Effect of Ice on the Generation of a Generalized Potential Vorticity

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ABSTRACT

In this work (using Hauf and Holler's entropy temperature) a potential vorticity is defined that generalizes the moist potential vorticity. This ''generalized'' potential vorticity is used to analyze the possible effect of ice on changes of potential vorticity. The authors find that the existence of spatial gradients of ice concentration supplies a mechanism to generate ''generalized'' potential vorticity. An estimate of this ''ice solenoid'' term shows that there are cases in which this new term and the classic solenoid term could have the same order of magnitude.

1. Introduction

Potential vorticity, first introduced by Ertel (1942) as dry potential vorticity, is a fundamental concept of atmospheric dynamics. Because of its conservation following fluid motion in a frictionless and adiabatic flow and its invertibility in a balanced system, the dry potential vorticity is very useful in both diagnostic and prognostic studies of the atmosphere.

When latent heat of condensation is taken into account, the dry potential vorticity is not conserved. However, by replacing the potential temperature with the equivalent potential temperature, it is possible to define the moist potential vorticity. The moist potential vorticity becomes conserved in moist adiabatic processes; it has been extensively used in studies of conditional symmetric instability in baroclinic systems since first proposed as a possible mechanism for the formation of frontal rainbands by Bennetts and Hoskins (1979) and Emanuel (1979, 1983). These authors showed that a negative moist potential vorticity is a sufficient condition for two-dimensional frictionless conditional symmetric instability. Recently, Cao and Cho (1995) and Persson (1995) have investigated the generation of moist potential vorticity in extratropical cyclones.

In the cloudy systems associated with extratropical cyclones, negative Celsius temperatures and ice are usual. Moist potential vorticity is not conserved when latent heat of freezing is taken into account. Therefore, if we wish to investigate the possible effect of ice in the generation of potential vorticity, we have to define a potential vorticity that takes into account the possibility of latent heat of freezing release.

In this paper we propose a generalized definition of potential vorticity and the possible effect of ice as a source of generalized potential vorticity is investigated.

2. Generalized potential vorticity

Hauf and Holler (1987) defined the entropy temperature θ_s. The entropy temperature is an expression of entropy in terms of temperature that generalizes potential temperatures previously used. It should be noted that entropy temperature is the most general and exact potential temperature. It is defined by

$$\theta_s = T\left(\frac{p_0}{p_d}\right)^{R_d/C^*}\exp\left[\frac{L_v + A_v}{C^*T}q - \frac{L_l + A_l}{C^*T}i\right], \quad (1)$$

with

$$C^* = C_{pd} + C_l(q + l + i), \qquad q = \frac{m_v}{m_d},$$

$$l = \frac{m_l}{m_d}, \text{ and } \qquad i = \frac{m_i}{m_d}, \quad (2)$$

where p_0 is a constant pressure of reference; p_d the partial pressure for dry air; L_v (L_l) the latent heat of condensation (freezing); A_v (A_l) the affinity of condensation (freezing); C_{pd} the specific heat at constant pressure for dry air; C_l the specific heat for liquid water; $m_k = M_k/M$, with M_k the mass of component k and M the total mass of the system; and the subscripts $k = d, v, l, i$ are, respectively, dry air, water vapor, liquid water, and ice. Note that q is the mixing ratio of water vapor and l and i mixing ratios for liquid water and ice.

Since the dry air mass is greater than other component masses, it could be assumed that the dry air guides the

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motion of the system; that is, the center of mass velocity is equal to the dry air velocity. Under this assumption, changes of entropy temperature are caused just by entropy changes (Hauf and Holler 1987). Consequently, if reversibility is assumed, $d\theta_s/dt = 0$ and θ_s remains constant for any parcel of cloudy air.

Following Bennetts and Hoskins (1979), we can define a ''generalized'' potential vorticity, q_g, as

$$q_g = f\left(\frac{g}{\theta_0}\right)\boldsymbol{\zeta}\cdot\boldsymbol{\nabla}\theta_s = f\zeta N_g^2 + \left(\frac{g}{\theta_0}\right)\left\{\mathbf{k}\times\frac{\partial\mathbf{v}}{\partial z}\right\}\cdot\boldsymbol{\nabla}_h\theta_s, \qquad (3)$$

where $\boldsymbol{\zeta}$ is the three-dimensional absolute vorticity vector, ζ its vertical component, θ_0 a typical value of the potential temperature, $N_g^2 = (g/\theta_0)(\partial\theta_s/\partial z)$ a ''generalized'' static stability, $\mathbf{k}$ the unit vertical vector, $\boldsymbol{\nabla}_h$ the horizontal gradient operator, and $\mathbf{v} = (u, v, 0)$ the horizontal velocity vector. It should be noted that the ''generalized'' potential vorticity takes into account not only the possibility of freezing, but also the possibility of changes of phase in nonequilibrium conditions.

Using the well-known prognostic equation for the absolute vorticity vector (i.e., Bennetts and Hoskins 1979), we obtain a prognostic equation for the generalized potential vorticity:

$$\frac{dq_g}{dt} = f\left(\frac{g^2}{\theta_0^2}\right)\mathbf{k}\cdot(\boldsymbol{\nabla}\theta_s\times\boldsymbol{\nabla}\theta) + f\left(\frac{g}{\theta_0}\right)\boldsymbol{\zeta}\cdot\boldsymbol{\nabla}P + f\left(\frac{g}{\theta_0}\right) \times(\boldsymbol{\nabla}\times\mathbf{F})\cdot\boldsymbol{\nabla}\theta_s, \qquad (4)$$

where $P = d\theta_s/dt$. The first term on the right-hand side of (4) indicates a change in generalized potential vorticity when there is an angle between θ_s and θ surfaces in the horizontal. The last two terms of the right-hand side of (4) are due to irreversible and frictional effects, respectively. If the flow is assumed reversible and frictionless, then (4) reduces to

$$\frac{dq_g}{dt} = f\left(\frac{g^2}{\theta_0^2}\right)\mathbf{k}\cdot(\boldsymbol{\nabla}\theta_s\times\boldsymbol{\nabla}\theta). \qquad (5)$$

If it is assumed that water changes of phase are produced in equilibrium conditions, then affinities vanish, $A_v = A_l = 0$. On the other hand, as the dry air mass in cloudy systems usually presents a little variation, we can consider C^* constant (Rivas Soriano et al. 1994). Assuming additionally that L_l is constant, we find

$$\boldsymbol{\nabla}\theta_s = \exp\left[-\frac{L_l i}{C^*T}\right]\left\{\boldsymbol{\nabla}\theta_e - \frac{\theta_e L_l}{C^*}\boldsymbol{\nabla}\left(\frac{i}{T}\right)\right\}, \qquad (6)$$

where θ_e is the equivalent potential temperature. Combining (5) and (6) gives

$$\frac{dq_g}{dt} = f\frac{g^2}{\theta_0^2}\exp\left[-\frac{L_l i}{C^*T}\right]\mathbf{k}\cdot\left\{\boldsymbol{\nabla}\theta_e\times\boldsymbol{\nabla}\theta - \frac{\theta_e L_l}{C^*}\left[\boldsymbol{\nabla}\left(\frac{i}{T}\right)\times\boldsymbol{\nabla}\theta\right]\right\} = \left(\frac{dq_g}{dt}\right)_m + \left(\frac{dq_g}{dt}\right)_{\text{ice}}. \qquad (7)$$

The first term of the right-hand side of (7) is equal to the classical solenoid term. Since this term has been extensively investigated (Bennetts and Hoskins 1979; Cao and Cho 1995; Persson 1995), we shall address our attention to the second term of the right-hand side of (7). This term only contributes to the rate of change of the generalized potential vorticity when there is ice. It represents the differential advection of the existing ice concentration by the thermal wind, as described by Persson (1995) for water vapor and the classic solenoid term.

3. Effect of ice on the generation of generalized potential vorticity

Considering the second term on the right-hand side of (7), we can write

$$\left(\frac{dq_g}{dt}\right)_{\text{ice}} = -A\mathbf{k}\cdot\left[\boldsymbol{\nabla}\left(\frac{i}{T}\right)\times\boldsymbol{\nabla}\theta\right], \qquad (8)$$

where

$$A = f\frac{g^2}{\theta_0^2}\theta_e\frac{L_l}{C^*}\exp\left[-\frac{L_l i}{C^*T}\right] > 0.$$

Equation (8) indicates that there will be a decrease (increase) in the value of generalized potential vorticity if the vector $\boldsymbol{\nabla}(i/T)\times\boldsymbol{\nabla}\theta$ has a component along (against) the vertical direction, as is shown in Fig. 1.

The situation schematized in Fig. 1 may be produced in extratropical cyclones, where cloud bands are nearly parallels to the fronts. On the other hand, the ice content decreases from the inner zones to the boundaries of the ice regions in cloud bands. It seems clear that the vector $\boldsymbol{\nabla}(i/T)$ points in opposite directions in the two sides of the ice region that are nearly parallels to the front. For example, if cloud bands associated with a north–south oriented cold front are considered, Fig. 1b could represent the situation on the east side of ice region and Fig. 1a on the west side.

A reasonable mean value of ice content in the mesoscale bands associated with fronts could be 0.3 g m^{-3} (Bennetts and Ryder 1984). Considering that the thickness and temperature of ice region is about 10 km and -5°C, respectively, we find that $|\boldsymbol{\nabla}(i/T)| \approx 8\times10^{-8}$ (K km)$^{-1}$. Since $\theta_e L_l/C^* \approx 10^4$ K^2 and taking $|\boldsymbol{\nabla}\theta| \approx |\boldsymbol{\nabla}\theta_e| \approx 10^{-2}$ K km^{-1}, thus $\theta_e L_l/C^*|\boldsymbol{\nabla}(i/T)\times\boldsymbol{\nabla}\theta| \approx 8\times10^{-6}\sin\alpha_1$ K^2 km^{-2}, where α_1 is the angle between $\boldsymbol{\nabla}\theta$ and $\boldsymbol{\nabla}(i/T)$. On the other hand, $|\boldsymbol{\nabla}\theta_e\times\boldsymbol{\nabla}\theta| \approx 10^{-4}\sin\alpha_2$ K^2 km^{-2}, with α_2 the angle between $\boldsymbol{\nabla}\theta$ and $\boldsymbol{\nabla}\theta_e$. Thus the relationship between the ''ice solenoid'' term and the classic solenoid term is

$$\left(\frac{dq_g}{dt}\right)_{\text{ice}}\left(\frac{dq_g}{dt}\right)_m^{-1} \approx 8\times10^{-2}\frac{\sin\alpha_1}{\sin\alpha_2}. \qquad (9)$$

A reasonable value for α_2 is 0.1° (Bennetts and Hoskins 1979). This implies a rate of about 0.4–0.5 PVU/day for $(dq_g/dt)_m$. This magnitude is consistent with the

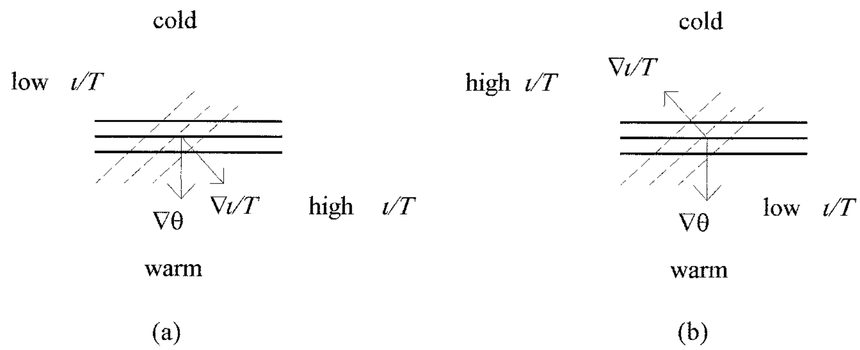

FIG. 1. A possible horizontal section showing potential temperature (θ) and i/T contours. In (a) $(dq_g/dt)_{\rm ice} > 0$; in (b) $(dq_g/dt)_{\rm ice} < 0$.

values shown in previous works (Bennetts and Hoskins 1979; Cao and Cho 1995; Persson 1995). If we choose the same value for α_1 as for α_2, the ice solenoid term is about 0.04 PVU/day. If $\alpha_1 = 1°$ is appropriate, the ice solenoid term contribution is about 0.4 PVU/day. Considering that a change of 0.3 PVU/day would be the minimum rate detectable in observations, the value for α_1 should be at least 0.7°. Therefore, an observable contribution of spatial gradients of ice concentration on changes of generalized potential vorticity requires an angle between θ and i/T surfaces about an order of magnitude higher than the angle between θ and θ_e surfaces. In consequence, the contribution of ice solenoid term would probably be smaller than the classic solenoid term contribution. However, because the spatial distribution of ice concentration is more irregular than the water vapor distribution, there will be cases where α_1 is considerably higher than α_2, especially for the 10-km scale we are considering. In these cases, the ice solenoid term should be taken into account.

4. Conclusions

The possible effect of ice on changes of potential vorticity is considered. For this purpose we define a ''generalized'' potential vorticity using Hauf and Holler's entropy temperature. We find that two terms contribute (if the flow is assumed reversible and frictionless) to change the generalized potential vorticity: the classic solenoid term and a new term that involves the spatial gradient of existing ice concentration. An estimate of this ''ice solenoid'' term shows that an observable contribution of ice on changes of generalized potential vorticity requires an angle between θ and i/T surfaces about an order of magnitude higher than the angle between θ and θ_e surfaces. So the contribution of the ice solenoid term would probably be smaller than the classic solenoid term in most of the cases.

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