The Potential Vorticity Budget of an Atmospheric General Circulation Model

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ABSTRACT

The flux form of the potential vorticity (PV) equation in isentropic coordinates is used to examine the effects of advection and the effects of parameterized mechanical and thermal forcing on the PV budget of the second generation Canadian Climate Centre general circulation model (CCC GCM). Model data corresponding to Northern Hemisphere winter are used. The simulated PV flux contains significant nonadvective contributions in the planetary boundary layer, in the gravity wave drag regions of the Northern Hemisphere, and in the tropical midtroposphere in regions of intense latent heat release associated with persistent moist convection. The model advective PV flux is compared to the advective PV flux calculated from a National Centers for Environmental Prediction observational dataset. Large discrepancies are seen where parameterized gravity wave drag dominates the mechanical forcing field in the model.

The zonally averaged model PV flux in the upper troposphere and lower stratosphere is characterized by a balance between the meridional transport of isentropic absolute vorticity and dissipation from parameterized gravity wave drag. A simulation not including gravity wave drag shows stronger poleward transport of relative vorticity and stronger equatorward transport of planetary vorticity in the Northern Hemisphere, compared to a run including this parameterization. The PV budget along two isentropic surfaces, one in the "overworld" and the other in the "middleworld" as defined by Hoskins, is examined. On the overworld (lower stratospheric) isentrope, the effect of parameterized gravity wave drag in the Northern Hemisphere is a predominantly southward transport of PV. This is balanced by northward advection of PV by the lower-stratospheric meridional circulation. Assuming a similar balance in the atmosphere, an estimate of the observed mean mechanical forcing field is obtained by calculating the advective PV flux on the 390 K surface from assimilated data. On the middleworld isentrope, the PV budget exhibits an approximate three-way balance between the advective, mechanical, and thermal parts of the PV flux in midlatitudes. The implications for stratosphere–troposphere exchange are discussed. The sign of the meridional component of thermal PV flux is used to deduce that on average, radiative cooling (diabatic heating) is located in regions of positive (negative) vertical wind shears in the model.

1. Introduction

Two interesting theoretical properties of Ertel potential vorticity (PV) have been demonstrated by Haynes and McIntyre (1987, 1990). Treating PV as the mixing ratio of a quantity they called potential vorticity substance (PVS), these authors proved that isentropic surfaces are impermeable to PVS and that PVS can neither be created nor destroyed within a layer bounded by two isentropic surfaces. The global integral of PVS per unit volume in such a layer was shown to vanish. These results distinguish PV in a definitive manner from the mixing ratio of atmospheric chemical constituents. While acknowledging the apparent similarities, Haynes and McIntyre cautioned against a direct analogy between PV and the mixing ratio of ozone, for example, one which had previously been expounded by Danielsson (1968) in studies of stratosphere–troposphere exchange. The theoretical framework provided by Haynes and McIntyre suggests a useful role for PV in studies of large-scale atmospheric flow, from both a dynamical and a chemical perspective.

Hoskins (1991) examined several interesting aspects of the large-scale circulation using the PV–θ framework. He suggested a subdivision of the atmosphere into an "overworld," in which all isentropes lie above the tropopause, a "middleworld" where isentropes intersect the tropopause but not the earth's surface, and an "underworld," consisting of those tropospheric isentropes that intersect the ground. Thus, middleworld PV budgets are useful for studies of stratosphere–troposphere exchange, while underworld PV budgets can be used to identify sources and sinks of PVS in the atmosphere. One of the conclusions of Hoskins' study was that on large scales, surface westerlies should appear in regions of diabatic cooling, whereas easterlies should be associated with diabatic heating. He derived this underworld result by considering an areally and temporally averaged form of the Lagrangian evolution...
equation for PV, and used observational data to support his contention.

The local tendency of PVS per unit volume can be expressed in terms of the divergence of a vector flux. Haynes and McIntyre (1987) argued that the contributions of both mechanical and thermal forcing to this flux are important in the global PV budget. Danileisen (1990) disputed this claim, suggesting that above the planetary boundary layer, the contribution to the PV flux from mechanical forcing could be ignored altogether. Haynes and McIntyre specifically mentioned the significant role of gravity wave drag effects in the lower-stratospheric PV budget. These effects can be examined within the GCM framework, since the model used here incorporates a subgrid-scale gravity wave drag parameterization (McFarlane 1987).

Hoerling (1992) considered the upper-tropospheric PV budget using global NMC data. Utilizing a special form of the PV evolution equation in isentropic coordinates, he demonstrated that the PV balance in midlatitudes was strongly linked to the distribution of tropical heating during Northern Hemisphere winter. Specifically, Hoerling showed that PV advection by the divergent outflow from the upper branch of the Hadley circulation was responsible for a midlatitude PV sink over much of southeastern Asia.

In view of the above studies and the steadily increasing focus on PV as a fundamental variable for the research of tropospheric and stratospheric dynamics (e.g., Hoskins et al. 1985; WMO 1986; Holopainen and Kaurola 1991), the aim of the present article is to examine several aspects of the large-scale PV budget simulated by the second generation Canadian Climate Centre general circulation model (CCC GCM) (McFarlane et al. 1992). The model uses triangular truncation at wavenumber 32 (T32) with 10 vertical levels spanning the region between the surface and the middle stratosphere. An advantage of using GCM output is that the mechanical and thermal forcing terms appearing in the momentum and thermodynamic equations can be obtained directly, allowing explicit illustration of some of the theoretical properties of the PV flux described by Haynes and McIntyre (1987, 1990).

Boer (1991) examined the vorticity budget of the first generation CCC GCM (Boer et al. 1984a,b) and found that on the vertical average, vorticity flows approximately from regions associated with high mean sea level pressure to regions associated with low mean sea level pressure. His study was based on the vorticity equation in isobaric coordinates, involving an advective horizontal transport term as well as explicit sources/sinks of vorticity. The evolution equation for PV differs from that for vorticity in the sense that the local tendency of density-weighted PV (PVS per unit volume) can be expressed entirely in terms of the divergence of a flux, consisting of both advective and nonadvective parts. Analysis of the PV budget also entails consideration of diabatic forcing effects, whereas Boer’s analysis explicitly involved mechanical forcing effects only. Thus, the potential vorticity budget equation should provide the framework for a more thorough examination of the GCM dynamics.

The primitive equations in an arbitrary vertical coordinate system are presented in the following section. From these an expression for PV is derived analogous to Ertel’s (1942) result and a flux form of the PV conservation equation obtained. In isentropic coordinates, the latter reduces to an evolution equation for the absolute vorticity on isentropic surfaces. The third section describes the model simulated PV fluxes for the Northern Hemisphere winter (December–February) season. Particular attention is devoted to balances between the advective, mechanical, and thermal parts of the PV flux and the underlying physics. Fluxes obtained from the standard model are compared to those calculated from a run without parameterized gravity wave drag and, where possible, to fluxes computed from observational data. The main conclusions of the study are summarized in section 4.

2. The PV equation in an arbitrary vertical coordinate system

The equations required to derive a Lagrangian conservation law for PV in terms of a generalized vertical coordinate $s = s(x, y, z, t)$ are the momentum, mass continuity, and thermodynamic equations, written respectively as follows:

$$\frac{dv}{dt} + 2\Omega \times v + \nabla \cdot \Phi + \frac{1}{\rho} \nabla \cdot p - F = 0 \quad (1a)$$

$$\frac{d\theta}{dt} - H = 0 \quad (1b)$$

$$\frac{d}{dt} \left( \frac{\partial p}{\partial s} \right) + \left( \frac{\partial p}{\partial s} \right) \nabla \cdot V = 0. \quad (1c)$$

These are similar to (2.1), (2.2), and (2.4) in Laprise and Girard (1990) and were derived previously by Kasahara (1974). In (1), $v = (u, v, 0)$ is the horizontal velocity, $\Omega = (0, 0, \omega \sin \phi)$ where $\omega$ is the angular frequency of earth’s rotation, $\Phi = gz$ is the geopotential height of local coordinate surfaces above mean sea level, $\rho$ is density, $p$ is pressure, $V = (u, v, s)$ is a generalized three-dimensional velocity vector, and $\theta$ is the potential temperature. The vector $F = (F_x, F_y, 0)$ and the scalar $H$ are mechanical and diabatic forcing terms, respectively. The operators appearing in (1) are defined as follows:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla,$$

$$\nabla_s = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial s} \right).$$
Using these definitions, the \( s \) component of the momentum equations yields the hydrostatic relation.

Following Ertel (1942), it can be shown that the expression for the evolution of PV in the \((x, y, s)\) coordinate space is

\[
\frac{dq}{dt} = -g \left( \frac{\partial p}{\partial s} \right)^{-1} \nabla_{s} \times \mathbf{F} \cdot \nabla_{s} \theta + \tilde{\zeta}_{as} \cdot \nabla_{s} H, \quad (3)
\]

where

\[
q(x, y, s, t) = -g \left( \frac{\partial p}{\partial s} \right)^{-1} \tilde{\zeta}_{as} \cdot \nabla_{s} \theta \equiv \frac{\tilde{\zeta}_{as} \cdot \nabla_{s} \theta}{\sigma_{s}} \quad (4)
\]

is the Ertel potential vorticity to \( s \) within the hydrostatic approximation, \( \sigma_{s} = -1/g(\partial p/\partial s) \) is the “density” in the \( s \) coordinate system, and \( \tilde{\zeta}_{as} = \nabla_{s} \times \mathbf{v} + 2\Omega \) is the absolute vorticity. Equation (3) is derived in step with Ertel’s original derivation but beginning with (1) instead of the analogous inelastic equations that use \( z \) as the vertical coordinate. Substituting \( s = z, p, \) or \( \theta \) into (4) and invoking the hydrostatic relation yields the following familiar expressions for \( q \):

\[
q(x, y, z, t) = \frac{\tilde{\zeta}_{az} \cdot \nabla_{z} \theta}{\rho} \quad (5a)
\]

\[
q(x, y, p, t) = -g \tilde{\zeta}_{zp} \cdot \nabla_{p} \theta \quad (5b)
\]

\[
q(x, y, \theta, t) = -g \tilde{\zeta}_{z\theta} \frac{\partial \theta}{\partial p} \quad (5c)
\]

where the scalar \( \tilde{\zeta}_{az} \) is the Rossby “isentropic vorticity,” defined by

\[
\tilde{\zeta}_{az} = \tilde{\zeta}_{a} + f = \left( \frac{\partial u}{\partial x} \right)_{\theta} - \left( \frac{\partial u}{\partial y} \right)_{\theta} + f \quad (6)
\]

and is approximately equal to the component of absolute vorticity perpendicular to local isentropic surfaces. Equations (5a)–(5c) represent expressions for the same quantity, but in terms of different vertical coordinates.

The Lagrangian evolution equation (3) can be written in flux form with the aid of the mass continuity equation (1c). Noting that the divergence of the curl of a vector vanishes and setting

\[
J_{s} = \sigma_{s} \mathbf{V} - \mathbf{F} \times \nabla_{s} \theta - \tilde{\zeta}_{as} H \quad (7)
\]

the flux form of (3) is

\[
\frac{\partial}{\partial t} (\sigma_{s} q) + \nabla_{s} \cdot J_{s} = 0. \quad (8)
\]

The quantity defined by (7), hereafter referred to as the PV flux, comprises advective, mechanical, and thermal parts, respectively. The first term on the left-hand side of (8) represents the local rate of change of PV per unit volume of \((x, y, s)\) space. In the \((x, y, z)\) coordinate system \( \sigma_{s} q \) reduces to the expression \( \rho q \) found in Haynes and McIntyre (1990).

The PV flux \( J_{s} \) can be written as the sum of a divergent and a rotational component,

\[
J_{s} = J_{s}^{d} + J_{s}^{r}, \quad (9)
\]

where

\[
J_{s}^{d} = -\nabla_{s} \chi_{s}
\]

\[
J_{s}^{r} = \mathbf{k} \times \nabla_{s} \psi_{s} \quad (10)
\]

for some scalar functions \( \chi_{s} \) and \( \psi_{s} \). Equation (8) indicates that only \( J_{s}^{d} \) contributes to local changes in \( \sigma_{s} q \). Inserting (9) and (10) into (8) yields

\[
\frac{\partial}{\partial t} (\sigma_{s} q) = \nabla_{s}^{2} \chi_{s}. \quad (11)
\]

The scalar \( \chi_{s} \) is denoted the PV potential function; in the Northern (Southern) Hemisphere, cyclonic PV flows from regions of high (low) potential to regions of low (high) potential.

For the case \( s = \theta \), (8) becomes

\[
\frac{\partial \tilde{\zeta}_{s\theta}}{\partial t} + \nabla_{\theta} \cdot J_{\theta} = 0, \quad (12)
\]

which is the absolute vorticity equation in isentropic coordinates (e.g., Hoskins 1991). The PV flux in (12) is partitioned into three parts

\[
J_{\theta} = J_{\theta}^{a} + J_{\theta}^{m} + J_{\theta}^{t}, \quad (13)
\]

where

\[
J_{\theta}^{a} = \mathbf{v} \tilde{\zeta}_{s\theta} = \mathbf{u}_{\theta}^{a} j + \mathbf{v}_{\theta}^{a} j \quad (14a)
\]

\[
J_{\theta}^{m} = \mathbf{k} \times \mathbf{F} = -F_{s} j + F_{\theta} j \quad (14b)
\]

\[
J_{\theta}^{t} = -\mathbf{k} \times \frac{\partial \mathbf{v}}{\partial \theta} H = \frac{\partial u}{\partial \theta} H_{j} - \frac{\partial u}{\partial \theta} j H_{j}. \quad (14c)
\]

The vanishing of the vertical component of \( J_{\theta} \) is a consequence of the impermeability of \( \theta \) surfaces to PVS (Haynes and McIntyre 1987). In (13), the vectors \( \mathbf{A}_{\theta} \), \( \mathbf{M}_{\theta} \), and \( \mathbf{T}_{\theta} \) denote the advective, mechanical, and thermal parts of the PV flux in isentropic coordinates, respectively. Each part is further decomposed into a divergent and a rotational component, and the divergent part written as the gradient of a potential function denoted \( \chi_{A}, \chi_{M}, \) and \( \chi_{T} \), respectively (e.g., \( \mathbf{A}_{\theta} = A_{\theta} a + A_{\theta} r \), where \( A_{\theta} = -\nabla_{s} \chi_{A} \)).

3. Simulated and observed PV fluxes

Model data used in this section are extracted from a 10-yr simulation employing observed SSTs for the period January 1979–December 1988 and correspond to the 1983–84 winter season (December–February). Vorticity, divergence, temperature, and surface pressure are sampled every 6 h and stored together with daily average values of the parameterized mechanical
and thermal forcing fields. The effect of data sampling frequency on the calculation of PV fluxes is discussed in the appendix.

Observed data used in this section are from National Centers for Environmental Prediction (NCEP) analyses and consist of twice daily vorticity, divergence, and temperature fields on 12 pressure levels between 50 and 1000 mb. Data from the three winter seasons between 1 December 1988 and 28 February 1991 are included.

The temporally and zonally averaged potential temperature distribution calculated from the (a) model dataset and (b) observed dataset is shown in Fig. 1. This figure provides an approximate correspondence between isobaric and isentropic surfaces in the model and atmosphere, respectively, and should be used for reference throughout this section. It should be noted that the largest differences between Figs. 1a and 1b appear at upper levels near the south pole, where model potential temperatures are much lower. This is a manifestation of the "cold pole" problem found in this and other general circulation models.

a. Zonally averaged PV budget

The zonally averaged form of (12) is

$$\frac{\partial [\zeta_p]}{\partial t} + \left( \frac{\partial}{\partial y} \right)_g \left[ v_{z=0} + F_x - \frac{\partial u}{\partial \theta} H \right] = 0,$$

(15)

where

$$\left( \frac{\partial}{\partial y} \right)_g = \frac{1}{a \cos \phi} \left( \frac{\partial}{\partial \phi} \right)_g \cos \phi$$

(16)

and $\phi$ is latitude. Equation (15) is valid only for isentropic surfaces that do not intersect the ground. The zonally averaged budget equation on underworld isentropes contains an additional contribution from the zonal component of $J$, evaluated at points of intersection of the isentropic surface with the ground.

Averaging (15) over an interval that satisfies the condition $\overline{\partial [\zeta_p]} / \partial t \approx 0$ yields

$$\left( \frac{\partial}{\partial y} \right)_g \left[ v_{z=0} + F_x - \frac{\partial u}{\partial \theta} H \right] \approx 0,$$

(17)

where the overbar denotes a temporal average. Figure 2 displays (a) $[v_{z=0}]$, (b) $[F_x]$, and (c) $[-\partial u / \partial \theta H]$ calculated from model simulated data. A substantial amount of cancellation between the three fields occurs. In middle- and upper-level Northern Hemisphere regions, the equatorward mechanical flux in (b) is balanced by northward advective PV flux in (a). The former flux, $[F_x]$, is composed of contributions from gravity wave drag and vertical diffusion. The effects of horizontal diffusion are not included in the expression for $F_x$ since they are negligible.

An approximate balance between advective and mechanical PV fluxes is also apparent in the tropical boundary layer. The lower branch of the Hadley circulation (e.g., McFarlane et al. 1992, Fig. 10) is responsible for the equatorward ($v < 0$) flux of cyclonic (positive) PV in the Northern Hemisphere of (a). This is balanced by the contribution of boundary layer friction to $[F_x]$ in (b), which opposes the easterly flow in this region. A similar balance is evident in the Southern Hemisphere, where negative values in (a) represent the equatorward ($v > 0$) advection of cyclonic (negative) PV.

The most pronounced contribution to the zonally averaged PV budget from diabatic forcing occurs in the tropical mid-upper troposphere ($\approx 330$ K) in Fig. 2c. There, a negative correlation exists between heating and the vertical shear in the zonal wind, yielding a positive contribution to $[-\partial u / \partial \theta H]$. This positive thermal flux is partially balanced by negative advection in Fig. 2a, although the latter feature extends over a greater depth of the atmosphere.

The balance between advective and mechanical parts of the PV flux seen in Figs. 2a and 2b can be traced to the horizontal momentum equations in isentropic coordinates, written as

$$\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} + \mathbf{k} \times (f \mathbf{v}) + \nabla \psi \mathbf{M} = \mathbf{F} - \frac{\partial v}{\partial \theta} H,$$

(18)

where $M$ is the Montgomery streamfunction (e.g., Andrews et al. 1987, 138). Using the identity

$$\mathbf{v} \cdot \nabla \psi = \nabla \psi \left( \frac{v^2}{2} \right) + \mathbf{k} \times (\nabla \psi),$$

(19)

(18) becomes

$$\frac{\partial v}{\partial t} + \nabla \psi \left( \frac{v^2}{2} \right) + \mathbf{k} \times (\nabla \psi) + \nabla \psi \mathbf{M} = \mathbf{F} - \frac{\partial v}{\partial \theta} H.$$  

(20)

Temporally averaging (20) and considering the rotational part of this equation gives

$$\{ \mathbf{k} \times (\nabla \psi) \}, \approx \mathbf{F}_r, - \left\{ \frac{\partial v}{\partial \theta} H \right\}, \quad (21)$$

or

$$\left\{ \nabla \psi \right\}_d + \mathbf{k} \times \mathbf{F}_r, - \mathbf{k} \times \left\{ \frac{\partial v}{\partial \theta} H \right\}, \approx 0, \quad (22)$$

where the subscripts $d$ and $r$ denote divergent and rotational parts of the given vector, respectively. (Note: $\{ \mathbf{k} \times \mathbf{B} \}, = \mathbf{k} \times \mathbf{B}_d$ for any vector $\mathbf{B}$.)

The expression on the left-hand side of (22) represents the divergent part of the temporally averaged total PV flux defined in (13) and (14). In regions where the
thermal PV flux is small, the zonally averaged $y$ component of (22) is

$$[u_{\phi}^T] + [F_y] \approx 0. \quad (23)$$

Equation (23) is obeyed in mid-upper levels of the Northern Hemisphere and in the tropical boundary layer of Fig. 2. Its first term can be further partitioned to yield

$$[u_{\phi}^T] + f[u] \approx -[F_y], \quad (24)$$

which is the isentropic coordinate analog to the result obtained by Boer (1991) in isobaric coordinates [see his Eq. (14)] assuming $u_{\phi}^T \approx u_{\phi}^{T2}$. Figure 3 shows the fields (a) $[u_{\phi}^T]$ and (b) $f[u]$ that the left-hand side of (24) comprises. These fields approximately cancel each other in the Southern Hemisphere and in Northern Hemisphere regions where $[F_y]$ is small [cf. Boer (1991) Eq. (15)]. In the tropical boundary layer, however, Figs. 2b and 3b indicate

$$f[u] \approx -[F_y], \quad (25)$$

analogous to Boer’s result in pressure coordinates, whereas in the Northern Hemisphere middle troposphere/upper stratosphere ($\theta \approx 330 \text{ K}$), Figs. 2b and 3c indicate that the three terms in (24) are of similar magnitude (note that the contour interval in Fig. 2b is half that in Fig. 3).

Equation (23) suggests that the term $[F_y]$ can be calculated from observational data, assuming (23) holds in the atmosphere. Several previous studies have attempted to provide estimates for mechanical dissipation in the atmosphere from budgets of momentum (e.g., Klinker and Sardeshmukh 1992; Holopainen et al. 1980). One of the problems encountered with such an approach is the quality of the data and the assimilating model involved. It is also possible that (23) is not satisfied in the atmosphere (and/or assimilating model) where this equation holds in our model. Finally, the advective PV flux is sensitive to the sampling frequency of the data used to compute it; twice daily data may provide somewhat inaccurate estimates of this quantity in certain regions (see appendix). Nevertheless, it is instructive to compare the terms on the left-

![Fig. 1. Temporally and zonally averaged potential temperature calculated from (a) model data for a single winter and (b) NCEP data averaged over three winters (contour interval = 15 K).](image1)

![Fig. 2. Seasonally and zonally averaged $y$ components of (a) the advective PV flux, $[A_y]$; (b) the mechanical PV flux, $[M_y]$; and (c) the thermal PV flux, $[T_y]$, as calculated from the model with gravity wave drag. Contour interval = 0.5 m s$^{-1}$ day$^{-1}$ in (a) and (b) and 0.25 m s$^{-1}$ day$^{-1}$ in (c).](image2)
in both northerly and southerly directions (note that the contour interval in Figs. 2a and 4c is half that in Figs. 4a and 4b. The negative-valued region centered along 55°N in Fig. 4c) suggests a region of momentum generation in the atmosphere in which the thermal PV flux is significant.

To gain insight into the role of parameterized gravity wave drag, a second simulation was performed without this parameterization for the same model December–February period used to produce Fig. 2. The fields (a) \( \overline{v'w'} \), (b) \( f [v'] \), and (c) \( \overline{F_v} \) for this simulation are shown in Fig. 5. In contrast to Fig. 3b, Fig. 5b shows a region of poleward relative vorticity transport in Northern Hemisphere midlatitudes that extends well above 380 K. This is a direct result of the removal of gravity wave drag in the model, leading to an approximate balance between figures 5a and 5b, as indicated by Fig. 5c. Although this yields better qualitative agreement between Figs. 5b and 4b than between Figs. 3b and 4b, as computed from model and objective analyses of observed data.

Figure 4 shows the fields (a) \( \overline{v'w'} \), (b) \( f [v'] \), and (c) their sum \( \overline{v'w'} \), calculated from NCEP data. In Southern Hemisphere midlatitudes, the meridional advection of both relative and planetary vorticity is stronger in the model. Figures 3b and 4b indicate that this is mainly the result of stronger simulated equatorward velocities in this region. Thus, although an approximate balance exists between \( \overline{v'w'} \) and \( f [v'] \) in both the simulation and objective analyses, the actual balanced state is quite different. In the Tropics, Figs. 3a and 4a and Figs. 3b and 4b show good qualitative and reasonable quantitative agreement.

In Northern Hemisphere midlatitudes, the simulated and objectively analyzed meridional circulations differ at heights \( \theta \approx 380 \text{ K} \), where the model predicts a poleward transport of planetary vorticity (Fig. 3b) while equatorward transport is seen in the analyzed data (Fig. 4b). Equatorward transport in the model is cut off above 380 K or so, where parameterized gravity wave drag makes an important contribution to (24). Comparison of Figs. 2a and 4c also shows poor agreement in this region. Transport of absolute vorticity is strictly poleward in the model, while the objective analyses show relatively weaker transports of absolute vorticity.
and 4b in northern midlatitudes, there is less agreement in the Tropics. Moreover, the quantitative agreement between model and objective analyses is clearly worse in Northern Hemisphere midlatitudes for the run without gravity wave drag. Removal of parameterized gravity wave drag leads to stronger poleward transport of relative vorticity and stronger equatorward transport of planetary vorticity in the model over most of the Northern Hemisphere depicted in Fig. 5. As expected, the two model runs show similar features in the Southern Hemisphere, where effects of orographic gravity wave drag are weak (e.g., Fig. 2b).

**b. Overworld results: The 390 K PV budget**

The vectors (a) $\vec{A}_{\phi_d}$, (b) $\vec{M}_{\phi_d}$, and (c) $\vec{T}_{\phi_d}$ along with their respective potential functions $\overline{X_A}$, $\overline{X_M}$, and $\overline{X_T}$ are shown on the 390 K surface in Fig. 6 for the simulation with gravity wave drag. The PV budget at this level is dominated by the advective and mechanical parts of the PV flux. The contribution from the thermal flux is negligible. The most interesting features of the figure are the dipoles located in gravity wave drag regions above the Rocky Mountains in the Western Hemisphere, and above the Himalaya in the Eastern Hemisphere. These are associated with predominantly southward mechanical flux of PV, from regions of high potential to low potential in (b), balanced by northward advective flux in (a).

Comparison of Figs. 6a and 6b and inspection of (22) suggest that the zonally averaged result (24) can be extended to two dimensions as follows:

$$\{\nabla \overrightarrow{\psi}\}_d + f \overrightarrow{\nabla} \approx -k \times \overrightarrow{F}_r.$$  \hspace{1cm} (26)

Figure 7 shows the two terms on the left-hand side of (26): (a) $\{\overrightarrow{V}_{\psi}\}_d$ and (b) $f \overrightarrow{\nabla}$. The sum of these fields

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Fig. 7. Seasonally averaged fields (a) $\langle \mathbf{v}_{\alpha} \rangle_d$, and (b) $\mathbf{v}_{\alpha}$, and their associated potential functions on the 390 K surface for the model with gravity wave drag. The distance between the tails of adjacent vectors corresponds to a magnitude of $10 \text{ m s}^{-1} \text{ day}^{-1}$ and the contour interval = $30 \text{ m}^2 \text{s}^{-2}$.

is approximately zero except in the regions of strong orographic gravity wave drag in Fig. 6b. Thus (26) provides an estimate of the rotational part of the two-dimensional mechanical forcing field given the horizontal wind field in an isentropic surface. It is valid provided that the thermal part of the PV flux is small compared to the advective and mechanical parts.

Figure 8 shows the fields (a) $\langle \mathbf{v}_{\alpha} \rangle_d$, (b) $\mathbf{v}_{\alpha}$, and (c) their sum, $\langle \mathbf{v}_{\alpha} \rangle_d$, calculated from NCEP data. Several discrepancies are seen when the observed fields in Figs. 8a and 8b are compared to corresponding quantities calculated from model data in Figs. 7a and 7b. Although there is qualitative agreement in the orientation of the transport patterns in the Northern Hemisphere, sources and sinks of both planetary and relative vorticity are stronger in the objective analyses. In the Southern Hemisphere, agreement between simulated and objectively analyzed fields is rather poor. Comparing Figs. 8c and 6a suggests that effects of parameterized gravity wave drag in the model may be over-emphasised, since Fig. 8c shows much less evidence of the dipole structures apparent in Fig. 6a. This result is consistent with the conclusions of Klinker and Sar-deshmukh (1992), who attempted to obtain an estimate of mechanical dissipation in the atmosphere using assimilated European Centre for Medium-Range Weather Forecasts (ECMWF) data.

The temporally averaged PV field in potential vorticity units (PVU = $10^{-6} \text{ K m}^2 \text{ kg}^{-1} \text{ s}^{-1}$) is shown on the 390 K isentropic surface in Fig. 9, as calculated from (a) the model with parameterized gravity wave drag, (b) the model without parameterized gravity wave drag, and (c) NCEP data. The model fields are calculated from (4) with $s = \eta$, the vertical coordinate in the model, and subsequently interpolated to isentropic surfaces. The PV field at high northern latitudes in Fig. 9a takes much smaller values than the field in Fig. 9b. This is related to a much stronger circumpolar west-
erly flow (and a much colder pole) in the simulation without gravity wave drag. Although the simulation with gravity wave drag compares better quantitatively with objective analyses in the Northern Hemisphere, the meridional PV gradient in the subtropics appears too large in the model. Part of this is a result of excessive PV values (\(\geq 12\) PVU) in midlatitude regions just west of 90°W and just east of 90°E. These are the regions of the model where gravity wave drag is responsible for a strong equatorward PV flux (Fig. 6b).

In the Southern Hemisphere, Figs. 9a and 9b are similar. Both simulations overestimate the cyclonic PV at high southern latitudes because of the “cold pole problem” seen in this and other atmospheric general circulation models (e.g., McFarlane et al. 1992, Fig. 8a). For example, Fig. 1 shows that at 90°S and 100 mb, the simulated potential temperature is approximately 400 K, whereas in the NCEP data it is near 450 K. The cold pole is associated with a stronger circumpolar flow in the model and hence, greater values of PV (note that \(\partial T/\partial z\) averaged between 100 and 200 mb is similar in the model and objective analyses). This cold pole problem is a common feature of model simulations, there being no parameterized process that corrects it as effectively as orographic gravity wave drag does in the Northern Hemisphere.

c. **Middleworld results: The 330 K PV budget**

The 330 K surface lies in the lower stratosphere at polar and middle latitudes and in the troposphere in the subtropics and Tropics as indicated in Fig. 1. Thus it is a useful surface on which to examine stratosphere–troposphere exchange. Figure 10 shows (a) the divergent part of the total PV flux, \(\mathbf{J}_{pv}\), along with the associated potential function, \(\chi\), and (b) the PV itself in PVU, on the 330 K surface as calculated from the simulation.

Fig. 9. The 390 K potential vorticity (PVU) calculated from (a) the model with gravity wave drag, (b) the model without gravity wave drag, and (c) NCEP data. The fields shown in (a) and (b) are averaged over a single model winter, whereas the field shown in (c) is averaged over three winters. Contour interval = 2 PVU.

Fig. 10. Seasonally averaged (a) divergent part of the total PV flux, \(\mathbf{J}_{pv}\) and associated potential function and (b) potential vorticity (PVU) on the 330 K surface for the model with gravity wave drag. Regions where the potential function has values \(\leq -20\) m² s⁻² are lightly shaded, and those where its values are \(> 10\) m² s⁻² are heavily shaded. The distance between the tails of adjacent vectors corresponds to a magnitude of 1 m s⁻¹ day⁻¹. Contour interval for the potential function = 10 m² s⁻² and for the potential vorticity = 0.5 PVU.
with parameterized gravity wave drag. The 2 PVU contour in Fig. 10b approximates the position of the tropopause in the Northern Hemisphere (e.g., Hoskins 1991).

In previous subsections it was assumed

$$\frac{\partial \zeta_{\sigma_0}}{\partial t} = \frac{\Delta \zeta_{\sigma_0}}{T} = \nabla_\theta \cdot \vec{J}_\sigma \approx 0 \quad (27)$$

as implied by (22). In (27), $T$ corresponds to the length of the averaging interval (December–February) and $\Delta \zeta_{\sigma_0}$ to the change in $\zeta_{\sigma_0}$ over this time interval. The approximate equality in (27) implies that the sum of the three terms in (22) is much smaller than any of the individual ones. Although (27) is valid on the 390 K surface, the net change in $\zeta_{\sigma_0}$ over the winter season on the 330 K surface is not insignificant. In fact, Fig. 10a shows two distinct PV sinks indicated by light shading, and one source, indicated by heavier shading in the Northern Hemisphere. The same regions are also shaded in Fig. 10b. The sinks and source show strong meridional PV gradients along their southern flanks, associated with the advection of low PV contours northward and high PV contours southward, respectively.

From (27)

$$\Delta \zeta_{\sigma_0} = T \nabla_\theta \cdot \vec{J}_\sigma, \quad (28)$$

so that changes in the 330 K isentropic vorticity field over the winter season under consideration can be obtained by calculating the divergence of the vector field in Fig. 10. Such a calculation yields $\Delta \zeta_{\sigma_0} = O(10^{-5} \text{ s}^{-1})$ in the shaded regions, which for typical values of $\sigma_0$ at 330 K gives a change in $q$ of a few PVU. For comparison, a similar calculation using data on the 390 K surface, where values of $q$ are on average much greater, gives $\Delta \zeta_0$ values that are an order of magnitude smaller (i.e., $O(10^{-1} \text{ PVU})$).

The vectors that $\vec{J}_\sigma$ comprises, (a) $\vec{A}_\theta$, (b) $\vec{M}_\theta$, and (c) $\vec{T}_\theta$ along with their respective potential functions $\chi_a$, $\chi_m$, and $\chi_T$ are shown in Fig. 11. The strong source in the Northern Hemisphere between 0° and 180°E of Fig. 11a is balanced on its northern edge by northerly flux in b and on its southern edge by southerly flux in c. Dipole structures in the Northern Hemisphere similar to those in Fig. 6b are evident in Fig. 11b. The latter are comparatively weaker but are still associated predominantly with gravity wave drag. In these regions, gravity wave drag is a mechanism for bringing PV rich stratospheric air from the lower stratosphere to the vicinity of the tropopause (roughly coincident with the 2 PVU contour in the Northern Hemisphere). In reality, such a process may be accompanied by development of mesoscale tropopause folds, which are not resolved by our model.

The northward thermal flux of PV between 0° and 90°E in the Northern Hemisphere of Fig. 11c results from a negative correlation between the diabatic forcing field $H$, and the vertical shear in the zonal wind, $\partial u/\partial \theta$. These fields are shown in Figs. 12a and 12b, respectively. In the tropical eastern hemisphere, relatively large positive values in the $y$ component of $\vec{T}_\theta$ in Fig. 11c result from a correlation between convective heating associated with the Australasian monsoon and negative vertical shear in the zonal wind, shown in Fig. 12. Negative values of $\partial u/\partial \theta$ in Fig. 12b are in turn associated with easterly winds.

The negative temporal correlation between $H$ and $\partial u/\partial \theta$ seen in Fig. 12 supports Hoskins' (1991) conclusion. He suggested a link between westerlies and cooling and easterlies and heating based on an observed underworld PV budget. Although the link between westerlies and cooling is perhaps not conclusive based upon Fig. 12, positive values in the $y$ component of $\vec{T}_\theta$ over most of the domain in Fig. 11c indicate that
the 330 K level than, for example, at higher stratospheric levels like 390 K.

4. Conclusions

The flux form of the PV equation in isentropic coordinates has been used to examine the PV budget of the Canadian Climate Centre GCM for Northern Hemisphere winter. The divergent part of the PV flux was partitioned into its advective, mechanic, and thermal parts. A balance between advective and mechanical parts of the PV flux in the overworld was revealed when examining the model PV budget along the 390 K surface. The thermal flux of this level was found to be negligible. Assuming a similar balance in the atmosphere (an assumption that could not be validated here), the advective PV flux was calculated from NCEP data in order to obtain some estimate of the ob-

westerlies (covering most of the extratropics in the figure) are in fact associated with cooling in these regions.

The balance between advective, mechanical, and thermal PV fluxes seen in Figs. 6 and 11 demonstrates an important difference between large-scale dynamics on the model 390 K and 330 K surfaces. On the upper surface, the rotational flow in the model is essentially maintained through a balance between advection of absolute vorticity and gravity wave drag. On the lower surface a strong thermal component is also present, suggesting that the dynamics of the model's rotational flow are different in the two regions.

Figure 13 shows the PV field in PVU on the 330 K surface for (a) the simulation with parameterized gravity wave drag, (b) the simulation without parameterized gravity wave drag, and (c) NCEP data. Discrepancies between Figs. 13a and 13b are associated mainly with the strength of the meridional PV gradient at northern midlatitudes. This gradient is weaker in the simulation including gravity wave drag because the circumpolar vortex is somewhat weaker in this run. Good qualitative and quantitative agreement is found between Figs. 13a and 13c, suggesting that the atmospheric circulation is better simulated by the model at

![Fig. 12. Seasonally averaged (a) adiabatic forcing, $\vec{A}$ (K day$^{-1}$), and (b) vertical wind shear $\partial w/\partial \theta$ (m s$^{-1}$ K$^{-1}$) on the 330 K surface for the model with gravity wave drag.](image)

![Fig. 13. The 330 K potential vorticity (PVU) calculated from (a) the model with gravity wave drag, (b) the model without gravity wave drag, and (c) NCEP data. The fields shown in (a) and (b) are averaged over a single winter, whereas the field shown in (c) is averaged over three winters. Contour interval: 1 PVU.](image)
served mean mechanical forcing field. Discrepancies between advective flux calculations based on objectively analyzed and model data in the upper troposphere and lower stratosphere appear to be associated with effects of the gravity wave drag parameterization in the model. A simulation without gravity wave drag shows better qualitative agreement with observationally based results in that region but poorer agreement elsewhere, in general.

Although objectively analyzed data rise questions about the magnitude and spatial structure of the effects of parameterized gravity wave drag, they do suggest that the effects of mechanical forcing are not negligible in the middle latitude lower stratosphere. Moreover, the results presented here based on NCEP objectively analyzed data are subject to possible inaccuracies associated with interpolation to isentropic surfaces and insufficiently frequent data sampling. The extent to which sampling is a problem is discussed in the appendix, where it is shown that a twice daily frequency may be inadequate for calculating the divergent part of the advective PV flux.

The PV budget along a middleworld isentropic surface was also examined. In contrast to the overworld (390 K) budget, the middleworld budget was characterized by a three-way balance between advective, mechanical, and thermal PV fluxes. Model simulated PV on the 330 K surface was relatively insensitive to the inclusion of parameterized gravity wave drag and compared better to observations than the 390 K PV. The simulated meridional component of the thermal PV flux is associated with a correlation between westerlies and radiative cooling and easterlies and diabatic heating. This result is consistent with the conclusions of Hoskins (1991), who demonstrated the existence of a similar pattern in the atmosphere.

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APPENDIX

Influence of Data Sampling Frequency on the Temporally Averaged PV Flux

The influence of data sampling frequency on the temporal correlations appearing in the time-averaged expressions for $A_\theta$ and $T_\theta$ is illustrated here, using the results of a special 10-day model run. The data correspond to 1–10 January, year 19 of a long-term GCM climate integration and consist of the vorticity, divergence, and diabatic heating fields sampled hourly. Since only the divergent parts of $A_\theta$ and $T_\theta$ contribute to local changes in $T_{\omega\theta}$ as shown by (12), only these parts, $A_{\theta\omega}$ and $T_{\theta\omega}$, respectively, are considered here.

The influence of data sampling frequency on the mechanical part of the PV flux, $M_{\theta\omega}$, is not considered, since temporal correlations do not appear in the time-averaged expression for this quantity.

a. The advective flux, $A_\theta$

The field [\(\bar{\omega}\)] corresponding to the zonally averaged y component of $\bar{A}_\theta$, is shown in Fig. A1. The overbar represents a temporal average over the 10-day period described above, using winds and isentropic vorticity sampled (a) hourly, (b) four times daily, and (c) twice daily. The panel for twice daily data represents the result obtained when data from the standard GCM history file corresponding to the 10-day period is used.

The fields in Figs. A1a and A1b are very similar to one another, indicating that 6-h sampling of the
model vorticity and divergence fields provides an accurate determination of $\langle \nu_{\infty} \rangle$ for the 10-day period under consideration. Figure A1c differs somewhat from the previous panels, especially in the vicinity of the 390 K surface, where the local maximum of $\approx 2.5 \text{ m s}^{-1} \text{ day}^{-1}$ underestimates the corresponding value of $\approx 3 \text{ m s}^{-1} \text{ day}^{-1}$ seen in panels a and b. The local minimum at upper levels near 50S in Fig. 1c is also deeper than in the previous fields.

Figure A2 shows the field $\tilde{A}_\theta$ and its associated potential function $\tilde{\chi}_\theta$ on the 390 K surface. As in Fig. A1, the fields are calculated using data sampled (a) hourly, (b) four times daily, and (c) twice daily. Consistent with the results of Fig. A1, Figs. A2a and A2b are very similar to one another, while clear differences exist when either is compared to Fig. A2c. These results indicate that sampling data four times daily provides a more accurate (and generally satisfactory) approximation to the advective PV flux than is obtained from data that has been acquired through sampling only twice daily.

b. The thermal flux, $T_\theta$

Standard GCM history files include only temporal averages of the diabatic heating field, $H$ over consecutive 24-h intervals; instantaneous values are not archived. Here $H$ is accumulated this way in order to facilitate the examination of simulated heat budgets. Thus, for any given 24-h period, only that part of $\tilde{T}_\theta$ associated with the mean flow of $T_\theta^m$ can be calculated from standard output, where

$$T_\theta^m = \frac{\tilde{\partial}_\theta}{\tilde{\partial}_\theta} \tilde{\xi} - \tilde{\partial}_\mu \tilde{\xi} j,$$

and the tilde in (A1) represents a diurnal average.

Although not shown here, we have found that data from the standard GCM archive provide an acceptable alternative to using data sampled with much finer time resolution when calculating $[-\tilde{\alpha}_i \tilde{\partial}_H]$. A similar conclusion is drawn when comparing the two-dimensional thermal PV flux fields obtained using different archiving procedures, suggesting that high sampling rates are not required for an accurate determination of $T_\theta$.

REFERENCES

—, —, and —, 1984b: The climatology of the Canadian Climate Centre general circulation model as obtained from a 5 year simulation. Atmos.–Ocean, 22, 430–473.