NOTES AND CORRESPONDENCE

The Stability of Cloud Top with Regard to Entrainment: Amendment of the Theory of Cloud-Top Entrainment Instability

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ABSTRACT

The stability of a uniformly saturated cloud layer separated from an overlying nonturbulent unsaturated layer by a thin inversion is considered. The stability of the interface can be described by entraining a parcel of air from above the inversion into the cloud layer below and by subsequently studying the effect of the mixing on the buoyancy of the parcel. From the relevant momentum equation for the parcel it is shown that the important quantity to consider is the total buoyancy (total mass of the parcel times the virtual potential temperature difference between parcel and environment) of the parcel per unit mass of entrained air. The total buoyancy is a more general and useful concept than all other parameters discussed in the literature.

1. Introduction

Stratocumulus fields typically form under strong subsidence inversions associated with subtropical and midlatitude high pressure systems. The stability of the interfacial layer between a cloud deck and the overlying cloud-free air is of great importance for predicting the horizontal extent of these stratocumulus-topped boundary layers. As a result of the cloud deck the stability analysis of this interfacial layer is complicated due to the occurrence of evaporation and radiation.

Cloud edges generally form a very sharp interface between vigorous turbulence inside the cloud and the quiet air outside the cloud. At the cloud edge this turbulent motion mixes cloud-free air from outside the cloud into the cloud, a process called entrainment. Turbulent entrainment ordinarily cancels out turbulent kinetic energy, because the buoyancy force counteracts vertical motion in the statically stable layer at the edge of the turbulent region.

Under dry-adiabatic conditions entrainment can generate turbulent kinetic energy if the interface is unstable. Under cloudy conditions entrainment can be influenced by radiative cooling and evaporative effects. Radiative effects are thought to have a small direct effect on the entrainment. The importance of the radiative effect is mainly indirect—it drives the convection and as such produces turbulence, which will finally promote the entrainment (Nicholls 1984; Nicholls and Turton 1986; Nicholls 1989). Lilly (1968) pointed out that the evaporative cooling of unsaturated air that has been entrained into the cloud can, under some conditions, cause this air to sink unstably as a convective down-draft. The process is referred to as cloud-top entrainment instability (CTEI) (Randall 1980; Deardorff 1980; Hanson 1984; Kuo and Schubert 1988; Siems et al. 1990; MacVean and Mason 1990; Weaver and Pearson 1990).

In the cloud-free case the stability can be characterized by the difference between the virtual potential temperature above and below the interface. Lilly (1968) constructed a simple model for the stratocumulus-topped boundary layer. He argued that the interface between the cloud layer and the overlying air will be stable only if the equivalent potential temperature remains constant or increases at the cloud top. The shortcoming of Lilly's stability parameter was that it did not include the effects of water vapor and liquid water on buoyancy. These additional effects were taken into account by Randall (1980) and Deardorff (1980); however, they restricted their study to the case in which a parcel after mixing remained just saturated. This yielded a theory that states that instability will only occur whenever the thermodynamic properties across the interface are unstable with respect to the wet-adiabatic values.

In this paper we modify the thermodynamic theory of cloud-top evaporative instability. We will use arguments similar to those given by Nicholls and Turton (1986) and Albrecht et al. (1985). It will be shown that this process, as well as all stability criteria discussed in the literature, can be depicted in a simple diagram. The limitations of all the criteria will be discussed and an improved stability criterion will be presented. The new stability criterion applies to both cloud-free and cloudy cases. The theory also yields a parameter for
the degree of stability of the interface, which can therefore be used in entrainment studies.

2. Theory

We consider a turbulent uniformly saturated cloud layer separated from an overlying nonturbulent unsaturated layer by a thin transition layer. In the analysis we will assume that this transition layer (the inversion) is infinitesimally thin. Across this transition layer the temperature, specific humidity, liquid water content, and turbulence properties change sharply. The overlying layer will be denoted by 1 and the cloud layer by 2.

We study the entrainment of a parcel with mass $m_1$ and subsequent mixing with a parcel of mass $m_2$. The mixed parcel with mass $m_1 + m_2$, with $m_1$ constant and $m_2$ a function of time, has a virtual potential temperature that is $\delta \theta_m$ higher than its surroundings. Moreover, if the mass $m_2$ has no initial momentum (entrainment of momentum is neglected), then the simplified momentum equation can be written as

$$\frac{dw(m_1 + m_2)}{dt} = \frac{g}{T}(m_1 + m_2)\delta \theta_m, \quad (2.1)$$

where $g$ is the acceleration due to gravity and $T$ is the temperature of the reference state. Normalizing the masses with $m_1$ we obtain

$$\frac{dw/X}{dt} = \frac{g}{T} \delta \theta_m/X, \quad (2.2)$$

where $X = m_1/(m_1 + m_2)$ is the mass mixing ratio. The important quantity to consider is thus the total buoyancy of the parcel $[(m_1 + m_2)\delta \theta_m]$ per unit mass of the entrained air ($m_1$). This contrasts with the buoyancy per unit mass ($\delta \theta_m$) that has been studied by most previous investigators (Nicholls and Turton 1986; Siems et al. 1990). It is not entirely clear what process they had in mind but $\delta \theta_m$ is the important quantity to consider when the initial momentum ($w_2$) of the surrounding air is the same as that of the parcel under consideration. In this case the relevant momentum equation can be written as

$$(m_1 + m_2) \frac{dw}{dt} = \frac{g}{T}(m_1 + m_2)\delta \theta_m. \quad (2.3)$$

We are interested, however, in how the momentum of an entrained and mixed parcel changes relative to its environment; therefore, (2.1) and (2.2) are the relevant momentum equations. Integration of (2.2) with respect to time gives

$$(m_1 + m_2(t))w(t) = m_1w(0) + m_1 \int_0^t \frac{g}{T} \delta \theta_m/X dt, \quad (2.4)$$

so the important quantity to consider is the integral of the total buoyancy of the parcel $(m_1 + m_2)\delta \theta_m$ per unit mass of entrained air ($m_1$) with respect to time.

The difference between (2.2) and (2.3) is thus the result of the assumption concerning the vertical velocity ($w_2$) of the cloudy air surrounding the parcel. If the vertical velocity of the cloudy air is at all times equal to that of the parcel, (2.3) is the equation to use. Physically, however, this assumption is quite unrealistic. It is more realistic to assume that the cloudy air surrounding the parcel is motionless. Therefore, the air that is mixed with the parcel has no initial momentum and as a result, (2.2) is the better equation.

a. Buoyancy change due to entrainment and mixing

In this section we will consider the entrainment phase and mixing phase separately, as depicted in Fig. 1. We will take the parcel from above the interface Ø and drag it into the lower layer Ø. We call this the entrainment phase. The mixing takes place after this phase. It is assumed that the mixed parcel always remains well mixed; that is, the thermodynamical properties do not vary within the parcel.

We calculate the relative increase in the buoyancy of the system (we consider only layer 2) from the dif-

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**FIG. 1.** Schematic illustration of the entrainment and mixing of a parcel.
In the virtual potential temperature after and before a physical process has occurred:
\[
\frac{1}{m_1} \left( \int_m (\theta_e)_a dm - \int_m (\theta_e)_b dm \right),
\] (2.5)

where \(m\) stands for mass, \(m_1\) is the mass of the entrained parcel, and \((\theta_e)_a\) and \((\theta_e)_b\) are the virtual potential temperatures \((a)\) after and \((b)\) before the physical process took place.

For the entrainment phase we substitute \((\theta_e)_a = \theta_{e1}\), \((\theta_e)_b = \theta_{e2}\), and \(m = m_1\) into (2.5), which gives
\[
\Delta \theta_e,
\] (2.6)

where the operator \(\Delta\) stands for the value of the virtual potential temperature above the inversion minus the value below the inversion. For the mixing process we obtain for the two integrals between the square brackets in (2.5): \((m_1 + m_2)\theta_{em}\) and \(m_1\theta_{e1} + m_2\theta_{e2}\), respectively. As a result for the mixing process (2.5) becomes
\[
\frac{\delta \theta_{e2}}{x} = \Delta \theta_e,
\] (2.7)

where \(x\) is the mass mixing ratio \(m_1/(m_1 + m_2)\) and \(\delta \theta_{e2} = \theta_{e2} - \theta_{em}\), in which \(\theta_{em}\) is the virtual potential temperature of the mixture. The total change in buoyancy due to entrainment and mixing is therefore given by
\[
\frac{\delta \theta_{e2}}{x}.
\] (2.8)

It is this measure that is important for the stability of the inversion layer. Therefore, this quantity will be discussed further in the next section.

We will now summarize the physical properties of the various quantities obtained. The quantity \(\delta \theta_{e2}\) is the virtual potential temperature difference between the parcel and the cloudy environment, whereas \(\delta \theta_{e2}/x\) is the buoyancy of the whole parcel per unit mass of entrained air \((m_1)\). For example, in the case in which both layers consist of moist (without liquid water) air and no condensation develops during the mixing, the temperature difference between the parcel and its surroundings is \(\delta \theta_{e2} = x\Delta \theta_e\). In this case the quantity \(\delta \theta_{e2}/x\) equals \(\Delta \theta_e\). The total buoyancy per unit mass of entrained air is thus independent of the mass mixing ratio \(x\).

b. Stability diagram

Before going into details of how to define a stability criterion for the cloud top, we will take a closer look at the properties and limits of (2.8). For the case in which both layers consist of moist air without any liquid water and no condensation develops during the mixing \((\delta \theta_{e2} = x\Delta \theta_e)\), it is immediately clear from (2.7) that no buoyancy change occurs as a result of the mixing, so the only buoyancy change we have to consider is due to the entrainment.

In the general case where phase changes occur it is obvious that there is a buoyancy change due to the mixing. This is because the virtual potential temperature of the mixture \((\theta_{em})\) is not simply a mass-weighted average of the virtual potential temperature above and below the inversion. A derivation of \(\theta_{em}\) and \(\delta \theta_{e2}\), for the case where the lower layer is saturated and the upper layer contains no liquid water, is given in appendix A. In Fig. 2 we have shown \(x\Delta \theta_e\), \(\delta \theta_{e2}\), and \(\delta \theta_{e2}/x\) as a function of \(x\) for the case where \(\Delta \theta_e = -5\, \text{K}, q_2 = 0.5\, \text{g kg}^{-1}\), and \(\Delta q = -2.8\, \text{g kg}^{-1}\) \((T_2 = 273\, \text{K} \text{ and } p_2 = 900\, \text{hPa})\). All three curves pass through the point \(x = 1\), \(\Delta \theta_e\) because in this case \(m_2 = m_1\) and the mixed parcel will retain the original properties it had when it was situated above the inversion. The buoyancy excess is therefore simply \(\Delta \theta_e\). For \(x = 0\) it can be shown that
\[
\left( \frac{\delta \theta_{e2}}{x} \right)_{x=0} = \Delta \theta_e = \frac{\Gamma_m}{\Gamma_d} \Delta \theta_e - \theta_{e2} \Delta q_T,
\] (2.9)

where \(\Gamma_m\) is the wet-adiabatic lapse rate at \(T_m\) (Duynkerke and Driedonks 1987), \(\Gamma_d\) is the dry-adiabatic lapse rate, and \(q_T\) is the total water specific humidity \((q_T = q + q_1)\). Randall (1980) and Deardorff (1980) assumed that instability would occur if
\[
\Delta \theta_e < 0.
\] (2.10)

The values \(\delta \theta_{e2}/x\) for small values of \(x\) (up to about 0.2 in Fig. 2) are nearly independent of \(x\) because the ratio of the wet to dry adiabatic lapse rate is nearly independent of temperature, in this case the temperature of the mixed parcel \((T_m)\). In this range of mass mixing ratios the mixed parcel remains saturated \((q_{1m} > 0)\). It can be seen that the stability parameter \((\Delta \theta_e)\) represents the stability for only a small range of mass.

![Fig. 2. Virtual potential temperature difference as a function of the mass mixing ratio x for \(\delta \theta_{e2} = -5\, \text{K}, q_2 = 0.5\, \text{g kg}^{-1}\), and \(\Delta q = -2.8\, \text{g kg}^{-1}\).](image-url)
mixing ratios. Therefore, it is not surprising that this
stability parameter is not adequate to explain cloud-
top entrainment instability. In the next section we will
discuss how this parameter can be improved.

c. Stability parameter

The functions $x \Delta \theta_{e2}$ and $\Delta \theta_{o2}$ in Fig. 2 have already
been shown by Nicholls and Turton (1986). It is clear
from Fig. 2 that any indicator of cloud-top interfacial
stability should take into account the density fluctua-
tions resulting from evaporative cooling during mixing.
As discussed in section 2b, the range of possible values
cannot be adequately represented by only two points
on the curve, $\Delta \theta_e$ and $\Delta_2$, which correspond to only
two special circumstances—no mixing and mixing
whereby saturation occurs.

Nicholls and Turton (1986) assumed that all values
of $x$ are equally possible within the entrainment inter-
face layer (EIL). On the assumption that all values
of $x$ are equally important in determining the interfacial
stability, they defined

$$\Delta_m = 2 \int_0^1 \delta \theta_{o2} dx. \quad (2.11)$$

The numerical factor 2 ensures that $\Delta_m = \Delta \theta_{o2}$ in the
cloud-free case. A disadvantage of $\Delta_m$ is that even in
the cloudy case its value is very close to $\Delta \theta_{o2}$, which
means that a cloud-top entrainment instability criterion
based on $\Delta_m$ will predict that instability will occur under
conditions that closely resemble those found in a moist
but unsaturated atmosphere.

Because, as shown in section 2b, $\delta \theta_{e2}/x$ (rather than
$\delta \theta_{o2}$) is the important quantity, we define a new stability
parameter:

$$\Delta_a = \int_0^1 \delta \theta_{e2}/xdx. \quad (2.12)$$

This can be interpreted as follows. We entrain a parcel
with mass $m_1$. At the time of entrainment no mixing
has taken place ($x = 1$). As soon as mixing takes place,
$x$ decreases until so much surrounding air has mixed
with the parcel that $x$ approaches zero. During the
whole mixing process the buoyancy excess of the parcel
per unit mass of entrained air ($m_1$) is given by $\delta \theta_{e2}/x$.
The stability parameter $\Delta_a$ therefore represents the time
integral of the buoyancy excess during the lifetime of the
parcel from its unmixed state to a state in which its
properties are overwhelmed by the properties of the
surrounding air.

Under the condition that $\Delta_a < 0$, entrainment and
evaporative mixing together promote convection. If $\Delta_a < 0$, typically the parcel still needs energy in order to
be entrained, as shown in (2.3). The cloud layer there-
fore has to be turbulent to supply this energy. After the
entrainment the mixing (due to turbulence) gives way
to evaporative cooling and the potential energy can be
converted to kinetic energy and thus to convection.
When $\Delta_a < 0$, there is a net conversion of potential
energy into kinetic energy throughout the transformation
during which the parcel is entrained and changes from its unmixed to its completely mixed state. Typically, some level of active turbulence is necessary
to maintain entrainment and mixing. Once $\Delta_a < 0$,
however, the entrainment and mixing increase spontaneous-
y.

d. Comparison with other CTEIs

MacVean and Mason (1990) also studied the cloud-
top entrainment instability mechanism. They were not
so much interested in the general stability of the in-
terface as they were in the conditions under which
cloud-top entrainment instability occurs. They did not
really investigate entrainment but they studied the
possible conditions for free mixing at an interface with
different thermodynamical properties. Their criterion
for instability is a net conversion of potential energy
to kinetic energy.

The difference between their approach and mine is
that they investigated what I would call free mixing.
They assumed that not only a parcel with mass fraction
$x$ from layer 1 is exchanged across the interface with
layer 2, but also a parcel with mass fraction $x$ from
layer 2 is exchanged with layer 1. This process is not
typical for entrainment in which only a parcel from
the overlying layer is pulled downwards into the lower
layer 2. The entrainment process depends upon the
turbulence that supplies the kinetic energy required
to pull the parcel down through the interface.

The process that MacVean and Mason (1990) studied
is also depicted in Fig. 2. The curve $\delta \theta_{e2}/x$ repre-
sents the parcel that is entrained from the upper layer
into the cloud layer and subsequently mixed. For the
parcel that is "entrained" from the cloud layer into the
overlying layer a similar expression can be obtained:
$\delta \theta_{e1} = \theta_{e1} - \theta_{en}$. The derivation of this expression is
given in appendix A and $\delta \theta_{e1}/x$ is shown in Fig. 2.
MacVean and Mason (1990) considered the limit of
$x \rightarrow 0$ and assumed that CTEI would occur if there
were a net conversion of potential energy into kinetic
energy, which is fulfilled under the condition that (for
their $H_1 = H_2$ case)

$$(\delta \theta_{e2}/x)_{x=0} + (\delta \theta_{e1}/x)_{x=0} < 0, \quad (2.13)$$

which can, with the substitution of (2.5), be written as

$$\Delta_2 + (\delta \theta_{e1}/x)_{x=0} < 0, \quad (2.14)$$

which can, if $q_{li} = 0$, be rewritten as

$$\Delta \theta_e < \Delta q_T \left[ \frac{1 + \frac{1}{c_p} \left( \frac{\partial q_T}{\partial T} \right)}{2 + \left( \frac{\partial q_T}{\partial T} \right)} \right].$$

(2.15)

This is Eq. (13) of MacVean and Mason (1990).
The result obtained by MacVean and Mason (1990) is thus based on the exchange of small mass fractions. It does not cover all the \( x \) values that can and will occur in reality. Moreover, they assume that a parcel is exchanged from the cloud layer into the overlying layer, a process that has nothing to do with entrainment. Therefore, from their analysis it is not possible to derive a general stability parameter to describe entrainment. Even if, according to their criterion (2.11), the inversion was unstable, energy would be needed to get the parcel from the cloud layer into the overlying air. Moreover, once the parcel is in the overlying layer it will be unable to mix with the surrounding air, because the overlying layer is not turbulent.

The most likely process leading to CTIE thus seems to be entrainment (and not free mixing) in which parcels of air are pulled into the cloud. This requires energy, but due to the evaporative cooling of the parcel there are situations under which the parcel can gain more energy than it needed to become entrained.

In Fig. 3 we have shown the criteria of MacVean and Mason (1990) and \( \Delta_x < 0 \) for \( q_{12} = 0.5, 1.0, \) and \( 1.5 \) g kg\(^{-1}\). For \( q_{12} = 0 \) the criterion \( \Delta_x = 0 \) reduces to the dry-adiabatic criterion (dotted line in Fig. 3). As \( q_{12} \) increases, the stability line moves farther to the right of the diagram. In the upper right corner the instability line merges with the wet adiabatic criterion (\( \Delta_x = 0 \)) because under these conditions (\( \Delta q, \Delta q_T \)) the upper layer is saturated too. If the liquid water content in the lower layer were to approach large values (\( q_{12} \rightarrow \infty \)) the new stability criterion would reduce to that for the wet-adiabatic criterion (\( \Delta_x = 0 \)). The new stability criterion can be used to determine the stability of the interface under all conditions (cloudy and cloud-free) and is applicable to the whole range of possible states.

Siems et al. (1990) used the argument that if the minimum in the curve of \( \Delta \theta_{e2} \) versus \( x \) (Fig. 2) decreases below a certain value entrainment instability should occur. If we denote this minimum by \( \Delta \theta_{e2} = (\Delta \theta_{e2})_{\text{min}} \) and \( x = (x)_{\text{min}} \) their stability criterion can be written in our notation as

\[
D = -\frac{(\Delta \theta_{e2})_{\text{min}}}{\Delta \theta_{e2}} > D_c. \tag{2.16}
\]

Siems et al. (1990) present physical arguments why \( D_c \) should be of order unity and they determined a specific value of 1.3 from laboratory experiments with liquids. In their numerical experiments, which were of more relevance to cloud-top entrainment instability, the critical value was much less sharply defined. Moreover, due to instability the entrainment velocity increased by only a factor of 2.

It is of interest to see how their criterion relates to the criterion discussed in this paper. If \( 0 < x < (x)_{\text{min}} \), the curve for \( \Delta \theta_{e2} \) can be written as

\[
\Delta \theta_{e2} = x \Delta_2. \tag{2.17}
\]

Therefore, (2.16) can be rewritten in terms of \( (x)_{\text{min}} \) and \( \Delta_2 \):

\[
D = -\frac{(x)_{\text{min}} \Delta_2}{\Delta \theta_{e2}} > D_c. \tag{2.18}
\]

It should first be noted that \( D > 0 \) corresponds to the criterion (2.10) originally proposed by Randall (1980) and Deardorff (1980). The curve for \( \Delta \theta_{e2} \) in Fig. 2 can be approximated by two straight lines, from which \( (x)_{\text{min}} \) can be calculated. It is straightforward to show that \( D = D_c \) corresponds to

\[
-D_c I_1 I_2 (\Delta q_T)^2 + \left( D_c \left( I_1 (\Delta \theta_e - q_{12} I_1) - \frac{I_m}{I_d} - 1 \right) \Delta \theta e I_2 \right) + \theta \frac{q_{12} I_1}{I_d} \Delta q_T + D_c \left( \frac{I_m}{I_d} - 1 \right) \Delta \theta e (\Delta \theta_e - q_{12} I_1)
\]

\[
- \frac{I_m}{I_d} \Delta \theta e q_{12} I_1 = 0, \quad \text{with:} \quad I_1 = I_1 / c_p
\]

\[
I_2 = I_2 / c_p - \psi \theta e. \tag{2.19}
\]

Given the values of \( \Delta \theta_e \) and \( q_{12} \), the value of \( \Delta q_T \) can be calculated analytically from (2.19), the results of which (for \( D_c = 1.3 \) and \( q_{12} = 0.5, 1.0, \) and \( 1.5 \) g kg\(^{-1}\)) are shown, from left to right, in Fig. 4. The criterion proposed by Siems et al. (1990) is therefore more restrictive than that proposed in this paper.

3. Discussion

The stability criterion divides the \( \Delta \theta_e, \Delta q_T \) plane as shown in Fig. 3. If (2.12) is the correct stability condition, we should find that all observations taken under persistent stratocumulus conditions lie to the right of
the dividing line. In order to give an idea of where the data are typically situated we have plotted the data of Weaver (1987) in Fig. 3. These data were all collected in solid stratocumulus. The data are grouped into two categories: soundings (circles) and horizontal legs (triangles). In the first case the aircraft took soundings while flying at constant heading. High vertical resolution profiles were obtained, and the data were plotted in a mixing diagram (a mixing line analysis was used to find the jumps in thermodynamical properties). The second method of obtaining the vertical structure is to consider horizontal legs at cloud top. The plane flies in and out of the cloud and provides multiple samples of the EIL. During all the observations presented in Fig. 3 the stratocumulus clouds showed no sign of breakup and as such are representative for clouds with a thermodynamically stable inversion.

It is now tempting to plot all observational data in the diagram and check whether we have found the correct stability criterion (Kuo and Schubert 1987; MacVe and Mason 1990). We should be very careful in doing this, however, because there are many other mechanisms that can cause the cloud to dissipate. One mechanism is subsidence, which, if sufficiently strong, can lower cloud top below the lifting condensation level (LCL) and make the cloud disappear (Weaver and Pearson 1990). Another mechanism is the solar absorption in the cloud, which can cause a diurnal variation in cloud thickness (Bougeault 1985; Duykerke 1989; Betts 1990; Hignett 1991; Blaskovic et al. 1991). Therefore, the existence or dissipation of a stratocumulus cloud cannot depend simply on whether the CTEI criterion is met or not.

One might also try to use the CTEI mechanism to characterize the boundary between the stratocumulus and (trade) cumulus regime. This boundary should then be characterized by points close to or just to the left of the stability line in Fig. 3. The process one has in mind is as follows: If the air flows toward the equator over increasingly warm water, boundary-layer \( \theta_e \) values will increase faster than the above-cloud values and the \( \Delta \theta_e \) jump will eventually cross from the stable to the unstable side of the stability line. As a consequence cloud patterns will change from stratocumulus to (trade) cumulus.

One should be very careful about drawing such a comparison, however, because in the case of (trade) cumulus it is very difficult to define the cloud-top jump properties. Typically (Kuo and Schubert 1988), these properties are found from the difference between the values, in temperature and moisture, of the above-cloud- and subcloud-layer air. In light of the theory presented in section 2 of this paper (for a solid cloud layer only) it is not clear whether this is a valid approach, especially because in the cloud layer the horizontally averaged \( \theta_e \) and \( q_v \) values are usually not constant with height. A comparison of these observations with theories for solid cloud layers should therefore be made with some caution. Moreover, there are other theories that can explain such a transition, such as the organization of convection whereby there is an optimal conversion of potential energy into kinetic energy (Asai and Kasahara 1967; Randall 1989).

4. Conclusions

By analyzing the simplified momentum equation for a parcel that is entrained and subsequently mixed with the surrounding air, we have found a buoyancy parameter that is relevant for this process. This parameter is the total buoyancy of the parcel per unit mass of entrained air (\( \Delta \theta_{em}/x \)). The parameter presented in this paper differs from the one used by investigators up till now, namely the buoyancy per unit mass (\( \Delta \theta_{em} \)).

Using the new buoyancy parameter (\( \Delta \theta_{em}/x \)) the process of evaporative cooling of an entrained parcel is studied as a function of the mass mixing ratio. All the other parameters discussed in the literature can be shown to be simplifications of this more general parameter.

On the assumption that all mass mixing ratios are equally likely, we propose a new stability parameter for the interface. If the lower layer contains no liquid water the stability criterion reduces to the criterion for the moist but unsaturated case, the dry-adiabatic criterion. If the liquid water content in the lower layer is infinitely large (\( q_{12} \to \infty \)) the stability criterion reduces to that found by Randall (1980) and Deardorff (1980) (\( \Delta \theta < 0 \)), the wet-adiabatic criterion. Thus, as the liquid water content of the lower layer increases from zero upwards, the entrainment instability criterion changes gradually from the dry-adiabatic criterion to the wet-adiabatic criterion.

From the analysis given in section 2 it is clear that the total buoyancy conversion is given by the integral of \( \Delta \theta_{em}/x \) with time (2.4). The outcome of this integral...
will probably depend upon the mixing speed and thus upon the turbulence intensity and scales within the boundary layer. It is not clear how all these details can be incorporated in the relatively simple criteria discussed in this paper. It might also be a difficult to describe these processes with a numerical model. The model should at least resolve scales that are one or perhaps several orders of magnitude smaller than those of the entrained parcel, because the most effective mixing will probably occur at these scales. Therefore, one would need a large eddy simulation (LES) model with a high resolution.

The instability criterion needs to be carefully verified with data. Stratocumulus clouds can dissipate due to a variety of mechanisms: subsidence, diurnal variation, and decoupling. Data from the transition zone between the stratocumulus and cumulus region should be used with caution, because other processes, such as boundary-layer mixing, might cause this transition (Albrecht 1991). It is apparently insufficient to relate the type and amount of cloud to the thermodynamical structure at the inversion only.

There is clearly a need for more definitive observational data in order to validate the various criteria. We should not only try to solve the instability problem, but we should also be looking for parameters (for example, in terms of dimensionless Richardson numbers) that describe the stability of the interface and as such control the entrainment and evaporative cooling (Nicholls and Turton 1986; Turton and Nicholls 1987; Nicholls 1989).

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APPENDIX A

Virtual Potential Temperature Difference as a Result of Mixing

a. Mixing a parcel entrained from above the inversion into the cloud layer

In the absence of precipitation the variables $\theta_e$ and $q_T$ are conserved during isobaric mixing involving evaporation and condensation. The subscripts 1 and 2 refer to above and below the interface, respectively, and the subscript $m$ refers to the resulting mixture. Thus, with the mass mixing ratio $\chi = m_1/(m_1 + m_2)$, we get

$$\theta_{em} = \theta_{e1} + (1 - \chi)\theta_{e2}, \quad (A.1)$$
$$q_{Tm} = \chi q_{T1} + (1 - \chi)q_{T2}. \quad (A.2)$$

The virtual potential temperature of the mixture becomes

$$\theta_{vm} = \theta_{e1} + \psi q_{m} - \chi \theta_{e2} + \psi q_{e2}.$$  

where the terms of the order $1_q/c_p$ have been neglected compared to those of the order $\theta$. Replacing $q_m$ in (A.3) yields

$$\theta_{em} = \theta_{e1} - q_{m1}[1 + \psi] \theta_{e1} - 1/c_p]$$

$$+ \chi q_{Tm}(\psi \theta_{e1} - 1/c_p). \quad (A.4)$$

The buoyancy of the mixed state relative to state 2 is proportional to the difference

$$\theta_{em} - \theta_{e2} = \theta_{e1} - \theta_{e2} - (1 + \psi)[\theta_{em} - \theta_{e2}q_{12}2]$$

$$+ (q_{m1} - q_{12}) v/c_p + \psi[q_{Tm} - q_{T1}q_{e2} - q_{T2}q_{e2}]$$

$$- q_{Tm}v/c_p. \quad (A.5)$$

The term $\theta_{em}q_{em}$ can be expanded, using (A.1) and (A.2), as

$$\theta_{em}q_{em} - \theta_{e2}q_{12} = \chi[\mp (x - 1) \theta_{e2} - \chi \theta_{e1}]$$

$$\times \{q_{T2} - q_{T1} + q_{T1}(\theta_{e1} - \theta_{e2})\}. \quad (A.6)$$

Substitution of (A.6) into (A.5) gives

$$\theta_{em} - \theta_{e2} = \chi(\theta_{e1} - \theta_{e2})\{1 - q_{m1} + \psi(q_{T2} - q_{12})\}$$

$$+ \{q_{m1} - q_{12}\}(v/c_p - \psi \theta_{e2})$$

$$+ \chi[\mp (x - 1) \theta_{e2} - \chi \theta_{e1}](q_{m1} - q_{12})$$

$$+ \psi\chi^2(\theta_{e2} - \theta_{e1})(q_{T2} - q_{T1}). \quad (A.7)$$

Defining the difference $\Delta \theta = \theta_{e1} - \theta_{e2}$ (where $\theta_{e1}$ stands for $\theta_e$, $\theta_{T1}$, etc.) and neglecting terms of the order $\Delta \theta$ but retaining those of the order $\theta$ and $v/c_p$ gives

$$\theta_{em} - \theta_{e2} = \chi \Delta \theta_e - \chi \Delta q_{T1}(v/c_p - \psi \theta_{e1})$$

$$+ \{q_{m1} - q_{12}\}(v/c_p - \psi \theta_{e2}) \quad (A.8)$$

b. Mixing a parcel from the cloud layer with the air above the inversion

A similar expression can be derived if a mass $m_2$ is mixed with air above the interface. The mass mixing ratio is then defined as $\chi = m_2/(m_1 + m_2)$, and $\theta_{em}$ becomes

$$\theta_{e1} - \theta_{em} = \chi \Delta \theta_e - \chi \Delta q_{T1}(v/c_p - \psi \theta_{e1})$$

$$- \{q_{m1} - q_{12}\}(v/c_p - \psi \theta_{e2}). \quad (A.9)$$

We typically consider the upper layer unsaturated; that is, $q_{12} = 0$.

REFERENCES


