Comments on ''A Convective Transport Theory for Surface Fluxes''

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The purpose of this note is to comment on a simple parametrization scheme for evaluation of surface fluxes in convective conditions, proposed by Stull (1994), and referred to by the author as the ''convective transport theory'' (CTT). My intention is to show that CTT can be evaluated based on the surface layer similarity theory, developed by Monin and Obukhov about five decades ago, and verified experimentally on many occasions since than.

According to Monin–Obukhov similarity, thermal stability regimes within the surface layer are characterized by the length scale L, called the Monin–Obukhov length (e.g., Sorbjan 1989),

$$L = -\frac{u_*^3}{\beta H_0}, \tag{1}$$

where $H_0 = \langle w'\theta' \rangle$ is the surface kinematic flux of the virtual temperature, $\beta = g/\Theta$ is the buoyancy parameter, and u_* is the friction velocity. Based on the definition of L, the ''purely'' free-convection regime (no mean wind, and consequently $u_* = 0$) can be associated with the nil value of L. The case that takes place when the mean wind is weak, and L is small but nonzero, will be hereafter referred to as ''forced'' free convection. I will briefly comment on both convective regimes.

Let us first discuss the purely free-convection case. The Monin–Obukhov prediction for the virtual potential temperature Θ in the surface layer can be expressed as

$$\frac{d\Theta}{dz} = -c_\theta \frac{H_0^{2/3}}{\beta^{1/3}} z^{-4/3}, \tag{2}$$

where z is height and c_θ is a universal constant that is positive in value. The above formula was first obtained by Prandtl (1932) and independently by Obukhov (1946). Equation (2) indicates that near the earth's surface the temperature lapse rate $d\Theta/dz$ is very large. Near the top of the surface layer (typically at about $z = 100$ m), $d\Theta/dz$ is very small (e.g., Kaimal 1966; Businger et al. 1971). For example, assuming in (2) that $z = 100$ m, $H_0 = 0.1$ K m s^{-1}, $\beta = 0.03$ m s^{-2}K^{-1}, and $c_\theta = 0.23$ (the value obtained by curve fitting the Kansas data presented by Businger et al. 1971), one can obtain $d\Theta/dz = -0.03$ K/100 m.

The expression (2) is often presented in a different form, which can be obtained by multiplying it by z/T_*, where $T_* = -H_0/u_*$ is the temperature scale

$$\frac{z}{T_*}\frac{d\Theta}{dz} = c_\theta \left(\frac{z}{L}\right)^{-1/3}. \tag{3}$$

Obviously, the above expression makes sense only during forced convection. During free convection ($L = 0$, $T_* = -\infty$) the expression (3) is singular. It could be noted that in some previously presented empirical formulations, the exponent ''$-1/3$'' in (3) has been replaced by the exponent ''$-1/2$'' (e.g., Businger 1988). Such an approach is inconsistent with the Monin–Obukhov similarity and implies that $d\Theta/dz \sim u_*$. Therefore, Stull's (1994) notion that the traditional bulk aerodynamic approach fails during purely free convection applies to formulation (3), and also to all empirical expressions with the exponent different than $-1/3$, but not to formulation (2).

Integrating (2) with respect to height leads to

$$\Theta_1 - \Theta_2 = 3c_\theta \frac{H_0^{2/3}}{\beta^{1/3}} (z_1^{-1/3} - z_2^{-1/3}), \tag{4}$$

where $z_1 < z_2$. The above expression can be rearranged to yield

$$H_0 = (3c_\theta)^{-3/2} \frac{\beta^{1/2}}{(z_1^{-1/3} - z_2^{-1/3})^{3/2}} (\Theta_1 - \Theta_2)^{3/2}. \tag{5}$$

Equation (5) indicates that for the evaluation of the surface heat flux, it is sufficient to measure temperature at any two levels within the free-convection surface layer. When z_2 is chosen near the top of the free-convection surface layer, where the temperature lapse rate $d\Theta/dz$ practically vanishes, Θ_2 is expected to be very close to the mixed layer temperature Θ_{ml}. Because (4) cannot be extended to the earth's surface (e.g., Brutsaert 1982),

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Θ_1 is expected to differ from the surface skin temperature Θ_S.

Defining

$$\Delta\Theta = \Theta_1 - \Theta_2,$$
$$w_B = (\beta z_i \Delta\Theta)^{1/2},$$
$$b_H = \{3c_\theta[(z_1/z_i)^{-1/3} - (z_2/z_i)^{-1/3}]\}^{-3/2}, \quad (6)$$

where z_i is the mixed layer height, allows (5) to be rewritten in the form

$$H_0 = b_H\, w_B\, \Delta\Theta. \quad (7)$$

Equation (7) is consistent with the result obtained by Stull (1994) [i.e., with his equation (18)], who used different arguments for its derivation. Stull assumed that b_H is an empirical constant, which leads to several inconsistencies discussed below.

First of all, as it follows from (6), the parameter z_i is only formally included in (7); that is, even though w_B and b_H are dependent on the mixed layer height, their product is independent of z_i. If b_H is assumed constant (as in Stull's formulation), this consequently implies an erroneous conclusion that H_0 is uniquely dependent on z_i. The surface heat flux is caused by small eddies near the surface and cannot be influenced by large eddies of the scale z_i. An additional argument demonstrating that such a relation of H_0 on z_i is generally incorrect can be obtained from analyses of "large eddy simulations" (e.g., Sorbjan 1995), which exhibit an intense growth of the mixed layer in cases when the surface heating is constant with time and also almost no change of the mixed layer height z_i when the surface heating decreases with time during late afternoons. In the formulation (6)–(7) derived from the Monin–Obukhov similarity, b_H is dependent on z_1, z_2, and z_i. When we assume that $(z_2/z_i)^{-1/3} \ll (z_1/z_i)^{-1/3}$, it will become apparent that b_H, approximately equal to $3c_\theta\,(z_1/z_i)^{1/2}$, is quite sensitive to the choice of the lower observation level. For $c\theta \sim 0.23$, and for the value $b_H = 5 \times 10^{-4}$ suggested by Stull (1994), we find that the equivalent altitude of the lower level would be $z_1/z_i = (b_H/3c_\theta)^2 \sim 5.5 \times 10^{-7}$, which is at the bottom of the molecular sublayer. This means that the free-convection temperature profile is extended to the earth's surface. Such an assumption is not appropriate for two reasons. First of all, (2) is not valid in the very close vicinity of the earth's surface (e.g., Townsend 1959; Brutsaert 1982). Second, Eq. (7) with constant b_H is not general, as it does not include any direct information about thermal characteristics of the viscous (canopy) sublayer. Such information is relevant since for a given value of the surface heat flux one would expect smaller $\Delta\Theta$ over rougher surfaces.

In order to include the roughness parameter for heat in the scheme, b_H has to be assumed to be a function of z_1/z_i, and the temperature Θ_1 at $z = z_1$ has to be calculated based on the temperature profile in the viscous (canopy) sublayer (e.g., Mahrt and Ek 1984). Nevertheless, it should be mentioned that for complicated heterogeneous surfaces such a simple approach might not be sufficient (e.g., Sun and Mahrt 1995).

Let us now turn our attention to the case of forced free convection. In this case (L very small and negative), Monin and Obukhov suggested that the turbulent Prandtl number, $\mathrm{Pr} = k_m/k_h$, can be assumed constant. This assumption leads to

$$\frac{dU}{dz} \sim \frac{u_*}{T_*}\frac{d\Theta}{dz},$$

which together with (2) yields

$$\frac{dU}{dz} = c_u \frac{u_*^2}{(\beta H_0)^{1/3}} z^{-4/3}, \quad (8)$$

where c_u is a universal constant that is positive in value. Integrating (8) with respect to height leads to

$$U_2 - U_1 = 3c_u \frac{u_*^2}{(\beta H_0)^{1/3}}(z_1^{-1/3} - z_2^{-1/3}), \quad (9)$$

where $z_1 < z_2$. After some rearrangements and with the help of (5), we will arrive at

$$u_*^2 = (3c_u)^{-1}(3c_\theta)^{-1/2}\frac{(\beta\Delta\Theta)^{1/2}}{(z_1^{-1/3} - z_2^{-1/3})^{3/2}}(U_2 - U_1). \quad (10)$$

The obtained result implies that for the evaluation of the surface fluxes in the case of forced convection, it is sufficient to measure temperature and wind velocity at two levels within the surface layer. When z_2 is chosen near the top of the free-convection surface layer, U_2 is expected to be close to the mixed layer wind speed U_{ml}.

In analogy to (7), defining

$$b_D = (3c_u)^{-1}(3c_\theta)^{-1/2}[(z_1/z_i)^{-1/3} - (z_2/z_i)^{-1/3}]^{-3/2}, \quad (11)$$

we obtain

$$u_*^2 = b_D\, w_B\, (U_2 - U_1). \quad (12)$$

Equation (12) is equivalent to the result obtained by Stull (1994), who in addition assumed that $U_1 = 0$ [see his Eq. (21)] and also that b_D is an empirical constant.

It can be easily verified that even though w_B and b_D are dependent on the mixed layer height, their product is independent of z_i. If b_D is assumed to be an empirical constant, this leads to the conclusion that u_*^2 is directly dependent on z_i. To my knowledge, such a direct dependence has not been documented empirically.

In the formulation (11)–(12), the coefficient b_D is dependent only on z_1, z_2, and z_i. When we assume that $(z_2/z_i)^{-1/3} \ll (z_1/z_i)^{-1/3}$, then approximately $b_D = (3c_u)^{-1}(3c_\theta)^{-3/2}\,(z_1/z_i)^{1/2}$. This shows that, analogously to b_H, b_D is sensitive also to the choice of z_1 and consequently cannot be considered the universal constant. For the value $c_u \sim 0.54$ [the value obtained by curve fitting the Kansas data of Businger et al. (1971)], and for $b_D = 1.83 \times 10^{-3}$ suggested by Stull (1994), it can be found that the equivalent value of z_1/z_i is practically zero. This means that the free-convection wind profile is extended to the earth's surface. Such an assumption is not appropriate because (9) is not valid in the very close vicinity of the earth's surface. Moreover, such an assumption excludes any information on the properties of the underlying surface.

In the case of forced free convection, surface characteristics such as the roughness parameter and the surface temperature could be included in the scheme by matching the convective profiles (4) and (9) with the profiles in the logarithmic layer and the viscous sublayer below it (e.g., Zilitinkevich 1970; Kraus and Businger 1994).

Concluding, the classic Monin–Obukhov similarity theory gives constructive hints regarding the form of coefficients b_H and b_D. Knowing the form of these coefficients, as well as understanding their validity in the surface layer (and in the viscous sublayer), allows one to estimate the expected accuracy and universality of the CTT. Based on the above presented analysis one might state that the CTT functional dependencies of H_0 on $\Delta\Theta^{3/2}$, and also of u_*^2 on the product $\Delta\Theta^{1/2}U_{ml}$, agree with the Monin–Obukhov similarity. The introduced by the CTT-dependence of the surface fluxes on $z_i^{1/2}$ (through w_B), as well as a lack of dependence on parameters characterizing the surface roughness and conduction in the viscous sublayer, should contribute to errors in situations different from conditions for which CTT has been calibrated.

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