Part II: Parameterization of Wave Forcing and Variability

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ABSTRACT

The purpose of this paper is to suggest a scheme for the parameterization of gravity wave propagation and effects in the lower and middle atmosphere that is tied as closely as possible to the spectral character of the observed gravity wave field. This effort begins with spectral expressions for gravity wave energy and momentum fluxes and prescribes the manner in which such idealized spectra respond to variations in the gravity wave environment. Also suggested are means of specifying spectral amplitudes in response to specific wave sources at lower levels and the spread of wave influences horizontally as the spectrum propagates vertically through the lower and middle atmosphere. Application of this scheme in a one-dimensional model yields wave amplitudes and fluxes in reasonable agreement with observations.

1. Introduction

A large body of research conducted during the last few decades has revealed a near-universal character of the gravity wave spectrum and substantial influences of these motions on the large-scale circulation and structure of the lower and middle atmosphere. These efforts, reviewed in the companion paper by Fritts and VanZandt (1993, hereafter FV93), suggest the ability to define a canonical gravity wave spectrum and the utility of a parameterization of gravity wave fluxes and effects based on the observed statistical structure of the motion field throughout the atmosphere. Such a scheme, it is hoped, will provide clear advantages relative to previous schemes based on the Lindzen–Holton representation of the wave spectrum as a collection of discrete and noninteracting components (Holton 1982; Garcia and Solomon 1985). It will also permit an extension of the parameterization schemes for orographic wave stresses used in a number of general circulation models (Palmer et al. 1986; McFarlane 1987; Rind et al. 1988) to other wave sources and their lower and middle atmospheric effects.

The scheme developed in this paper relies on the canonical model of the observed gravity wave spectrum presented by FV93. As such, it assumes that the gravity wave spectrum evolves in a continuous fashion throughout the atmosphere, with gradual changes in wave energy, characteristic scales, anisotropy, wave energy, and momentum fluxes, and the implied flux divergences, body forces, and turbulent diffusion. The advantages of such a scheme derive from the consistency of spectral amplitudes and fluxes with observed variations in the atmosphere rather than with artificial and/or simplistic concepts of monochromatic wave propagation.

We begin with a review of the model spectrum, its variations with altitude, atmospheric stability, and anisotropy, and its implied fluxes of energy and momentum in section 2. For simplicity, the wave spectrum is assumed to be confined to four discrete propagation directions. We then describe in section 3 the manner in which the spectrum is assumed to vary with changes in the local mean wind. Our criteria for the evolution of the gravity wave spectrum and its associated fluxes of energy and momentum are reviewed in section 4. Section 5 discusses the selection of spectral amplitudes and initial anisotropy for various wave sources at lower levels and the application of this parameterization scheme in general circulation models. The responses of the proposed parameterization scheme to representative midlatitude winter and summer mean wind profiles are presented in section 6. Our results are summarized in section 7. In a second companion paper (Lu and Fritts 1993), this gravity wave parameterization is employed to examine both gravity wave filtering for canonical mean wind profiles with a superposed steady tidal component and gravity wave–tidal interactions with a time-dependent tidal forcing and interactive gravity wave drag.
2. Model gravity wave spectra, fluxes, and divergences

a. Energy spectra and variations with $z$ and $N$

Following FV93, we assume initially a separable spectrum in vertical wavenumber, intrinsic frequency, and azimuthal direction of propagation for total wave energy given by

$$E(\mu, \omega, \phi) = E_0 A(\mu) B(\omega) \Phi(\phi)$$

(1)

with

$$A(\mu) = A_0 \frac{\mu}{1 + \mu^2}$$

(2)

and

$$B(\omega) = B_0 \omega^{-p},$$

(3)

where $\mu = m/m_*$, $m_*$ is the characteristic vertical wavenumber describing the transition between the spectral forms at large and small wavenumbers, $A_0 = 4/\pi$, $B_0 = (p - 1)f_0^{p-1}[1 - (f/N)^{p-1}]^{-1}$ with $p = 5/3$, $\int A(\mu) d\mu = 1$, $\int B(\omega) d\omega = 1$, $\int \Phi(\phi) d\phi = 1$, and $\Phi(\phi)$ describes the azimuthal distribution of wave propagation. The total energy includes horizontal and vertical kinetic and potential energy, with these spectra related as described by FV93. In order to relate energy densities to representative intrinsic phase speeds in the presence of wave field anisotropy, however, we will relax our assumption of separability of the azimuthal dependence and require only that the spectral forms remain the same for each direction of wave propagation in the discussion below.

The variations of total energy and $m_*$ with increases in $N$ and altitude may be written, following VanZandt and Fritts (1989) and FV93, as

$$E(m) \propto N^{1/2} e^{ziH_E} A(\mu)/m_*$$

(4)

and

$$m_* \propto N^{3/4} e^{-ziH_*},$$

(5)

where $H_E$ and $H_*$ are scale heights describing variations in total energy and $m_*$ with altitude. The dependencies on $N$ were derived assuming $(s, t) = (1, 3)$ and $N$ increasing with height, while the variations with altitude reflect the observed secular increase of energy and decrease of $m_*$ with altitude.

Consistent with observations and theory (Dewan and Good 1986; Smith et al. 1987; Fritts et al. 1988; Tsuda et al. 1989), we assume a saturated gravity wave spectrum at large vertical wavenumbers, with a spectral energy density $E(m) = 2N^2/15m^2 \propto e^{ziH_E}$ (for $H_s \to \infty$) at $m \gg m_* (\mu \gg 1)$, from Eq. (21) of FV93, and growth of wave energy for constant $N$ as $E(m) \propto e^{ziH_0} \propto H_0 \gg H$ at $m \ll m_* (\mu \ll 1)$. Then $H_E$ and $H_*$ can be related to $H_s$ and $H_0$, following FV93, as

$$\frac{1}{H_E} = \frac{1}{2} \left( \frac{1}{H_s} + \frac{1}{H_0} \right)$$

(6)

and

$$\frac{1}{H_*} = \frac{1}{4} \left( \frac{1}{H_s} - \frac{1}{H_0} \right).$$

(7)

Because $H_s \gg H$ and $H_0$, the scale heights for energy and $m_*$ are $H_E \sim 2H_s$ and $H_* \sim 4H_0$. The observed increase of energy (per unit mass) by a factor of $\sim 100$ from the tropopause to the mesopause implies a mean value of $H_E \sim 2.3H$. As noted by FV93, however, $H_E$ and $H_*$ appear not to be strictly constant with height. Instead, there is a tendency for less growth (larger scale heights) in the troposphere and mesosphere, larger growth (smaller scale heights) in the stratosphere (Justus and Woodruff 1973; Balsley and Garello 1985), and a cessation of growth above $\sim 100$ km (Fritts et al. 1993).

Finally, to simplify our parameterization as much as possible while retaining sensitivity to both zonal and meridional variations in the local mean wind profiles, we assume that the full spectrum is composed of four components with discrete directions of propagation toward east, west, north, and south. With this assumption, the total energy may be written $E_0 = 2E_x = E_e + E_w + E_n + E_s$, with the fraction for each direction of propagation having the same spectral form and with the total spectrum having a saturated amplitude at $m \gg m_* (\mu \gg 1)$.

b. Energy and momentum fluxes

Considering now the component of total energy propagating in only one direction $E_j$, the corresponding vertical energy and momentum fluxes are (FV93)

$$F_{E_j}(\mu, \omega) = c_{xj} E_j(\mu, \omega) = \frac{\omega}{m} \delta_{-\gamma} E_j(\mu, \omega)$$

(8)

and

$$F_{p_j}(\mu, \omega) = \frac{\omega}{N} E_j(\mu, \omega) (\delta_{\pm \gamma})^{1/2},$$

(9)

where $\delta_+$, $\delta_-$, and $\gamma$ are defined by FV93 and we have taken $\omega$ and $m$ positive to correspond to upward energy propagation. Integrating over all $\mu$ and $\omega$ yields, to a good approximation,

$$F_{E_j} = C \frac{N E_j}{m_*},$$

(10)

and

$$F_{p_j} = D E_j,$$

(11)

where we have written $\int j = f_j N$, neglected terms of order $f^2$, defined $m^{*}_{s,j} \approx N^2/40 E_j$ with $m^{*}_{s,j}$ the char-
acteristic vertical wavenumber for the fraction of total energy \( E_j \), and obtained \( C \approx 1/18 \) and \( D \approx 1/22 \) by integration of Eqs. (8) and (9) with \((s, t, \rho, f) = (1, 3, 5/3, 1/200)\). The definition of \( m_{aj} \) follows from Eq. (22) of FV93 and the assumption that each component spectrum contributes equally to the saturated spectral amplitude at large \( m \).

The cumulative energy and momentum fluxes may then be expressed as

\[
F_E = \sum F_{E_j} = CN \frac{E_j}{m_{aj}},
\]

\[
F_{F_x} = F_{F_e} - F_{F_w} = D(E_e - E_w) = Db_x E_0,
\]

(12)

(13)

and

\[
F_{F_y} = F_{F_n} - F_{F_s} = D(E_n - E_s) = Db_y E_0,
\]

(14)

where \( b_x \) and \( b_y \) are the zonal and meridional wave field anisotropies defined by FV93 as

\[
b_{x,y} = \left( b_x, b_y \right) = \int_0^{2\pi} \Phi(\phi) (\cos \phi, \sin \phi) d\phi.
\]

(15)

In our case this distribution is assumed to be composed of delta functions in the cardinal directions with amplitudes proportional to the component energies, \( E_j \). With this definition \( b_x \) and \( b_y \) are constrained by \(-1 \leq b_x + b_y \leq 1\) because of their expression in terms of the total energy and our specification of four discrete directions of propagation. Thus, neither term may exceed 0.5 in magnitude if energy is distributed evenly between zonal and meridional directions of propagation.

c. Energy and momentum flux divergences

Flux divergences are likewise computed following FV93. Equating the energy dissipation rate to the vertical divergence of wave energy flux, we obtain

\[
\epsilon = -\frac{N}{\rho} \frac{\partial}{\partial z} \left( \frac{\rho}{N} F_E \right)
\]

\[
= F_E \left[ \frac{1}{H} - \frac{1}{H_e} - \frac{1}{H_s} + C_{Ne} \frac{\partial}{\partial z} \ln(N) \right],
\]

(16)

where we have assumed that variations of \( H_e \) and \( H_s \) are much smaller than those of \( E_0 \) and \( m_* \). As discussed by FV93, the coefficient \( C_{Ne} \) contains a weak dependence on \( N \) and obtained by differentiation of Eq. (10). Likewise, the presence of \( N \) in the definition of \( \epsilon \) in Eq. (16) is due to our assumption of a universal intrinsic frequency spectrum and its dependence on \( N \) in turn. Alternatively, using the limiting value for \( H_* \) with \( H_* \rightarrow \infty \), Eq. (16) may be written

\[
\epsilon = F_E \left[ \frac{1}{H} - \frac{3}{2H_e} + C_{Ne} \frac{\partial}{\partial z} \ln(N) \right].
\]

(17)

Under the same assumptions, the corresponding momentum flux divergences are given by

\[
D_{Fx} = -\frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho F_{Fx} \right) = -\frac{1}{\rho} \frac{\partial}{\partial z} \left[ \rho D(E_e - E_w) \right]
\]

\[
= -\frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho Db_x E_0 \right) = Db_x E_0
\]

\[
\times \left[ \frac{1}{H} - \frac{1}{H_e} + C_{Ne} \frac{\partial}{\partial z} \ln(N) - \frac{\partial}{\partial z} \ln(b_x) \right]
\]

(18)

and

\[
D_{Fy} = -\frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho F_{Fy} \right) = -\frac{1}{\rho} \frac{\partial}{\partial z} \left[ \rho D(E_n - E_s) \right]
\]

\[
= -\frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho Db_y E_0 \right) = Db_y E_0
\]

\[
\times \left[ \frac{1}{H} - \frac{1}{H_e} + C_{Ne} \frac{\partial}{\partial z} \ln(N) - \frac{\partial}{\partial z} \ln(b_y) \right],
\]

(19)

where \( C_{Ne} \) expresses the dependence on \( N \) and obtains from differentiation of Eq. (11).

Apart from the sensitivity to variations in \( N \) discussed by VanZandt and Fritts (1989), these expressions display a dependence on both the background variations of wave energy at large and small vertical wavenumbers (through \( H_e \) and \( H_* \)) and, in the case of the momentum fluxes, the anisotropy and its variations with height. The wave energy flux and energy dissipation rate are not influenced by wave field anisotropy, of course. Momentum fluxes are, however, and observations suggest that wave field anisotropy is highly variable and dependent on both the mean and low-frequency components of the motion field through which the higher-frequency motions accounting for the majority of momentum and energy fluxes must propagate. A discussion of the manner in which these effects are parameterized is presented in the following section.

3. Evolution of energy spectra with a varying mean wind

Gravity waves respond to variations in density and atmospheric stability as they propagate vertically, as discussed in the previous section. They also are very sensitive to variations in the local mean wind in the plane of wave propagation. Indeed, the variations in wave energy per unit mass with increasing altitude are due primarily to the former, while changes in the anisotropy of the motion field are due largely to the latter. Our intent here is to anticipate the effects of mean wind variations on the component energy spectra and the corresponding variations in energy and momentum fluxes in a manner as consistent as possible with both model and physical constraints.

The most significant approximation in the development of our parameterization scheme is our as-
sumption that the shape of the vertical wavenumber spectrum is invariant with altitude, intrinsic frequency, and direction of propagation. This is suggested by numerous observations and simplifies our development substantially. However, it also includes implicit assumptions about the processes, the scales, and the effects of wave saturation at large and small $m$.

Consider, for example, a single element of the eastward-propagating gravity wave spectrum $E_c(\omega)$ at height $z_i$ with initial intrinsic frequency

$$\omega_i = kc_i = k(c - \bar{u}_i) > 0,$$

where $k$ is the horizontal wavenumber, $c_i$ is the initial intrinsic horizontal phase speed, $c$ is the observed horizontal phase speed, and $\bar{u}_i$ is the background zonal wind at $z_i$. The corresponding initial vertical wavenumber is

$$m_i \approx kN/\omega_i = N/c_i = N/(c - \bar{u}_i),$$

assuming the motion to be approximately hydrostatic and two-dimensional (irrotational), and $m$ is positive downward.

In a horizontally stratified medium with a steady background wind $\bar{u}(z)$, $k$ and $c$ are approximately independent of altitude, leading to variations of $\omega$ and $m$ with altitude given by

$$\omega(z) = k[c(z) - \bar{u}(z)] = k[c_i - \Delta \bar{u}(z)] = kc_i[1 - m_i/m_c(z)],$$

and

$$m(z) = N/c(z) = \frac{m_i}{1 - m_i/m_c(z)},$$

where $\Delta \bar{u}(z) = \bar{u}(z) - \bar{u}_i$ and $m_c(z) = N/\Delta \bar{u}(z)$. This wave will encounter a critical level with $\omega(z) = 0$ and $m(z) = \infty$ at the altitude where $c = \bar{u}(z)$, $c_i = \Delta \bar{u}(z)$, or $m_i = m_c(z)$. Likewise, initial wave energy occurring at $m_i > m_c$ will also experience critical-level encounters (and dissipation) below height $z$, while initial wave energy at $m_i < m_c$ will be shifted to larger, but finite, wavenumbers.

If we were to assume that this spectral energy transfer at $m_i < m_c$ occurs subject to conservative wave propagation (and neglecting variations due to changes in $N$ and altitude), the initial energy density $E_i(m)$ would increase in wavenumber and decrease in amplitude to conserve wave action flux,

$$c_{_{tg}} \frac{\rho E_i(m)dm}{\omega} = \frac{\rho E_i(m)dm}{m}.$$  

Thus, the transformed energy density per unit mass would vary as $E_c(m) = E_i(m_i)(m/m_i)(dm_i/dm)$ with $dm_i/dm = (m_i/m_c)^2$, from Eq. (23), such that

$$E_c(m) = \frac{E_i(m_i)m_i}{m} = \frac{E_i}{\pi m_i} \left(1 + \beta \mu\right)^2 \mu, (25)$$

where $m_i \approx N/2\sqrt{10} E_c^{1/2}$, $\mu = m/m_i$, and $\beta = \Delta \bar{u}/c_i = m_i/m_c$. At large wavenumbers, $m \gg m_i$, and $m_c$, however, this energy density, with a limiting amplitude and slope given by

$$E_c(m \gg m_i) = \frac{E_i}{\pi m_i} \frac{\beta^2}{(\beta^4 + 1)^{1/2}},$$

far exceeds the saturation limit for each component spectrum, $E_i(m) \approx N^2/30m^3$ [see Eq. (21) of FV93 and the assumption leading to the definition of $m_i$, following Eq. (11) of this paper], and implies an infinite integrated energy density.

Alternatively, Eq. (25) describes the evolution of a spectrum propagating against the mean flow with $\beta < 0$ for which conservative wave propagation demands a decreasing vertical wavenumber and an energy density that decreases to zero at $m = m_c$. The initial and transformed spectra given by Eq. (25) for $\beta = m_i/m_c = \pm \beta_0$ are shown for reference in Fig. 1. These reveal the difficulties in dealing in a simple parameterized manner with a complete description of the spectral evolution in the presence of mean wind shear. In view of these difficulties, we choose not to use the detailed spectral evolution implied by wave action conservation at each wavenumber as a basis for computation of spectral energy changes and must devise an alternative procedure that more accurately reflects the observed spectral evolution at large $m$.

Clearly, saturation processes must limit spectral amplitudes continuously, preventing such conservative

![Fig. 1. Initial (solid), upshifted (long dash), and downshifted (short dash) vertical wavenumber spectra showing changes implied by conservative wave propagation for $\beta = 0$ and $\pm 0.5$.](image-url)
amplitude growth at large $m$, as spectral amplitudes far in excess of saturated values are not observed. Conversely, at small vertical wavenumbers, $m_i \ll m_z$, only small changes in $E(m)$ and $m$ occur and are more likely to take place in a conservative fashion (see Fig. 1). Thus, we do not expect $\beta > 0$ to alter substantially either the spectral shape or the integrated spectral energy density, except possibly very near $m_i$. With these constraints at large and small $m$, an assumption that $m_z$ does not increase with $\beta > 0$ provides a condition largely consistent with both observations and theory. Likewise, for $\beta < 0$ and small in magnitude, we anticipate that the spectrum will evolve in a largely conservative fashion, with the major changes occurring near $m_i$. Larger variations in $m_z$ (for $\beta < 0$) will be associated, instead, with increases in altitude or $N$. In both cases, changes in $m_z(z)$ will be described approximately by

$$m_z(z) = \left( \frac{N(z)}{N_i} \right)^{3/4} e^{-\Delta z / m_z} \left( \frac{m_i}{1 - m_z/m_c(z)} \right), \quad (27)$$

with $\Delta z = z - z_i$, where Eq. (27) includes the systematic variations with height from Eq. (5) and those due to variations in the mean wind from Eq. (23).

One additional condition is needed, however, to ensure as much consistency as possible with theoretical limits on wave energy increases with height. Referring to Eq. (27), we note that large negative $\beta = m_z/m_c$ will dominate the height variations of $m_z(z)$ for small $\Delta z$ unless the decrease of $m_z$ (and the corresponding increase in wave energy) is constrained in another manner. We also know from theory that wave action (or wave momentum flux) cannot increase with altitude for steady, conservative wave motions. With the component momentum fluxes (per unit volume) given by $pDE$, this imposes an upper limit on any component spectrum with $\beta < 0$ given by

$$E_j \leq e^z / H_z. \quad (28)$$

Consistent with the energy spectrum defined by FV93 and our discussion of component spectral energies and transfers above, we write the total energy

$$E_0 \approx \frac{1}{10} \frac{N^2}{m_{z0}^2} = \Sigma E_j \approx \frac{1}{40} \Sigma \frac{N^2}{m_{zj}^2} = \frac{1}{40} c_{zj}^2, \quad (29)$$

with $E_j$ the component energies defined previously,

$$m_{z0}^2 = \frac{1}{4} \Sigma m_{zj}^2, \quad (30)$$

and $c_{zj} = N/m_{zj}$ the characteristic intrinsic phase speeds of the component spectra. This ensures that equally energetic component spectra share a common characteristic vertical wavenumber $m_{z0} = m_{z0}$ with the total energy spectrum, that each component contributes equally to the saturated spectral amplitude at large $m$, and that each component can also be characterized by only one variable. We also assume, based on the approximate isotropy of the motion field throughout the atmosphere, that the total energy remains equipartitioned between zonal and meridional directions of propagation with $E_x + E_y = E_n + E_s = E_0/2$, with $E_0 \propto N^{1/2} e^z / H_z$ from Eq. (4).

Finally, we impose a constraint on the maximum anisotropy within the zonal and meridional motion fields to ensure that no component spectrum decreases relative to the oppositely propagating component to an artificially small value. This constraint is motivated by the insensitivity of motions at small $m$ to mean wind changes and observations of large and oscillatory momentum fluxes in response to large and variable wind shears. For this purpose, we assume that no less than a fraction $\alpha$ of the zonal or meridional energy may be associated with any one component. This implies that the maximum momentum flux is limited to $(1 - 2\alpha)$ of the total possible if all wave energy in one plane were propagating in the same direction and results in a constraint on the anisotropies expressed as $(-0.5 + \alpha) \leq b_{x,y} \leq (0.5 - \alpha)$, given equal zonal and meridional energies. The consequences of this constraint are examined in greater detail in the companion paper by Lu and Fritts (1993).

4. Review of constraints on energy and flux variations with height

The physical and model constraints on the evolution of the energy spectrum and its fluxes of energy and momentum with height are reviewed here. In the following sections, we determine canonical values of the model parameters and use these conditions to test our parameterization scheme in several environments representative of the lower and middle atmosphere.

The primary constraint arises as a consequence of gravity wave saturation and the resulting amplitude limits at large vertical wavenumbers. This implies both an approximately universal vertical wavenumber spectrum and a variation of wave energy density with height and atmospheric stability that may be expressed as

$$E_0 \propto N^{1/2} e^z / H_z, \quad (31)$$

where $H_E \approx 2H_e$ and $H_n \approx H$ for a saturated spectrum. In practice, a value $H_E \approx 2.3H$ appears to describe the variation of energy density (per unit mass) observed in the lower and middle atmosphere (Balsley and Carter 1982). The variation with $N$ is a consequence of the enhanced saturation that is induced in regions where $N$ increases with altitude (VanZandt and Fritts 1989).

For simplicity of representation, the total energy is assumed to comprise four component spectra propagating east, west, north, and south, denoted $E_x$, $E_y$, $E_n$, and $E_s$. Each component spectrum has a total energy related to its characteristic vertical wavenumber and intrinsic phase speed as
with $m_{ij}$ as defined in section 2 and $E_j$ varying with altitude, apart from variations in the mean wind, as $E_0$. Thus, only one variable is required to specify each component spectrum. The energies $E_j$ (or alternatively, the characteristic vertical wavenumbers or intrinsic phase speeds, $m_{ij}$ or $c_{ij}$) are specified in order to represent the gravity wave energy and anisotropy associated with various wave sources at tropospheric altitudes, as described further in section 6.

The component spectra are assumed to vary with changes in $N$ and the mean wind in both zonal and meridional directions, with the final values $m_{ij}(z)$ for components $E_j$ given by

$$m_{ij}(z) = \left[ \frac{N(z)}{N_i} \right]^{3/4} e^{-\Delta z/\lambda} \left( \frac{m_{ij}}{1 - m_{ij}/m_c(z)} \right),$$

where subscripts $i$ and $f$ denote initial and final values, $\Delta z = z - z_i$, $m_{ij}(z) = N/\Delta \bar{u}_j(z)$, and $\Delta \bar{u}_j(z) = \bar{u}_j(z) - \bar{u}_{ij}$, subject to the constraints described below. This expression also assumes that $\beta_j = \Delta \bar{u}_j/c_{ij} < 1$ (or $m_{ij} < m_c$), where $\Delta \bar{u}_j$ and $c_{ij}$ are the mean wind change in the height interval $\Delta z$ (positive or negative) and the characteristic phase speed of the component energy spectrum in that direction (positive to the east and north). For $\beta_j > 0$, $m_{ij}$ is assumed not to increase with height to reflect saturation at large $m$ and insensitivity to mean wind variations at small $m$. For $\beta_j$ large and negative, Eq. (33) implies a component energy that increases with height more rapidly than allowed for conservative propagation. In this case, Eq. (33) is replaced by the condition

$$E_j \propto e^{z/\lambda},$$

with the opposite component constrained to decay as

$$E_k = \frac{E_0}{2} - E_j,$$

and $E_0$ varying as above.

A final condition is imposed to ensure that no component spectrum becomes so small that it cannot achieve significant effects at greater altitudes. This is expressed by $\alpha < 2 E_j/E_0 < (1 - \alpha)$ where we have taken $\alpha = 0.2$, permitting as much as 80% of the energy in each plane to propagate in one direction, imposing a constraint on the anisotropies (and momentum fluxes) given by

$$-0.5 + \alpha \leq b_{x,y} \leq 0.5 - \alpha.$$  

Further motivation for this condition is presented in section 6 below.

Given initial values of the component energies, the above conditions allow computation of the variations of these quantities with altitude, including changes in atmospheric stratification and the local mean wind. It is then straightforward to obtain the corresponding energy and momentum fluxes and their vertical divergences from the profiles of $E_j$. With our scheme for anticipating spectral energy changes with altitude, the energy and momentum fluxes may be expressed most directly as

$$F_E = \Sigma F_{E_j} = CN \Sigma \frac{E_j}{m_{ij}},$$

$$F_{px} = F_{pe} - F_{pw} = D(E_e - E_w),$$

and

$$F_{py} = F_{pn} - F_{ps} = D(E_n - E_s),$$

with $C$ and $D$ as defined in Eqs. (10) and (11) and the inferred flux divergences given by

$$\epsilon = -\frac{N}{\rho} \frac{\partial}{\partial z} \left( \frac{\rho}{N} F_E \right),$$

$$D_{Fx} = -\frac{1}{\rho} \frac{\partial}{\partial z} (\rho F_{px}),$$

and

$$D_{Fy} = -\frac{1}{\rho} \frac{\partial}{\partial z} (\rho F_{py}).$$

5. Specification of model parameters

Observational studies have provided a number of estimates of the magnitude and variability of gravity wave energies and momentum fluxes at various altitudes throughout the lower and middle atmosphere. The intent in this section is to discuss the manner in which these observations may be used to pose representative initial conditions and constraints on the motion field within the framework of our parameterization.

a. Observations of velocity variances

Radar observations near the tropopause have revealed horizontal velocity variances that vary considerably with location and time. Inferred variances are typically $\sim 10$ m$^2$ s$^{-2}$ over mountainous terrain under moderate wind conditions (Balsley and Carter 1982; VanZandt et al. 1990; Nastrom and Fritts 1992). However, these variances are often enhanced, relative to background levels, due to orographic wave motions (Nastrom et al. 1987; Jasperson et al. 1990; Nastrom and Fritts 1992), and are likely to be even larger above higher or more rugged terrain. Mean variance estimates over terrain, then, may pose a reasonable upper limit on the mean tropopause velocity variance.

Variances in association with frontal systems, convection, and wind shear are likewise enhanced over background levels and are comparable to or slightly
smaller than mean variances over significant terrain, with typical values of $\sim 3 - 8$ m$^2$ s$^{-2}$ (Vincent and Eckermann 1990; Fritts and Nastrom 1992). In contrast, variances tend to be much smaller away from obvious wave sources, with representative values $\sim 1$ m$^2$ s$^{-2}$ or less and $\sim 6$ times less energetic, in general, than the average for identified gravity wave sources (Fritts and Nastrom 1992).

Velocity variances increase in a nearly uniform manner with altitude, achieving values of $\sim 400 - 1000$ m$^2$ s$^{-2}$ near the mesopause at $\sim 86$ km, based on observations of wind and density fluctuations (Balsley and Carter 1982; Vincent 1984; Balsley and Garell 1985; Meek et al. 1985; Fritts and Yuan 1989; Rüster and Reid 1990; Wang and Fritts 1990). Also noted is a tendency for variances to be somewhat smaller over oceans than over major landmasses (Fritts et al. 1989; Vincent and Lesicar 1991; Fritts and Isler 1992), suggesting that orographic excitation or enhanced convection over land may contribute to statistically more energetic gravity waves at mesospheric heights. Finally, observations at greater altitudes suggest that gravity waves may propagate to considerable altitudes in the lower thermosphere (Fejer et al. 1985; Gavrilo et al. 1992, personal communication) but that wave amplitudes apparently cease growth or grow more gradually above $\sim 100$ km (Fritts et al. 1993).

b. Observations of momentum fluxes

Momentum flux measurements have been made in a variety of ways, including surface pressure analyses, aircraft measurements of mountain wave structure in the troposphere and lower stratosphere, constant volume balloon measurements of wave fluctuations, and radar measurements of the anisotropy within the motion field throughout the lower and middle atmosphere. Surface pressure measurements have revealed typical pressure differences across mountainous terrain of $\sim 0.1 - 0.2$ mb, with maxima of $\sim 0.5$ mb, corresponding to implied momentum fluxes of $\sim 1 - 6$ N m$^{-2}$ (Smith 1978; Hoina 1984, 1985). Measurements with aircraft, however, reveal substantially smaller maximum values, $\sim 0.3 - 1.5$ N m$^{-2}$ (Lilly and Kennedy 1973; Lilly 1978; Kennedy and Shapiro 1979; Hoina 1984, 1985; Nastrom and Fritts 1992), suggesting that much of the surface flux is associated with trapped lee wave activity that influences only the lowest levels of the troposphere (Smith 1978; Palmer et al. 1986).

More typical values are reported to be $\sim 0.1$ N m$^{-2}$ or less and are nearly constant throughout the troposphere (Massman 1981; Lilly et al. 1982; Hoina 1985). Corresponding Reynolds stresses, $u\bar{w}$ and $\bar{v}\bar{w}$, are $\sim 0.1 - 0.2$ m$^2$ s$^{-2}$ for typical flows and $\sim 10$ times larger under more extreme conditions (Lilly and Kennedy 1973; Lilly and Lester 1974; Nastrom and Fritts 1992), providing reasonable bounds on our parameterization of these fluxes.

Momentum flux measurements using a variety of atmospheric radars have provided estimates of the mean values and indications of the degree of variability in several altitude regions. Measurements in the upper troposphere and lower stratosphere using the MU radar have revealed fluxes (per unit mass) of $u\bar{w} \sim -0.1$ to $-1.0$ m$^2$ s$^{-2}$, with a mean of $\sim -0.2$ m$^2$ s$^{-2}$ (Fritts et al. 1990), in good agreement with inferences from aircraft measurements under typical conditions (Lilly et al. 1982; Nastrom and Fritts 1992). These measurements also imply a deceleration due to vertical momentum flux divergence of $\sim 1 - 2$ m s$^{-1}$ day$^{-1}$ above the tropospheric jet core, in reasonable agreement with that required to close the tropospheric jets and balance the Coriolis torques acting on the mean meridional motions at these heights (Palmer et al. 1986; McFarlane 1987).

Radar measurements in the mesosphere and lower thermosphere likewise suggest significant anisotropy in the motion field and Reynolds stresses, or momentum fluxes per unit mass, $\sim 100$ times those values obtained near the tropopause. Mean momentum fluxes are as large as $\sim 10 - 20$ m$^2$ s$^{-2}$ and generally are anticorrelated with the local mean flow (Vincent and Reid 1983; Reid and Vincent 1987; Fritts and Vincent 1987; Fritts and Yuan 1989; Tsuda et al. 1990; Wang and Fritts 1990; Hitchman et al. 1992). In contrast, flux estimates for intervals of $\sim 1 - 8$ h exhibit substantial variability and maxima of $\sim 60$ m$^2$ s$^{-2}$ at times of strong modulation by low-frequency motions (Fritts and Vincent 1987; Reed et al. 1988; Fritts and Yuan 1989; Rüster and Reid 1990; Fritts et al. 1992), while estimates of anisotropy are also large (Ebel et al. 1987; Fritts and Wang 1991).

c. Specification of energies, anisotropies, and variability

Spectral parameters may be specified at any altitude that is useful for modeling purposes and representative of the major gravity wave sources. For example, it will likely be useful to specify a wave spectrum due primarily to orography at the mean terrain height and in terms of the mean wind at that level, where this is possible. Likewise, spectra arising from convection and wind shear will need to be specified, for greatest sensitivity to meteorological conditions, at the heights at which these sources occur for best results at lower levels. At greater altitudes, we anticipate that the character of the wave spectrum will be determined increasingly by the filtering influences defined in section 3. Finally, it may be valuable to distribute the influences of localized sources at lower levels over increasing areas as altitude increases, recognizing the horizontal spread of wave activity due to different propagation directions. The constant $D$ in Eq. (11) implies a characteristic frequency $\sim N/22$ and a corresponding horizontal spread of $\sim 22\Delta z$ at an altitude $\Delta z$ above the source level.
Assuming for our purposes here that \( E_h = 5 \text{ m}^2 \text{s}^{-2} \) represents a mean horizontal velocity variance near the tropopause (~10 km), consistent with the measurements summarized previously, this implies a mean wave energy \( E_0 = (p + 1)E_h/2p = 4 \text{ m}^2 \text{s}^{-2} \) (with \( p = 5/3 \)) and a characteristic intrinsic phase speed at the surface of \( c_s \sim 5 \text{ m s}^{-1} \). Also recognizing that our parameterization will need to accommodate a range in initial source energies to account for smaller background energies and the larger energies accompanying strong wave forcing, we consider the behavior of our parameterization scheme for initial phase speeds of 3, 5, and 8 m s\(^{-1}\). Consistent with observations, we assume a mean scale height for energy growth (per unit mass) of \( H_E \sim 2.3H \) below 85 km and a transition to constant energy above 100 km. We also recognize, however, that a slowly varying \( H_E \) would provide more quantitative agreement with observed energy variations with height (Justus and Woodrum 1973; Baisley and Garelo 1985) and examine the implications of this variability as well. Additionally, we note that constant wave energy above 100 km prevents changing anisotropies at these heights unless the \( c_{ij} \) are allowed to decrease with altitude. Thus, this condition is relaxed above 85 km in order to ensure continued sensitivity to variations in the mean wind profiles.

Likewise, we base our test values of initial anisotropy on atmospheric observations at lower levels. These suggest maximum values, again primarily in association with orography (Fritts et al. 1990; Nastrom and Fritts 1992), of \( b_{x,y} \sim \pm 0.2 \). Thus, we choose \( b_{x,y} = -0.2, 0.0, \) and 0.2 to span the range of anisotropies representative of convective, orographic, and shear excitation processes. At greater heights, considerably greater anisotropy is suggested by large momentum fluxes and evidence of strong Doppler shifting in vertical velocity frequency spectra (Vincent and Reid 1987; Fritts and Vincent 1987; Reid and Vincent 1987; Reid et al. 1988; Fritts and Yuan 1989; Fritts and Wang 1991). This suggests a higher degree of anisotropy than imposed at source levels as a consequence of gravity wave filtering and is the motivation for our tolerance of larger anisotropy at greater altitudes. To illustrate this sensitivity, we present results for values of \( \alpha = 0.1, 0.2, \) and 0.3. As noted in Eqs. (36)–(42), a larger \( \alpha \) reduces the maximum possible momentum fluxes, while a smaller value permits greater anisotropy at all heights, larger variations in \( c_{ij} \) and \( E_j \) at lower levels, and less sensitivity to mean wind variations at greater altitudes where \( c_{ij} \) is large.

6. Variations of wave energies and fluxes in canonical mean profiles

We now examine the manner in which the parameterization scheme described above responds to density, wind, and stratification profiles representative of the lower and middle atmosphere using a one-dimensional model. Representative winter and summer mean wind, temperature, and stability profiles obtained from the CIRA 1986 model (January and June at 40° latitude) and used for our test purposes are shown with solid and dashed lines in Fig. 2. These were selected to permit a meaningful comparison of the predicted energy and momentum fluxes with available observations. We also assume equal zonal and meridional energy propagation, recognizing that the atmosphere is much less constrained to zonal motions than models, but we display only the zonal energy variations as these respond much more dramatically to mean wind variations.

The zonal mean wind profiles (left panel of Fig. 2) exhibit a tropospheric jet with eastward maxima of ~30 m s\(^{-1}\) in winter and ~20 m s\(^{-1}\) in summer at

![Fig. 2](image-url)
altitudes of \( \sim 12-13 \) km, eastward and westward mesospheric jets with maxima of \( \sim 60 \) m s\(^{-1}\) at \( \sim 68-70 \) km, and further reversals of the zonal mean motion above \( \sim 96 \) and \( 86 \) km during winter and summer. The CIRA 1986 temperature profiles exhibit tropopause minima of \( \sim 210 \) K at \( \sim 16-18 \) km, stratopause maxima of \( \sim 260-270 \) K at \( \sim 46-48 \) km, and mesopause minima of \( \sim 190 \) and \( 170 \) K at \( \sim 96 \) and \( 92 \) km, respectively. Corresponding values of \( N^2 \) range from \( \sim 10^{-4} \) rad\(^2\) s\(^{-2}\) in the troposphere to \( \sim 5 \cdot 10^{-4}, 3 \cdot 10^{-4}, \) and \( > 6 \cdot 10^{-4} \) rad\(^2\) s\(^{-2}\) in the stratosphere, mesosphere, and lower thermosphere, while density varies in both seasons with scale heights ranging from \( \sim 5 \) to \( 8 \) km and a mean value of \( \sim 7 \) km. Despite these height and seasonal variations, density does not vary by more than a factor of 2 from that given approximately by the mean scale height for this altitude interval.

**a. Dependence on initial wave energy**

The behavior of the zonal phase speeds, together with the eastward, westward, and net zonal momentum fluxes, is shown for the canonical winter and summer.

![Graphs showing phase speeds and momentum fluxes](image)

**Fig. 3.** Profiles of mean wind (solid), \( c_{aw} \) (long dash), and \( c_{aw} \) (short dash) obtained for canonical (a) mean winter and (c) summer profiles and their associated component and total (solid) momentum fluxes (b) and (d) with \( c_{aw} = c_{aw} = c_{aw0} = 5 \) m s\(^{-1}\), \( b_s = 0.0 \), and \( \alpha = 0.2 \).
profiles from 0 to 120 km in Fig. 3. Details of these responses at lower levels are shown below 40 km in Fig. 4 to display variations more clearly where phase speeds and fluxes are small. As described above, the zonal phase speeds, \( c_{wx} \) and \( c_{wy} \), are related to the zonal energies, \( E_x \) and \( E_y \), by Eq. (32) and we have assumed that \( c_{wx} = c_{wy} = 5 \text{ m s}^{-1} \), \( b_x = 0.0 \), and \( \alpha = 0.2 \).

As anticipated from the prescribed growth of wave energies with height, we note a tendency for phase speeds and fluxes to grow and vary in response to changes in the mean wind. Phase speeds increase from their initial value of 5 m s\(^{-1}\) to over 100 m s\(^{-1}\) above 100 km, while momentum fluxes, initially zero because of our initial choice of \( b_x = 0 \), increase to maximum values of \( \sim 10-20 \text{ m}^2 \text{ s}^{-2} \) at altitudes above \( \sim 60 \) km. Throughout the altitude range, intrinsic phase speeds are consistent with those inferred from vertical wavenumber spectra because of our basing of the parameterization scheme on observational data. Phase speeds also exhibit large local variations, particularly where propagation is opposite to changes in the mean wind with height, due to the assumed nearly conservative wave propagation of these spectral components.

Both winter and summer momentum flux profiles

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**Fig. 4.** Same as Fig. 3 but for altitudes of 0–40 km.
display a tendency to be anticorrelated with the zonal mean motion at upper levels and for maximum values to occur near and below reversals in the zonal mean motion, in reasonable agreement with observations. Similar tendencies are noted at lower levels (see Fig. 4), with momentum fluxes and the zonal mean motion generally anticorrelated and with momentum fluxes in the upper troposphere and lower stratosphere of \( \sim -0.2 \text{ m}^2 \text{ s}^{-2} \), in good agreement with measurements cited earlier.

The responses of the parameterization to varying initial phase speeds for both winter and summer zonal mean winds are displayed for upper- and lower-altitude ranges in Fig. 5. For this purpose, we assume phase speeds of 3, 5, and 8 m s\(^{-1}\) to cover a range of energies from \( \sim 2 \text{ to } 12 \text{ m}^2 \text{ s}^{-2} \) near the tropopause and intended to be representative of weak to very strong wave forcing at lower levels. Smaller initial phase speeds better describing tropopause energies away from obvious wave sources of \( \sim 1-2 \text{ m} \text{ s}^{-1} \) result in very weak responses throughout the altitude range and are not shown.

The momentum flux profiles obtained for different initial phase speeds exhibit an approximately linear re-

![Momentum Flux Profiles](image)

**Fig. 5.** Momentum flux profiles obtained with \( c_\phi = 3 \) (short dash), 5 (solid), and 8 (long dash) m s\(^{-1}\), \( \beta_\phi = 0.0 \), and \( \alpha = 0.2 \) for canonical winter and summer profiles at (a) and (b) 40-120 km and (c) and (d) 0-40 km.
response to initial energy at lower levels where phase speeds are strongly influenced by mean wind variations. With few exceptions, especially near the zonal wind minimum in the lower stratosphere, the momentum fluxes reflect the ratio of initial energies below ~80 km. At greater altitudes, however, the fluxes depart from this relationship because the smaller phase speeds are more strongly influenced by mean wind variations than are the larger phase speeds. This accounts for the tendency for momentum fluxes obtained for an initial phase speed of 8 m s\(^{-1}\) to be relatively more negative in winter and more positive in summer than those for an initial phase speed of 5 m s\(^{-1}\) at altitudes above ~80 km. Despite these variations, the general consistency of the flux profiles, whether large or small, with the mean values anticipated from observations lends confidence that variable forcing will nevertheless produce a reasonable mean response.

b. Dependence on initial and maximum anisotropies

Momentum flux profiles obtained for winter and summer zonal mean winds with initial anisotropies of \(b_x = -0.2, 0.0, \text{ and } 0.2\) and an initial energy corresponding to a mean phase speed \(c_{\omega_0} = 5 \text{ m s}^{-1}\) are compared in Fig. 6. A similar comparison with \(b_x = 0.0\) for \(\alpha = 0.1, 0.2, \text{ and } 0.3\) is provided in Fig. 7 for altitudes below 40 km. The profiles displayed in Fig. 6 reveal that initial anisotropies have very little impact on the momentum fluxes if they are consistent with the tendency for creation of anisotropy due to mean wind changes with height. Indeed, the profiles displayed in Fig. 6 for \(b_x = -0.2\) and 0.0 are nearly identical at all altitudes, while that for \(b_x = 0.2\) yields a more positive momentum flux for both winter and summer wind profiles up to ~50 km because of a delayed suppression of \(E_r\) relative to \(E_n\) by lower-level shears. Results are not presented above 40 km because all values of \(b_x\) yield virtually identical momentum fluxes at greater altitudes due to the limits placed on the anisotropy by finite \(\alpha\). Nevertheless, the initial anisotropy may be expected to play a larger role in cases where the mean wind at lower levels in that direction of propagation is not strongly sheared.

In contrast to the negligible influences of \(b_x\) at greater altitudes, the maximum allowed anisotropy expressed through \(\alpha\) has significant effects at all levels, with decreasing \(\alpha\) increasing both the maximum fluxes and the altitudes at which these maxima occur. These departures arise because a large allowed anisotropy implies a potentially very small fraction of the total energy propagating in one direction and a correspondingly large depth required for that component, even propagating upward in a conservative fashion, to recover a competitive energy and momentum flux. Assuming a fraction of zonal energy \(\alpha\) growing at the maximum rate, with the zonal energies constrained by Eqs. (35) and (36), the height interval \(\Delta z\) needed for the smaller component to reach equal energy may be expressed as

\[
\alpha E_0 = \alpha e^{\Delta z/H} = (1 - \alpha)E_0 = e^{\Delta z/H_E} - \alpha e^{\Delta z/H},
\]

yielding

\[
\Delta z = \left( \frac{1}{H} - \frac{1}{H_E} \right)^{-1} \ln(2\alpha)^{-1}.
\]

This implies recovery depths of ~0.9H, 1.62H, and 2.85H for \(\alpha = 0.3, 0.2, \text{ and } 0.1\), assuming \(H_E = 2.3H\), and suggests that large anisotropies are extremely difficult to overcome once established. This accounts for the maintenance of larger negative and positive momentum fluxes at greater heights in the winter and

![Momentum Flux Profiles](image)

**FIG. 6.** Momentum flux profiles obtained with \(b_x = -0.2\) (short dash), 0.0 (solid), and 0.2 (long dash), \(c_{\omega_0} = 5 \text{ m s}^{-1}\), and \(\alpha = 0.2\) below 40 km for (a) winter and (b) summer mean winds. Profiles are virtually identical at greater altitudes due to constraints on anisotropy.
summer zonal mean wind fields for $\alpha = 0.1$, despite the reversals of the zonal winds and the rapid growth of the smaller spectral components at upper levels. Similarly, a larger $\alpha$ implies smaller and generally less variable momentum fluxes at all altitudes.

c. Implications of variable $H_E$

We now consider briefly, for completeness, the consequences for energy and momentum fluxes of a variable $H_E$ with height, as suggested by the observations of Justus and Woodrum (1973) and Balsley and Garrotto (1985). Such variations, if large, may impact significantly the distribution of energy and momentum flux divergences and should likely be included in any attempt to incorporate gravity wave forcing in large-scale models.

We estimate the degree of variability of the scale height $H_E$ from the observed decrease of energy per unit volume with height and the corresponding scale height for density in the same altitude interval. This yields a range of $H_E$ varying about a mean of $\sim 2.3H$ by $\sim H$. Thus, to illustrate the effects of this variability, we model the altitude variations as
\[ H_E = \left[ 2.3 + \eta \sin \left( \frac{2\pi z}{60} \right) \right] H, \] (45)

with \( z \) in kilometers. The momentum fluxes obtained for \( H_E \) profiles with \( \eta = 0 \) and 0.8, \( c_{e_0} = 5 \text{ m s}^{-1} \), \( b_x = 0.0 \), and \( \alpha = 0.2 \) are shown in Fig. 8. These reveal similar momentum flux profiles, with a tendency for maxima to occur at slightly lower altitudes and with slightly reduced and enhanced amplitudes at lower and upper altitudes, respectively. The more significant differences between the results for constant and variable \( H_E \) obtain from the stronger constraints on wave ener-
gies (and fluxes) where \( H_E \) is large and are more visible in the vertical flux divergences discussed below. Note also that Eq. (17) implies \( \epsilon = 0 \) at 45 and 105 km for \( \eta = 0.8 \), assuming that \( N \) is constant.

\[ d. \text{ Profiles of energy dissipation rate and induced zonal acceleration} \]

We turn, finally, to the vertical profiles of energy dissipation rate and momentum flux divergence implied by the spectral variability and responses to mean wind variations discussed above. These are illustrated

\[ \text{Fig. 8. Momentum flux profiles obtained with } c_{e_0} = 5 \text{ m s}^{-1}, b_x = 0.0, \text{ and } \alpha = 0.2 \text{ for canonical winter and summer profiles at (a) and (b) 40–120 km and (c) and (d) 0–40 km with constant (solid) and variable (dash) } H_E. \]
for the canonical winter and summer profiles for the standard case with $c_{ao} = 5$ m s$^{-1}$, $b_x = 0.0$, $\alpha = 0.2$, and both constant and variable $H_E$ in Figs. 9 and 10.

Profiles of energy dissipation rate $\epsilon$ (Fig. 9) for constant and variable $H_E$ display approximately exponential increases with altitude. Departures from exponential growth for constant $H_E$ are due entirely to variations in $N$ and cessation of wave amplitude growth at upper levels. For variable $H_E$, these departures result additionally from variations in $H_E$ itself. The implications at lower levels are an energy dissipation rate for variable $H_E$ that falls below that for constant $H_E$ because the less rapid growth of wave energy (per unit mass) dominates the increased vertical divergence of wave energy flux where $H_E$ exceeds its mean value. At intermediate levels, the energy dissipation rate for variable $H_E$ increases much faster with increasing altitude because here $H_E$ is less than its mean value and $\epsilon$ is dominated by increases in the energy flux. This preferential growth of $\epsilon$ for variable $H_E$ causes it to exceed the values for constant $H_E$ into the mesopause region and above. The energy dissipation rates achieve values of $\sim 0.1-0.2$ m$^2$ s$^{-3}$ near the mesopause, in credible agreement with the larger values reported for in situ and radar estimates.

Fig. 9. Profiles of energy dissipation rate obtained with $c_{ao} = 5$ m s$^{-1}$, $b_x = 0.0$, and $\alpha = 0.2$ for canonical winter and summer profiles at (a) and (b) 40–120 km and (c) and (d) 0–40 km with constant (solid) and variable (dash) $H_E$ ($\eta = 0$ and 0.8).
(Hocking 1985; Blix et al. 1990a,b). Maxima are achieved near and above 100 km due to the greater constraints on wave amplitudes at greater altitudes, with somewhat larger values implied during summer with our choices of wind profiles and initial spectral energies.

Zonal acceleration profiles implied by momentum flux divergences (Fig. 10) display values at lower levels for both winter and summer mean winds that vary from $\sim +0.3 \text{ m s}^{-1} \text{ day}^{-1}$ below the tropospheric jet to $\sim -2 \text{ m s}^{-1} \text{ day}^{-1}$ near 20 km, in close agreement with observed values and those needed to provide the requisite decelerations in large-scale models (Fritts et al. 1990; Palmer et al. 1986; McFarlane 1987). Because of the finite intervals over which divergences were computed, these profiles were smoothed with a 3-point running average. As noted above there is a tendency for those profiles obtained for variable $H_E$ to maximize at lower levels and to have slightly smaller amplitudes due to the slower growth (larger $H_E$) of wave energies at these heights. Referring to the momentum flux profiles for variable $c_{e0}$ (Fig. 5), we see that the same gen-
eral profiles are implied, but with correspondingly smaller or larger accelerations associated with weaker or stronger sources.

At greater altitudes, there is a tendency for the induced accelerations, like the momentum fluxes themselves, to oppose the mean zonal winds for both winter and summer profiles. The peak accelerations for the $c_{z0} = 5 \text{ m s}^{-1}$ source strength are $\sim \pm 100 \text{ m s}^{-1} \text{ day}^{-1}$ below the mesopause, while the altitude profiles of momentum flux and divergence exhibit responses to the variable mean motions that are in good agreement with the limited observations that are available (Vincent and Reid 1983; Reid and Vincent 1987; Fritts and Vincent 1987; Fritts and Yuan 1989; Wang and Fritts 1990). As at lower levels, maxima occur at somewhat lower altitudes for variable $H_E$ although maximum values tend to be slightly larger. Especially encouraging, perhaps, is that the momentum flux and induced acceleration profiles correspond closely with observations in that region of the atmosphere that is believed to be most strongly forced by such wave motions. This suggests that the parameterization scheme has accounted, at least approximately, for the spectral energy changes caused by variable mean winds at lower altitudes.

7. Summary and conclusions

We have used a spectral model of gravity waves in the atmosphere introduced by FV93 to develop a parameterization for gravity wave fluxes of energy and momentum arising due to variable stability and mean winds. This parameterization relies on a separable and invariant spectral form that is consistent with observations throughout the atmosphere. For representative gravity wave parameters, the scheme yields predictions of mean fluxes of momentum for winter and summer CIRA 1986 profiles that agree well with mean flux observations at lower and upper levels.

In the troposphere and lower stratosphere, both winter and summer profiles imply a negative zonal mean flux (per unit mass) of $\sim -0.2 \text{ m}^2 \text{s}^{-2}$, consistent with airborne and radar measurements and the needs of large-scale models. In the mesosphere, mean fluxes tend to oppose the zonal mean wind with flux maxima above the wind maxima and vertical profiles in general agreement with radar estimates at these heights. Extreme values allowed by the model are likewise in reasonable agreement with the maximum values measured under conditions of strong wave forcing and modulation.

Vertical divergences of energy and momentum fluxes lead to mean energy dissipation rate and zonal acceleration profiles that exhibit structure that agrees well with observations, with $\epsilon \sim 0.1$–0.2 $\text{m}^2 \text{s}^{-3}$ near the mesopause and accelerations of $\sim -2$ and $\pm 100 \text{ m s}^{-1} \text{ day}^{-1}$ near the tropopause and mesopause, respectively.

Energy and momentum flux profiles for varying initial spectral energies and isotropies, intended to permit sensitivity to variable sources and source strengths at lower levels, show that the parameterization scheme responds as expected to increases and decreases in the intrinsic phase speeds of the gravity wave spectrum. This is manifested as lesser (greater) sensitivity to varying mean winds for larger (smaller) initial phase speeds. Initial anisotropy was found to be unimportant at greater altitudes because of the strong influences of mean wind shears on the spectral evolution at lower levels where phase speeds are small. In contrast, the maximum allowed anisotropy within the wave field was found to have large effects on the momentum flux profiles, both because it limits the maximum permitted fluxes and because a large anisotropy permits very disparate component energies and potentially large recovery depths for those components suppressed by strong adverse wind shears.

Finally, the effects of a variable scale height $H_E$ for gravity wave energy were examined because of the evidence for such variations in the atmosphere. This was found to suppress fluxes and divergences at lower levels and to increase the magnitudes and lower the altitudes of maximum responses at upper levels.

In summary, we have introduced a scheme for the parameterization of gravity wave propagation, energy and momentum fluxes, and effects in the atmosphere that appears to respond in a physical manner to variations in atmospheric density, stability, and mean winds. The scheme has the ability to describe the responses to variable source strengths and anisotropies, to variable (local and maximum) anisotropies at greater heights, and to time-dependent as well as mean motions. The primary parameters are initial energy $E_q$ (or phase speed) that is specified based on local source strength, initial and maximum anisotropies, $b_{x,y}$ and $\alpha$, specified based on source type (i.e., orographic, convective, wind shear) and model sensitivity, and the scale height $H_E$ for gravity wave energy variations with altitude. With these parameters specified, the gravity wave field at any point requires only four variables, the $E_j$, for complete specification of its variations and effects at greater altitudes. As a further example of this scheme, the response of the parameterization to canonical tidal structures, both stationary and time dependent, superposed on the CIRA 1986 winter and summer profiles, are used to examine the gravity wave–tidal interaction in a companion paper by Lu and Fritts (1993).

The parameterization scheme introduced in this paper is highly simplified in its present form in order to ensure a minimum number of parameters needed for its application. There are, nevertheless, other processes that depend on gravity wave fluxes and divergences that cannot be described by the present scheme but that are likely to lead to important effects throughout the atmosphere. Examples include 1) the possible tidal
modulation of gravity wave energy fluxes and the associated diurnal variability of turbulent diffusion and constituent transports, 2) variations of $H_g$ with the local mean wind environment and/or season, and 3) latitudinal variations of the canonical spectra due to the influences of varying $f$, among others. To account for these effects, it will be necessary to expand the number of parameters describing the gravity wave fluxes and develop new schemes for relating them as physically as possible to variable wave sources and environments. Such additional efforts are unlikely to be productive, however, until the limitations of the present scheme have been identified through applications in existing models.

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