Spells of Low-Frequency Oscillations and Weather Regimes in the Northern Hemisphere

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ABSTRACT

The low-frequency variability in the midlatitudes is described through an analysis of the oscillatory phenomena. In order to identify nearly periodic components of the atmospheric flow, the multichannel version of the singular spectrum analysis (M-SSA) is developed and applied to an NMC 32-year long set of 700-hPa geopotential heights. In the same way that principal component analysis identifies the spatial patterns dominating the variability, M-SSA identifies dynamically relevant space-time patterns and provides an adaptive filtering technique.

Three major low-frequency oscillations (LFOs) are found, with periods of 70 days, 40–45 days, and 30–35 days. The 70-day oscillation consists of fluctuations in both position and amplitude of the Atlantic jet, with a poleward-propagating anomaly pattern. The 40–45-day oscillation is specific to the Pacific sector and has a pronounced Pacific/North American (PNA) structure in its high-amplitude phase. The 30–35-day mode is confined over the Atlantic region, and consists of the retrogression of a dipole pattern. All these oscillations are shown to be intermittently excited, and M-SSA allows the localization of their spells. The two Atlantic oscillations turn out to be frequently phase locked, so that the 30–35-day mode is likely to be a harmonic of the 70-day mode. The phase locking of the Pacific 40–45-day with the Atlantic 30–35-day oscillations is also studied.

Next, the relationships between LFOs and weather regimes are studied. It is shown in particular that the occurrence of the Euro-Atlantic blocking regime is strongly favored, although not systematically caused, by particular phases of the 30–35-day mode. The LFOs themselves are able to produce high-amplitude persistent anomalies by interfering with each other.

The transition from a zonal regime to a blocking regime is also shown to be highly connected to the life cycle of the 30–35-day mode, indicating that regime transitions do not result only from the random occurrence of particular transient eddy forcing. There are preferred paths between weather regimes. This result leaves us with the hope that at least the large-scale environment-favoring weather regimes may be forecast in the long range. Conditional probability of occurrence of blocking, 30 days ahead, is enhanced, relative to climatological probability, by a factor of 2 if the phase of the 30–35-day oscillation is known. This also emphasizes the necessity of operational models to represent correctly the extratropical LFOs in order to produce skillful long-range and even medium-range forecasts of weather regimes.

1. Introduction

The atmospheric extratropical low-frequency variability has long drawn the attention of meteorologists. The increasing number of comprehensive studies of the variability over time scales beyond the synoptic range is motivated by its two extremely challenging properties: nonlinearity and unpredictability. Another motivating factor, revealed by observational studies (Blackmon 1976), is that the extratropical atmosphere is dominated by low-frequency motions, although synoptic fluctuations are more noticeable in our daily lives. The existence of such a dominant variability is intriguing, at first sight, since it involves planetary waves that generally have e-folding times much longer than baroclinic transient waves in midlatitudes. The key phenomenon is that these synoptic transients are able to maintain low-frequency motions by themselves (Egger and Schilling 1983; Metz 1987) through nonlinear feedback.

During the last decade, the general trend has been to describe the low-frequency variability by classifying the associated large-scale anomalies into a small number of patterns. Wallace and Gutzler (1981) examined the spatial structures of the cross-correlation functions and deduced a small number of teleconnection patterns describing a large part of the variability. These few important patterns are more explicitly formulated using empirical orthogonal functions (EOFs hereafter) (Barnett and Preisendorfer 1978; Horel 1981; Mo and Ghil 1987; Preisendorfer 1988). Recently, the reduction of the low-frequency variability has been addressed in
terms of clusters (Legras et al. 1987; Mo and Ghil 1988; Molteni et al. 1990) that define recurrent states of the atmosphere. Also, the identification of quasi-stationary weather patterns has been performed with simple models (Reinhold and Pierrehumbert 1982; Vautard and Legras 1988) and then from observations (Vautard 1990). Although there are significant differences among the results, all these studies indicate that the low-frequency extratropical atmosphere "prefers" a small number of states, and a major part of the variability is due to the alternating between such states, generally called weather regimes.

So far, the emphasis has been put on the spatial aspects of the low-frequency variability, and we have a good "static" representation of the atmospheric attractor. Nevertheless, there is still little knowledge of the low-frequency time behavior. For instance, the transitions between the weather regimes are poorly understood. Still, with the development of the complex principal component analysis (CPCA), some propagative patterns have been detected (Horel 1984; Branstator 1987; Lanzante 1990), but with little regularity in time. The first motivation of the present work is to document the most regular space–time coherent patterns of the low-frequency variability, and eventually their relationships with the weather regimes.

A nice way to formulate the problem is to use the dynamical systems theory. It is generally agreed that, for forced-dissipative dynamical systems, the set of possible states occurring after a long time of evolution, called the attractor, is small, of probability zero in the phase space. This set may be a simple object, such as a point, or a periodic orbit. In these cases, the behavior of the system is asymptotically stationary or periodic. From the pioneering work of Lorenz (1963), we now know that aperiodic (chaotic) behavior is to be expected in general, even when the dimension of the phase space is as low as three, and when the system is entirely deterministic. In the latter case, the attractor is a strange attractor and has a complex topological structure.

Chaos, however, does not mean that the behavior of the trajectories in phase space is random. Regularities can still be present, such as intermittent spells of regular oscillations or quasi-stationary evolution. This typically happens when weakly unstable periodic orbits or fixed points are contained within the attractor, as sketched in Fig. 1. The phase-space trajectory spends some time in the neighborhood of the fixed point, before being ejected, and captured for a while by the periodic orbit. A typical example of this intermittent behavior is given, for instance, in the Lorenz (1963) attractor, where two unstable fixed points compete in the attraction of the trajectories (Sparrow 1982).

In the midlatitudes, quasi-stationary behavior is observed, more generally, when some particular patterns persist well beyond the synoptic time scale. The European blocking dipole is one candidate for this persistence property. This problem was investigated first by Charney and Devore (1979) and then by subsequent authors (Legras and Ghil 1985; Itoh 1985), who defined weather regimes as the stationary solutions of simplified general circulation models. A slightly different approach was adopted by Vautard and Legras (1988), where multiple quasi-stationary states result from pure interaction between the large-scale flow and the synoptic transients, so that the weather regimes are not stationary in a strict sense, but do represent attracting phase-space areas where the large-scale part of the flow is quasi-stationary, whereas the synoptic activity continues. Quasi-stationary behavior is therefore not always related to the existence of unstable fixed points.

Low-frequency oscillations (LFOs) became a major subject of interest since the observation made by Madden and Julian (1971) of a 40–50 day oscillation in the tropics. For the sake of clarity, we shall distinguish here three LFO families:

(a) intramonthly oscillations, with periods of a week to a month;
(b) intermonthly oscillations, with periods between a month and a year;
(c) interannual oscillations, with periods greater than a year.

A striking example of an oscillation belonging to the first category is the Branstator–Kushnir high-latitude retrogressive traveling wave that was particularly enhanced during winter 1979/80 (Branstator 1987; Kushnir 1987). In the midlatitudes, intermonthly oscillations have been evidenced in the angular momentum (Weickmann et al. 1985; Penland et al. 1991). Lau and Phillips (1986) found two extratropical wave trains exhibiting coherence with the tropical OLR in the same frequency range. The trace of periodic activity in the 30 to 60 day range is also found from the entirely independent measurements of the length of day by Dickey et al. (1991). Recently, Ghil and Mo (1991) and Kimoto et al. (1991) made an extensive analysis.

![Unstable fixed point](image)

**Fig. 1.** Schematic diagram representing the evolution of the trajectory of a dynamical system in which there is an unstable fixed point and an unstable periodic orbit.
of such extratropical oscillations. The interannual variability, as well, with its pronounced regularities such as the El Niño–Southern Oscillation that displays a nearly periodic behavior has been the subject of numerous studies.

We shall focus in this observational study on the intermonthly oscillations. The multichannel singular spectrum analysis (M-SSA) is presented as an analysis tool and is applied to a set of 700-hPa geopotential heights. As we shall see, M-SSA reveals the existence of a strong mode with a period of 70 days, and also two different modes in the 30–60-day range. We shall also study the relationships between the LFOs. The companion study of Kimoto et al. (1991), using the same methodology, focuses on the three-dimensional structure associated with the 30–60-day extratropical oscillation and its links to the tropical circulation. Here, we want to give an insight into the various LFOs using the same data with a wider range of time scales, and we shall particularly focus on their relation to weather regimes.

One particularly interesting question is whether these regimes are not simply slow phases of some particular oscillations. In other words, if LFOs correspond to periodic orbits, one is led to imagine that weather regimes are pieces of the orbit. If not, the proximity of particular regimes to periodic orbits may still create interactions. This is the second major point addressed in this paper. An important issue is whether blocking occurs at particular phases of the LFOs. If the answer is positive, then three major conclusions can be drawn. First, it would help to understand the problem of the onset of blocking: to be triggered, blocking needs a favorable large-scale environment. Second, this answer would have an impact on prediction of blocking, since oscillations are the easiest kinds of phenomena to forecast, and therefore higher probability of occurrence of blocking could be predicted well in advance. Finally, it would also show that a necessary condition for a model to correctly predict blocking is that it predicts the underlying oscillation.

In section 2, emphasis is put on the mathematical formulation of the M-SSA method. We insist there on technical details so that this article reads as a manual. We show particularly that M-SSA produces data-adaptive space–time filters able to isolate oscillation spells. In section 3, M-SSA is applied to a 32-year series of 700-hPa geopotential heights and the resulting LFOs are presented for various experiments. Then, a composite analysis is conducted to study the impact of these oscillations onto the large-scale flow. Phase and amplitude relationships between LFOs are also examined. In section 4, the influence of the LFOs onto the weather regimes, as identified by Vautard (1990), is investigated. A short summary and discussion follows in section 5. We present there some tentative dynamical explanations of the LFOs.

2. The multichannel singular spectrum analysis

The aim of multichannel singular spectrum analysis (M-SSA) is to identify coherent space–time patterns, given a regularly sampled archive of maps. It is mathematically equivalent to the extended EOF analysis of Weare and Nasstrom (1982). The spirit of extended EOFs, however, is different and aims at including temporal information in the EOFs, by adding a few lags in the state vectors. M-SSA essentially differs by the use of a large number of lags, from which spectral properties can be drawn. More precisely, given a time scale \( \tau \), one would like to represent the variability of \( \tau \)-long sequences of maps as the sum of a relatively small number of "relevant" space–time patterns. We review here the methodology of M-SSA.

\textbf{a. The eigenvalue problem}

Assume, in a general way, that we are to analyze a dataset consisting of a multichannel time series \( X_i \), \( 1 \leq i \leq N \), \( 1 \leq l \leq L \), \( l \) representing time and \( i \) the channel number. Index \( j \) may, for instance, represent a point number on a specified grid. Each channel being assumed to have a zero mean, classical PCA gives the principal axes of this set by expanding each vector \( X_i \) onto an orthonormal basis \( (E^k, 1 \leq k \leq L) \):

\[
X_i = \sum_{k=1}^{L} a_{ik}^j E_j^k, \quad 1 \leq l \leq L. \tag{2.1}
\]

The coefficient \( a_{ik}^j \), called the \( k \)th spatial principal component (S-PC, hereafter), is the orthogonal projection of the original vector \( X_i \) at time \( i \) onto the \( k \)th spatial EOF (S-EOF) vector \( E^k \). The vectors \( E^k \) are the eigenvectors of the covariance matrix of the time series \( X \).

In the single-channel singular spectrum analysis (SSA; Fraedrich 1986; Broomhead and King 1986a; Vautard and Ghil 1989; Vautard et al. 1992), a scalar series \( (x_i) \), \( 1 \leq i \leq N \), is analyzed. The SSA expansion is

\[
x_{i+j} = \sum_{k=1}^{M} a_{ik}^j E_j^k = X_{ij}, \quad 1 \leq j \leq M. \tag{2.2}
\]

The analogy with PCA is made by augmenting the scalar time series \( x_i \) into the multichannel time series \( X_i = (x_{i+1}, x_{i+2}, \ldots, x_{i+M}) \). Aside from this definition, there is no formal difference between the two expansions (2.1) and (2.2). SSA allows the identification of time patterns in time series. The user-prescribed parameter \( M \) in Eq. (2.2) is called the window length, or embedding dimension. The vectors \( E^k \) (time empirical orthogonal functions, T-EOFs hereafter), which are \( M \)-long scalar time sequences, are the eigenvectors of
the Toeplitz matrix of \( x\), \( T_x\), that contains in column \( j'\) and row \( j \) the autocovariance of \( x \) at lag \( j - j' \). The coefficients \( a_{i}^{j} \) are called here the time principal components (T-PCs). SSA has been applied to numerous climatic time series, such as paleoclimatic isotopic records (Vautard and Ghil 1989; Yiou et al. 1992), the El Niño–Southern Oscillation index (Rasmussen et al. 1990; Keppenne and Ghil 1992), global surface temperature records (Ghil and Vautard 1991; Vautard et al. 1992), and geopotential heights (Ghil and Mo 1991).

By analogy, the M-SSA expansion (see also Broomehead and King 1986b; Kimoto et al. 1991) of \( L \)-dimensional data vectors \( X_{i,l}, 1 \leq l \leq L, 1 \leq i \leq N \), is

\[
X_{i,l+j} = \sum_{k=1}^{L \times M} a_{i}^{j} E_{i}^{j,k}, \quad 1 \leq l \leq L, \quad 1 \leq j \leq M. \quad (2.3)
\]

Here, the state vector considered at time \( i \) is \( (X_{i,1}, X_{i,1+L}, \ldots, X_{i,1+L-1}, X_{i,1+L}, \ldots, X_{i,1+M}) \). Here, \( M \) is the window length, but now the eigenvalue problem is of embedding dimension \( L \times M \). The coefficients are called the space–time principal components (ST-PCs). The \( k \)th basis vector (ST-EOF) is the eigenvector of the block–Toeplitz \( (L \times M) \times (L \times M) \) matrix \( T_X \) containing the cross-covariance coefficients of the different channels \( l \) at lags 0 to \( M - 1 \):

\[
T_X = \begin{pmatrix}
T_{11} & T_{12} & \cdots & T_{1L} \\
T_{21} & T_{22} & \cdots & T_{2L} \\
\vdots & \vdots & \ddots & \vdots \\
T_{L1} & T_{L2} & \cdots & T_{LL}
\end{pmatrix} \quad (2.4)
\]

where \( T_{ll'} \) is the \( M \times M \) lag covariance matrix between channel \( l \) and channel \( l' \). As discussed in Vautard et al. (1992), the least biased estimate of the element in row \( j \) and column \( j' \) of \( T_{ll'} \) is

\[
(T_{ll'})_{j,j'} = \frac{1}{N-|j-j'|} \sum_{i=1}^{N-|j-j'|} X_{i,j}X_{i,j'+j-j}. \quad (2.5)
\]

Both PCA and SSA are therefore particular cases of M-SSA. PCA can be derived from M-SSA with \( M = 1 \), and SSA with \( L = 1 \). All these expansions are applications of the general Karhunen–Loève theorem. The eigenvalues \( \lambda_k \) of the symmetric, nonnegative covariance matrix \( T_X \) of the problem are sorted in decreasing order. The orthogonality in both “time” (zero cross-covariance of two different ST-PCs at lag 0) and “space” (orthogonality of the ST-EOFs) implies in particular that \( \lambda_k \) is the variance of the \( k \)th ST-PC. The ST-EOFs are \( M \)-long time sequences of vectors, describing space–time patterns of decreasing importance as their order \( k \) increases.

b. Oscillations and pairs of eigenelements

Like for SSA (Vautard and Ghil 1989; Vautard et al. 1992), the fundamental property of M-SSA lies in the fact that when

1. two consecutive eigenvalues are nearly equal,
2. the two corresponding time sequences described by the EOFs are nearly periodic, with the same period, and in quadrature,
3. the associated PCs are in quadrature,

then there is in the series an oscillation whose period is the same as that of the ST-EOFs themselves and whose spatial pattern is the same as the one of the ST-EOF. In order to illustrate this property, let us consider the progressive wave, on the finite interval \(-Y \leq y \leq Y\)

\[
W(y, t) = A \cos(\mu y - \omega t), \quad (2.6)
\]

where \( A \) is the amplitude, \( \mu \) the wavenumber, and \( \omega \) the frequency. Assume that the signal is discretized in time by \( t_i = i\Delta t \), and in space by \( y_j = j\Delta y \). The M-SSA expansion is easy to determine, if one notices that for any real number \( \phi \),

\[
W(y, t + s) = A \cos(\omega t + \phi) \cos(\mu y - \omega s + \phi) + A \sin(\omega t + \phi) \sin(\mu y - \omega s + \phi). \quad (2.7)
\]

Indeed, in discrete form, Eq. (2.3) is satisfied by taking

\[
X_{i,l} = W(y_i, t_i), \quad (2.8a)
\]

\[
E_{l,j} = F_{1} \cos(\mu L \Delta y - \omega j \Delta t + \phi), \quad (2.8b)
\]

\[
E_{l+1,1} = F_{2} \sin(\mu L \Delta y - \omega j \Delta t + \phi), \quad (2.8c)
\]

\[
a_{l}^{1} = \frac{A}{F_{1}} \cos(\omega i \Delta t + \phi), \quad (2.8d)
\]

\[
a_{l}^{2} = \frac{A}{F_{2}} \sin(\omega i \Delta t + \phi), \quad (2.8e)
\]

\[
\phi = (\mu (L + 1) \Delta y - \omega (M + 1) \Delta t)/2. \quad (2.8f)
\]

In Eqs. (2.8b–e), \( F_1 \) and \( F_2 \) are normalization constants depending on the parameters \((M, \Delta y, \Delta t)\), calculated by imposing that the ST-EOFs \( E^1 \) and \( E^2 \) be of unit norm. The phase \( \phi \) is governed by the orthogonality condition. Under these definitions, the two ST-EOFs are orthogonal and in phase quadrature. The covariance, at lag zero, of the two PCs vanishes clearly, and they are also in phase quadrature. The nonvanishing part of the eigenvalue spectrum is restricted on the first two eigenvalues \( \lambda_1 = A^2/(2F_1^2) \) and \( \lambda_2 = A^2/(2F_2^2) \). These eigenvalues, although not equal, are equivalent as \( M \) becomes large: the ratio \((\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2)\) goes to zero as \( M \) goes to infinity.

Similar results hold when the signal is a more general harmonic oscillation of the form \( W(y, t) = A(y) \cos\omega t + B(y) \sin\omega t \). When the signal is a finite sum of \( q \)
harmonic oscillations (quasi-periodic signal), the number of nonvanishing eigenvalues is $2q$; the eigenvectors are gathered by pairs satisfying (i), (ii), and (iii), separating each oscillation, provided the window length is large enough. In the general case, when a quasi-periodic signal, with $q$ harmonic oscillations, is superimposed to a chaotic or noisy signal, M-SSA gives $q$ oscillatory pairs, but the rest of the eigenvalue spectrum does not vanish. Again, in this case, a rigorous proof is given only when $M \rightarrow \infty$ (Devijver and Kittler 1982). In all applications so far performed, the separation property has never failed, given a suitable choice of the window length (see the discussion of section 2e).

In the multichannel case, the separation property acts both in time and space. M-SSA is capable of distinguishing two oscillations with the same spatial patterns but with different periods, as well as oscillations with the same period and spatially orthogonal patterns. If an oscillation contains some harmonics, M-SSA will capture the harmonics as other oscillatory pairs. In other words, M-SSA is unable to recognize phase-locked oscillations as a single phenomenon. The harmonics are recovered by composite analysis keyed to the fundamental period.

c. Reconstruction of oscillatory components

M-SSA provides a useful way of extracting from the analyzed signal the components associated with some eigenelements. Indeed, the sum in the right-hand side of Eq. (2.3), restricted to one or several terms, describes the part of the signal behaving as the corresponding EOFs. By analogy with the single channel case (Vauhtard et al. 1992), we define the $k$th reconstructed component (RC, hereafter) at time $i$, and for channel $l$, by the formulas

$$X^k_i = \frac{1}{M} \sum_{j=1}^{M} a^k_{i,j} E^k_j,$$

where $M \leq i \leq N-M+1,$ (2.9a)

$$X^k_i = \sum_{j=1}^{i} a^k_{i,j} E^k_j,$$

when $1 \leq i \leq M-1,$ (2.9b)

$$X^k_i = \sum_{j=1-N}^{M} a^k_{i,j} E^k_j,$$

when $N-M+2 \leq i \leq N.$ (2.9c)

The series $X^k$ are obtained, as in SSA, from a least-squares problem. Unlike the ST-PCs, which are scalar series, RCs are multichannel series, representing the part of the original signal corresponding to the associated eigenelements. The original signal is exactly the sum of all the RCs. A reconstructed oscillation given by the pair $(k, k+1)$ of eigenelements is the sum $X^k_i + X^{k+1}_i$. In the example developed above [Eqs. (2.6)-(2.8)], the reconstruction of the two nonvanishing eigenelements gives the original wave. In the general case, the reconstruction process allows one to isolate the part of the signal involved with an oscillation. The efficiency in recovering both the amplitude and the phase of an oscillation, even at the ends of the series, is shown in section 3 (Fig. 3 below).

d. Spectral properties

From the spectral point of view, the difference between M-SSA and SSA lies in that one ST-PC covers the spectral properties of all the channels. Denoting by $P^0(f)$ the power spectrum of the time series in channel $l$, and by $P^l(f)$ the cross spectrum of the two series in channels $l$ and $l'$, the power spectrum $P^k(f)$ of the $k$th ST-PC is

$$P^k(f) = \sum_{l'=1}^{L} P^l(f) \Phi^l_k(f) \Phi^l_k(f),$$

(2.10)

where

$$\Phi^l_k(f) = \sum_{j=1}^{M} E^k_j e^{2 \pi i j f}$$

(2.11)

is the reduced Fourier transform of the $k$th EOF in channel $l$. Unlike for SSA, ST-PCs cannot be interpreted directly as filtered versions of the original series, since the coherence between all the channels is taken into account in the transform. Nonetheless, a similar global property occurs: if $P(f)$ denotes the total spectrum of the series,

$$P(f) = \sum_{i=1}^{L} P_i(f),$$

(2.12)

then the sum of the spectra of individual ST-PCs is proportional to $P(f)$,

$$P(f) = \frac{1}{M} \sum_{k=1}^{L \times M} P^k(f).$$

(2.13)

Equation (2.13) is particularly useful to identify the variance explained by one or several components in the spectral domain. As pointed out above, a major difference between scalar series and multichannel series is that several spatial patterns may respond in the same frequency band. Therefore, a strong advantage of M-SSA with respect to any other method is its ability to distinguish them. For instance, if an oscillation is superimposed with some colored noise, having also power around the frequency of the oscillation, then M-SSA does distinguish between the part due to the oscillation and the part due to the noise. For SSA, an oscillatory pair usually explains about 100% of the variance near the frequency of the oscillation, whereas for M-SSA, only a fraction of the variance may be explained at the peak (see Fig. 5 below).
There are links between SSA or M-SSA and maximum entropy (ME) spectral estimates. In the single channel case, for instance, the same Toeplitz matrix is used for diagonalization and estimating coefficients of an $M-1$-order autoregressive model. Penland et al. (1991) gave a first insight into these links. Vautard et al. (1992) developed an ME spectral approach that is fully consistent with SSA. For M-SSA, the problem is more complex, as we are dealing with multichannel ME autoregressive models. The method is as follows. A multichannel autoregressive model is fitted to the data time series,

$$Y_i = A_1 Y_{i-1} + A_2 Y_{i-2} + \cdots + A_{M-1} Y_{i-M+1} + W_i,$$

(2.14)

where $Y_i$ is the “state” vector of dimension $L$ at time $i$, and $A_j$ are regression matrices, instead of regression coefficients in the single-channel case, and $W$ is a multichannel white noise process. In the Appendix, we show how to compute the matrices $A_j$. The model is built in such a way that the covariance matrix $T_Y$ of $Y$ is equal to $T_X$. Therefore, the multichannel process $Y$ has the same second-order statistics as $X$ up to lag $M - 1$. In particular, a M-SSA of $Y$ gives the same EOFs and eigenvalues as for $X$.

The series in each channel of $Y$ has ME power spectra and cross-spectra taking the form of rational fractions of the variable $z = e^{2i\pi f}$. Then, one estimates the power spectra $P_y(f)$ of each ST-PC by the power spectra of the ST-PCs of the autoregressive process $Y$, which take also the form of complex rational fractions of the variable $z = e^{2i\pi f}$ (see the Appendix). The ME spectral estimates satisfy the additivity property given by Eq. (2.13). Spectra shown in section 3 are calculated in this way. A major weakness of ME spectral estimates is their lack of statistical significance estimators. Here, we will assess statistical significance by checking the stability of the results when the values of the parameters of the method and the length of the data are varied.

e. Choice of the window length

A major problem when using SSA or M-SSA is the choice of the window length $M$. Generally, the larger the window length, the sharper the spectral resolution. On the one hand, with large windows, one is able to distinguish between two close spectral peaks. The price one pays is a poorer temporal localization of the intermittent oscillation spells in the reconstruction, as a consequence of Heisenberg’s principle. If the window length $M$ is longer than the duration of an intermittently enhanced oscillation, then the reconstruction process will underestimate the amplitude of the oscillation within the spell and overestimate it off in its neighborhood (Gibbs effect). On the other hand, M-SSA does not distinguish between different oscillations of period longer than the window length. Vautard et al. (1992) suggested that

$$\frac{1}{f} \leq M \leq \frac{1}{2df}$$

(2.15)

is a proper choice of $M$ if one wishes to accurately resolve an oscillation of frequency $f$ and spectral bandwidth $2df$. Unfortunately, these bounds cannot be determined a priori. The numerous experiments performed with SSA and M-SSA taught us that the use of a window length $M$ typically allows the distinction of oscillations with periods in the range $(M/5, M)$.

f. Qualitative comparison with other methods

When one is interested in recognizing propagative patterns, a commonly used method is the complex principal component analysis (CPCA; Horel 1984; Preisendorfer 1988). CPCA also provides an expansion like M-SSA, where the basis functions are propagative patterns. However, unless some band filtering is performed beforehand, no particular time scale is attached to a pattern. Regular oscillations in general correspond to eigenelements of the CPCA problem, as shown in the example of Branstator (1987), but propagative patterns found with CPCA are not always associated with oscillations. Also, when spatially similar oscillations cover a wide frequency range, CPCA cannot separate several possible peaks. M-SSA discriminates between different frequency peaks in a natural way. In CPCA, a commonly applied technique is the prefactoring of data (another possibility is the complex principal component analysis in the frequency domain: Horel 1984). M-SSA does this filtering adaptively.

Another commonly used analysis method is the principal oscillation pattern analysis (POP; Hasselman 1988; Penland 1989). The POP analysis relies basically on the diagonalization of a lag covariance matrix. The POPs correspond to oscillations as well, although only one time scale is involved in the diagonalization process. Therefore, the POP analysis is equivalent to an M-SSA with $M = 1$, with a suitably chosen sampling time. In other words, POP analysis uses a first-order autoregressive model, whereas M-SSA uses a model of order $M - 1$ [see Eq. (2.14)]. According to these arguments, the POP analysis should act on a less wide range of time scales. A more quantitative comparison of these methods is left for future study.

g. Dynamical relevance and statistical reliability

Two major problems of interpretation occur when any kind of EOF analysis is performed on an observational dataset: how does one attach a dynamical meaning to the EOF patterns, and how does one estimate the statistical confidence of the EOF patterns?

The first problem lies in the fact that the S-EOF patterns are determined by some orthogonality constraints, and therefore are difficult to interpret in dynamical terms. The S-EOFs do not correspond to the
most unstable modes, nor to physically important directions in phase space. One serious improvement to PCA is done by rotating the basis functions (rotated principal component analysis), therefore relaxing the orthogonality constraint to obtain more physically relevant patterns (Richman 1986). Although the non-rotated S-EOFs do not necessarily point toward important features of the atmospheric dynamics, such as weather regimes, they should at least span subspaces containing these important phase-space areas (Mo and Ghil 1987; Molteni et al. 1988). In the case of ST-EOF patterns, rotation would lead to a loss of the key spectral properties. An alternative approach to this relevance problem is to make use of a physically based scalar product, such as the pseudomomentum (Brunet 1993). These improvements are left for future studies.

In order to show that ST-EOFs should still resemble observed recurrent patterns, let us consider a portion of the series of length $M$, starting at time $i + 1$, ending at time $i + M$, and a M-SSA component $k$. Over this portion, the expansion (2.3), combined with the orthogonality properties of the ST-EOFs show, after a few lines of algebra, that the ST-PC $a_i^k$ is the value of the coefficient $a$ that minimizes the quantity

$$H_i(a) = \sum_{i=1}^{M} \sum_{l=1}^{L} (X_{i+l} - aE_{0})^2.$$  

Therefore, $a_i^k$ is the coefficient achieving the least-squares fit of the $k$th ST-EOF onto the original multichannel series. If the $k$th ST-PC $a_i^k$ is large compared with other ones during some period of time, one expects therefore the time series to resemble the associated ST-EOF. When, moreover, conditions (i) and (ii) are satisfied, for components $k$ and $k + 1$, the least-squares fit property (2.16) remaining valid with two coefficients, a dominant oscillation is expected in the signal, with the same space–time behavior as the corresponding ST-EOFs, provided that the pair does not show up too far in the eigenvalue spectrum.

The least-squares problem (2.16) also leads to the definition of the local variance fraction. The sum of squares of the ST-PCs is equal to the sum of squares of the multichannel series over the window length,

$$\sum_{i=1}^{L} \sum_{j=1}^{M} (X_{i+j})^2 = \sum_{k=1}^{M} \sum_{i=1}^{L} (a_i^k)^2.$$  

Therefore, it is possible to measure the local variance fraction of one or several components. In the case of an oscillatory pair $(k, k + 1)$, the local variance fraction is

$$V_i = \frac{(a_i^k)^2 + (a_i^{k+1})^2}{\sum_{k=1}^{M} (a_i^k)^2}.$$  

This index is particularly useful to measure the relative amplitude of some oscillatory activity within a portion of the same length as the one used in the analysis.

The second major issue is the statistical reliability of the resulting patterns. Up to what order can we trust the patterns, provided that the process is stationary in the statistical sense? In EOF problems, it is possible to attach a statistical confidence to eigenvalues, when the data are normally distributed. The 95% confidence interval of an eigenvalue $\lambda_k$ is given by the heuristic variance formula, $\lambda_k \pm \sqrt{2/\nu} N$, where $N$ is the number of independent samples. Ghil and Mo (1991) gave a simple way to estimate $N$, in the SSA case, that can be used in M-SSA as well, $N = N/M$. This tends to overestimate the confidence interval (Vautard et al. 1992). On the contrary, there is no way, to our knowledge, to estimate the confidence of the EOF patterns themselves. When rotation is applied to EOFs, their stability seems to be enhanced (Richman 1986). For the SSA problem, Vautard et al. (1992) showed, using Monte Carlo simulations on synthetic examples, that the order of the patterns is much less statistically stable than the patterns themselves. In our examples, we estimate the stability of the EOF patterns by separating the data into two parts, or changing the values of the parameters of the method (section 3d).

When the data contain some measurement or numerical noise, assumed to be white, the tail of the SSA or M-SSA eigenvalue spectra saturates at a given value, depending on the noise variance and the sampling rate (Broomhead and King 1986a). This break in the eigenvalue spectrum allows the separation between significant and noisy components. The significant components, however, may not be stable through a change of dataset of the same length. The reconstructions of all significant components are, on the contrary, remarkably similar from one dataset to another (Vautard et al. 1992). With the data we shall use here, the noise level is never reached, and only the first few EOFs will be considered.

A more tricky problem is to assess the statistical significance of an oscillatory pair of eigenvalues. The problem has no general solution, and the existence of oscillations shall be confirmed by more classical spectral methods. All the oscillations identified in the temperature series of Ghil and Vautard (1991) were confirmed by the multitaper spectral method of Thomson (1982), which also provides a statistical significance test. Our methodology here is to extract pairs using more or less subjective criteria (see section 3a) and to confirm the existence of the oscillations by other means.

3. The low-frequency oscillations

a. Data and experiments

We analyze a 32-year set of National Meteorological Center (NMC) analyses of the geopotential heights at 700 hPa, covering the Northern Hemisphere extratropics [also used by Ghil and Mo (1991) and Kimoto et al. (1991)]. The dataset starts on 1 May 1954 and
ends on 30 April 1986. The values are originally given on the NMC diamond grid, consisting of two shifted \(10^\circ \times 10^\circ\) grids. Three sectors are analyzed separately: the Atlantic sector (ATL), extending from 30\(^\circ\)N to 70\(^\circ\)N and 80\(^\circ\)W to 40\(^\circ\)E, the Pacific sector (PAC; 30\(^\circ\)N-70\(^\circ\)N, 140\(^\circ\)E-100\(^\circ\)W), and the whole extratropical Northern Hemisphere (NH) between 30\(^\circ\) and 80\(^\circ\)N. The first two sectors contain 113 grid points, while the NH sector contains 396 points.

The main numerical difficulty lies in the fact that M-SSA diagonalizes the big matrix \(T_X\). The NH domain, for instance, contains 396 grid points. With a window length of, say, \(M = 100\) days one would have to diagonalize a matrix of dimension 39 600. Therefore, M-SSA has to be applied on only a small number of spatial variables, and with the number of lags as small as possible. The most efficient way to compress as much information as possible into a small number of spatial variables is to perform a prior PCA in each domain. The procedure is essentially as in Kimoto et al. (1991).

In order to reduce as much as possible the numbers of spatial variables, and to focus on large-scale phenomena, only S-PCs having most of their variance in the low-frequency range are retained using an objective criterion: each S-PC, \(a(t)\), for each domain, is decomposed as

\[
a(t) = a^*(t) + a'(t),
\]

(3.1)

where the asterisk denotes the 7-day centered moving average operator, and the prime is the deviation from this average. Then, an S-PC \(a(t)\) is kept as a low-frequency S-PC when the variance of \(a^*(t)\) is larger than the variance of \(a'(t)\). This choice is motivated by the fact that one wants to eliminate the part of the variability resulting from synoptic weather systems. For the ATL sector, one keeps the first ten S-PCs. For the PAC sector, S-PCs 1 to 7 and 9–10 are retained. However, for NH, this choice would lead to retaining too many S-PCs (about 25), so, for the NH experiments only, we keep S-PCs \(a(t)\) for which the variance of \(a^*\) is twice as big as the variance of \(a'\). The first 13 NH S-PCs are retained. In fact, our results are really insensitive to the number of PCs kept, as we shall see in section 3c. Thus, for each domain, the multichannel dataset consists of L S-PCs, with \(L = 10\) for ATL, \(L = 9\) for PAC, and \(L = 13\) for NH.

In order to reduce as much as possible the temporal information, we choose a large sampling rate of 5 days with a window length of 200 days, which is a good compromise for the identification of oscillations with periods from two months to one-half year, according to the discussion in section 2. The resulting value of \(M = 40\), and then matrices of sizes \(400 \times 400\) (ATL), \(360 \times 360\) (PAC), and \(520 \times 520\) (NH) are diagonalized. These experiments are denoted hereafter ATL200, PAC200, and NH200.

Due to the long window length, these experiments do not allow an accurate localization of the one- to two-month oscillations, according to the discussion in section 2e. Therefore, a second series of experiments is performed with a window length of 80 days and a sampling rate of 2 days, thus keeping \(M = 40\) (ATL80, PAC80, and NH80). In order to improve the accuracy in the localization of the spells, a low-pass filter is applied beforehand. Instead of a prescribed filter, we use here the adaptive filters provided directly by the first series of experiments. More precisely, using Eqs. (2.9a-c), the original multichannel series \(X\) (of dimensions 10, 9, and 13 for ATL, PAC, and NH) are split into

\[
X = Y + Z,
\]

(3.2)

where

\[
Y_{ij} = \sum_{k=1}^{P} X_{ij}^k,
\]

(3.3a)

and

\[
Z_{ij} = \sum_{k=P+1}^{MN} X_{ij}^k.
\]

(3.3b)

In Eqs. (3.3), \(P\) is a suitably chosen order, after which ST-EOFs start to respond with periods smaller than 60 days. Therefore, \(Z\) is the adaptively filtered series that will be used for studying the 30–60-day extratropical variability. The choice of \(P\) will be discussed after the analysis of the first experiment in section 3b. We take \(P = 8\) for ATL, and \(P = 9\) for PAC and NH. This choice also removes adaptively the annual cycle in the series \(Z\). These experiments will be denoted ATL80, PAC80, and NH80 (see section 3b below).

To summarize, before M-SSA is applied, data are projected onto a reduced subspace of large scale where baroclinic activity is removed; then data are undersampled to reduce further the size of the eigenvalue problem; and finally an adaptive low-pass filter is applied in the second series of experiments, in order to focus on the one-to-two month variability. All these manipulations may seem artificial but are necessary. We check in section 3d that the results are indeed insensitive to the way data are processed before the analysis. After the analysis is performed, and in order to recover daily values, a cubic spline interpolation is applied. Since the sampling rate is 5 or 2 days, this interpolation procedure does not affect the statistics of the processes varying with time scales larger than a month.

In order to select the significant oscillatory pairs, we need a criterion quantifying the conditions (i), (ii), and (iii) of section 2a. Ghil and Mo (1991) used a criterion based on the lag correlation between successive ST-PCs. Here, we follow their approach, and apply a somewhat more severe method: we require, for a pair \((k, k + 1)\) to be an oscillatory pair, that the lag correlation between PCs of order \(k\) and \(k + 1\) has itself an oscillatory behavior. In the example of section 2b,
the lag correlation is a sine function. If the oscillation is only intermittent, the lag correlation should decrease toward 0 as the lag goes to infinity. We require that at least one cycle is coherent, that is, that the lag correlation remains close to a sine function one period ahead: we keep the pair \((k, k+1)\) as an oscillatory pair when the two successive extrema, on each side of 0, of the lag correlation are larger than 0.5 in absolute value.

b. Periods and variances
A) EXPERIMENTS ATL200, PAC200, AND NH200

The eigenvalue spectra for the experiments ATL200, PAC200, and NH200 are displayed in Fig. 2, up to order 30. The eigenvalues are normalized in such a way that they represent fractions of the total variance. Except at some orders, the spectra are monotonically decreasing. The tail of the spectra (not shown) also decreases monotonically, with apparently no noise plateau, indicating either the good quality of the data or the presence of colored noise. The first pair of eigenvalues clearly stands out of the spectra. It corresponds to the annual cycle. Figure 3 shows the reconstructed oscillation, \(X_{1,0}^* + X_{1,1}^*\), together with the raw data \(X_{1,0}\), for the first S-PC (channel 1) for the ATL domain during the last three years of data. This figure shows how accurately the adaptive filter extracts the annual cycle, even at the end of the series. Note that with a window as short as 200 days, the annual cycle cannot be separated from the interannual variability, which is therefore contained in the reconstruction displayed on Fig. 3. The annual and interannual variability explains roughly 50% of the total low-frequency variability (see Fig. 2).

In all spectra, eigenvalues of order 3 and greater are more than one order of magnitude below the first two.

A visual inspection of the spectra suggests that some eigenvalues gather in pairs, like the ST-EOFs 3-4, 7-8, and 12-13 for the ATL domain. We shall hereafter discuss only pairs satisfying the lag correlation criterion of section 3a, and having a period greater than 30 days.

**Table 1.** Characteristics of the oscillatory pairs identified with the criterion described in section 3b. Each row corresponds to a pair \((k, k+1)\). The first column contains the name of the experiment. The second column the order of the pair. The third column contains the period of the oscillation calculated as the peak value of the sum of the SSA-EOFs (see the Appendix) of the two PCs. The fourth column, the total percentage (left number) of variance explained and the percentage, annual cycle removed (right number), are reported, calculated from the eigenvalue spectrum. The fifth column contains the peak value of the variance ratio \(r_k(f) + r_{k+1}(f)\) [cf. Eq. (A8)] in the frequency domain. The last column shows the maximum, along the dataset, of the local variance ratio \(V_k\) [cf. Eq. (2.18)]; the latter values refer to data where the annual cycle is removed, for experiments ATL200, NH200, and PAC200, and for the data where low frequencies are adaptively filtered, for the experiments ATL80, NH80, and PAC80. For the latter experiments, all variance fractions therefore are not calculated relative to the total low-frequency variance, but rather to the variance in the range of periods from a week to two months.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Pair</th>
<th>Period (days)</th>
<th>Variance (%)</th>
<th>Peak variance (%)</th>
<th>Local variance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATL200</td>
<td>1-2</td>
<td>365</td>
<td>41</td>
<td>100</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>7-8</td>
<td>70</td>
<td>1.6/2.7</td>
<td>29</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>12-13</td>
<td>32</td>
<td>1.2/2.0</td>
<td>29</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>15-16</td>
<td>40</td>
<td>1.1/1.9</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td>PAC200</td>
<td>1-2</td>
<td>365</td>
<td>53</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>3-4</td>
<td>182</td>
<td>2.7/5.7</td>
<td>76</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>6-7</td>
<td>122</td>
<td>1.5/3.2</td>
<td>34</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>10-11</td>
<td>52</td>
<td>1.0/2.1</td>
<td>30</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>12-13</td>
<td>35</td>
<td>1.0/2.1</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td>NH200</td>
<td>1-2</td>
<td>365</td>
<td>55</td>
<td>100</td>
<td>72</td>
</tr>
<tr>
<td></td>
<td>3-4</td>
<td>182</td>
<td>2.1/4.7</td>
<td>73</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>6-7</td>
<td>122</td>
<td>1.1/2.4</td>
<td>25</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td>8-9</td>
<td>70</td>
<td>1.0/2.2</td>
<td>26</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>20-21</td>
<td>33</td>
<td>1.0/2.2</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>ATL80</td>
<td>1-2</td>
<td>33</td>
<td>4.4</td>
<td>31</td>
<td>26</td>
</tr>
<tr>
<td>PAC80</td>
<td>1-2</td>
<td>43</td>
<td>5.1</td>
<td>35</td>
<td>28</td>
</tr>
<tr>
<td>NH80</td>
<td>2-3</td>
<td>42</td>
<td>3.6</td>
<td>24</td>
<td>17</td>
</tr>
</tbody>
</table>
They are listed in Table 1, together with their main characteristics; the period of the oscillations is estimated as the peak location of the sum $P(f) + P^{k+1}(f)$ of the ME power spectra of the two ST-PCs (see section 2d and the Appendix). When dividing this sum by the total spectrum $P(f)$ [see the Appendix, Eq. (A8)], one obtains the fraction of the variance $r_{k,k+1}(f)$ at each frequency $f$, explained by the oscillation. Particularly interesting is the value of $r_{k,k+1}(f)$ at the peak. The variance explained is calculated from the eigenvalue spectrum, and the variance explained, annual cycle removed, is deduced by removing the contribution of the first pair. Local variance fractions, with annual and interannual variability removed, of all oscillatory pairs are calculated from Eq. (2.18), with a sum in the denominator restricted to $k \geqslant 3$, and its maximum, corresponding to the highest-amplitude spell, is reported in Table 1.

The first ATL oscillatory pair identified, after the annual cycle, is a 70-day oscillation. The corresponding ST-EOFs 7 and 8 are displayed as multichannel functions of time on Fig. 4. In a given channel $l$, the ST-EOF may be interpreted as an $M$-long time series of the $l$th S-PC. The quadrature between the two ST-EOFs is striking. The fact that the period is identical in all channels is quite general to all oscillations, and reflects precisely the spatial coherence detected at this period. The phase shift between the channels indicates that, as for the example of section 2, this is a nonstanding oscillation. The amplitude in channel 2 is larger than in other channels; hence the associated oscillation is dominated by fluctuations of the second S-PC, which is mostly the NAO pattern of Wallace and Gutzler (1981). This mode explains up to 20% of the variance locally with annual and interannual variability removed. The maximum relative amplitude is reached during winter 1977/78, but the oscillation is also enhanced during several other winters (1961/62, 1984/85).

The next pair identified on Table 1 has a 30–35-day period (pair 12–13), with a smaller variance fraction, and the next one is a 40-day mode (pair 15–16). As explained in section 2e, the distinction of these two modes may result from the too large window used, compared with the small length of the spells; only one mode is identified with the ATL80 experiment (see below). The average variance explained is much smaller (a few percent) than the local maxima of the variance fraction, indicating that the ATL LFOs actually do occur in localized spells. Figure 5 shows the variance fraction explained by components 1 to 16 in the frequency domain. The curves associated with oscillatory pairs exhibit similar pronounced peaks, and explain individually about 15% of the variance at the peak. The contribution of these pairs is therefore about 30% of the variance at their peak frequencies (Table 1).

Over the PAC domain, a semiannual mode is identified (pair 3–4), explaining a relatively large variance fraction. The next pair (6–7) corresponds to an oscillation with a period of about 120 days. A careful examination of the PCs shows that this mode is the third harmonic of the annual cycle. Then, two oscillations with periods of 50–55 days (pair 10–11) and 35 days are identified (pair 12–13). There is no signature of a 70-day mode such as in the ATL domain, suggesting that this mode does not affect the PAC area. Despite the low average variance fraction explained by the intraseasonal modes, the local variance may be quite significant (about 10%–15%), showing again that these oscillations are very intermittent. The PAC intraseasonal LFOs also explain about 30% of the variance at their peak frequency.

When M-SSA is applied to the NH domain, a semiannual component is also dominant (pair 3–4), as well as a third harmonic (pair 6–7). The next oscillatory pair (8–9) is the 70-day oscillation. The lag correlation between the ATL ST-PC 7 and the NH ST-PC 8 has an oscillatory behavior, with a maximum of 0.87,
Fig. 5. Variance fraction, in percent, of the first 16 PCs (experiment ATL200) in the frequency domain, as calculated from Eq. (A8) of the Appendix. Pairs of eigenelements are emphasized (see legend in the figure).

showing that the two modes correspond to the same phenomenon. This also confirms the robustness of this mode. The next pair (20–21) corresponds to a 33-day oscillation, with much less variance explained than the PAC and ATL modes of one to two months. The relations among all the 30–60-day modes are studied in section 3c.

Before commenting on the results for the 80-day window experiments, let us justify our choice for the adaptive filtering we used. Figure 6 shows the fraction of the variance of the ST-PCs adaptively removed in the frequency domain [Eqs. (3.2) and (3.3a,b)], in order to study the 30–60-day variability. These curves are obtained by dividing the sum of the ME power

Fig. 6. Variance fraction, in percent, of the cumulated PCs 1–8 (ATL200), 1–9 (NH200 and PAC200), in the frequency domains. These curves represent the fraction of the variance removed by the adaptive filter used for the experiments with a window length of 80 days.
spectra of ST-PCs 1 to 8 (ATL) and 1 to 9 (PAC, NH) by the total ME power spectrum $P(f)$. For all domains, these first slow components explain from 20% to 90% of the variance beyond 60 days, so that there are still some low frequencies in the filtered data. In the 30–60-day band, less than two percent of the variance is affected. The window of 80 days should therefore resolve accurately all oscillations in this frequency band (as discussed above, the window should be larger than the range of periods to study).

2) **EXPERIMENTS ATL80, PAC80, AND NH80**

For the 80-day window experiments, the percentages of variance in Table 1 refer to the total variance of the adaptively low-pass filtered datasets. Only one dominant oscillation is found (pair 1–2) with a period of 30–35 days over the ATL domain. The lag correlation between the ATL80 ST-PC 1 and the ATL200 ST-PC 12 has a maximum of 0.73, showing that they correspond to the same oscillation. This in turn indicates
that the oscillation is highly significant: it is recovered with two very different window lengths. The fraction of variance is higher, due to the removal of low frequencies. In the PAC domain, a 40–45-day oscillation is found (pair 1–2), again well correlated with the PAC200 modes. The NH80 experiment gives a global oscillation with a period of 40–45 days (pair 2–3), but with a smaller variance ratio than for the 30–60-day ATL80 and PAC80 oscillations. Ghil and Mo (1991) also observed several modes responding in the 30–60-day band, using SSA.

The conclusions that can be drawn from these preliminary figures are

(i) The annual cycle together with the interannual variability occupy about 50% of the low-frequency variance. Second and third harmonics of the annual cycle are present, especially over the Pacific domain.

(ii) Besides the harmonics of the annual cycle, there is a dominant 70-day oscillation, which does not affect the Pacific domain. Then, one oscillation with a period in the 30–60-day range is found in each domain. A careful examination of the amplitude shows that these modes, as the rest of the low-frequency variability, are particularly enhanced during winter periods [see also Ghil and Mo (1991)].

Accordingly, the next section is devoted to a detailed description of the space–time structure of the oscillations during the winter periods, winter starting on 15 November and lasting 136 days. We shall thereafter focus on the 70-day mode by using only the ATL200 experiment, and on the three 30–60-day modes, using the ATL80, PAC80, and NH80 experiments.

c. Space–time structure

We analyze the space–time behavior of the oscillations by performing a composite analysis keyed to the phases of the oscillations, which provides reliable information about the amplitude of the phenomenon, whereas ST-EOFs do not. Ghil and Mo (1991) and Kimoto et al. (1991) also carried out a composite analysis, defining phase categories by the use of ST-PCs. Here we use the RCs as a basis for our composites, since RCs can be directly compared with the data. The RCs $Y = X^k + X^{k+1}$ of the oscillatory pairs $(k, k+1)$ are computed. Then, the channel $f$ for which the variance of $Y_f$ is maximal is chosen. Its time derivative $Y_f$ is estimated by centered finite differences. The instantaneous phase $\phi_i$ of the oscillation at time $i$ is defined as the angle between the instantaneous two-dimensional vector $(Y_{i-1}, Y_i)$ and the vector $(0, 1)$. The phases $\phi_i$ are classified into eight equally populated categories, by splitting the interval $(0, 2\pi)$ into eight intervals in such a way that the same number of values of $\phi_i$ falls within each interval. In this way, the atmosphere spends approximately a constant time in each phase category. For each oscillatory pair, the time behavior of the category number turns out to be very regular, with successive values of 1, 2, 3, . . . , 8, 1, . . . , except for some rare irregularities (typically once or twice within the 32 years studied).

The 95% statistical confidence limits, for each composited quantity, are calculated using a nonparametric Monte Carlo method (Dole and Gordon 1983; Livezey and Chen 1983; Vautard et al. 1990). First, consecutive
days belonging to a given phase category are gathered into periods. These periods are shuffled 100 times, thus providing 100 independent Monte Carlo sets of randomly distributed category periods with the same distribution of lengths. Each statistical quantity keyed to the phase category is also calculated with the 100 Monte Carlo sets of category numbers. For each phase category, one obtains therefore one "true" value and 100 "random" values. The 95% confidence interval for the random case is given by the 6th and the 95th random values, after being sorted into ascending order. A statistical estimate is significantly higher (lower) than normal when it lies above (below) this interval.

1) THE ATLANTIC 70-DAY OSCILLATION

The composites of the raw 700-hPa geopotential height keyed to categories 5, 6, 7, and 8 of the 70-day oscillation are displayed in Figs. 7a–d, representing half of the oscillation cycle. The propagative pattern is similar to that of the corresponding ST-EOFs 7 and 8 (not shown). The pattern is, in fact, not confined to the

FIG. 10. Same as Fig. 7 but for categories 4–7 of the ATL80 30–35-day oscillation. (a) Category 4; (b) category 5; (c) category 6; (d) category 7.
Atlantic sector, since, on categories 6 and 7, an anomaly takes place over Siberia. No significant anomaly is observed over the Pacific sector. Over the North Atlantic area, this oscillation consists of the poleward propagation of an elongated dipole across the Atlantic, associated with fluctuations in the location and amplitude of the Atlantic jet. Such anomaly patterns are comparable with the dominant North Atlantic Oscillation (NAO) mode of low-frequency variability of Wallace and Gutzler (1981), as well as geopotential anomalies associated with the largest variations of the Atlantic storm track (Lau 1988). Here, we show not only the spatial coherence but also the temporal regularity of this phenomenon.

The amplitude of the composite is maximal in category 8, with a positive anomaly of nearly 80 m southeast of Greenland. In the opposite category (4; not shown), the anomaly is much weaker, with an amplitude of about −50 m, suggesting some nonlinearity in the dynamical process underlying the oscillation: if an unstable periodic orbit corresponds to this oscillation, its geometrical shape is not purely ellipsoidal. In other categories (1 to 3; not shown), the composites are approximately the opposite patterns from those in categories 5 to 7. Note that categories 5 and 6 have a rather similar composite, indicating that the phase velocity is reduced during these phases: the northward propagation is not regular within the cycle.

The 70-day oscillation is now confirmed by two other means: (i) classical spectral analysis and (ii) Hovmöller diagrams.

(i) We display on Fig. 8 the power spectrum of the difference between the unfiltered daily values of the geopotential at point (60°N, 35°W) and at point (40°N, 35°W). This spectrum clearly peaks near 70 days, even though lower frequencies are also involved. Note also the pronounced peak near 30 days, showing that the other ATL oscillation also affects this signal (see also Fig. 10). SSA experiments performed on this time series, without any prior filtering, also show a dominant 70-day cycle with similar composite patterns.

(ii) The Hovmöller diagrams of Figs. 9a–b show the time behavior of the geopotential along the 35°W longitude line for the two winters 1961/62 and 1977/78, for which the amplitude was particularly strong. The transient fluctuations have been removed by applying a priori 30-day moving average. The slow phase velocity period occurs when the jet is strongest or weakest, as discussed above.

One may argue that M-SSA is a rather heavy tool to analyze the oscillations that could be recovered by simple spectral analysis at selected points. The point here is precisely that M-SSA allows a systematical exploration without any prior knowledge of the data, and these selected points are not known a priori.

2) THE ATLANTIC 30–35-DAY OSCILLATION

Figures 10a–d show the composites keyed to categories 4 to 7. In category 4, the anomaly consists of an enhancement of the Atlantic jet stream east of its average position. The anomaly pattern drifts westward in categories 5 and 6 and is replaced by a strong positive anomaly over Scandinavia and Scotland. The resulting flow is characterized in categories 7 and 8 by a diffuence of the jet. The opposite categories exhibit opposite anomalies of about the same amplitudes. This oscillation is therefore characterized by the retrogression of a dipole pattern, with a high amplitude over Scandinavia and the Atlantic domain. No significant anomaly is found over the Pacific area. The traveling pattern of this mode bears similarities to the composites made by Ghil and Mo (1991) and Kimoto et al. (1991), who analyzed the geopotential height in the global Northern Hemisphere.

In phase categories 7 and 8, the anomaly has the structure of the Euro–Atlantic blocking pattern, although the amplitude of the composite is not strong enough to produce a closed cell when the winter average flow is added. The retrogression of blocking-like patterns has already been pointed out by Rex (1950), and associated with explosive upstream cyclogenesis by Colucci (1985). This association of low-frequency retrogression with synoptic activity indicates that baroclinic transients definitely play an important dynamical role in the maintenance of this oscillation.

An important dynamical question is whether the 30–35-day oscillation is a harmonic of the 70-day oscillation. Recently, Tribbia and Ghil (1993, personal communication) showed, using a barotropic model with realistic parameter values, that their 40-day extratropical mode is a harmonic of an 80-day periodic orbit. This periodic orbit is nonlinearly produced, since in the neighborhood of all fixed points, no instability with such period is found. In order to investigate this possibility from our dataset, we calculate the histogram of simultaneous categorical occurrences of the two oscillations. The results are shown in Table 2. The distribution is far from uniform, with simultaneous phase
categories significantly more coincident than if we make the assumption that the two oscillations are independent. The phase locking is clear since one 70-day oscillation phase category corresponds to two preferred categories of the 30–35-day oscillation. This tends to confirm the recent results of Tribbia and Ghil. Nevertheless, as revealed by Table 2, this phase locking does not occur systematically.

Simultaneous categories 8 of the two oscillations occur frequently and correspond to roughly the same pattern, that is, a positive anomaly centered between Greenland and Iceland, whereas simultaneous opposite categories are much less likely to occur. This explains the higher amplitude of the 70-day composite during phase category 8, and the lower amplitude of the opposite phase category. No correlation was found with the annual cycle, so that the two oscillations are internally driven LFOs.

3) THE PACIFIC 40–45-DAY OSCILLATION AND THE GLOBAL OSCILLATION

The composites of the PAC 40–45-day oscillation are displayed in Figs. 11a–d for phase categories 4 to...
7. The oscillation is confined over the Pacific domain, and consists of a monopole pattern drifting westward. Both amplitude and phase velocity are varying. The slow phases occur in categories 4 to 6, when the amplitude is higher, and when the pattern has a substantial positive PNA projection. In fact, the oscillation is almost a standing wave. These patterns are again similar to the restriction to the Pacific of the global composites by Ghil and Mo (1991) and Ferranti et al. (1990). This addresses the question of the globality of the 30–60-day extratropical oscillation. The histogram of the simultaneous ATL 30–35-day oscillation and the PAC 40–50-day oscillation is calculated as in Table 2, and reported in Table 3. There is no significant preference for simultaneous categories. Therefore, one is led to conclude that the two oscillations are independent.

The problem becomes more puzzling when the pair 2–3 (period 40–45 days) of the NH80 experiment is submitted to the same composite analysis, displayed on Figs. 12a–d. When the whole Northern Hemisphere is considered, one obtains an oscillation sharing the characteristics of both the PAC and the ATL ones. The amplitude, however, is significantly reduced over the Pacific domain. The joint histograms of this oscillation with the ATL 30–35-day oscillation and the PAC 40–50-day oscillation (not shown) exhibit a significant correlation with both sectoral oscillations.

One possible explanation is that there are indeed two physical mechanisms for the oscillations, with intermittent phase locking. In order to investigate this possibility, we calculate the local variance fraction explained by the three 30–60-day modes, using Eq. (2.18). A portion of this local variance is displayed on Fig. 13 for the three oscillations (last 6 years). Visual inspection of this variance fraction shows that the high-amplitude spells of the sectoral oscillations are uncorrelated. Spells of the Atlantic oscillation may exist even when the Pacific oscillation is not particularly excited and vice versa (like in winters 1984/85 and 1985/86). The amplitude of the hemispheric oscillation shows no particular correlation with the sectoral oscillations. The 20 distinct spells of higher amplitude of the hemispheric oscillation are localized by isolating the 80-day periods around the highest peaks of the variance fraction. Based only on these 1600 days, the simultaneous histogram of the sectoral oscillations is calculated and reported in Table 4. A clear phase locking is observed between the sectoral oscillations, when the amplitude of the hemispheric oscillation is large, and only in this case. This latter result, together with the independence of phase and amplitude of the sectoral oscillations, leads to the conclusion that the hemispheric oscillation results from the phase locking of the Pacific and the Atlantic modes of different physical origins.

The influence of the tropical Madden–Julian wave on the extratropical oscillations is not well understood. All investigations designed to study the connection between tropics and extratropics show evidence of correlation. The composite of the high-latitude circulation keyed to phases of the tropical OLR oscillation (Ferranti et al. 1990) displays a more similar pattern in the Pacific area than in the Atlantic area. From our analysis, the period of the Pacific dominant mode is closer to the 50-day period found for the tropical oscillation, as revealed by the angular momentum (Penland et al. 1991) and also by the length of day (Dickey et al. 1991). This, together with the results of the modeling study of Simmons et al. (1983), suggests that the Pacific sector is much more involved in the tropical–extratropical interactions, whereas the Atlantic oscillation is purely extratropical. The question whether the tropical oscillation drives the extratropical one is a difficult question. Several modeling experiments (Marcus 1990; Molteni et al. 1993) indicate that, in fact, the extratropical oscillation is driving the tropical one.

d. Sensitivity experiments

In order to test the sensitivity to parameter changes in the method, we apply M-SSA in the different domains in the various following ways:

(i) Same method parameter values as for the ATL200, PAC200, and NH200 experiments, but with the first half (experiments ATL200.1, PAC200.1, and NH200.1) and the second half (ATL200.2, PAC200.2, and NH200.2) of the data.

(ii) Same M-SSA parameter values as for ATL200 and PAC200, but with only the first five channels, thus taking $L = 5$ (ATL200.5, PAC200.5, and NH200.5).

The criterion used for the selection of oscillatory pairs is the same as above. The results are summarized in Table 5. First of all, the annual cycle is found in all experiments as the pair 1–2. Then, its harmonics are mostly recovered by all experiments. The internal LFOs are also quite stable; the 70-day oscillation is absent only in the NH200.5 experiment. This results from the fact that the small number of S-PCs (five) used does not allow a correct spatial localization of the phenomenon confined over the Atlantic and Siberian area. In all ATL experiments, the oscillation is identified, with similar periods and variance explained. Its existence is
also confirmed by the analysis of the simple index based on two grid points (see section 3c). Note that the rank of the pair is not stable, since it occurs in pair 5–6 in the ATL200.1 experiment. Some lower frequencies are also found in the ATL200.1 and ATL200.2 experiments, but their significance is rather poor.

One- to two-month ATL modes are also recovered in each experiment, with ranks and periods varying from one experiment to another. Two Pacific modes are again found in the 30–60-day band. Like the other oscillations, the NH 40–45-day mode is recovered in all sensitivity experiments but the NH200.2 one.

4. Low-frequency oscillations and weather regimes

The similarity between the Euro-Atlantic blocking pattern and a particular phase of the ATL 30–35-day oscillation is suggestive of an interaction between the two dynamical phenomena. Vautard (1990; V90 hereafter) identified four quasi-stationary weather regimes
over the same ATL area by requiring that the average, over all the occurrences of neighboring patterns, of the instantaneous tendencies vanishes. The first weather regime, BL, is the Euro–Atlantic blocking. The second, ZO, is an enhanced zonal flow across the Atlantic. The third, GA, consists of a strong positive anomaly over Greenland, and the last, AR, is a ridge off the European coasts. Then, the regime events were temporally localized and listed in the Appendix of V90. We use here these events, except some of them in the beginning of the record, as they occur before 1 May 1954, which is the beginning of our record. There are 64 BL events, 93 ZO events, 85 GA events, and 87 AR events.

In Table 6, we display the number of days of each weather regime falling into the phase categories of the ATL 70-day and ATL 30–35-day oscillations. Statistical significance is assessed in the same way as before, that is, by shuffling the periods of constant phase categories one hundred times, and performing the same counting algorithm for all the Monte Carlo realizations. The BL occurs significantly more than average in categories 7–8 of the 70-day oscillation, and in categories 6–7–8 of the 30–35-day oscillation. This is consistent with the similarity between BL and the composite (category 7) of the 30–35-day oscillation. This preference is more surprising for the 70-day oscillation since the similarities between the composites and the BL pattern are not evident. Preferences are also found for the other weather regimes. Note, however, that the 30–35-day oscillation does not apparently interact with the AR regime. These results are recovered, with the NH200 70-day oscillation, with the same degree of significance, and for the NH80 40–45-day oscillation, although less significant in this case. On the contrary, no significant figures are found for the interaction with the PAC oscillations.

On Figs. 14a–b, the regime events are displayed, as pieces of the reconstruction of the ATL 30–35-day and 70-day oscillations [using Eqs. (2.9a–c)], projected onto two selected pairs of S-EOFs. The preference of weather regimes to particular phases of the oscillation is evident: BL events gather in the upper half of Fig. 14a, whereas GA events are gathered in the right part, and ZO events in the lower left part of the diagram. In Fig. 14a, the trajectory turns clockwise, and anticlockwise in Fig. 14b. It is noteworthy that regimes do not occur systematically in particular phases of the LFOs. Therefore, weather regimes are not simply due to slow phases of some nearly resonant Rossby waves. The BL event occurring in the lower-left corner of Fig. 14a is a piece of the famous January 1963 persistent blocking event; very persistent events such as this one cannot stick to one phase of the oscillation. In such a case, the part of the signal not described by the reconstruction is dominant. The only interpretation of Table 6 and Figs. 14a–b is that LFOs influence the development of weather regimes, but in a nonsystematical way.

The phase-space picture of Fig. 1 cannot explain the interaction between LFOs and regimes, since, even during persistent blocking events, LFOs are still active, as in the example of January 1963. In fact, during this period, the S-PC 2 has high-amplitude fluctuations, with periods of about one month, whereas S-PCs of higher order are nearly constant. Therefore, the system LFOs regimes is more likely to behave like two weakly

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Table 4. Same histogram as in Table 3 but counting only days belonging to one of the 20 highest-amplitude spells of the NH 40–45-day oscillation. Note that here the days counted do not necessarily belong to winter periods.

Fig. 13. Time evolution of the local variance (centered with respect to the window of 80 days) of the 30–60-day modes, calculated from Eq. (2.18). Solid curve: ATL80 30–35-day mode; dashed curve: PAC80 40–45-day mode; dotted curve: NH80 40–45-day mode.
Table 5. Summary of the pairs found in the sensitivity experiments. The columns correspond to the experiments (see text). Each row corresponds to a pair of eigenelements. In each cell, the period (in days) appears first (top) and the pair order follows (bottom).

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coupled dynamical systems, such as the example proposed by Molteni et al. (1993), who coupled the Lorenz equations (representing the regimes) with an oscillator (representing the LFO).

An important question is also whether the oscillations are able by themselves to produce high amplitude persistent anomalies. The composites of Figs. 7 and 10 are not strong enough to modify deeply the atmospheric circulation. We display on Fig. 15 the histogram of BL days occurring in simultaneous categories of the two oscillations. The preference for category 7 of the 30-35-day oscillation occurs only when the 70-day oscillation is in category 8 or in neighbor categories, the two simultaneous categories being phase locked, according to Table 2.

The global composite of the 700-hPa geopotential keyed to simultaneous phase 7 of the 30-35-day oscillation and phase 8 of the 70-day oscillation is displayed in Fig. 16a (average over 107 maps). The amplitude of the anomaly is able to nearly produce the dipole pattern with the closed cell characterizing the Euro-Atlantic blocking. When only the 20 higher amplitude spells of each oscillation are considered (see previous section), the resulting composite is shown in Fig. 16b. Although this composite is an average over 23 maps, it exhibits a strong blocking pattern. Therefore, the interference between the two ATL oscillations

Table 6. Number of days corresponding to each regime (BL, ZO, GA, AR; columns) and simultaneously belonging to the categories (rows) of the two ATL oscillations. As in Tables 2-4, figures exceeding the 95% significance level are in boldface characters (higher values) or in italics (lower values).

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Fig. 14. Projection, onto the plane spanned by two S-PCs, of the reconstruction of the 30-35-day oscillation and the 70-day oscillation, described, respectively, by the pairs 1-2 (ATL80) and 7-8 (ATL200). The trajectory is drawn only during weather regime events. (a) 30-35-day oscillation. Regimes represented are BL (heavy curve), GA (light curve), and ZO (dotted curve). S-PCs 2 and 4 are used for projection. (b) 70-day oscillation. Regimes represented are ZO (dotted curve), AR (light solid curve), and BL (heavy solid curve). S-PCs 1 and 2 are used for projection.
is able to produce blocking events. Again, there are a number of blocking events that cannot be explained by this interference, suggesting that blocking may originate from different physical and dynamical mechanisms.

Going back to Figs. 14a–b and Table 6, it is also interesting to notice that the preferred phases of the 30–35-day oscillation are consistent with the preferred transitions between the regime events found in V90, that is, ZO $\rightarrow$ BL $\rightarrow$ GA. This chain of retrograding patterns is already present in the composites of the 30–35-day oscillation (Figs. 10a–d), ZO corresponding to categories 4–5, BL to category 7, and GA to categories 8–1. A careful examination of individual event dates shows that most of the ZO $\rightarrow$ BL and BL $\rightarrow$ GA transitions correspond to BL events falling in the preferred categories 6, 7, or 8. This suggests that the transitions between Atlantic weather regimes are strongly influenced by the 30–35-day oscillation. This provocative statement contradicts the view of transitions as being randomly produced by synoptic transients. Transitions are indeed favored by regular phenomena. However, the time scales involved in the LFOs are such that very rapid transitions (1–3 days) cannot be localized accurately.

These latter remarks, together with the fact that the LFOs are able to produce high amplitude anomalies and influence weather regimes, have important consequences for the extended-range predictability. The oscillations are regular, predictable phenomena. Therefore, on the one hand, nontrivial probabilities of occurrence may be given a long time in advance. On the other hand, our results show that the operational forecasting models must represent correctly the extratropical LFOs in order to forecast blocking events. The particular failure of general circulation models in forecasting ZO $\rightarrow$ BL transitions (Tibaldi and Molteni 1990) could be due to a bad representation of the Atlantic 30–35-day oscillation.

In order to illustrate the potential of predictability of the weather regimes by taking the phases of the ATL LFOs as predictors, we display on Figs. 17a–d the conditional probability of occurrence of the four regimes, in winter, given the phase category of the 30–35-day oscillation $\tau$ days before, as a function of $\tau$, with the 95% Monte Carlo confidence intervals. The unconditional probability of occurrence of BL is of about 6%, and the conditional probability at $\tau = 0$ is about 2–3 times larger in category 7. The conjugate occurrence of BL and phase 8 of the 30–35-day oscillation 30 days before is still about twice as probable as the “random” occurrence of BL. The conditional probabilities remain significant up to $\tau = 40–50$ days for BL, ZO, and GA. In particular, for GA, the conditional probabilities one period ahead ($\tau = 35$) are not very different from the conditional probabilities at lag $\tau = 0$. This indicates that GA is the regime most influenced by the oscillation.

For AR, no significant probabilities occur at lag $\tau = 0$, whereas there is a significant increase in the range $\tau = 20–30$ days. Such a behavior could be due to purely nonlinear effects: the phase-space trajectories lying in the neighborhood of the regime AR, uncorrelated with the oscillation, were stuck to the 30–35-day unstable periodic orbit 20 days before, a behavior illustrated in Fig. 1 (with time reversal). This explanation is, however, somewhat speculative and the phenomenon has to be studied in more detail.

The conditional probabilities are also calculated for the 70-day oscillation (not shown). The same behavior is observed, with figures significant up to $\tau = 60$ days for BL, and 100 days for ZO and GA. For AR, the same phenomenon as for the 30–35-day oscillation is observed, with maximal probabilities occurring near $\tau = 40$. Finally, all the results for BL shown in this section also hold, when a more synoptically based definition of blocking is used. When the criterion of Tibaldi and Molteni (1990), based on the approach of Lejenä and Økland (1983), is used, we find the same preferred categories for the two oscillations, significant at the 95% confidence level. This shows the robustness of our results to a change in the method of identification of weather regimes. For this latter definition of blocking, the dataset used and covering period are different, since the dataset consists of 500-hPa NMC analyses starting in 1963.

5. Summary and discussion

We have presented a descriptive analysis of the low-frequency variability of the Northern Hemisphere. Our
purpose was to focus on regular phenomena observed in the 700-hPa geopotential height fields. During the last decade or so, particularly important spatial patterns have been found in the form of quasi-stationary weather regimes (Charney and DeVore 1979; Legras and Ghil 1985; Vautard 1990) or recurrent patterns (Legras et al. 1987; Mo and Ghil 1988; Molteni et al. 1990). Here, we studied the regular recurrent space-time patterns. The simplest form of such regularities is the low-frequency oscillations (LFOs), possibly due to the existence of unstable periodic orbits in phase space.

The method used to identify LFOs is the multi-channel singular spectrum analysis (M-SSA). We have presented the theoretical property of this method, its ability to distinguish at the same time various dominant modes of the variability. M-SSA gives a rather complete space–time and spectral picture of the oscillating phenomena in a wide range of time scales. We focused on the intermonthly variability (periods larger than 30 days). The intermonthly oscillations are:

(i) An Atlantic oscillation with a 70-day period, whose pattern is mostly related to the NAO pattern of Wallace and Gutzler (1981), and which therefore consists of fluctuations in the position of the Atlantic jet. This oscillation propagates poleward, with a standing component over Siberia.

(ii) Another Atlantic oscillation with a period of 30–35 days consisting of the retrogression of a dipole pattern across the Atlantic.

(iii) A Pacific oscillation with a period of 40–50 days, consisting of the retrogression of a monopole anomaly, with a particularly high amplitude when the pattern is in phase with the PNA teleconnection pattern.

(iv) A hemispheric 40–45-day oscillation, with patterns similar in each sector to the two previous ones.

Despite the particular attention paid to the extratropical 30–60-day variability in recent studies, the dominant oscillatory mode turns out to be the 70-day mode. This mode and the 30–35-day Atlantic oscillation are frequently phase locked. The analysis performed in section 3 shows that the 30–35-day mode may be a harmonic of the 70-day mode, as also found in the barotropic model of Tribbia and Ghil (1993, personal communication). The Atlantic and Pacific 30–60-day modes are uncorrelated, despite their relatively similar periods. However, when the analysis is carried out for the entire hemisphere, a global oscillation is found with a period of 40–45 days. A detailed examination of high-amplitude spells of the global oscillation indicates that there are indeed two distinct oscillations that are intermittently phase locked.

Thus, we have studied the interactions between the LFOs and the North Atlantic weather regimes BL (blocking), ZO (zonal), GA (Greenland anticyclone), and AR (Atlantic ridge), as defined by Vautard (1990). Weather regimes are significantly influenced by the phase of the LFOs. The Euro–Atlantic blocking (BL) occurs preferentially in a particular phase of the Atlantic 30–35-day oscillation. On the one hand, this indicates that LFOs act as favorable environments for the occurrence of weather regimes. On the other hand, regimes do not occur systematically in certain phases of the LFOs, showing that they do not simply represent...
slow phases of some regular waves. The retrogression characterizing the 30–35-day ATL mode is consistent with the preferred ZO → BL → GA transition chain, observed by Vautard (1990), using an entirely different approach. In particular, the transition from a zonal to a blocking pattern is strongly favored by the Atlantic 30–35-day oscillation, which contradicts the hypothesis of random transitions between these regimes.

We have also shown that high amplitude spells of the LFOs are able to produce high amplitude anomalies by themselves. This suggests, for instance, that blocking may originate from several different dynamical mechanisms, one being a particular phase of the phase-locked 70-day and 30–35-day oscillations, especially when the amplitude is high (see Figs. 16a–b). Other blocking instances are not linked with phases of the LFOs where the pattern is similar, indicating that there must be at least one different dynamical origin of blocking.

The impact of these results for long-range forecasting is twofold. On the one hand, regular oscillations are predictable phenomena. We showed in section 4 that the knowledge of the current phase of the 30–35-day oscillation increases the knowledge of the probability of occurrence of blocking by a factor of 3 (for phase category 7 of the oscillation) relative to its climatological probability, and by a factor of 2, 30 days ahead. Therefore, if the phase of the LFOs is correctly estimated, nontrivial probabilities of regime occurrences may be given. On the other hand, general circulation models have to represent correctly the LFOs in order to correctly forecast transitions. Most general circulation models now simulate quite well the tropical Madden and Julian (1971) oscillation, but there is little knowledge about their skill in reproducing the extratropical oscillations.

The physical origin of the LFOs is a more difficult question. First, as shown by several authors (Ferranti et al. 1990; Kimoto et al. 1991), there are interactions between the tropical Madden–Julian oscillation and the extratropical 30–60 day variability. The question concerning whether the tropics drive the extratropics or the opposite is still a controversial one. As shown by Dickey et al. (1991), the period of the Madden–Julian wave is close to 50 days. Taking into account the fact that the Pacific area is more sensitive to the
tropical circulation than the Atlantic (Simmons et al. 1983), we conjecture that the Pacific 40–50-day mode found here is linked with the tropical oscillation, whereas the Atlantic 30–35-day mode is an internally sustained extratropical oscillation. The phase locking between this mode and the 70-day one shows that these phenomena have to be studied together.

From the extratropical point of view, oscillations may originate from two competing mechanisms. Previous modeling studies (Jin and Ghil 1991; Tribbia and Ghil 1992) indicate that the LFOs may result from the barotropic interaction between the zonal circulation and the topography. This mechanism, however, is unlikely or insufficient for the Atlantic domain, due to the lack of mountains. There are also good reasons to believe that the synoptic transients are playing a crucial role in the maintenance of these oscillations. First, transients contribute to a large part of the observed low-frequency variability (Egger and Schilling 1983; Metz 1987, 1989) and the maintenance of weather re-

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**Fig. 18.** Same composite as in Fig. 7 (thin contours). The thick contours surround the areas of significantly high transient activity (solid thick contours), and significantly low transient activity (dashed thick contours). Transient activity is measured by the variance of high-frequency fluctuations (periods below 7 days).
gimes (Hoskins et al. 1983; Shutts 1983; Vautard and Legras 1988). Second, the time scales involved in LFOs such as the 70-day oscillation are much larger than the e-folding times and periods of the baroclinic waves. Therefore, transients "have time" to force "on average" the low-frequency oscillation patterns.

An interesting problem could be to formulate a time-averaged equation governing the low-frequency evolution where the transient feedback is slaved to the low-frequency pattern:

$$\frac{dX}{dt} = F(X, X) + G(X),$$  \hspace{1cm} (5.1)

where $X$ represents the low-frequency part of the atmospheric flow, $F$ the interaction of $X$ with itself, and $G(X)$ the parameterization of transient feedback. Given a proper parameterization $G(X)$, the linear stability analysis of Eq. (5.1) could provide the modes of the low-frequency variability. For instance, Eq. (5.1) has been studied within the framework of a two-layer baroclinic model and yielded realistic stationary solutions (Vautard and Legras 1988).

In order to establish the modification in the transient activity during spells of LFOs, we study the composite anomalies of the storm tracks keyed to the same categories of the 70-day oscillation as displayed on Figs. 7a–d. More precisely, the variance of the deviation of the 700-hPa geopotential height from a 7-day moving average are calculated and composites of this variance keyed to the categories are calculated. On Figs. 18a–d, we display areas where this composite variance is significantly higher and lower than the average variance, at the 90% confidence level, together with the composite themselves (same as in Fig. 7). Results for the opposite categories are approximately opposite. The baroclinic activity is enhanced in the areas where the jet is intensified, and shifted in the trough regions. Therefore, the convergence of transient eddy momentum fluxes should increase in the same areas, displacing the Atlantic jet northward. Reduced transient activity is, on the contrary, observed over the southern parts of the positive anomaly centers, giving rise to the same global effect. Thus, transients tend to propagate the Atlantic anomaly pattern northward. This result must, however, be confirmed by direct calculation of eddy fluxes, which is left for future studies. Our results are consistent with the studies of Lau (1988) and Metz (1989), who correlated transient activity modes with the low-frequency patterns, and with those of Cai and Van den Dool (1991), who reported important relationships between transient feedback and low-frequency traveling patterns.

M-SSA not only provides new directions of research in the understanding of the midlatitude circulation, but is also easily coupled with linear forecasting methods, such as maximum entropy methods (ME). Such an application of SSA has been performed by Kepenne and Ghil (1992), who gave 36-month valid forecasts of the Southern Oscillation index. When applied to the short IPCC global surface temperature series, SSA-ME forecasts seem to remain valid up to a five-year lead time (Vautard et al. 1992). Thus, there is a hope that at least a nonnegligible part of the variance, associated with the oscillations, may be forecast easily by simple linear models (dynamically or empirically derived). If the drift of the general circulation models in the long run distorts the parameter values (frequency, amplitude) of these regular oscillations, it could be interesting to consider the outputs of the simple linear empirical models as constraints on the GCMs to produce better long-term forecasts.

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**APPENDIX**

**Multichannel SSA-ME Spectral Estimates**

In this Appendix, we show how to compute numerically the regression matrices defined in section 2d [Eq. (2.14)], and how to estimate spectral quantities associated with the multichannel data series $X$. For a single-channel process, these matrices become single elements, and are easily calculated by solving the Yule-Walker equations (Ulrych and Bishop 1975). The equations satisfied by the regression matrices ($A_i$, ..., $S$) in the multichannel case, in order to fit the data lag covariances, up to lag $M$ become

$$\begin{pmatrix} G_0 & G_1 & \cdots & G_{M-1} \\ G_1 & G_0 & \cdots & \cdot \\ \vdots & \vdots & \ddots & \vdots \\ G_{M-1} & \cdots & \cdot & G_0 \end{pmatrix} \times \begin{pmatrix} I \\ -A^T_0 \\ -A^T_1 \\ \vdots \\ -A^T_{M-1} \end{pmatrix} = \begin{pmatrix} S \\ 0 \\ 0 \\ 0 \end{pmatrix}, \hspace{1cm} (A1)$$

where $G_j$ is the cross-covariance matrix between all channels at lag $j$, whose element in row $l$ and column $l'$ is

$$(G_j)_{l,l'} = \mathcal{C}(X_lX_{l'+j}); \hspace{1cm} (A2)$$

$I$ is the identity matrix, $S$ is the symmetric positive-definite covariance matrix of the noise forcing $W$, and...
the expectation operator. This system is a nonsingular linear system, which can be solved numerically by classical methods. Note that Eq. (A1) is not put exactly in the form of a classical linear system for the sake of simplicity. Note also that the large matrix on the left-hand side of (A2) is nothing else but the block-Toeplitz matrix $\mathbf{T}_X$, defined in Eq. (2.4), with some reordering of rows and columns. Once the regression and noise covariance matrices are estimated the spectral quantities associated with the autoregressive fitted process $Y$ can be computed, and they provide estimates of the same quantities for the observations $X$ under study since $Y$ and $X$ have the same second-order moments, up to lag $M - 1$. If $B$ denotes the backward shift operator, Eq. (2.16) can be rewritten

$$\mathbf{H}(B)Y = \mathbf{W},$$

(A3)

where $\mathbf{H}(z)$ is a matrix whose coefficients are polynomials in the variable $z$,

$$\mathbf{H}(z) = 1 - z\mathbf{A}_1 - z^2\mathbf{A}_2 - \cdots - z^{M-1}\mathbf{A}_{M-1}.$$  

(A4)

Taking Fourier transforms of Eq. (A3), one gets the cospectrum $P_{ll'}(f)$ between channels $l$ and $l'$ of the process $Y$ as the coefficient in row $l$ and column $l'$ of the $L \times L$ matrix

$$\mathbf{Q}(f) = \mathbf{H}^{-1}(z)\mathbf{S}\mathbf{H}^*^{-1}(z),$$

(A5)

where

$$z = e^{2\pi f},$$

(A6)

and the asterisk denotes the adjoint operator. In particular, the total spectrum $P(f)$ [Eq. (12.1)] is the trace of the matrix $\mathbf{Q}$ and the power spectra in each channel are the diagonal elements of $\mathbf{Q}$.

The power spectra of the PCs of $Y$, approximating those of $X$, may also be evaluated, by combining Eqs. (2.10)–(2.11) and (A5)–(A6):

$$P^k(f) = \Phi^k(f)\mathbf{Q}(f)\Phi^k(f),$$

(A7)

where $\Phi^k(f)$ is an $L$-dimensional vector whose elements are defined in Eq. (2.11) and correspond to the Fourier transforms of the ST-EOFs. Once all these quantities are estimated, the distribution of the fraction of the variance occupied by the $k$th PC along the frequency axis is given by the ratio

$$r^k(f) = \frac{P^k(f)}{MP(f)}.$$  

(A8)

This ratio is used in section 3b, and particularly to construct Figs. 5 and 6.

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