally symmetric model were chosen arbitrarily, and no attempt was made to adjust either to achieve greater agreement between M and M'. The spectra of M and M' are also similar (Fig. 3), though M' shows a distinct peak at the frequency of the oscillations in the impulse response.

A simpler zonally symmetric model, one without a Hadley cell, can be constructed by starting from a motionless initial condition and leaving out the mean eddy forcing. The response of M in this simpler model to the same initial rearrangement of vorticity as before is shown by the thin line in Fig. 10. In this case, the upper-level zonal wind anomaly remains centered on the equator, and the anomaly in M decays very slowly and without oscillations. This implies that the oscillations found above result from the meridional advection of angular momentum by the Hadley cell and that these oscillations are not the "ringing" of a zonally symmetric tidal mode.

4. Summary and discussion

The preceding sections demonstrate that variations in the angular momentum of our simple model atmosphere are dominated by upper-layer winds in the tropics, that these variations have very long time scales, and that variations in M result from variations in the eddy fluxes of vorticity in the tropics that are low-pass filtered by the dynamics of the model's Hadley circulation.

Several questions are raised by these results. First, why do perturbations survive for 90 days (Fig. 10) in a model in which the dissipative time scales are 5 days for the lower-layer winds and 15 days for the thermal field? A perturbation confined to the upper layer is not directly influenced by the drag and is subject only to thermal dissipation. Consider, as a simple example, the dissipation by Newtonian cooling of a baroclinic jet in a β channel, with zonal winds given by

$$u(y, z) = U\frac{z}{H}\sin\left(\frac{\pi y}{L}\right), \quad 0 \leqslant y \leqslant L. \tag{3}$$

The quasigeostrophic potential vorticity q of this flow is

$$q(y, z) = -\frac{U}{H}\left(\frac{z\pi}{L} + \frac{\epsilon L}{H\pi}\right)\cos\left(\frac{\pi y}{L}\right), \tag{4}$$

where H is the scale height and $\epsilon = f_0^2/N^2$, the square of the ratio of the Coriolis and buoyancy frequencies. If this flow is acted upon by Newtonian cooling with a rate D_N, then the rate of dissipation of potential vorticity is

$$q_t = \frac{D_N U \epsilon L}{H^2 \pi}\cos\left(\frac{\pi y}{L}\right). \tag{5}$$

If we define a dissipative time scale $\tau = |q/q_t|$, then at $z = H$

$$\tau = D_N^{-1}\left[\left(\frac{H\pi N}{Lf_0}\right)^2 + 1\right]. \tag{6}$$

The time scale for dissipation gets longer as the horizontal scale of the jet decreases relative to the internal Rossby deformation radius, or equivalently, for fixed L as the equator is approached, sending f_0 to zero.

The behavior of the zonally symmetric model is consistent with (6) in that zonal-wind anomalies decay rapidly outside of the tropics, whereas tropical anomalies are advected away from the equator by the Hadley circulation before decaying (Fig. 9). If the zonally symmetric model is perturbed from a state of no motion, then there is no Hadley circulation and zonal-wind anomalies can remain close to the equator indefinitely. In this case, anomalies in M persist for hundreds of days.

A second question is whether the present results can explain the very long time scales found by James and James (1989). The two-level model has not been integrated long enough to determine if it supports strong variability on interdecadal time scales. It is expected that the spectrum would be flat or even slightly blue at periods longer than a few hundred days, because within 100 days the Hadley circulation moves zonal flow anomalies to the subtropics. Possibly, this is an artifact of having only two model layers. In a five-layer model, such as that of James and James, advection by the Hadley circulation may nearly vanish at some latitudes in the middle model layers. In a perpetual solstice run, the center of the Hadley circulation could be on or close to the equator. In this case, vorticity deposited by eddies in middle model levels at the equator could persist nearly indefinitely. (As in our model, mechanical friction only acts on the lowest layer of James and James' model.) The spectrum would then resemble that of an undamped response to white noise, that is, the spectrum of a random walk, which is red at all frequencies.

Assuming that the same mechanism is responsible for the variability in these different models, the question arises of whether or not this mechanism can operate in the atmosphere. The lack of effective dissipation in the tropical upper layer is the key to the low frequencies exhibited by the two-layer model. When the model is run with stronger diffusion, Fickian diffusion with a diffusivity of 2.5×10^5 m^2 s^{-1}, there is less variability in the gravest spherical harmonics and the spectrum of this variability is less red. Our models, and those of James and James, exclude two processes that could mechanically damp zonal winds in the interior of the tropical atmosphere: the vertical redistribution of momentum by moist convection (cumulus friction) and the vertical transport of momentum by internal gravity waves. Whether or not these processes would provide enough drag on the tropical winds to suppress ultralow-frequency variability can only be resolved by analyzing extended runs of general circulation models that include these processes.

Finally, it is perhaps worth noting a fundamental difference between the variability described here and other types of internally generated low-frequency variability. Other examples of low-frequency variability, be they blocks (Mullen 1986, 1987), Rossby wave trains (Robinson 1991a; Cai and Van den Dool 1991; Qin and Robinson 1992), or variations in the zonal index (Karoly 1990; Robinson 1991b), involve a positive feedback between the low-frequency feature itself and the fluxes of heat and vorticity by higher-frequency transient eddies. The variability of M in the present model, however, appears to be nothing more than the response to synoptically generated noise, filtered by the zonally symmetric dynamics of the Hadley circulation, and no dynamical feedbacks are required.

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