Balanced Dynamics of Mesoscale Vortices Produced in Simulated Convective Systems

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ABSTRACT

Long-lived, mesoscale convective systems are known to occasionally produce mesoscale convective vortices (MCVs) in the lower to middle troposphere with horizontal scales averaging 100–200 km. The formation of MCVs is investigated using fully three-dimensional cloud model simulations of idealized, mesoscale convective systems (MCSs), initialized with a finite length line of unstable perturbations. In agreement with observations, the authors find that environmental conditions favoring MCV formation exhibit weak vertical shear confined to roughly the lowest 3 km, provided the Coriolis parameter \( f \) is chosen appropriate for midlatitudes. With \( f = 0 \), counterrotating vortices form on the line ends, positive to the north and negative to the south with westerly environmental shear.

The MCV and end vortices are synonymous with anomalies of potential vorticity (PV). Using PV inversion techniques, the authors show that the vortices are nearly balanced, even with \( f = 0 \). However, the formation of mesoscale vortices depends upon the unbalanced, sloping, fronto-to-rear and rear inflow circulations of the mature squall line. End vortices form partly from the tilting of ambient shear but more from the tilting of the perturbation horizontal vorticity inherent in the squall line circulation. With the addition of earth’s rotation, an asymmetric structure results with the cyclonic vortex dominant on the northern end of the line. The key to this MCV formation is organized convergence above the surface cold pool and associated mesoscale ascent and latent heating. A simulated MCV can even form in an environment with no ambient shear.

Using a balanced model, the authors perform extended time integrations and show that the MCV produced in a sheared environment remains largely intact because the shear is confined to low levels and is relatively weak. In addition, the interaction of the vortex with the shear produces sufficient, mesoscale vertical motion on the downshear side of the vortex to trigger convection in typical, observed thermodynamic environments.

Results suggest that balanced dynamical arguments may elucidate the long-term behavior of mesoscale vortices. However, because the balance equations neglect the irrotational velocity contribution to the horizontal vorticity, the formation of the mesoscale updraft that leads to an MCV and the generation of vertical vorticity through vortex tilting are both treated improperly. Thus, the authors believe that existing balanced models will have serious difficulty simulating MCS evolution and mesoscale vortex formation unless mesoscale environmental forcing determines the behavior of the convective system.

1. Introduction

a. Background

One of the most interesting aspects of convection is its tendency to organize into structures much larger than individual convective elements. In practice, this organization is not spontaneous but builds upon some preexisting feature, such as a frontal boundary or variations in terrain. In the case of mesoscale convective systems (MCSs), the structure of the mature system may bear little physical resemblance to structures present in its incipient stage. Furthermore, the mesoscale features that evolve often persist for periods of time that exceed that of the ensemble convection from which they were spawned. It is this transition to much longer time and spatial scales that makes MCSs such interesting dynamical entities.

Observational radar studies have documented the now-familiar mature MCS structure (Houze 1989) consisting of a leading, nearly continuous line of convection, sometimes severe, and a trailing region of generally stratiform precipitation with embedded convective elements. The terms “leading” and “trailing” are relative to the motion of the convective line. The surface cold pool is a nearly ubiquitous and well-documented feature of MCSs. Its importance for enhancing the longevity of convective systems can be understood in the context of nearly two-dimensional flow (Rotunno et al. 1988, hereafter RKW). The maintenance of an intense system results from a quasibalance between the horizontal vorticity generated at the leading edge of the cold pool and vorticity of the mean shear. The cold pool by itself produces lifting of air parcels,
but it is the vorticity balance that allows strong updrafts to be continuously regenerated (for nonsupercellular squall lines). If the cold pool vorticity generation exceeds that available in the mean, low-level shear, the updraft will acquire an upshear tilt, and a stratiform precipitation region develops. This is common in environments of weak shear and large convective available potential energy (CAPE). The RKW formalism also predicts, as has been observed (Bluestein and Jain 1985; see also the interpretation by Houze et al. 1990), that squall lines forming in environments with strong or deep shear may not develop trailing stratiform regions, and these may be the most intense lines, perhaps being composed of supercell thunderstorms. Long-lived convective systems may arise through other mechanisms, where persistent ambient vertical motion provides both the destabilizing and focusing mechanisms for convection and the system proceeds through a sequence of regenerations (Trier et al. 1991).

Occasionally documented within mature MCSs is a cyclonic vortex in the lower or middle troposphere with a horizontal scale of between 50 and 200 km, a vertical extent of 2–8 km, and horizontal velocity perturbations of 10–15 m s\(^{-1}\) (Zhang and Fritsch 1987; Menard and Fritsch 1989; Brandes 1990; Bartels and Maddox 1991). This circulation is highlighted in the “asymmetric” conceptual model of convective systems presented by Houze et al. (1989), in which a mesoscale, cyclonic vortex dominates the flow over the northern half of the MCS. This implies a fully three-dimensional MCS structure, in contrast to the nearly two-dimensional “symmetric” MCSs discussed above. Although the presence of a vortex (hereafter referred to as mesoscale convective vortex, or MCV) is occasionally detected in standard rawinsonde data, the spacing of stations is often inadequate to detail its structure. The scale is often too large to be captured by radar data (although see Fig. 15 from Brandes 1990); hence, one often sees fragments of vortices instead of the complete structure because part of the circulation may extend outside the region of appreciable radar reflectivity. In addition, the existence of smaller-scale vortices can often obscure the larger-scale signal in radar data. Wind profilers have had some success in at least producing time-height cross sections of MCVs (Brandes 1990; Johnson and Bartels 1992), however, there are few well-documented, published cases thus far.

Bartels and Maddox (1991) appears to be the only published study of the climatology and mean behavior of MCVs. Because satellite data was used to identify MCVs, there are questions, as the authors acknowledge, about the actual frequency with which they occur. It has been concluded by several investigators that MCVs form within the stratiform region of MCSs, preferentially on the northern end of the line (Leary and Rappaport 1987; Zhang and Fritsch 1987; Brandes 1990; Johnson and Bartels 1992). However, the cirrus shield, which typically overlies the stratiform region, makes MCV detection problematic. Only in those cases where the cirrus shield dissipates are MCVs detected in satellite data. This is perhaps not an issue for the longest-lived vortices, but it is still not known how often development occurs within the stratiform region and is never detected. Bartels and Maddox identified only about three cases per year, but it is quite possible that MCVs occur much more frequently.

An interesting aspect of MCVs is their ability to focus convective activity and perhaps promote subsequent MCS development downstream. While later convection appears in the vicinity of MCVs only about half the time, Bartels and Maddox noted that if it does occur, it is much more likely to evolve into another MCS rather than remain scattered, disorganized convection. It has not been deduced from observations whether this behavior is caused by the MCV itself or whether it reflects the fact that if an MCS forms on a particular day, the environmental conditions downstream on the next day may also be favorable for MCS formation. If MCVs are shown to be a viable mechanism for initiating MCS development, there are implications for the predictability of MCSs and the forecasting of heavy rainfall or severe weather events one or two days in advance. It is also possible that MCVs may provide a necessary finite-amplitude initial disturbance to foster tropical cyclogenesis (Bosart and Sanders 1981; Fritsch et al. 1994).

Observations suggest that the environment of MCSs that produce MCVs is characterized by relatively weak shear, perhaps averaging 10 m s\(^{-1}\) over the lowest few kilometers (Menard and Fritsch 1989; Brandes 1990). The convective available potential energy computed from environmental soundings is large, often exceeding 2000 J kg\(^{-1}\). This is not a distinguishing feature of convective systems that produce MCVs, however, because CAPE is climatologically large over the central United States during late spring and summer, the time period when MCVs are observed.

A broad spectrum of vortex structures has been observed in organized convective systems. Vortices on scales of less than 20 km are common, but these tend to be transient or associated with single convective elements. Vortices on scales of 20–40 km have been seen in Doppler radar data by Stirling and Wakimoto (1989) and Verlinde and Cotton (1990), the latter even suggesting the presence of a vortex pair in the lower and middle troposphere. In general, MCVs may form along with these other vortices in MCSs, but observations have sometimes been unable to distinguish between different vortex types and mechanisms of formation.

The reason for the formation of MCVs has been speculated upon by Zhang and Fritsch (1987), Hertenstein and Schubert (1991), Raymond and Jiang (1990), and Raymond (1992, hereafter R92), among others. Zhang and Fritsch performed simulations using The Pennsylvania State University/National Center for Atmospheric Research mesoscale model (MM4)
parameterized convection and found a cyclonic vortex formed within the stratiform precipitation region between 1 and 5 km AGL. Their explanation centered on enhanced vortex stretching because of the diabatic heating, either by a preexisting disturbance or by circulations within their simulated MCS.

The formation of mesoscale vortices is fundamentally a three-dimensional process. Skamarock et al. (1994, hereafter S94) show that the effect of having finite-length convective lines is very important and allows the formation of vortices on the ends of the line even for weak environmental shears (10 m s$^{-1}$ over the lowest 2.5 km). For westerly shear, a cyclonic vortex forms on the north end of the line, and an anticyclonic vortex on the south end. This pattern has also been seen for higher environmental shears in the bow echo simulations of Weisman (1993), but in our study the vortices are larger, with a scale of at least 50 km.

The addition of ambient cyclonic rotation ($f > 0$) favors more rapid intensification of the vortex on the northern end of the line. An MCV can even form without ambient shear (S94). In this case, there is little signature of the end vortex prior to the dominant cyclonic vortex forms, although convective lines do form with enhanced vorticity at their ends. The end vortices should not be viewed as completely independent from the cyclonic mesovortex, however, because the cyclonic end vortex provides larger vorticity locally and enhances the stretching effect.

b. Balanced dynamics

Diagnosis of mesoscale phenomena has often focused on the action of unbalanced body forces on air parcels using the momentum equations, or on the generation of flows by horizontal gradients of buoyancy through the vorticity equation. A less traditional view of mesoscale dynamics has recently been offered by R92, who puts forth the idea that much of the evolution of mesoscale circulations may be thought of as a fundamentally balanced process. This view is different in that it circumvents the actual balance adjustment process and views flow evolution as proceeding through a series of quasi-equilibrium balanced stages. This is because fast modes (gravity waves) are presumed to have infinite phase speed and perform the necessary adjustment to balanced flow instantaneously. Phenomena such as convection, inertial (and symmetric) instability, and gravity currents are not explicitly included in a balanced model.

In a balanced model, there is only one prognostic variable, usually the potential vorticity (PV), which carries information about the flow. The advantage of needing only one variable is that the dynamics may be simplified conceptually. Because the PV [Eq. (1.1)] contains both potential temperature and vorticity, a balance constraint relating the mass and wind fields is necessary to close the system. Having obtained the wind field from the PV, one may advect the PV itself and advance the system in time (Hoskins et al. 1985). The use of PV and its evolution to deduce dynamical behavior has been termed "PV thinking." To describe the behavior of a system completely in terms of balanced dynamics, two items are essential. First, the motion must be nearly balanced at all times. Second, PV anomalies must be produced by the balanced part of the flow, including vertical circulations calculable from the PV evolution alone, the so-called "slaved" divergent and vertical motions. This means that to include the effects of gravity waves and convection in a balanced model (other than implicit balance adjustment), their behavior must be parameterized in terms of the balanced motion.

The definition and prognostic equations for the unapproximated PV ($q$) may be written

$$
\rho q = \nabla \cdot (\eta \theta) 
$$

(1.1)

and

$$
\frac{\partial (\rho q)}{\partial t} = -\nabla \cdot (-\eta \theta + \rho u q + F \times \nabla \theta),
$$

(1.2)

where $\eta$ is the absolute vorticity vector, $\rho$ the density, $\theta$ the potential temperature, and $F$ the frictional or applied force. The PV defined in (1.1) is Rossby–Ertel PV (Rossby 1940; Ertel 1942), and (1.2) is Ertel’s theorem, from which one can deduce that in the absence of heating and friction, PV is conserved. Because the rhs of (1.2) can be written as the divergence of a vector, there is a volume integral conservation relation implied, even when heating and friction are present. In addition, the particular form of the "flux" vector in (1.2) has implications, discussed in Haynes and McIntyre (1987, 1990; hereafter HM87 and HM90), which will be detailed in section 3. Heating variations along vortex lines will generate PV anomalies, and in general, positive PV anomalies are associated with cyclonic circulation and negative PV anomalies are associated with anticyclonic circulation regardless of their horizontal scale. On large scales the balance is geostrophic, but can approach cyclostrophic in such systems as tropical cyclones.

Because Ertel’s theorem applies to the full primitive equations, PV can be used as an approximate tracer for diagnosing atmospheric flows independent of whether the flow is nearly balanced (Danielsen 1968). However, when the conservation properties of PV cannot be combined with an accurate balance constraint, PV evolution captures the essential dynamics. The choice of a balance constraint for a given problem is often motivated by the character of the flow. On large scales, quasigeostrophy (QG), with its pseudo-potential vorticity and associated conservation law, is valid for low

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1 Henceforth, the acronym PV refer only to Rossby–Ertel PV.
Rossby number flow. For flows with large shears but small curvature (fronts), semigeostrophy (SG) is very accurate (Hoskins 1975). The underlying balance constraint in both QG and SG is geostrophic balance. To accurately describe flows with large curvature as well, a more sophisticated balance constraint is needed, such as nonlinear balance (similar to gradient wind balance; see section 3). For each of these systems, one defines an appropriate PV such that PV thinking can be qualitatively applied the same way, regardless of the complexity of the balance constraint. The issue then becomes one of accuracy versus conceptual simplicity because more accurate balance conditions are inherently more complicated.

The application of PV “thinking” to larger-scale flows with longer intrinsic timescales has produced some useful interpretations of particular phenomena such as cyclogenesis (Hoskins et al. 1985; Davis 1992). An open question is whether similar, balanced dynamical concepts can be applied to observed mesoscale phenomena, especially in regimes where the Rossby number may be very large or infinite. As pointed out by McWilliams (1985) and R92, the concept of balanced flow is not necessarily limited by scale. A vortex may be in cyclostrophic balance on scales of a few kilometers even though this flow is almost completely ageostrophic.

This paper seeks to apply some of the existing concepts of PV thinking to the problem of mesovortex formation in MCSs and the long-term behavior of vortices. Our approach will be to perform numerical integrations of squall lines in idealized environments (described in section 2), thereby separating the ability of internal squall line dynamics to organize mesoscale vortices from vortex formation that might depend on preexisting structures such as upper-level troughs, fronts, or inhomogeneous terrain. These two factors are inseparable in the simulation of observed cases unless a large number of cases are considered. Because we are not considering systems developing in response to mesoscale forcing, as they are often observed to do, there will be differences between the MCS structure we obtain and what is observed in individual cases. Neglecting environmental variation allows us to concentrate on the underlying physics in the hope of facilitating the interpretation of observed MCS dynamics in a general and fundamental context.

In section 3, we summarize the formation of mesoscale tropospheric vortices from both the vorticity and PV perspectives. Therein, we show that these features are balanced and demonstrate that their formation fundamentally depends on unbalanced motion. Section 4 deals with the long-term behavior of the model-generated MCV. We examine its evolution after the convection is presumed ceased, and show that the vertical shear does not act to weaken the vortex substantially. We also calculate the vertical displacement of air parcels ahead of the vortex and show that such displacement are sufficient to destabilize typical sounding profiles. Section 5 summarizes results and discusses issues for future research.

2. Numerical simulations

a. Model specifics

The Klemp–Wilhelmson cloud model (Klemp and Wilhelmson 1978) is used for all the simulations reported herein. The numerical details are as described by Weisman (1992). The model is integrated on a grid with 4-km horizontal resolution, 600 km on a side. At least through 6 h of integration, our simulations reproduce all significant system-scale features seen in the higher-resolution runs by S94, who used a 2-km resolution in a domain extent of 1000 km. The smaller domain and coarser resolution in our case allow a greater number of sensitivity experiments to be performed with acceptable cost.

The vertical resolution for all experiments is 700 m. A free-slip condition is imposed at the lower boundary and a gravity wave radiation condition is used at the top of the model (about 17 km). A crude tropopause is placed at about 12 km, with the thermodynamic profile being the sounding used by Weisman et al. (1988). The CAPE for boundary-layer parcels is about 2400 J kg\(^{-1}\). The lateral boundaries employ open conditions, which minimize reflections of gravity waves. The model has no ice physics, radiation, terrain variation, or surface fluxes of heat, moisture, or momentum. These limitations prevent us from simulating the entire life cycle of MCSs, which may encompass 12 hours or more. The decay phase of an MCS is perhaps critically dependent upon physical processes we neglect; hence, we will not be discussing that part of the life cycle. Concerning ice physics, a preliminary study by Skamarock (personal communication) supports the two-dimensional simulations by Fovell and Ogura (1988), and shows that the addition of a parameterization of ice processes does not alter the basic results we will present, at least through 6 hours of integration.

The initial perturbation for all simulations consists of a line of warm thermals, each assigned a buoyancy of 3 K, horizontal radius of 10 km, and vertical extent of 2800 m. The initial line length in our control simulation is 200 km, consisting of six thermals. The effect of varying the initial line length is discussed in S94, but it does not significantly affect MCV formation.

The mean wind profile, unless specified otherwise, contains a constant shear of 10 m s\(^{-1}\) over the lowest 2.5 km and constant wind above. This shear is not balanced by a horizontal temperature gradient, which would amount to a little over 1°C/100 km. The mean wind does not lead to large accelerations because the Coriolis force acts only on perturbations to the mean state. If we were to include a temperature gradient to balance the mean shear, we would possibly introduce
a variation of CAPE along the convective line. For unsaturated conditions, however, moisture can be specified independently from temperature and could be chosen to make the CAPE uniform. Even if this is not done, the variation in buoyancy over the length of the line is about 2.4°C, which is smaller than the amplitude of the thermals used to initiate convection.

Figure 1 shows the winds at \( z = 2.1 \) km and the outflow boundary at 3 h and 6 h in the simulation with the Coriolis force \( (f = 10^{-4} \text{ s}^{-1}) \) included (henceforth referred to as our control simulation). The outflow extent is defined as the \(-0.5°C\) contour of perturbation temperature at \( z = 350 \) m. This evolution closely follows the conceptual model of an asymmetric system discussed by Houze et al. (1989). During the first few hours of simulation, the squall line circulation is controlled by the interaction of the cold pool and the shear, much as in a two-dimensional simulation. Most of the air motions are in the east–west plane and would not directly sense the meridional temperature variation. Only after a time corresponding to \( 1/f \) (3 h) does the low-level flow acquire a significant meridional component on a system scale. A crude estimate of the buoyancy that would have been contributed by a mean thermal gradient in balance with the shear can be obtained by noting that over 6 h (Fig. 1b), the southern flank of the surface cold pool has spread about 100 km southward from its initial position, implying an additional perturbation of only \(-1°C\). Because this temperature perturbation is small compared to the magnitude of the cold pool itself (about \( 6°C \)), the neglect of the large-scale thermal gradient should have small consequences at least through 6 h.

b. Sensitivity studies

Given environmental conditions favorable for producing a long-lived convective system (one that lasts for many hours), the most important parameter affecting the mesovortex development is rotation (Coriolis force \( f > 0 \)). Figure 2 shows the wind and PV [see Eq. (1.1)] fields at \( z = 2.1 \) km, 3 and 6 h into the simulations with and without a Coriolis force. The fields have been coarsened by a factor of 5 (to a 20-km grid) to more clearly show the mesoscale features, and a \( \Delta \delta_x \) smoother—desmooth has been applied to the PV alone. The coarsening procedure preserves the area integral of quantities. The winds in the figures in the left-hand column are simply coarsened versions of those in Fig. 1. Both simulations feature a westerly shear of 10 m s\(^{-1}\) confined to the lowest 2.5 km.

Without rotation, there is an obvious antisymmetry in the PV (symmetry in the case of thermodynamic variables) about the domain center.\(^2\) One can also notice that the end vortices evident at 3 h appear to migrate toward the center with time, achieving a separation of about 100 km by 6 h. Strong westerly perturbation flow is apparent between the end vortices, exceeding 20 m s\(^{-1}\). With rotation, the symmetry is broken and the cyclonic vortex dominates the circulation by 6 h. The companion anticyclonic vortex is much weaker and actually begins revolving around the cyclonic vortex by 6 h.

Variations in the vertical shear can exert an important influence on the structure of the mesoscale vortices. In Fig. 3, we show results at 6 h for four different profiles of ambient vertical wind shear but the same value of Coriolis parameter \( (f = 10^{-4} \text{ s}^{-1}) \). Although some cyclonic circulation is evident in each simulation, the most organized mesovortex appears when the shear is weak to moderate and confined to low levels. When the shear is strong, a large-scale circulation exists, but it is not well separated from the convective line and smaller-scale circulations appear within it.

The above points are in general agreement with observations (Bartels and Maddox 1991), and are consistent with the fact that stronger, deeper shears are more favorable for maintaining an erect ascent plume that fails to tilt rearward with time (Weisman et al. 1988). Put another way, stronger shears inhibit the formation of a stratiform region, which has been viewed as the birthplace of long-lived, mesoscale vortices, based on case studies listed in the introduction.

3. Vortex formation

We now consider the issue of whether the vortices that develop in our simulations are balanced. It has been suggested that the ability of a vortex to achieve a state of balance bears directly on its longevity. It is entirely possible, however, that fully balanced structures may be short-lived due to the deleterious action of vertical shear and horizontal deformation. Therefore, perhaps a more meaningful motivation for determining the degree of balance is to assess whether one can apply concepts of balanced dynamics to study the system evolution. Underlying this desire is the notion that the constraint of balance affords a conceptual simplification of the dynamics.

a. Balance equations

Determining whether a flow is balanced requires a precise definition of a balance constraint, which may be used to invert PV. Balance is then assessed by comparing the flow so obtained with the total flow, in this case from the model. Perhaps the most widely used set of balance equations for motions with large

\(^2\) Careful inspection of model fields reveals that the symmetry is not perfect, especially concerning the position of individual convective cells. The general symmetry is maintained throughout the simulation in spite of this, suggesting that it is stable to small perturbations.
Fig. 1. Horizontal velocity at 2.1 km and temperature perturbation at z = 0.7 km (−0.5°C contour) at (a) 3 h and (b) 6 h in the simulation with earth’s rotation included (f = 10⁻⁴ s⁻¹). Vector scale at lower right is 30 m s⁻¹. The mean wind at this level has been subtracted, as is the case in all other constant elevation vector plots. The solid line is the outflow boundary at z = 350 m.
local rotation are the nonlinear balance equations, phrased either in terms of PV conservation (Charney 1962; R92) or as a system with an approximate vorticity equation and energy conservation (Lorenz 1960; Gent and McWilliams 1983). In the former, the elliptic system for inverting the PV contains a truncated divergence equation and an approximation to the definition of PV. These equations are obtained by systematically neglecting the irrotational wind with respect to the nondivergent wind (the latter being written in terms of a streamfunction, $v_\psi = k \times \nabla \psi'$). In the PV definition, this amounts to replacing the horizontal vorticity components with the vertical shear of the nondivergent wind. The anelastic, hydrostatic form of the invertibility statement for PV may be written,
\[
\frac{\bar{\rho}(z)}{\theta(z)^2} q' = f \frac{\partial^2 \pi'}{\partial z^2} + \frac{\partial^2 \pi'}{\partial z^2} \nabla^2 \psi' \\
+ \nabla^2 \psi' \frac{\partial^2 \pi'}{\partial z^2} - \frac{\partial^2 \psi'}{\partial x \partial z} \frac{\partial^2 \pi'}{\partial x \partial z} - \frac{\partial^2 \psi'}{\partial y \partial z} \frac{\partial^2 \pi'}{\partial y \partial z} \\
\theta(z) \nabla^2 \pi' = f \nabla^2 \psi' \\
+ 2 \left( \frac{\partial^2 \psi'}{\partial x^2} \frac{\partial^2 \psi'}{\partial y^2} - \left( \frac{\partial^2 \psi'}{\partial x \partial y} \right)^2 \right),
\]

where \( \psi' \) is the nondivergent streamfunction, \( \pi' \) is the perturbation Exner function, and \( \bar{\theta}(z) \) the mean potential temperature. Primes denote departures from a resting state. The full Exner function is defined as \( \pi(p) = C \pi \left( \frac{p}{p_0} \right)^{- \alpha \pi} \). In (3.1), \( \bar{\theta}(z) \) has replaced \( \theta \) where it is not differentiated. To derive (3.2), \( \bar{\theta}(z) \) replaces \( \theta \) as the coefficient of \( \nabla^2 \pi' \) in the equations for horizontal momentum, as in the cloud model. For boundary conditions, we specify the potential temperature perturbations on horizontal boundaries (Neumann conditions for \( \pi' \)), and we use an extrapolation to evaluate the vertical derivative of \( \psi' \) at the boundary. Lateral boundary conditions are homogeneous Dirichlet conditions on \( \pi' \) and \( \psi' \). Inversion of \( q' \) proceeds...
by an iterative method, similar in approach to the method discussed by R92. In all cases, PV is computed from (1.1) using the full wind.

In Fig. 4 we show the nondivergent winds obtained from solving (3.1) and (3.2) at 3 h and 6 h, and in Fig. 5, we display the difference between these fields and the total winds for the same times. This difference shows almost entirely irrotational motion; hence, the PV contains nearly all of the rotational information. As is evident from Fig. 6, showing the upper-level anticyclone and outflow at 9.8 km, the anticyclonic rotation is well captured by the balance equations, however, the divergence is large.\(^3\) The situation is similar near the

\(^3\)As pointed out by a reviewer, the depth of our anticyclone is about 1 km greater than observed in some published studies, such as
surface (Fig. 7), where the PV inversion accounts for the anticyclonic rotation over the southern half of the system and the reflection of the vortex on the northern end, despite large irrotational velocities behind the gust front. The accuracy of the inversion is not a product of the grid coarsening. Inversions on the full model 4-km grid produce similar results, but with more small-scale structure in the rotational wind. The balance equations are obtained from a systematic neglect of the irrotational wind with respect to the nondivergent wind. Even at 6 h, it is not obvious that this condition holds even qualitatively. We must then ask how the flow can appear balanced. One possibility, suggested by the scaling arguments of McWilliams (1985) is that, at large Rossby and small Froude numbers, layers of fluid become isolated, and the relevant conservation law applies to the absolute vorticity. This implies that temperature perturbations are small, and that the PV and vorticity are equivalent (related by the mean stratification). It also requires that PV and circulation anomalies develop simultaneously, rather than there being a period of adjustment for the wind following the PV change. Such behavior was observed in our simulations (see Figs. 4 and 5). In this regime, inverting the PV is equivalent to inverting the vorticity; hence, the nondivergent wind obtained must equal the model’s nondivergent wind.

We can assess whether the flow is in this parameter regime by evaluating the separate terms in the definition of PV,

\[
\left( \frac{d}{d \theta} + \eta \nabla \theta \right) q' = f \theta' + \theta' \zeta' + \eta \nabla \theta'.
\]

The subscript \( h \) refers to horizontal vector components and the subscript \( z \) denotes a partial vertical derivative. Figure 8 shows these terms as an average over roughly the area of the lower-tropospheric PV anomaly that defines the vortex at 6 h. The dominant term is the linear vorticity term (term 2), and within the cyclonic vortex the nonlinear term involving perturbation vorticity and stability (term 3) is about half as large. The fact that term 3 dominates the Coriolis term (term 1) in the vortex (between \( z = 1 \) km and \( z = 4 \) km) suggests that the dynamical balance in the vortex is approximately cyclostrophic. This is consistent with the fact that term 2 dominates the mean PV (\( \propto \delta \theta / \delta z \)), which is not larger than 0.3 PVU in the low and middle troposphere. However, even though the Coriolis term is a small term in the instantaneous balance, its integrated effect over time contributes significantly toward developing the relative vorticity in the MCV. Aloft, term 2 still dominates in the anticyclone, but the balance is gradient balance owing to the comparable size of terms 1 and 3.

The dominance of term 2 is consistent with the idea that the PV is approximately a surrogate for relative vorticity in this flow regime. This fact likely stems from the constraint imposed by moist thermodynamics in the sloping updraft and downdraft regions where the PV anomalies form. Here the temperature profile is nearly moist adiabatic, leading to a cancellation between adiabatic and diabatic potential temperature changes. Though the heating, vertical motion, and PV anomaly generation rates are large, temperature perturbations remain small. Therefore, it is not surprising that the wind appears balanced in the presence of large divergence. While the flow regime has similar properties to the large Rossby number, small Froude number regime of the McWilliams scale analysis, it is fundamentally different because of the central role of diabatic heating.

As we show in Figs. 9a and 9b, the model’s potential temperature perturbations approach the balanced potential temperature perturbation field by 6 h such that the model’s tropospheric temperature and nondivergent winds are nearly in balance. Significant departures from balance exist in the lower stratosphere. The initial negative temperature perturbation in the stratosphere is probably induced by adiabatic cooling in mesoscale ascent at the tropopause, but apparently there is little PV associated with this feature. The later subsidence of this feature might simply be associated with its negative buoyancy. Analogous lower stratospheric temperature perturbations have been observed (e.g., Johnson et al. 1990).

The balanced thermodynamic profile shows a cold core below and a warm core above the level of the maximum circulation. Nearly all of the cold perturbation below the vortex PV anomaly is associated with the cold pool, which is specified as the lower boundary condition for the PV inversion. If we artificially remove the lower-boundary potential temperature perturbation, that is, apply a homogeneous, Neumann lower-boundary condition on \( \pi' \) and invert only the interior PV, it turns out that there is almost no negative temperature

\[ \bar{\theta}' = (\partial \bar{\theta} / \partial z) \partial \pi' / \partial z, \]

\[ \text{where } \pi' \text{ is the balanced perturbation Exner function. In calculating the model potential temperature profile, the average perturbation over the region outside the } 200 \text{ km } \times \text{ 200 km central portion has been subtracted. This effectively eliminates the part of the perturbation field that does not vary horizontally. We do this because the lateral boundary conditions for the PV inversion are homogeneous, and therefore the balanced solution ignores any horizontally uniform perturbations.} \]
perturbation induced by the interior PV anomaly itself. The vortex is then warm core, by definition, because it features only a positive temperature perturbation that decays with height. The extent to which an MCV is warm core, however, is merely a function of the elevation of its associated PV anomaly. The more elevated the vortex, the more pronounced the cold anomaly beneath, but the temperature anomaly at the altitude of the vortex center is nearly zero. Because the existence of warm temperatures above and cold temperatures below a cyclonic vortex is characteristic of the balanced state that must accompany a positive PV anomaly, attempts to classify MCVs as warm core or cold core may not be fruitful.

Based on the results of this section, the development of a balanced state can be summarized as follows. In
the first few hours of simulation, buoyancy perturbations generate horizontal vorticity and vertical circulations and the associated diabatic heating produces PV anomalies. The constraint of near-neutral ascent implies that temperature perturbations are small, and thus PV anomalies translate directly into vorticity and circulation. As the circulation anomalies grow, the temperature and mass fields become more balanced. This process is consistent with the adjustment of the mass field to the balanced part of the wind field that occurs on small scales (Kuo et al. 1987). (Here “small” refers to systems with an aspect ratio, $L/H$, less than $N/f$.) That the balanced temperature perturbations are finite is significant. This can contribute to organized vertical motion during the balanced evolution of an MCV, primarily through its interaction with the shear flow. Section 4 will show how important this vertical motion can be for destabilizing the atmosphere on the mesoscale.
b. Time dependence

1) Background

The time dependence of PV (1.2) may be expressed in a more general form as

$$\frac{\partial (\rho q)}{\partial t} = -\nabla \cdot \mathbf{J},$$

(3.4)

where $\mathbf{J} = -\mathbf{\nabla} \theta + \rho \mathbf{u} q$, $\mathbf{\nabla}$ being the absolute vorticity vector. Here, we have ignored the effect of friction. Thus, where there is a convergence of $\mathbf{J}$, $\rho q$ (and hence $q$) increases and vice versa. As is the usual problem with equations of the form (3.4), any nondivergent vector field may be added to $\mathbf{J}$ and not affect the local change of PV. Therefore, the precise form of $\mathbf{J}$ is arbitrary, and we are free to choose the form that most succinctly illustrates the mechanisms responsible for vortex spinup.

In HM87 and HM90, it was shown that one can choose a perspective in which the component of $\mathbf{J}$ or-
thogonal to isentropic surfaces was zero. In our squall line simulations, the slopes of isentropic surfaces are small away from the gust front, especially before the mature MCV develops. Hence, the direction orthogonal to the isentropes is essentially vertical and therefore only the horizontal components of $\mathbf{J}$ need be considered. Thus, (3.4) may be written

$$
\frac{\partial (\rho q)}{\partial t} \approx -\frac{\partial}{\partial x} \left[ \partial \left( \frac{\partial \theta'}{\partial z} - \frac{\partial w}{\partial y} \right) + \rho v q \right] - \frac{\partial}{\partial y} \left[ -\beta \left( \frac{\partial v}{\partial z} - \frac{\partial w}{\partial x} \right) + \rho v q \right].
$$

(3.5)

The smallness of temperature perturbations relative to the heating rates observed in the mesoscale updraft implies, from the thermodynamic equation, that $w \approx \theta' / N^2$, $N^2$ being the mean stratification and $\theta' \approx g\theta / \theta_0$. This, in turn, means that the tilting of horizontal vortex lines into the vertical is intimately linked with the generation of PV anomalies associated with horizontal gradients of diabatic heating in the presence of vertical shear. The primary effect of both is to produce a vertical component of relative vorticity. In addition, because the vertical component of $\mathbf{J}$ is zero to the extent that the isentropes are flat, the diabatic creation and destruction of PV anomalies above and below a heating region will appear in the advective part of the flux as horizontal convergence. The fact that there is flow converging simply represents the constraint of mass continuity.

A consequence of the cancellation between adiabatic and diabatic potential temperature changes is that (3.5) reduces to the Boussinesq vorticity equation, which may be written,

$$
\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x} \left[ w \frac{\partial \nu}{\partial z} + u \left( f + \zeta \right) \right] - \frac{\partial}{\partial y} \left[ -w \frac{\partial u}{\partial z} + v \left( f + \zeta \right) \right].
$$

(3.6)

One may therefore define a vorticity flux vector without a vertical component

$$
\mathbf{K} = -w \left( \frac{\partial \mathbf{u}}{\partial z} \right) + \mathbf{u} \left( f + \zeta \right),
$$

(3.7)

such that

$$
\frac{\partial \zeta}{\partial t} = -\nabla \cdot \mathbf{K}.
$$

(3.8)

Because both the PV and vorticity equations may be written in divergence form, the change in area-average PV or vorticity may be calculated by performing the closed line integral of the component of $\mathbf{J}$ or $\mathbf{K}$ normal to the boundary of the area in question. It is also apparent that, in the limit of $w \approx \theta' / N^2$, PV thinking and “vorticity thinking” coalesce.

From (3.6) it is clear that the terms involving $w$ account both for vortex tilting and for vertical advection of relative vorticity. The relationship between the diabatic (nonadvective) part of $\mathbf{J}$ in (3.5) and vortex tilting is therefore not an identity. The analogy is complete, however, in the limit that temperature perturbations are small (diabatic and adiabatic potential temperature changes cancel out), and we only consider the linear terms in (3.5) and (3.6). For horizontally uniform mean shear, the terms $\partial (\bar{w} \bar{\theta}) / \partial y$ and $\partial (\bar{\theta} \bar{u}) / \partial y$ both represent the tilting of mean shear.

The second term within each bracket in (3.6) represents horizontal advection of vorticity as well as the generation of relative vorticity by vortex stretching. Again, if the thermal perturbations are small, these are the same processes as horizontal advection and the generation of PV anomalies through vertical gradients of diabatic heating.

The Eulerian formulation in (3.5) and (3.6) has advantages over the Lagrangian formulation, when we discuss a system as complicated as a three-dimensional MCS. One advantage is removing the need for defining “representative” parcel trajectories. Such trajectories

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Here, “small” means that $\theta' \approx \theta$, where subscript $h$ refers to a horizontal component. The derivation of (3.6) from (3.5) does not require $\theta' \approx H \theta$/$\partial z$, where $H$ is a height scale, as long as $\theta'$ is a function of $z$ only.
might be used to evaluate the final location of parcels that experience vorticity generation along their paths. Trajectory calculations become particularly cumbersome for air that is moving through regions with embedded convection. The mesoscale vortices we are examining are relatively fixed with respect to the system, and contain air with a variety of histories which makes for a complicated Lagrangian interpretation. In addition, because only the normal component of the vorticity flux at the boundaries of a specified area is necessary to evaluate changes in circulation, the need to calculate the higher-order derivatives in (3.5) and (3.6) is circumvented. As will be shown in the next subsection, this allows us to diagnose the development of mesoscale vortices in a straightforward way. We feel these facts make the Eulerian formulation somewhat simpler in this case, but certainly not "more correct."

2) APPLICATION

A further simplification of (3.5) is possible if we consider averaging zonally, across the squall line. This strategy is useful for isolating the effects of the line ends, which play a crucial role in the time evolution of these systems. By averaging in x such that the perturbation velocities and diabatic heating vanish at the end points of integration, only the meridional component of \( \mathcal{J} \) survives, and (3.5) may be written as

\[
\frac{\partial (\overline{\rho q})}{\partial t} = -\frac{\partial}{\partial y} \left( \overline{-\eta \overline{\theta} + \overline{u q}} \right) = -\frac{\partial}{\partial y} (\overline{\mathcal{J}}). \tag{3.9}
\]

The overbar implies a zonal average and subscript y denotes a meridional vector component, \( \eta_y \), being the meridional component of horizontal vorticity.

We now present a sequence of line-parallel cross sections that show \( \mathcal{J} \), at different times. The finescale fields have been coarsened to 20 km to more easily show where the coarse-grained convergence and divergence of this quantity are occurring. All of the coarsening and averaging were performed after the components of \( \mathcal{J} \) were computed from the 4-km model output. A single pass of a 2Δx smoother--desmoother has been applied to the resulting fields before plotting.

At 2 h (Fig. 10), the diabatic contribution to \( \mathcal{J} \), dominates the lower-tropospheric pattern, with convergence on the northern end of the line (at \( y = 400 \) km) and divergence on the southern end (\( y = 200 \) km). At this time, the front-to-rear flow has not developed to its full intensity; thus, this pattern results mostly from the low-level cooling in the presence of ambient shear.

By 3 h, the pattern of \( \mathcal{J} \) reflects the front-to-rear flow perturbation (Fig. 11). The change in PV (actually \( \rho q' \), Fig. 11d) between 2.5 and 3.5 h compares well with the implied change in Fig. 11a, based on the horizontal variations of \( \mathcal{J} \).\(^7\) The convergence of the advective flux (Fig. 11b) reinforces the nonadvective flux convergence (Fig. 11c) near \( y = 350 \) km, and this is where

\(^7\) At low levels, the maximum change in \( \rho q' \) is about 0.6 K m\(^{-1}\) s\(^{-1}\) h\(^{-1}\), or in terms of \( q' \), about 0.5 PVU h\(^{-1}\). The flux in Fig. 11a is contoured in intervals of \( 6 \times 10^{-7} \) K m\(^{-2}\). The divergence of this flux on the northern end of the line is about \( 5 \times 10^{-8} \) K m\(^{-2}\) over a distance of about 40 km, implying a rate of change of PV of about 0.4 PVU per hour.
the MCV begins to spin up. At upper levels, there is a very broad divergence of \( \bar{J}_y \) over the domain, which biases the PV perturbation toward negative values. This is the signature of the large-scale outflow from the system at the tropopause and leads to the decrease in PV and related anticyclone. The largest PV change seems to be occurring near the ground, but the strongest cyclonic circulation is actually elevated nearly 2 km. This is partly because of the anticyclonic perturbation associated with the surface cold pool, which partially cancels the cyclonic circulation near the ground.

An additional reason for apparently biasing the largest PV changes toward the surface results from the averaging process itself. This can be clearly seen in cross sections of \( J_y \), orthogonal to the north–south oriented line (Fig. 12). We have averaged the y component of the PV flux over an 80-km, nearly straight portion of the convective line, and present this along with model winds in the plane of the cross section on the 4-km grid. The dipole structure of \( J_y \), bracketing the front-to-

rear sloping updraft at 3 h is apparent. Below the maximum rearward flow, the shear is negative (mean shear is zero above 2.5 km); above it, the shear is positive with \( \bar{J}_y \) positive throughout (Fig. 12c). Because \( J_y \) must decay to zero beyond the line ends, there is a tendency for divergence of \( J_y \) at the southern end at low levels and convergence at the northern end, leading to an anticyclonic vortex to the south and cyclonic to the north. The pattern above the sloping updraft is reversed. Averaging across the line results in the largest positive values of \( J_y \) near the ground, as in Fig. 11a, even though the largest positive values of \( J_y \) are elevated about 2 to 3 km (Fig. 12).

From Figs. 11 and 12, it appears that the formation of the front-to-rear and rear-inflow jets by 3 h is crucial to the generation of mesoscale vortices, which begin to take shape after this time. The tilting of the mean shear, although dominant at 2 h, is later superceded by the tilting of the perturbation shear contained in the ascending front-to-rear and descending rear-inflow circulations. The latter is dominated by the tilting of easterly shear in the front-to-rear ascent (see schematic in Fig. 13). Consistent with this, the mode of PV transport changes from northward due to evaporative cooling in westerly shear to northward due to heating in reverse (easterly) shear. The tilting of mean shear was the dominant mechanism in the formation of "bookend" vortices in the bow-echo simulations of Weisman (1993). Weisman, however, obtained the strongest bow echoes with shears of at least 20 m s\(^{-1}\) over 2.5 km, twice the shear we use. It is consistent that he found a greater importance of mean shear tilting, both because of the greater mean horizontal vorticity and because strong environmental shear tends to limit the baroclinic generation of reverse shear, which is subsequently tilted in our simulation.

In the simulation without rotation, the distribution of \( \bar{J}_y \) (Fig. 14) is nearly the same as the nonadvective portion of the flux in the Coriolis simulation. Because the ambient PV is zero, the convergence and divergence patterns do nothing except within the end vortices themselves. Eventually, convergence of relative vorticity becomes a significant contributor to the circulation increase, but the original source of vertical vorticity is tilting. The pattern of the flux aloft is also much different with \( f = 0 \), for it lacks a large-scale divergence aloft characteristic of the spinup of an anticyclone near the tropopause in the simulation with rotation.

Beyond 3 h, the mesoscale vortices in both the simulations with and without ambient rotation amplify rapidly. Even with the modest environmental shears considered here and no rotation, strong end vortices develop in response to the horizontal nonadvective flux of PV from south to north in the region of the mesoscale updraft and downdraft. This flux is dominated by the anticorrelation of the shear and heating by 3 h, and operates much the same with or without ambient rota-

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**Fig. 10.** Vertical cross sections at 2 h showing the fields of (a) \( \bar{J}_y \) and (b) the nonadvective portion of \( J_y \). North is on the right. Averages are over a 200-km segment across the line. Contour interval for the PV flux is \( 6 \times 10^{-7} \) K s\(^{-1}\).
tion present. With rotation, however, the horizontal convergence into the line also spins up vorticity, biasing the circulation in the cyclonic sense and leading to the MCV.

Being a result of the generation of horizontal vorticity by buoyancy gradients (RKW), the mesoscale updraft and downdraft are not contained in the balance equation formalism considered in this section. The key error in the balance equations is an omission of the irrotational velocity in the definition of horizontal vorticity. The balance equations also lack a means of predicting the baroclinic production of horizontal vorticity. In the region of diabatic heating and cooling, the "balanced" vorticity vectors point in a direction opposite the true vorticity vectors. Therefore, the vortices generated by the tilting of balanced vorticity have the opposite sign as the actual vortices. For a balance model to properly reproduce the mesoscale vortices noted here, especially the counterrotating end vortices, it is insufficient to simply parameterize the diabatic heating and cooling; one must parameterize the flux of PV.

The development of mesoscale vortices in the cloud model illustrates how unbalanced motion leads to balanced circulations that tend to dominate the total flow with time. Perhaps an important factor in this trend is the strong thermodynamic constraint imposed by moist-adiabatic adjustment in the mesoscale updrafts and downdrafts along with the presence of stable stratification on a larger scale. Although potential temperature perturbations are constrained to be small, heating rates, and hence rates of PV anomaly generation are not. Therefore, heating over a mesoscale region efficiently generates balanced vortices because most of the
slightly ahead of MCVs and sometimes this convection does lead to another MCS. Even if another organized convective system does not result, convection still tends to occur near an MCV (Menard and Fritsch 1989). The purpose of this section is to examine whether the MCV obtained in our simulations can possibly trigger and focus convection.

There are two primary issues related to vortex longevity. The first is how the vortex persists for a full day, longer than the characteristic timescales involving the shear \((L/\Delta u)\) and the inertial frequency \((f + 2V/L)^{-1}\). Here, \(V\) and \(L\) are the characteristic tangential velocity and horizontal length scale, respectively, associated with the vortex, and \(\Delta u\) is the difference in the environmental flow between the bottom and top of the vortex (vertical shear), or between one side and the other (horizontal deformation). Zhang and Fritsch (1987) suggested that some MCVs persist because they are able to develop a balanced, warm core structure.

The implication is that circulations must persist long enough to achieve this balanced state and then become a persistent feature. For our simulated vortices, however, the nondivergent wind is nearly always in balance with the PV. There is no period of adjustment in the wind, only in the temperature.

Under a cyclostrophic balance condition, the necessary condition for inertial instability is that \(|\omega|^2\), the square of the angular velocity, decreases with radius. This criterion ignores the correction due to baroclinicity, which is small in the present case. For a point

PV anomaly generation is translated directly into vertical vorticity. A similar interpretation appears in Chen and Frank (1993).

4. Vortex longevity

One of the more important aspects of MCVs is their longevity compared to the lifetime of the convective system from which they originate. The typical MCS lasts 6–12 h and often dissipates entirely by morning on the day following its formation, while MCVs often last 24 h and sometimes last for days. It has been suggested that an MCV spawned in such a system can trigger convection necessary to regenerate an MCS. Convection has been observed to develop within or

---

**Fig. 12.** Zonal cross section showing \(u\), \(w\), and \(J\), averaged meridionally over an 80-km segment of the squall line at (a) 2 h and (b) 3 h; (c) diabatic heating rate at 3 h averaged meridionally, contour interval is 3 m s\(^{-1}\) for vertical velocity, 30 m s\(^{-1}\) for horizontal velocity. Contour interval for the PV flux is \(2 \times 10^{-5}\) K s\(^{-2}\). Negative values are contoured with thin lines; positive values contoured with thick lines. In (c), cooling is denoted by dashed contours.

**Fig. 13.** Schematic of vertical vorticity generation through vortex tilting. For westerly shear (a), vortex lines point toward the north, so that descending motion pushes the vortex line down in the center, resulting in cyclonic rotation on the north end and anticyclonic rotation on the south end. Localized ascent in easterly shear (b) also produces the same pattern of vertical vorticity through tilting. The flux of vorticity (and PV, since the heating is proportional to \(w\)) is northward in both cases.
vortex, the velocity decays as $1/r$, so the condition is not satisfied. Away from any finite-size vortex, the same will generally be true, so that there is a possibility for inertial instability only near the edge of a vortex. Longevity of the MCV would seem to depend more on the absence of vertical shear and horizontal deformation. Shear can be especially deleterious for small-scale structures because the advective timescale decreases with the lengthscale.

The other issue is how an MCV can persist for several days. An MCV persisting on this timescale is what is more traditionally referred to as long-lived, as opposed to persistence beyond the standard dynamical timescales listed above. In what follows, we will be referring to longevity on the time scale of 0.5 to 1 day rather than several days, unless otherwise stated. Persistence of a vortex for several days probably involves a regeneration mechanism, because almost any measurable background shear or deformation would tend to eliminate a vortex on these timescales. A feedback process has been suggested by Fritsch et al. (1994) in which an MCV initiates the development of a new MCS which reinvigorates the MCV. The subject of MCV regeneration is left for future work.

If we consider an MCV as an idealized circular vortex, vertical air motion is possible only if there is ambient shear (Raymond and Jiang 1990, hereafter RJ). An isolated vortex in a shear flow will tend to tilt downshear and weaken with time, however. At issue is whether the vertical displacement of air parcels that results from the interaction of the vortex with the shear is large enough to trigger convection before the MCV weakens beyond the point where it can induce significant upward motion (RJ). The character of the shear flow, as well as the vortex strength are therefore important parameters in determining the fate of an MCV and the vertical motions it produces. The subsequent occurrence of convection also depends on the downstream environmental conditions, which vary widely from case to case.

Fig. 14. As in Fig. 11 but for the simulation with no Coriolis force.
Because of downstream environmental variability, we cannot know a priori the vertical displacement of environmental air necessary to initiate convection. In the model simulation, the environment downstream is assumed uniform with the thermodynamic profile as shown in Weisman et al. (1988). This sounding contains a small region near 2 km AGL where lifted boundary-layer parcels become negatively buoyant. If one considers lifting this “cap” as a layer, then an upward displacement of only about 200 m is required to make the profile unstable. In some observed cases, this number can be considerably greater.

In general, upward motion is expected downshear from an idealized vortex and descending motion upshear. As described in R92, this pattern results primarily from the motion of air relative to the vortex along inclined isentropic surfaces. If a mean shear is geostrophically balanced by a horizontal temperature gradient, the flow downshear from a cyclonic vortex will ascend the sloping isentropes, while upshear, isentropic downgliding will occur. Because the isentropes are displaced upward beneath a vortex and downward above it, flow along the shear relative to the vortex will be forced to rise downshear and sink upshear from the PV anomaly that defines the vortex. Other vertical motion results from the time tendency of the vortex itself as its PV deforms in the shear flow, and from perturbation flow up or down isentropes distorted by the perturbation.

To address the question of vortex longevity and its ability to initiate convection downstream, extended numerical integrations are necessary beyond those simulated with the cloud model thus far. We have already established that the MCV in our simulations is essentially balanced at 6 h, and suggest that it should evolve in accordance with balanced dynamics thereafter, especially if we postulate that the spawning MCS decays. We may therefore consider subsequent vortex evolution solely in terms of PV using a balanced model.

Equations (3.1) and (3.2), coupled with a conservation statement for PV, form a balanced set appropriate for low Froude, large Rossby number flow (R92). Given an initial PV distribution, integration of this model proceeds by solving (3.1) and (3.2) for the non-divergent velocity, diagnosing consistent irrotational and vertical velocities and using the full wind to advect the PV at each time step. For a consistent set of equations, the full wind, including the vertical and irrotational velocities must be included in the advection. The method of diagnosing the irrotational and vertical velocities at each time step is outlined in the appendix.

The novelty of our approach is that we initialize the balanced model with the PV and balanced flow from the cloud model simulation at 6 h. We assume that the overall vortex structure at 6 h is representative of the vortex structure after the hypothesized MCS decay. This assumption is necessary because the cloud model does not contain ice physics, radiation, and boundary-layer processes necessary to simulate the diurnal cycle and the decay phase of the MCS. Justification for this assumption is that the vortex at 6 h shows many of the characteristics of MCVs observed 18–24 h following their formation, after the parent convective system has decayed. We also assume that the environment is mostly subsaturated and that the surface has reached a uniform temperature that is several degrees colder than the cloud model’s initial sounding. Similar conditions are observed in the vicinity of MCVs during the morning following their formation. The absence of large-scale saturation (especially at upper levels) is crucial for the identification of MCVs in satellite imagery (Bartels and Maddox), although some residual cloudiness is helpful for marking the circulation.

The balanced model employs a horizontal resolution of 20 km, vertical resolution of 700 m, and a domain 800 km in the x direction (along the mean shear) and 600 km in y. The semi-Lagrangian time integration scheme of Smolarkiewicz and Pudykiewicz (1992) is used with a time step of 600 s. We use homogeneous Dirichlet conditions on the pressure and streamfunction perturbations, velocity potential, and vertical velocity at lateral boundaries. An additional equation is integrated for the tracer $\eta(x, y, z, t)$, which represents the altitude of air parcels. This field is initialized $\eta(x, y, z, t_0) = z$, and the vertical displacement is obtained as $\eta(x, y, z, t) - z$.

As in the cloud model (control) simulation, the vertical shear is confined to the lowest 2.5 km. In the balanced model (with $f \neq 0$), we include a meridional temperature gradient to balance the vertical shear. This is done with one approximation; namely, we ignore the interior gradient of PV that results when the vertical shear varies with height. The mean thermal gradient appears in the thermodynamic equation, which is used to diagnose $w$, and is used as the lower boundary condition for the model. Thus, surface temperature anomalies can be generated and surface Rossby waves can propagate, but without an interior PV gradient, baroclinic modes cannot grow.

A cross section of the fields of vertical velocity and meridional velocity at 2 h is shown in Fig. 15. Because we have replaced the localized surface cold pool by uniformly cooler surface air, there is little weakening of the vortex signature near the ground. From Fig. 7, one could note that with the localized surface cold pool, the vortex was partially cancelled by anticyclonic circulation at low levels. Most of the balanced vertical motion occurs at low levels, where the vertical shear and meridional thermal gradient are nonzero. As with the idealized vortex mentioned above, ascent occurs on the downshear side of the cyclonic PV anomaly. This remains approximately true at all times during the integration, although the ascent tends to shift toward the southeast side of the vortex with time, as was the case in R92.
eliminate a very large inversion over a mesoscale region. The triggering of convection will also be enhanced if boundary-layer parcels penetrate inversions, given sufficient kinetic energy from buoyancy.

If we artificially double the depth of the shear layer to 5 km, keeping the shear equal to $4 \times 10^{-3}$ s$^{-1}$ as in the above simulation, we find that the vortex is strongly tilted downshear. While the local velocities and vertical displacements of air are still substantial, all fields show less coherence in the vertical. Thus, not only is deeper shear less favorable for vortex formation in the first place, but any coherent vortex structure will weaken more quickly in deep shear. Our results do not preclude the possibility of vortices existing and initiating convection in deeper shears, but the longest-lived vortices should occur in weak, shallow shear.

In the case with no ambient rotation (where no mean temperature gradient is required to balance the shear), there is also a coherent vertical motion signal downshear from the counterrotating end vortices, which can produce appreciable vertical displacements of air. By integrating the balanced model initialized with the vortices obtained after 6 h of the no-Coriolis simulation (Fig. 2), we obtain a mesoscale region of upward displacement ahead of the vortex pair, with values approaching 300 m (Fig. 18).

Unlike the idealized, cyclonic vortex, a vortex pair such as this can have appreciable mesoscale vertical

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**Fig. 15.** Balanced (a) nondivergent meridional wind and (b) vertical velocity at 2 h into the balance model integration. The cross section is along $y = 310$ km. Contour interval for $u_y$ is 2 m s$^{-1}$ and 1 cm s$^{-1}$ for $w$. The heavy, solid line is the zero contour.

**Fig. 16.** Balanced (a) nondivergent meridional wind after 12 h of balance model integration and (b) vertical displacement between 6 and 12 h. Contour interval for $u_y$ is 1 m s$^{-1}$ and 100 m for vertical displacement. The cross section is along $y = 350$ km. The heavy, solid line is the zero contour.
motion even in the absence of vertical shear, through the upgliding and downgliding of the vortex circula-
tions with respect to the perturbed isentropes. The flow
associated with each vortex decays with horizontal dis-
tance away, and the flow due to one vortex at the center
of the opposite vortex is the speed with which the vor-
tex pair propagates, about 5 m s$^{-1}$ in this case. The flow
farther away than the center of the opposite vortex is
slower than this speed; the flow nearer is stronger.
Thus, on the outer edge of the opposite vortex, the re-
relative flow is front to rear. Because the isentropes are
slightly depressed around each vortex, there is down-
gliding ahead and upgliding behind each vortex. Al-
though the weak vertical motion in this case might not
destabilize a typical midlatitude sounding, it could have
a profound effect in the Tropics.

5. Conclusions

The simulation of MCSs with finite-length initial
lines and an absence of imposed periodicity in the
along-line direction produces effects not seen in pre-
vious squall line simulations with cloud-resolving mod-
els. In environments of large CAPE and weak to mod-
erate low-level shear but no ambient rotation, strong
end vortices develop over 6 h. With rotation, a domi-
nant cyclonic vortex appears with many characteristics
of observed MCVs.

The formation of larger-scale features dominated by
rotation about a vertical axis signals a transition of the
convective system to larger spatial and longer temporal
scales. Balanced dynamics, as we have defined it, plays
an increasingly greater role in the system evolution. As
such, we have found that PV and its associated diag-
nostics can provide a fairly simple picture of the de-
velopment of these balanced flow features. However,
our results indicate that the formation of mesoscale vor-
tices depends on the front-to-rear and rear-inflow cir-
culations, features which result from the generation of
horizontal vorticity through gradients of buoyancy, a
process not contained in the present balanced dynam-
ical framework. Thus, it appears that much of the dy-
namics of MCSs, at least of the type simulated here, is
unbalanced until such time as the MCV grows to dom-
inate the flow.

The MCV formation is favored by system-scale con-
vergence in the presence of earth’s rotation, as shown
by S94. Without rotation, end vortices form at first
through the interaction of diabatic cooling in down-
drafts with the ambient shear. As the MCS matures,
however, the strong vertical circulations and associated
diabatic heating become more important. The primary
contribution to the low-level end vortices is the generation of PV anomalies through the correlation of heating with easterly shear beneath the maximum front-to-rear flow. Evaporative cooling in the westerly shear beneath the descending rear-inflow jet complements this process but has a secondary importance.

Weak, low-level shear appears most favorable for the formation of a dominant MCV. This feature also forms in environments without shear, and in environments with stronger shear to some extent. However, concerning the long-term behavior of an MCV and its ability to initiate convection, weak shear confined to
low levels seems an optimal state. If the shear is moderate and extends over the depth of the vortex, the perturbation weakens faster and vertical parcel displacements induced through the shear do not attain a coherent structure. Without shear, almost no vertical motion results, so although the vortex is perfectly preserved, it is dynamically inert. Most documented MCVs, especially those spawning new MCSs, exist in environments of weak but well-defined low-level shear.

The presence of balanced vortices after only a few hours of squall line evolution suggests that the main issue concerning vortex longevity is not the presence or absence of inertial stability. For instance, on scales of 50–100 km, anticyclonic vortices (with negative total PV) are equally balanced and as likely to persist as cyclonic vortices. The traditional arguments about vortex longevity based on inertial stability are perhaps most applicable to larger-scale structures. The existence of balance suggests that subsequent vortex evolution may be understood in a balanced dynamical framework by examining the action of shear and deformation on a vortex structure. A more complete treatment of this problem would also include latent heat release in the upward motion downshear from the vortex. This might serve to counter the tendency for vertical shear to tilt and weaken the vortex by creating a positive PV anomaly downshear at low levels. In general, our calculations support the original hypothesis put forth by RJ that the vertical motion induced by an MCV in shear is sufficient to initiate convection and possibly another MCS.

Among the daunting questions remaining is what effect, if any, do mesoscale vortices of either sign have on the spawning convection system. In two-dimensional systems, the westerly shear beneath an elevated rear-inflow jet may counter the negative vorticity generation at the leading edge of the cold pool and force a stronger, more erect mesoscale updraft (Weisman 1992). The strong westerly flow south of the MCV, or between the end vortices in the \( f = 0 \) case, may act similarly to counter the tendency for the mesoscale updraft to tilt upshear, producing more intense convection near the gust front. The tendency for convection to be more intense south of the MCV or between the counterrotating end vortices is seen in our simulations and in observations, as well as the tendency for convection to weaken to the north of the MCV.

The nature of rear inflow in a mature, three-dimensional system is fundamentally different than in a two-dimensional squall line. As seen from Fig. 4, the strong, balanced flow between the end vortices appears as a rear inflow, but it is horizontally nondivergent and thus is not associated with any vertical motion directly. In the two-dimensional case, the rear inflow decelerates just upshear from the surface gust front, and thus can be associated with strong upward motion. This difference between rear inflow in two and three dimensions is one manifestation of the pronounced effect of limiting the convective line length. It suggests that the application of results from two-dimensional squall lines to the analysis of observed or simulated mature three-dimensional systems must be done carefully.

Another avenue of study should detail the development of a new MCS under the influence of an MCV. An important question is how forced mesoscale ascent affects MCS formation, and also whether the development of a new MCS can intensify an MCV. This intensification is probably an important factor in allowing MCVs to persist for days. One possible approach is to initialize a cloud-resolving model with a balanced vortex structure in a shear flow and a conditionally unstable thermodynamic environment. Further research is also needed on the formation of vortices in environments more characteristic of the Tropics, with weaker background rotation and small values of CAPE. Understanding the formation of tropical MCSs and associated MCVs may be an important step toward explaining the genesis of tropical cyclones.

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APPENDIX

Integration of the Balanced Model

a. Diagnosis of vertical velocity

The balanced model consists of (3.1) and (3.2), combined with the conservation relation for PV (we assume adiabatic, inviscid conditions) and the thermodynamic equation, which is integrated to provide both the upper and lower boundary conditions and is also used to diagnose vertical velocity in the interior. The irrotational wind is obtained from the mass continuity equation

\[
\nabla^2 \chi = -\frac{1}{\rho(z)} \frac{\partial}{\partial z} (\bar{\rho}(z)w), \tag{A.1}
\]

where \( \chi \) is the velocity potential.

The method for diagnosing the vertical velocity uses the thermodynamic equation and the assumption that the majority of the advection of PV and potential temperature is by the nondivergent wind. This is consistent with the scaling for the balance equations in which one assumes that the parameter \( |v_\phi|/|v_\psi| \ll 1 \); that is, the irrotational wind is small compared to the nondivergent wind. One can think of an iterative process in which, at the first iteration, both the PV and \( \theta \) are advected only by \( v_\psi \) over one time step. We may then write

\[
q(t + \delta t) = q(t) - \delta t \int_{t}^{t + \delta t} v_\phi \cdot \nabla q \tag{A.2}
\]

\[
\theta(t + \delta t) = \theta(t) - \delta t \int_{t}^{t + \delta t} v_\phi \cdot \nabla \theta \tag{A.3}
\]
We denote the local PV and \( \theta \) changes over this time step as \( \delta q \) and \( \delta \theta \), respectively. Note that (A.2) and (A.3) do not make any reference to a specific advection scheme. We simply assume there is a means to advance fields from time \( t \) to time \( t + \delta t \). In the present case that is achieved through the semi-Lagrangian scheme of Smolarkiewicz and Pudykiewicz (1992). Some details of the implementation of this scheme in the present model will be discussed after the general technique is reviewed.

The field \( q(t + \delta t) \) is now inverted to obtain a new mass and streamfunction field at time \( t + \delta t \). From this a new potential temperature field can be calculated. We denote the difference between this field and \( \theta(t) \) as \( \delta \theta_q \).

The thermodynamic equation is now approximated as

\[
\frac{\delta \theta}{\delta t} = -\int_\gamma^{t+\delta t} \mathbf{v}_\phi \cdot \nabla \theta - \int_\gamma^{t+\delta t} w \frac{\partial \theta}{\partial z}.
\] (A.4)

At this point, we make the numerical approximation that \( w \) is valid at the midpoint of the time step, and the static stability is the average of the stability at \( t \) and \( t + \delta t \). This allows us to solve for \( w \);

\[
w = -\left( \frac{\partial \theta}{\partial z} \right)^{-1} (\delta \theta_q - \delta \theta)/\delta t.
\] (A.5)

Equation (A.5) uses \( \delta \theta_q \) to estimate the local time tendency of \( \theta \). After the estimate of \( w \) is obtained, (A.1) is solved for \( \chi \), the velocity potential.

Iterations on this proceed by solving (A.2) and (A.3) again for new tendency estimates using the vertical and irrotational advections to improve the estimate of the change in PV and hence improve \( \delta \theta_q \). The horizontal advection of potential temperature is also updated using the irrotational wind, which improves the estimate of \( \delta \theta \). These new tendency estimates may be substituted into (A.4) to obtain improved estimates of the vertical velocity.

b. Application of semi-Lagrangian scheme

The semi-Lagrangian integration scheme advances a conservative field in time by first performing a trajectory calculation that determines the upstream location of the parcel which will move to a given grid point over the next time step. Then the field to be advected is interpolated to the upstream location, and hence assigned to the grid point of interest at the end of the time step. Needed are the velocities at the current time step and an estimate of the velocity at the next time level in order to do the trajectory calculation.

In our application, the velocity at the next time step, \( \mathbf{v}^{n+1} \), is initially estimated by extrapolation using the previous two velocities, \( \mathbf{v}^n \) and \( \mathbf{v}^{n+1} \). Because the vertical and irrotational velocities are defined as the average over a time step, \( w^n = w^{n+1} \) and \( \mathbf{v}_x^n = \mathbf{v}_x^{n+1} \), and the extrapolation is trivial. These velocities are used to advect the PV using the semi-Lagrangian scheme, and the change in PV is used to determine the change in potential temperature. To solve for the vertical velocity, the horizontal advection of potential temperature, integrated over a time step is needed. This is obtained by using the semi-Lagrangian scheme to advance potential temperature with the vertical velocity set to zero. The thermodynamic equation (A.5) is then solved for \( w \).

Equation (A.1) is solved for the irrotational flow, and the above process repeated to refine the estimates of PV change, vertical motion and irrotational wind. Because we are continually improving the estimate of PV at the next time step, the estimate of \( \mathbf{v}_x^{n+1} \) is also improved at each iteration. Because the improvements to the velocity are smaller at each iteration, the inversion of PV at time step \( n + 1 \) requires successively fewer subiterations. (Iterations refer to the number of times at each time step that the PV advection is performed and a new estimate for \( w \) is obtained. Subiterations refer to the inversion of PV itself.) When the change in streamfunction and pressure is known within a specified amount everywhere in the domain, the iterations stop and we jump to the next time step. Typically, only two or three iterations are required to update the PV at each time step.

REFERENCES


