Comments on "A New Formulation of the Exchange of Mass and Trace Constituents between the Stratosphere and Troposphere"

VOLKMAR WIRTH

Meteorologisches Institut, Universität München, Munich, Germany

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A few years ago, Wei presented a paper (Wei 1987, refered to as W87 hereafter) in which she derives a general formula for the flux F of a specific physical property across the tropopause. In the meantime, the exchange of mass and trace constituents between the stratosphere and troposphere has received considerable attention, and Wei's formula is being used to quantify the relevant cross-tropopause fluxes (e.g., Hoerling et al. 1993). While the mathematically rigorous derivation is considered to be important and meritorious, the interpretation given in W87 (and repeated, e.g., in Hoerling et al. 1993) appears rather misleading to the present author. This is an attempt to clarify the issue.

Restricting attention to the special case of mass, Wei's general formula reduces to the following expression for the cross tropopause mass flux from the troposphere to the stratosphere:

$$F(\rho) = \rho J_\eta\left(\frac{d\eta}{dt} - \frac{\partial\eta}{\partial t_{TP}} - \mathbf{U}\cdot\nabla_{TP}\eta\right), \qquad (1)$$

where ρ is density, η is a generalized vertical coordinate, $J_\eta = |\partial z/\partial\eta|$ with altitude z, d/dt denotes the material rate of change following the parcel, and $\mathbf{U}$ is the horizontal wind. All terms on the right-hand side have to be evaluated at the local tropopause, and subscript TP with a partial derivative means that the respective derivative has to be taken on the local tropopause. The tropopause is defined as the surface of a threshold value of potential vorticity (PV), which is given by

$$P = \rho^{-1}\boldsymbol{\zeta}_a\cdot\nabla\theta, \qquad (2)$$

where $\boldsymbol{\zeta}_a$ is absolute vorticity and θ is potential temperature. The existence of a multiple tropopause (tropopause fold) is excluded from the present considerations.

Different choices for η in (1) yield different versions of the equation. For instance, using potential temperature θ as vertical coordinate, one obtains

$$F(\rho) = \rho J_\theta\left(\frac{d\theta}{dt} - \frac{\partial\theta}{\partial t_{TP}} - \mathbf{U}\cdot\nabla_{TP}\theta\right), \qquad (3)$$

which is identical to Eq. (18) in W87. In certain situations, as will be shown below, there can be a great deal of cancellation among the terms on the right-hand side. On the other hand, with potential vorticity P as vertical coordinate, (1) is reduced to a particularly simple expression,

$$F(\rho) = \rho J_P\frac{dP}{dt}, \qquad (4)$$

which does not suffer from the possible cancellation of terms. The formal simplicity of the latter formula is associated with the fact that now the tropopause is a coordinate surface and, hence, the second and third term in parentheses on the right-hand side of (1) vanish. Equation (4) reveals that motion across the PV-defined tropopause requires air parcels to gain or lose potential vorticity (see Hoerling et al. 1993). The material rate of change of PV, in turn, is given in terms of diabatic heating $d\theta/dt = \dot\theta_d$ and the horizontal component of the external forcing $\mathcal{F} = (\mathcal{F}_x, \mathcal{F}_y, 0)$ as follows (e.g., Hoskins et al. 1985):

$$\frac{dP}{dt} = \rho^{-1}\boldsymbol{\zeta}_a\cdot\nabla\dot\theta_d + \rho^{-1}\nabla\times\mathcal{F}\cdot\nabla\theta. \qquad (5)$$

Although the net mass flux $F(\rho)$ is, of course, independent of the choice of the vertical coordinate, the partition into several contributions (i.e., terms on the right-hand side) is dependent on such choice (W87). According to W87, the formulation (3) is particularly suitable for interpretation, and she attributes distinct physical mechanisms of cross tropopause mass exchange to the individual terms on the right-hand side. For instance, positive heating at the tropopause level or a local drop in tropopause potential temperature is viewed as yielding a positive mass exchange. On the other hand, the present author believes that this can be

Corresponding author address: Dr. Volkmar Wirth, Meteorologisches Institut, Universität München, Theresienstraße 37, D-80333 Munich, Germany.

rather misleading and that only (4) yields an unambiguous physical interpretation. To demonstrate this, three particular scenarios are considered. Figure 1 serves for illustration, showing an upper-level feature supposed to represent an idealized cutoff cyclone.

First scenario: Assume a situation characterized by adiabatic frictionless advection of a PV anomaly at the tropopause level. Such a situation can be obtained by translating the anomaly from Fig. 1 with a horizontal wind $\mathbf{U}_{\rm trans}$, which does not make an additional contribution to PV around the tropopause level (e.g., a constant $\mathbf{U}_{\rm trans}$). Equation (3) then reduces to

$$F(\rho) = \rho J_\theta\left(-\frac{\partial\theta}{\partial t_{TP}} - \mathbf{U}\cdot\nabla_{TP}\theta\right), \qquad (6)$$

that is, there are two terms on the right-hand side left that generally are nonzero. For instance, when the upper-level anomaly from Fig. 1 approaches, the tropopause potential temperature at a fixed horizontal location drops by some 25 K ($-\partial\theta/\partial t_{TP} > 0$), which according to (6) results in a positive contribution to the cross-tropopause mass flux. However, the second term in (6) turns out to exactly cancel the first term. The necessity of this cancellation follows from (4) with (5), which immediately yields $F(\rho) = 0$, since $\dot\theta_d = \mathcal{F} = 0$ by assumption. Thus, at no location has any mass physically crossed the tropopause. In such a situation it does not seem very helpful to interpret the terms $-\partial\theta/\partial t_{TP}$ and $-\mathbf{U}\cdot\nabla_{TP}\theta$ as independent upward and downward mass fluxes and to attribute physical meaning to them.

Second scenario: Consider again an upper-level anomaly like the one in Fig. 1. Assume a diabatic heating field $\dot\theta_d > 0$ with a maximum in the vortex center underneath the lowered tropopause; furthermore let $\mathcal{F}$ be zero. This scenario may represent the effect of latent heat release due to deep convection in a cutoff cyclone and has been studied in more detail by Wirth (1995). For a reasonable distribution of the heating field, (5) is approximated by $dP/dt \approx f\rho^{-1}\partial\dot\theta_d/\partial z$, with f being the Coriolis parameter. Let the heating field be such that its vertical gradient $\partial\dot\theta_d/\partial z$ is negative at the local tropopause. It follows that $dP/dt < 0$ and hence, from (4), $F(\rho) < 0$. In other words, owing to its particular form, the positive diabatic heating $\dot\theta_d$ leads to a negative mass flux $F(\rho)$. As physical mechanism one might consider the material loss of PV induced by the spatially nonuniform diabatic heating, allowing air parcels to cross the PV-defined tropopause. Interpreting the same situation with Eq. (3), on the other hand, leads one to the conclusion that the diabatic heating $d\theta/dt = \dot\theta_d > 0$ gives a positive contribution to $F(\rho)$, which just happens to be overcompensated by the other two terms so as to yield a net negative $F(\rho)$. (It turns out that this overcompensation is essentially due to the second term, $-\partial\theta/\partial t_{TP}$). Again, it is not obvious from (3) that this (over-) compensation must happen necessarily, and it appears somewhat artificial to interpret the (positive) heating $\dot\theta_d$ as an individual mechanism leading to a positive mass flux, since its overall effect in this scenario is a negative mass flux $F(\rho)$.

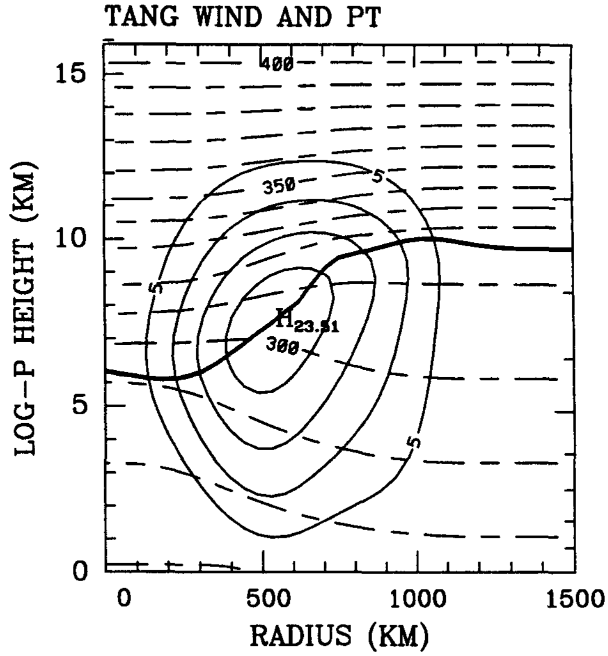

FIG. 1. Radius–altitude section of an idealized axisymmetric cutoff cyclone: tangential wind v (in m s^{-1}, solid lines, contours every 5 m s^{-1}, zero contour suppressed) and potential temperature θ (K, dashed lines, contours every 10 K). The heavy line indicates the PV-defined tropopause, which in this case is the surface where $P = 2$ K m^2 kg^{-1} s^{-1}.

Third scenario: Both previous scenarios were characterized by nonstationarity and the term $-\partial\theta/\partial t_{TP}$ on the right-hand side of (3) played the major role in balancing one of the other terms. On the other hand, in the long-term time mean this term vanishes, and one might ask the question whether at least in a climatological sense Eq. (3) gives a useful account of the underlying physical processes. However, this is not the case, and the third scenario provides a (counter-) example. Assume a specific region on the globe that is characterized by frequent advection of cutoff cyclones like in scenario 1, each of which decays diabatically at this location like in scenario 2. Conceptually, scenario 3 should be viewed as an alternating occurence of scenarios 1 and 2. Denoting the climatological time mean by angle brackets, apparently both the mean diabatic heating $\langle\dot\theta_d\rangle = \langle\dot\theta_d\rangle_1 + \langle\dot\theta_d\rangle_2 = \langle\dot\theta_d\rangle_2 > 0$ and the mean mass flux $\langle F(\rho)\rangle = \langle F(\rho)\rangle_1 + \langle F(\rho)\rangle_2 = \langle F(\rho)\rangle_2 < 0$ are governed by the contributions from scenario 2 (here the subscripts denote the contributions from the respective scenarios). Assume that the temporal variation of the mass densities ρJ_θ and ρJ_P are small compared with their time mean values, such that they can be treated as constant and taken in front of the angle brackets. Then the time mean version of (4) becomes

$$\langle F(\rho)\rangle \approx \rho J_P \left\langle \frac{dP}{dt} \right\rangle, \qquad (7)$$

which can be interpreted as was done for scenario 2. On the other hand, the time mean version of (3),

$$\langle F(\rho)\rangle \approx \rho J_\theta \left(\left\langle \frac{d\theta}{dt} \right\rangle - \langle \mathbf{U}\cdot\nabla_{TP}\theta\rangle \right), \qquad (8)$$

again features overcompensation of terms: as $\langle d\theta/dt\rangle = \dot{\theta}_d > 0$, this terms must be overcompensated by the horizontal advection term (second term on the right-hand side) in order to give $\langle F(\rho)\rangle < 0$. Thus, one obtains a negative climatological mean cross tropopause mass flux despite positive heating at the tropopause. As before, the interpretation of single terms on the right-hand side of (8) as physical mechanisms is considered to be problematic.

In summary, it has been argued here that the interpretation of individual terms in the isentropic formulation of Wei's general formula as physical mechanisms of stratosphere–troposphere mass exchange can be misleading. The problem arises because in certain physically relevant situations different terms necessarily cancel or even overcompensate. On the other hand, with PV as vertical coordinate, Wei's general formula reduces to a particularly simple form, which contains one single term involving the material rate of change of PV and which does not suffer from the possible cancellation of terms. Admittedly, the problem has been shifted to finding out how PV is being materially changed by the various physical processes. The latter step, however, seems necessary to provide an unambiguous physical interpretation.

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