Local Energetics of an Idealized Baroclinic Wave Using Extended Exergy

FRED KUCHARSKI* AND ALAN J. THORPE

Department of Meteorology, University of Reading, Reading, United Kingdom

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ABSTRACT

The concept of local extended exergy is here applied to an idealized, dry, and reversible-adiabatic cyclone development. The extended exergy as well as the kinetic energy are decomposed into a mean part, defined by a zonal average, and into a perturbation from the mean. The resulting local energy evolution equations provide an extension of the well-known Lorenz-type available energy equations. A term in the baroclinic conversion rate, connected with static stability anomalies, which is not usually considered, is of significance even in this idealized case study and contributes significantly to the nonlinear equilibration of the baroclinic wave.

1. Introduction

Applications of any available potential energy (APE) to atmospheric data are generally based on the global APE concept of Lorenz (1955). Lorenz defined APE as the difference between the mass-integrated total potential energy (that is the sum of internal and potential energy) and the total potential energy of a stably stratified hydrostatic reference state. The APE theory of Lorenz has been extended for the use in vertically integrated, but horizontally limited, area studies of the development of cyclones (e.g., Smith et al. 1977). But the local interpretation of production and conversion terms in these approaches revealed some problems because fluxes through the boundaries of local APE as well as its release can be shown to be orders of magnitudes larger than the generation of kinetic energy and diabatic production rates (e.g., Smith 1980). Therefore, only the small difference of a large conversion term and a large boundary term gives the actual generation of kinetic energy.

On the other hand there are local *approximate* versions of APE related to a decomposition of the state of the atmosphere into a mean state and deviations from that [e.g., Boussinesq-type models for convection (Dutton and Fichtl 1969) or quasigeostrophic theory (Pedlosky 1987)]. All these energetics depend, in their basic formulation, to a large extent on the approximations of the underlying thermohydrodynamic equations. Andrews (1981) derived an exact version of a locally positive definite available potential energy that is valid—provided there is stable mean stratification—for all scales of atmospheric motion. However, the considerations of Andrews were restricted to reversible-adiabatic processes. Therefore, no production and destruction processes of APE could be studied. Kucharski (1997) extended the ideas of Andrews (1981) so that diabatic and irreversible processes could be included in the local available energy formalism. It turned out that an extension of the exergy concept from technical thermodynamics can be used to formulate this new approach to local available potential energy.

The aforementioned ''cancellation effect'' of large terms in the energy conversion equations using the APE approach of, for example, Smith (1980) appears to be systematic and can be removed to a large extent by using the extended exergy as available potential energy representation (extended exergy is, in contrast to Smith's available potential energy, a second-order quantity) and reformulating the boundary flux terms and conversion terms between kinetic energy and available potential energy as formulated in Kucharski (1997).

In applications of the extended exergy concept to energy studies of the entire atmosphere a *globally* defined hydrostatic reference state is used to define locally a disturbance energy having global conservation properties for reversible-adiabatic processes. However, because all scales of atmospheric motion are included in the disturbances from the reference state, there is some practical and theoretical value in a further scale separation of extended exergy. Thus, the extended exergy is decomposed into a mean part and a perturbation, or ''turbulent'' part [as, e.g., in the approach of Orlanski and Katzfey (1991) in the case of APE]. The turbulent part is interpreted here to be the synoptic-scale disturbances, while the mean part is defined as a zonal mean extended exergy not resolving synoptic disturbances. As

* Current affiliation: The Met. Office, Bracknell, Berkshire, United Kingdom.

Corresponding author address: Dr. Fred Kucharski, NWP Division, The Met. Office, Room 248, London Road, Bracknell, Berkshire RG12 2SZ, United Kingdom.
E-mail: fkucharski@meto.gov.uk

a consequence the term *local* in this study means that the vertical and meridional directions are retained, whereas the results are independent of the zonal direction of the domain occupied by the analyzed cyclone.

The purpose of this paper is to show (i) how the extended exergy can be used to study the local energetics of the class of Boussinesq-simplified primitive-equation models of cyclone developments and (ii) to examine the insight provided by the extended exergy. The proposed turbulent extended exergy concept is therefore applied to an idealized dry and reversible-adiabatic baroclinic model of the development of a cyclone (Rotunno et al. 1994; Snyder et al. 1991). Various rates of the energy evolution equations are evaluated. A new term in the baroclinic conversion rate (compared to Lorenz-type approaches), which is related to static stability anomalies, is shown to be of importance even in this idealized, dry and reversible-adiabatic case study due to a developing tropopause fold. Its role in the nonlinear equilibration of the baroclinic wave is discussed.

It is speculated that the static stability contribution to the baroclinic conversion rate will be of even more importance in applications to more realistic models including diabatic effects such as latent heat release or sensible and latent heat fluxes.

2. Extended exergy concept

In this section the results from Kucharski (1997) are briefly summarized. Kucharski showed that under the constraint of stable mean stratification the classic concept of exergy can be extended to formulate a locally positive definite available potential energy. The sum of extended exergy and kinetic energy satisfies a global conservation law for reversible-adiabatic processes. The derivation of the extended exergy is achieved with three steps.

1) Define a hydrostatic, time-independent static equilibrium state:

$$T_R(z) \quad \text{and} \quad \frac{dp_R}{dz} = -\rho_R(z)g,$$

where $T_R(z)$, $p_R(z)$, $\rho_R(z)$ are reference temperature, pressure, and density, respectively, satisfying the perfect gas law. The g is the acceleration due to gravity.

2) Perform the exergy transformation, regarding the internal energy, $u(s, \alpha)$, as a function of specific entropy, s, and specific volume, $\alpha = 1/\rho$ (see Fig. 1): The internal energy $u(s, \alpha)$ is a convex function of s and α in phase space (i.e., the matrix of the second derivatives of u is positive definite). The reference state is just one point on the internal energy surface. The difference between the internal energy and the tangential plane, which touches the internal energy surface in the point of the reference state, defines—due to the convexity of internal energy—a locally *positive* measure of deviations from the reference state. This quantity is called (classic) static *exergy* if the temperature of the reference state, T_R, is constant. Its mathematical formulation is

$$e_{ex} := \Delta u - T_R\Delta s + p_R\Delta\alpha, \qquad (1)$$

where $\Delta\psi := \psi - \psi_R$.

3) Considering the balance of the (mass integrated) sum of exergy and kinetic energy, one derives a conservation law for an isothermal reference temperature $T_R(z) =$ const. for reversible-adiabatic motion. This suggests a physical meaning of the static exergy as a kind of available energy. However, the static exergy with $T_R(z) =$ const. is also positive in a stably stratified static equilibrium state where, for example, the temperature $T = T(z)$ is a nonconstant function of height only. Van Mieghem (1973) showed that in such a state there is no energy available for conversion into kinetic energy.

An arbitrary $T_R(z)$ distribution leads to production/destruction terms in the evolution equation of the sum of (1) and kinetic energy even in the case of reversible-adiabatic motion. But these terms can be shown to form an exact differential of an additional energy form that adds to (1) to give, when adding the kinetic energy, a new, conserved (in a mass integral) quantity: the *extended exergy e* [a more detailed discussion can be found in Kucharski (2000); the exact formulation of e is found in Eq. (17) of Kucharski (1997)]. An accurate approximation is (Kucharski 1997)

$$e \approx \overbrace{\frac{1}{2}\frac{g^2}{N_R^2}\left(\frac{\Delta\theta}{\theta_R}\right)^2}^{\text{I}} + \overbrace{\frac{1}{2}RT_R\frac{c_v}{c_p}\left(\frac{\Delta p}{p_R}\right)^2}^{\text{II}}, \qquad (2)$$

where $\theta_R(z)$ is the potential temperature of the reference state. Here, c_p and c_v are the specific heat values at constant pressure and volume, respectively. The symbol R is the individual gas constant. The variable N_R^2, the squared Brunt–Väisälä frequency of the reference state, is defined as

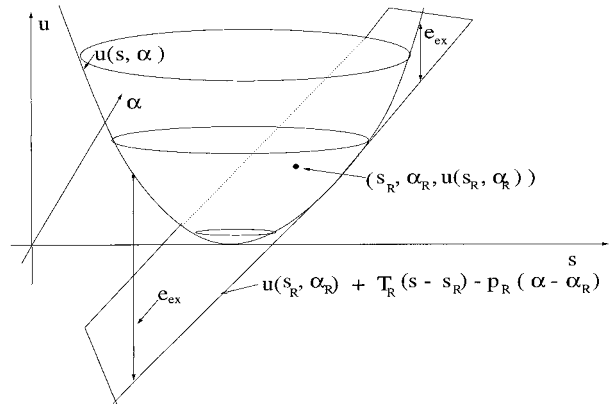

FIG. 1. The convex internal energy, u, as a function of specific entropy, s, and specific volume, α, in phase space—see text.

$$N_R^2 := \frac{g}{\theta_R}\frac{d\theta_R}{dz}. \tag{3}$$

Whereas the classic exergy according to Eq. (1) is always a positive measure of deviations from the reference state, the extended exergy (2) is only positive if the reference state is stably stratified, $N_R^2 > 0$. Stated differently, if the horizontal mean state of a system is stable, it is possible to extend the exergy concept by defining (2). According to Eq. (2) the extended exergy is principally only available for transformation into kinetic energy if there are deviations from the horizontal mean state.

Note that the classic static exergy approximation is included in (2) by defining the reference state with a constant temperature, that is, $T_R(z) = T_0$.

The (thermal) contribution (I) in Eq. (2) is essentially the available potential energy, which has also been found by Lorenz (1955), in the global mass-integral context. Contribution (II) has to be negligible or constant if an anelastic approximation is valid, or a Boussinesq incompressibility assumption is made. Indeed, contribution (II) can be estimated for typical atmospheric potential temperature and pressure deviations from the reference state to be only about 10% of the contribution (I) in the troposphere (using characteristic values of $\Delta\theta = 3$ K and $\Delta p = 10$ hPa), because of N_R^2 values of about 1×10^{-4} s^{-2}. In the stratosphere, however, it is the main contribution, because N_R^2 increases by a factor of at least 3 and the relative pressure deviations from the reference state tend to increase with height.

The reference state is determined—for the purpose of this paper—by a horizontal average, at an initial time, of the density of the entire atmosphere and an integration of the hydrostatic equation. However, the conservation law for the sum of the volume integrals of extended exergy and kinetic energy, $E + K$, is valid (assuming reversible-adiabatic motion) for any time-independent hydrostatic reference state.

3. Mean and turbulent extended exergy and kinetic energy

In this application of the turbulent extended exergy only zonal averages will be considered. The general turbulent extended exergy and its evolution equations including compressibility effects and diabatic processes as well as frictional dissipation is presented in the appendix using the extended exergy approximation (2). Here, we provide the approximate energy forms and evolution equations valid for the incompressible, Boussinesq-simplified, and reversible-adiabatic primitive-equation model used in our idealized baroclinic life cycle study.

The term turbulent is used here to make it clear that only (zonally) averaged energies can be considered. Otherwise, decomposing, for example, Eq. (2), and subsequently, averaging will not lead to a positive definite energy quantity.

Zonally averaged variables will be denoted as $\langle\psi\rangle$, with $\psi = \mathbf{v} = (u, v, w)$ (three-dimensional velocity vector), θ, p.

For the deviations from the averaged variables,

$$\langle\psi'\rangle = 0, \quad \text{with } \psi' := \psi - \langle\psi\rangle, \tag{4}$$

is valid.

The energies of the mean state, $\chi_m = \{e_m, k_m\}$, have to be calculated by means of the averaged variables,

$$\chi_m = \chi_m(\langle\psi\rangle), \tag{5}$$

with $\psi = \mathbf{v}$, θ, p. Thus we have for the mean kinetic energy

$$k_m = \frac{1}{2}\langle\mathbf{v}\rangle^2,$$

and for the mean extended exergy using Eq. (2)

$$e_m \approx \frac{1}{2}\left\{\frac{g^2}{N_R^2}\left(\frac{\Delta\langle\theta\rangle}{\theta_R}\right)^2 + RT_R\frac{c_v}{c_p}\left(\frac{\Delta\langle p\rangle}{p_R}\right)^2\right\}. \tag{6}$$

For the turbulent extended exergy the approximate expression is

$$\langle e_t\rangle \approx \frac{1}{2}\left\langle\frac{g^2}{N_R^2}\left(\frac{\theta'}{\theta_R}\right)^2 + RT_R\frac{c_v}{c_p}\left(\frac{p'}{p_R}\right)^2\right\rangle. \tag{7}$$

The turbulent kinetic energy is given by

$$\langle k_t\rangle = \frac{1}{2}\langle\mathbf{v}'^2\rangle. \tag{8}$$

Mean and turbulent energy evolution equations

The incompressible and reversible-adiabatic approximations of the turbulent kinetic and extended exergy evolution equations given in Kucharski (1997) are

$$\langle\rho\rangle\frac{\partial k_m}{\partial t} = -\boldsymbol{\nabla}\cdot\{\langle\rho\rangle[\langle\mathbf{v}\rangle k_m + \langle\mathbf{v}\rangle\cdot\langle\mathbf{v}'\mathbf{v}'\rangle]\} - \langle\rho\rangle C(k_m, \langle k_t\rangle) - \langle\rho\rangle C(k_m, e_m) \tag{9}$$

$$\langle\rho\rangle\frac{\partial e_m}{\partial t} = -\boldsymbol{\nabla}\cdot\left\{\langle\rho\rangle\left[\langle\mathbf{v}\rangle e_m + \langle T\rangle\eta_m\frac{c_p}{\langle\theta\rangle}\langle\mathbf{v}'\theta'\rangle + \frac{1}{\langle\rho\rangle}(\langle p\rangle - p_R)\langle\mathbf{v}\rangle\right]\right\} - \langle\rho\rangle C(e_m, \langle e_t\rangle) + \langle\rho\rangle C(k_m, e_m) \tag{10}$$

$$\langle\rho\rangle\frac{\partial\langle k_t\rangle}{\partial t} = -\boldsymbol{\nabla}\cdot[\langle\rho\rangle\langle\mathbf{v}k_t\rangle] + \langle\rho\rangle C(k_m, \langle k_t\rangle) + \langle\rho\rangle C(\langle k_t\rangle, \langle e_t\rangle) \tag{11}$$

$$\langle\rho\rangle\frac{\partial\langle e_t\rangle}{\partial t} = -\boldsymbol{\nabla}\cdot[\langle\rho\rangle\langle\mathbf{v}e_t\rangle + \langle p'\mathbf{v}'\rangle] + \langle\rho\rangle C(e_m, \langle e_t\rangle) - \langle\rho\rangle C(\langle k_t\rangle, \langle e_t\rangle), \tag{12}$$

with

$$C(e_m, \langle e_t\rangle) \approx \underbrace{-\langle \mathbf{v}'_H\theta'\rangle \cdot \frac{g^2}{\langle\theta\rangle^2}\frac{1}{N_R^2}\boldsymbol{\nabla}_H\langle\theta\rangle}_{\text{I}} \underbrace{- \langle w'\theta'\rangle\frac{g}{\langle\theta\rangle}\left(\frac{N_m^2 - N_R^2}{N_R^2}\right)}_{\text{II}} \underbrace{- \langle \mathbf{v}'_H\theta'\rangle \cdot \frac{1}{\langle\theta\rangle\rho_R}\boldsymbol{\nabla}_H\langle p\rangle}_{\text{III}} \quad (13)$$

$$C(\langle k_t\rangle, \langle e_t\rangle) \approx -\frac{1}{\langle\rho\rangle}\langle \mathbf{v}'_H \cdot \boldsymbol{\nabla}_H p'\rangle \approx -\frac{1}{\langle\rho\rangle}\boldsymbol{\nabla}\cdot\langle p'\mathbf{v}'\rangle + \frac{g}{\langle\theta\rangle}\langle w'\theta'\rangle + \frac{N_m^2}{g\langle\rho\rangle}\langle w'p'\rangle \quad (14)$$

$$C(k_m, \langle k_t\rangle) := -\langle\mathbf{v}'\mathbf{v}'\rangle \cdot \boldsymbol{\nabla}\cdot\langle\mathbf{v}\rangle := -\langle\mathbf{v}'\mathbf{v}'\rangle : \boldsymbol{\nabla}\langle\mathbf{v}\rangle \qquad C(k_m, e_m) := \frac{1}{\langle\rho\rangle}\langle\mathbf{v}_H\rangle \cdot \boldsymbol{\nabla}_H\langle p\rangle,$$

where the subscript H means horizontal vector components. The notation $\mathbf{A}:\mathbf{B}$ (double scalar product) means that the tensors $\mathbf{A}$, $\mathbf{B}$ are multiplied tensorially, and from the resulting tensor the trace has to be calculated to give the scalar $\mathbf{A}:\mathbf{B}$.

The mean Brunt–Väisälä frequency is defined as

$$N_m^2 := \frac{g}{\langle\theta\rangle}\frac{\partial\langle\theta\rangle}{\partial z}. \quad (15)$$

The factor η_m in the flux convergence of Eq. (10) is

$$\eta_m := \left\{\frac{g^2}{c_p}\frac{1}{N_R^2}\frac{\langle\theta\rangle - \theta_R}{\theta_R} + T_R\frac{R}{c_p}\frac{\langle p\rangle - p_R}{p_R}\right\}\frac{1}{\langle T\rangle}.$$

Meaning of terms in the evolution equations

The first terms in each equation are the divergences of fluxes of the corresponding energy. These terms are not discussed in detail. They vanish after integration over the entire volume, but locally they act to redistribute the energy. Indeed, Orlanski and Sheldon (1995) discussed the possibility (using horizontally local energetics in pressure coordinates) that many cases of cyclogenesis could be related to a downstream development due to turbulent geopotential fluxes from a decaying, preexisting cyclone (instead of baroclinic generation). The corresponding term in height coordinates, $\langle p'\mathbf{v}'\rangle$, is also apparent in the evolution equation of $\langle e_t\rangle$, Eq. (12).

- The expression $C(e_m, \langle e_t\rangle)$ is the reversible transformation between mean extended exergy and turbulent extended exergy. Term (I) represents the well-known (Lorenz (1955) type) effect of advection in the presence of a mean horizontal potential temperature gradient thus reducing it and increasing turbulent extended exergy (or vice versa). The common terminology of this effect is ‘‘baroclinic conversion.’’

  Term (II) is often not apparent in approaches of the Lorenz type, although it is present in, for example, Lorenz (1955). It is zero by definition in quasigeostrophic models in which N^2 is treated as a constant. This term seems to be important if relative anomalies in N^2 become of order 1. Then term (II) has an order of magnitude comparable to term (I). The physical interpretation of term (II) is as follows. Suppose there is a negative N^2 anomaly corresponding to a region of reduced static stability (this could be associated with a positive potential temperature anomaly at the ground and a negative one at upper levels). Then, if heat is transported from lower to higher levels ($\langle w'\theta'\rangle$ is positive), the mean stratification will be stabilized. This will reduce $N_m^2 - N_R^2$ and also $|\langle\theta\rangle - \theta_R|$, which means an increase in $\langle e_t\rangle$.

  Term (II) in Eq. (13) will enhance the contribution (I) if a *negative* static stability anomaly is apparent, because a baroclinic development relies on a positive vertical heat flux $\langle w'\theta'\rangle$ (e.g., Eady 1949). On the other hand, *positive* N^2 anomalies reduce the baroclinic conversion. Therefore, if a large area of horizontal potential temperature gradient is assumed to be interspersed with smaller areas of static stability anomalies, cyclone development might be focussed in areas of reduced static stability. This consideration is supported by numerical studies (Orlanski 1986) and observations (Reed and Duncan 1987). Orlanski (1986) concluded that preexisting areas of low static stability may be responsible for scale selection and positioning of a developing mesoscale storm. Reed and Duncan (1987) investigated the subsequent formation of four polar lows and showed that the disturbances formed in a shallow baroclinic zone of small static stability.

  The initial state of the primitive-equation model considered here is, however, geostrophic with constant potential vorticity in the troposphere and nearly constant N^2, so that term (II) in Eq. (13) is very small initially. But later in the nonlinear development of the baroclinic wave, it will become significant due to a developing tropopause fold.

  Term (III) is also absent in Lorenz's theory and is indeed small in the troposphere. However, near the tropopause and in the stratosphere term (III) could become significant because pressure gradients tend to increase with height due to the quasi-hydrostatic condition.

- The expression $C(\langle k_t\rangle, \langle e_t\rangle)$ is the reversible transformation between turbulent extended exergy and turbulent kinetic energy. The mean work done by the horizontal (ageostrophic) motion against the horizontal gradient of pressure deviation transforms turbulent extended exergy into kinetic energy (or vice versa).

- The expression $C(k_m, \langle k_t \rangle)$ is the reversible transformation between mean kinetic energy and turbulent kinetic energy. This effect is known as *barotropic conversion.*
- The expression $C(k_m, e_m)$ is the work of mean horizontal (ageostrophic) motion against the mean horizontal pressure gradient and transforms mean extended exergy into mean kinetic energy. This term is negligibly small in the idealized cyclone development under consideration.

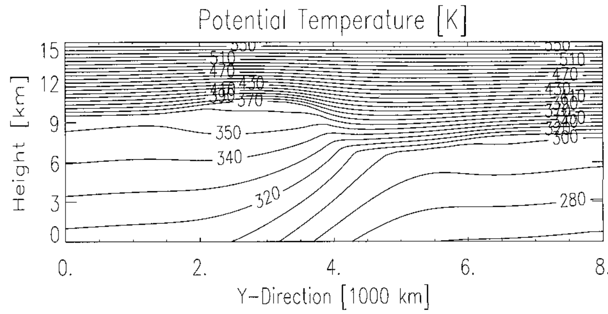

FIG. 2. The initial potential temperature distribution. Contour interval (CI) = 10 K—see text.

4. Application to a baroclinic wave life cycle

a. The model and the reference state

In this section the turbulent extended exergy concept will be applied to a primitive-equation model described in Rotunno et al. (1994) and Snyder et al. (1991) in order to show the consistency of the proposed theory and to examine in detail the insight provided by this concept. The model is dry and adiabatic and the model equations are posed in the Boussinesq framework, which means that a height-dependent hydrostatic reference state is subtracted from the thermodynamic variables in the equations of motion. Additionally, an incompressibility assumption is made in the continuity equation by using a constant density, and a constant characteristic potential temperature is used in the denominator of the bouyancy term in the vertical equation of motion. The boundary conditions are that the flow is periodic in the x direction, $v = 0$ at the meridional boundaries, and $w = 0$ at the horizontal bounding surfaces. The model has a troposphere and a stratosphere. The domain of integration is $y_L = 8000$ km, $x_L = 4000$ km, and $H = 15$ km. The Coriolis parameter $f = 1 \times 10^{-4}$ s^{-1} is taken to be constant. The x, y, and z directions are covered respectively by 40, 80, and 60 grid intervals.

The model shows the development of a cyclone with a typical scale of 4000 km, so that the classical quasigeostrophic APE theory (Lorenz 1955) should apply quite well. However, it will be shown that even in this dry case study there are some features that are not interpretable within the Lorenz framework.

The base-state potential temperature distribution of the model is shown in Fig. 2. The base state is designed to have constant potential vorticity in the troposphere that is joined smoothly to another constant, but larger potential vorticity in the stratosphere. There is a strong baroclinicity with maximal values in the middle latitudes. The overall temperature change between $y = 0$ and $y = 8000$ km is approximately 50 K.

Mean variables are defined through a zonal average. Since the model is periodic in the x direction this average fulfills the rules of averaging exactly.

As the model equations are cast in the Boussinesq framework, it is necessary to calculate the reference state, defined for our energetics analysis, using assumptions not directly available from the model fields. The reference state is calculated by integrating the quasi-hydrostatic equation using the potential temperature field of Fig. 2 and

$$\frac{\partial p}{\partial z} = -\frac{pg}{R\theta}\left(\frac{p}{p_0}\right)^{-R/c_p}, \tag{16}$$

where $p_0 = 1000$ hPa. The solution method is an iteration starting with a first guess pressure field and using the potential temperature field given in Fig. 2. The constant surface pressure of the reference state is taken as $p_s = 1013.25$ hPa. Then the implied density field obtained from $(p/R\theta)(p/p_0)^{-R/c_p}$ is averaged horizontally resulting in the reference density $\rho_R(z)$. The remaining reference-state variables $T_R(z)$, $\theta_R(z)$, $p_R(z)$ are found by integrating the hydrostatic equation

$$\frac{dp_R}{dz} = -\rho_R(z)g \tag{17}$$

and by using the perfect gas law. The so-defined reference state is held fixed in time during the development of the cyclone.

This reference state depending on height is used in the derivation of the Boussinesq equations. As has been computed here this reference state satisfies exactly a hydrostatic balance and the perfect gas law. However, additional approximations are often made in the resulting Boussinesq equations. Notably in the mass conservation equation, a constant density ρ_0 can be used to yield the continuity equation. Equally in the vertical momentum equation, or hydrostatic relation, a constant θ_0 can be used in the denominator of the bouyancy term. But note that this does not imply that the reference state has constant density and potential temperature because such a state would not be able to simultaneously satisfy the gas law and the hydrostatic equation.

The initial state of the baroclinic model consists of the base state described above with a small-amplitude perturbation added to it. The structure of the perturbation is the linearly most unstable mode.

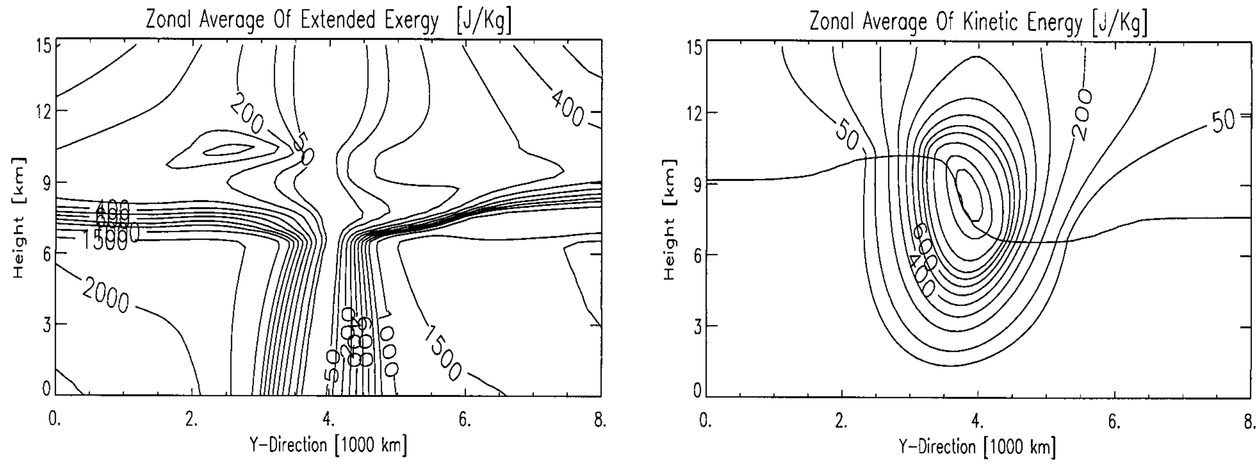

FIG. 3. The initial zonally averaged extended exergy $\langle e \rangle$ (left) and kinetic energy $\langle k \rangle$ (right) distributions. Also shown is the tropopause indicated by $N_m^2 = 3 \times 10^{-4}$ s^{-2} (right).

b. Total energies and their changes

The kinetic energy and extended exergy distributions, connected with the potential temperature distribution of Fig. 2 and with the reference state determined through Eq. (17), are presented in Fig. 3. All fields shown are zonally averaged. However, at initial time the zonal average of any field equals the mean fields, for example, Eq. (6) for extended exergy, because there are no perturbations from the the zonal average. There are large values of average kinetic energy (right panel) at the tropopause level above the strong baroclinic zone. At low levels the kinetic energy vanishes. Also shown is the tropopause [indicated by a mean static stability of $N_m^2 = 3 \times 10^{-4}$ s^{-2} or equivalently a potential vorticity of 1 PVU (=10^{-6} m^{-2} s^{-1} K kg^{-1})]. The tropopause is located approximately at a height of 10 km south of *y* distance 4000 km and decreases to a height of about 7.5 km north of *y* distance 4000 km. The average extended exergy [left panel, calculated using Eq. (2)] has been computed from the potential temperature distribution of Fig. 2 and the corresponding quasi-hydrostatic pressure distribution from Eq. (16). Due to the reduced static stability in the troposphere compared to the stratosphere, the pressure contribution [i.e., the term (II) in Eq. (2)] is in the troposphere locally very small compared to the potential temperature contribution. The extended exergy has generally large values in the troposphere and smaller values in the stratosphere. The reason is the change of the reference stability, N_R, from the troposphere to the stratosphere. However, extended exergy values of about 400 J kg^{-1} in high and low latitudes in the stratosphere are due to the pressure contribution [term (II) of Eq. (2)]. As will be shown later, these contributions are not available due to the model's incompressibe dynamics. In the troposphere, the averaged extended exergy has maxima at high and low latitudes, but in the region with the strongest temperature gradient, values of averaged extended exergy are nearly zero. The reason for this is that the reference state is independent of *y* (being a static equilibrium state), whereas the averaged state has a substantial north–south thermal gradient. Since the cyclone development is expected to be located just in the region where the average extended exergy is almost zero, there must be a transport of average extended exergy toward this region, because baroclinic conversion to turbulent extended exergy is an essential part of the cyclone development.

Here, only the extended exergy point of view of cyclone development will be discussed. As an example, we will look at a snapshot of the energetics after 162 h of development. This time has been chosen because the cyclone is already well developed, but turbulent kinetic energy values have not yet become unrealistically large as they do later in the simulation due to the lack of friction in the model equations. In Fig. 4 the surface pressure perturbation from the initial state and the surface potential temperature are shown. The minimum surface pressure perturbation of the cyclone exceeds 22 hPa in the center of the model.

Figure 5 shows the zonally averaged potential temperature distribution. The average baroclinicity has been reduced slightly at this time.

In Fig. 6 the change in the average extended exergy (left) and the average kinetic energy (right) over the 162-h period are shown. The average kinetic energy at the ground has now increased by about of 200 J kg^{-1} and decreased by about the same value, but with smaller area, at the tropopause level. The reason for the tripole structure in the average kinetic energy distribution in 7.5-km height near *y* distance 4000 km is that the maximum of mean kinetic energy is slightly shifted (due to transport) to the south and down (explaining the left dipole structure). The maximum on the right of this dipole is generated by transformation from turbulent extended exergy into turbulent kinetic energy (see Fig. 9, right panel). The average extended exergy has in-

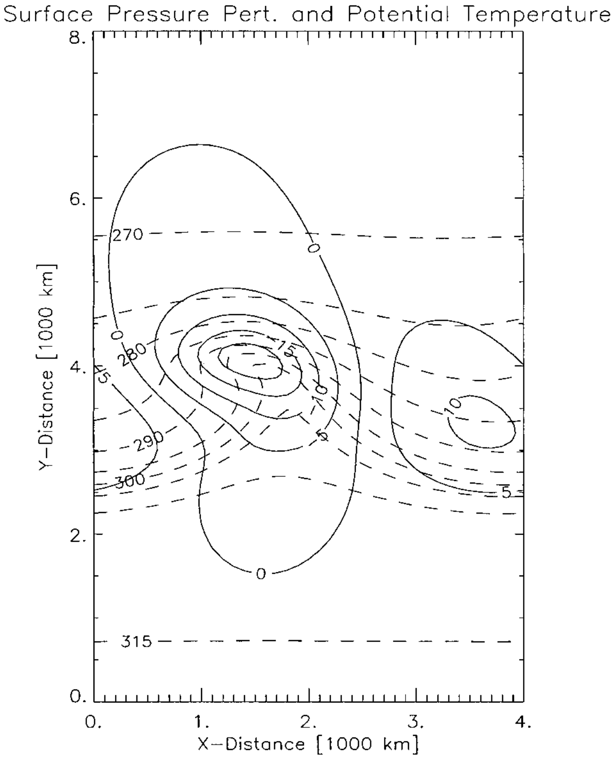

FIG. 4. The surface pressure perturbation form the initial state [solid contours, contour interval (CI) = 5 h Pa] and potential temperature (dashed contours every 5 K) after 162 h.

creased by about 150 J kg^{-1} in the baroclinic zone around y distance 4000 km. There are also areas with decreasing average extended exergy values whose maxima are located toward the equator (250 J kg^{-1}) at the bottom and poleward (250 J kg^{-1}) near the tropopause. This is an indication of the aforementioned transport of average extended exergy into the baroclinic zone with initially zero average extended exergy (see Fig. 3). The increased average extended exergy and average kinetic energy values near y distance 4000 km represent the developed cyclone. Average extended exergy changes

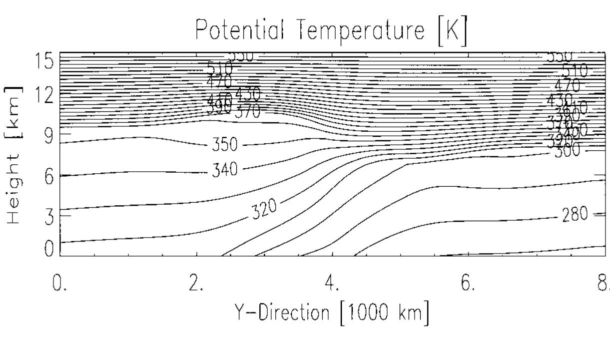

FIG. 5. The average potential temperature distribution after 162 h (CI = 10 K).

in the stratosphere are very small, which is consistent with the model's incompressible dynamics.

The negative volume-average change of average extended exergy should equal the change of average kinetic energy, because the model is (except for a small diffusion) reversible-adiabatic. After the 162-h integration the volume-average extended exergy has decreased by about 14 J kg^{-1} and the kinetic energy has increased by about 13 J kg^{-1}. Thus the overall balance requirement is satisfactorily fulfilled to better than 10%. The remaining error is due to the above mentioned diffusion and/or due to the Boussinesq approximation in the equations of motion.

c. Evolution of turbulent energies

In Fig. 7 the turbulent energies according to Eqs. (8) and (7) are shown. In the baroclinic zone the turbulent extended exergy (left) is now a maximum at the ground with nearly 180 J kg^{-1}, decreasing with height and vanishing at the tropopause level. Correspondingly, the turbulent kinetic energy (right) also has a maximum at the ground in the baroclinic zone (180 J kg^{-1}), but there is a second maximum slightly shifted to the north in the tropopause level (100 J kg^{-1}). The reason for the upper maximum of kinetic energy will be investigated by considering various rates of the energy evolution equations (9)–(12). The two possible mechanisms in question are (i) barotropic conversion from the mean kinetic energy, and (ii) transport of turbulent extended exergy generated by baroclinic conversion.

Figures 8–11 show various components of the energy evolution [equations (9)–(12)] after the 162-h development. The rates are presented in a mass-specific form (in J kg^{-1} h^{-1}). Since the model is incompressible, the values also represent the volume-specific counterparts (in J m^{-3} h^{-1} using a basic-state density of 1 kg m^{-3}). Figure 8, left panel, shows the baroclinic transformation rate [term (I) of Eq. (13)], which exceeds 30 J kg^{-1} h^{-1} at a height of 5 km at y distance 4000 km. This rate is the reason for the large increase in turbulent extended exergy shown in Fig. 7.

Term (II) of Eq. (13) is shown in Fig. 8, right panel. There is a large reduction (about 50%) of the baroclinic transformation rate because of the stabilization of temperature stratification due to a descent of the tropopause in a fold in the baroclinic zone (the descending tropopause is seen in Fig. 8, right panel, indicated by a mean static stability of $N_m^2 = 3 \times 10^{-4}$ s^{-2}, relative to the tropopause shown in Fig. 3, right panel). This effect does not seem to change the principle structure of baroclinic generation of turbulent extended exergy, but even in this idealized case study it needs to be taken into account because of balance requirements as shown below.

Term (III) of Eq. (13) (not shown) is negligible in this case.

Figure 9, left panel, represents the flux convergence

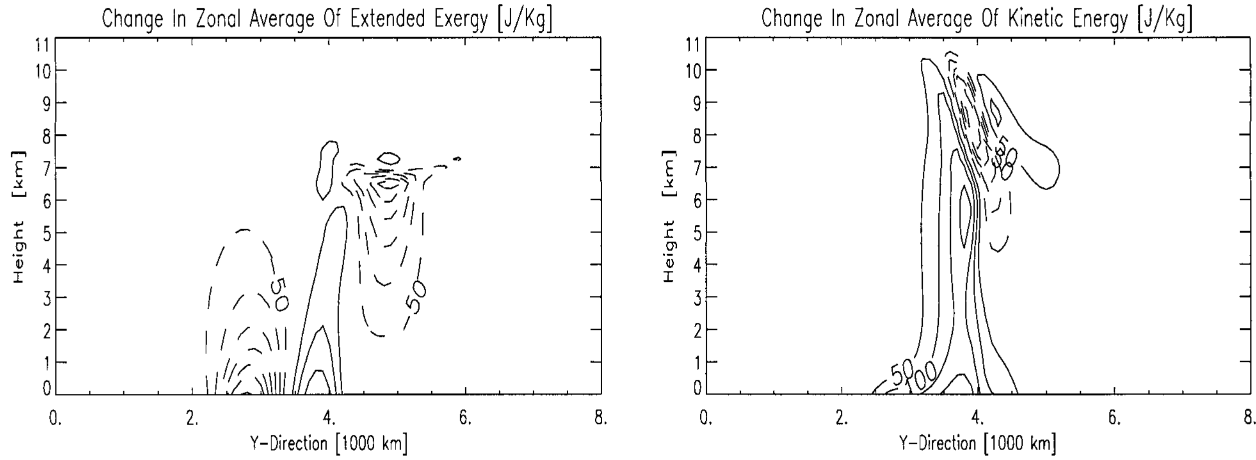

FIG. 6. The changes in average extended exergy $\langle e \rangle$ (left) and kinetic energy $\langle k \rangle$ (right) after 162 h compared to Fig. (2) (positive: solid contours; negative: dashed contours; CI = 50 J kg^{-1}).

of turbulent extended exergy [first term of Eq. (12); due mainly to pressure–velocity correlations $\langle p'\mathbf{v}' \rangle$]. Positive values indicate a local gain and negative values a local loss in turbulent extended exergy. The baroclinic generation shown in Fig. 8 is therefore transported upward to the tropopause and downward to the ground. There is also an indication of a small northward transport. In these areas the conversion into turbulent kinetic energy [term $C(\langle k_t \rangle, \langle e_t \rangle)$ in Eq. (11)] presented in Fig. 9, right panel, has its maximum values (16 J kg^{-1} per hour, respectively) and produces the bimodal structure of the turbulent kinetic energy distribution of Fig. 7.

Figure 10 shows the barotropic conversion rate between turbulent and mean kinetic energy $C(k_m, \langle k_t \rangle)$ [see Eq. (11)]. Negative values are centered at the bottom and at 7.5 km around y distance 4000 km and indicate a transformation from turbulent kinetic energy into mean kinetic energy. Maximum absolute values are 12 J kg^{-1} h^{-1} at the ground and 6 J kg^{-1} h^{-1} at the tropopause.

The flux convergence of turbulent kinetic energy (not shown) is near zero everywhere in the model domain.

Figure 11 shows the residuals (errors) in the turbulent extended exergy and turbulent kinetic energy equations, respectively [calculated by the rhs minus the lhs of Eqs. (11) and (12)]. Time rates of change of turbulent extended exergy and kinetic energy have been estimated by a centered difference at 162 h with an increment of 1 h. The maximal local errors rarely exceed 2 J kg^{-1} h^{-1} for turbulent extended exergy (at 6.5 km and the ground at y distance 4000 km) as well as for turbulent kinetic energy (at 5.5 km and the ground at y distance 4000 km). Thus the local relative errors compared to the locally largest transformation rates of the turbulent extended exergy and kinetic energy balance remain locally below 10% and 20% of the generation rates, respectively. This greater relative error in the turbulent kinetic energy equation compared to the turbulent extended exergy equation is due to the fact that the equation for turbulent kinetic energy contains the transfor-

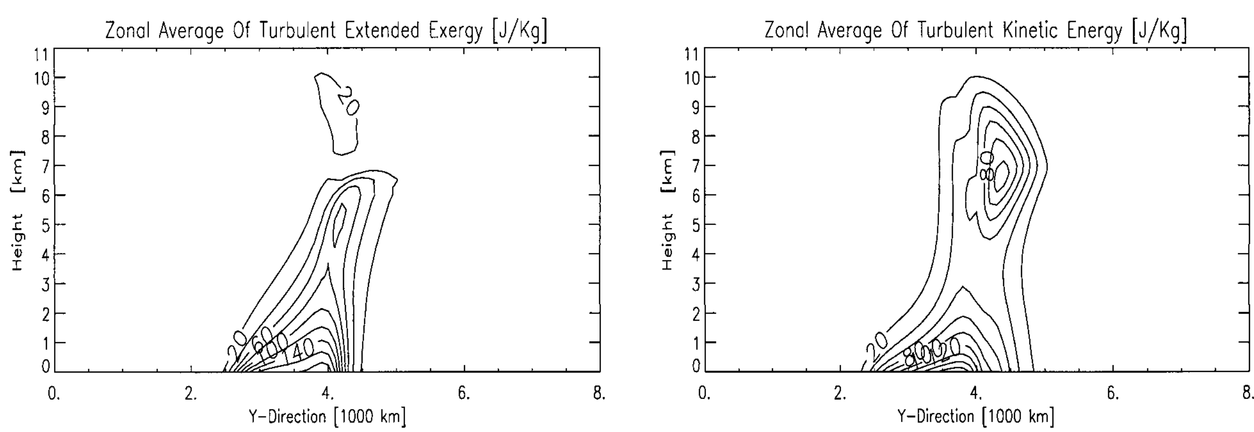

FIG. 7. The turbulent extended exergy $\langle e_t \rangle$ (left) and turbulent kinetic energy $\langle k_t \rangle$ (right) distributions after 162 h (CI = 20 J kg^{-1}).

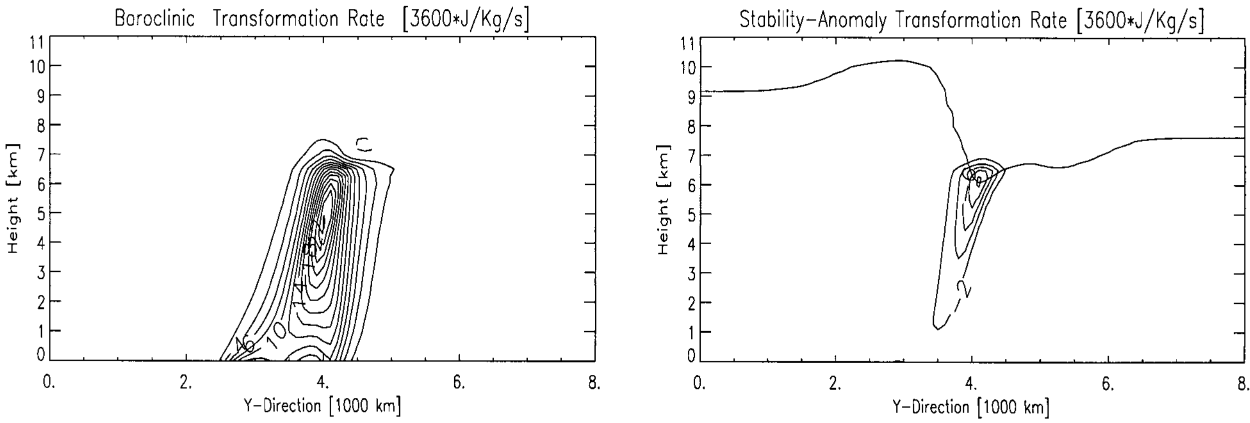

FIG. 8. The baroclinic transformation rate (left) and stability-anomaly transformation rate (right) after 162 h (CI = 2 J kg^{-1} h^{-1}). Also shown is the tropopause (descending in a fold at *y* distance 4000 km) indicated by $N_m^2 = 3 \times 10^{-4}$ s^{-2} (right).

mation rate (14) consisting of correlations between velocity perturbations and gradients of pressure perturbations. The computation of gradients of pressure perturbations is very sensitive to details of the numerical algorithm of its calculation, resulting in comparatively large errors. These errors do not enter the turbulent extended exergy balance (12), because the term $\nabla \cdot \langle p'\mathbf{v}' \rangle$ is calculated using the right-hand side of Eq. (14). However, the important result is that adding or subtracting the remaining errors from any of the shown transformation rates would not change their structure and thus their interpretation. Because of changes of the sign of the errors a volume integral would reduce these errors.

If the stability-anomaly contribution of the generation rate of extended exergy [term (II) of Eq. (13)] had been ignored, the local errors in its balance would have exceeded 50%. Therefore, even in this idealized case of dry cyclogenesis, the stability-anomaly contribution to the generation of extended exergy should be included in the analysis at least because of balance requirements.

5. The role of the static stability-anomaly energy conversion in the nonlinear equilibration

In this section, the role of the static stability-anomaly contribution to the baroclinic energy conversion in the nonlinear equilibration phase of the baroclinic wave life cycle will be discussed. Therefore we consider the time evolution of the volume-average conversion rates (over box: *x* distances 0–4000 km, *y* distances 2000–6000 km, *z* height 0–10 km) shown in Fig. 12. The time series starts after 150 h of development and focuses on the mature and equilibration phase of the cyclone's development. The notion *equilibration* means that there is no net energy gain for the baroclinic wave.

The baroclinic conversion [term (I) of Eq. (13)] is the major energy source, reaching its maximum (12 J kg^{-1} h^{-1}) after 192 h (=8 days) of development. After this time it starts to decay.

The static stability-anomaly contribution [term (II) of Eq. (13)] is initially small but inceases significantly from

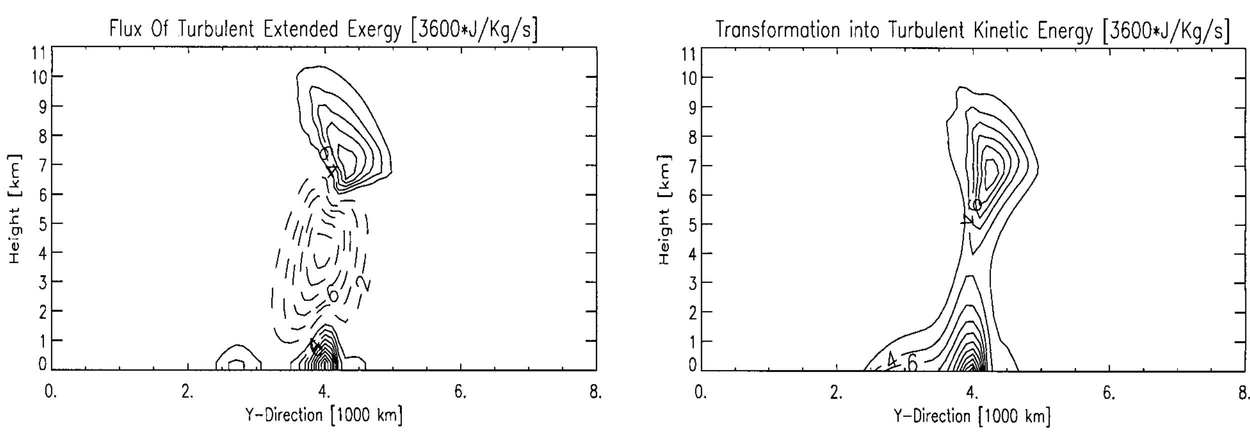

FIG. 9. The flux convergence of turbulent extended exergy (left) and the transformation rate into kinetic energy (right) after 162 h (positive: solid contours; negative: dashed contours; CI = 2 J kg^{-1} h^{-1}).

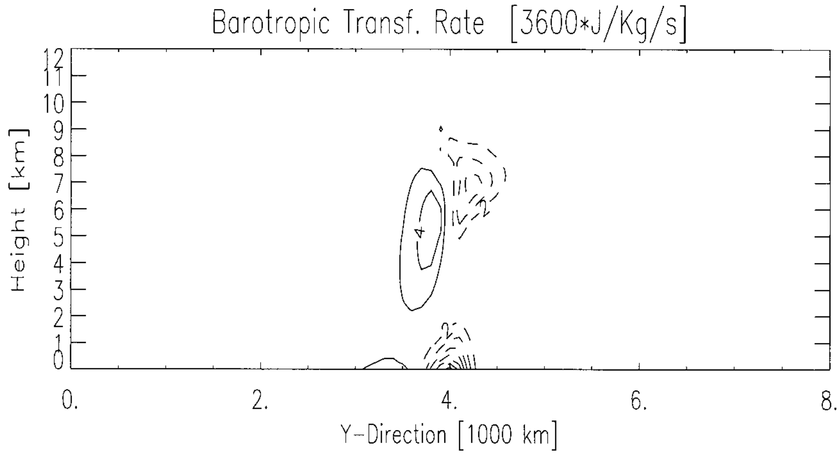

FIG. 10. The barotropic transformation rate after 162 h (positive: solid contours; negative: dashed contours; CI = 2 J kg^{-1} h^{-1}).

160 h of development on. Its maximum negative values are 4 J kg^{-1} h^{-1} at 210 h. As explained before this significant reduction of the net turbulent energy generation occurs because of a developing tropopause fold. But the N^2 anomaly would develop even without the existence of a tropopause in the model because of the positive vertical heat fluxes in the baroclinic wave.

This static stability-anomaly contribution can be interpreted as the manifestation in terms of the energetics of the nonlinear equilibration of baroclinic waves by the adjustment of vertical temperature profiles as discussed, for example, by Gutowski (1985).

Gutowski concluded that the mean-state Brunt–Väisälä frequency N_m can be modified by cyclone's (positive) vertical heat fluxes so that the flow is (approximately) baroclinically stable [by considering the Charney and Stern (1962) theorem]. Compared to the initial state, these vertical heat fluxes lead to positive anomalies in the static stability N_m^2. Therefore, the static stability-anomaly energy conversion shown in Fig. 12 can be interpreted as direct energetic response to the N_m modification baroclinic adjustment process proposed by Gutowski.

The barotropic conversion rate contributes only very little to the energy of the cyclone througout its early and mature stage of development, but from 210 h on it is reducing the energy of the cyclone and is thus serving as a second equilibration mechanism, which is well known from other studies (e.g., Simmons and Hoskins 1980).

After 225 h (=9¼ days) of development, the sum of the (negative) static stability-anomaly contribution and the barotropic conversion compensates for the (positive) baroclinic energy conversion, and there is no net energy gain of the cyclone from this time on.

6. Conclusions

The cyclone development under consideration has been shown to rely on a classical form of baroclinic energy conversion. Barotropic conversion transforms, on average, turbulent kinetic energy into mean kinetic

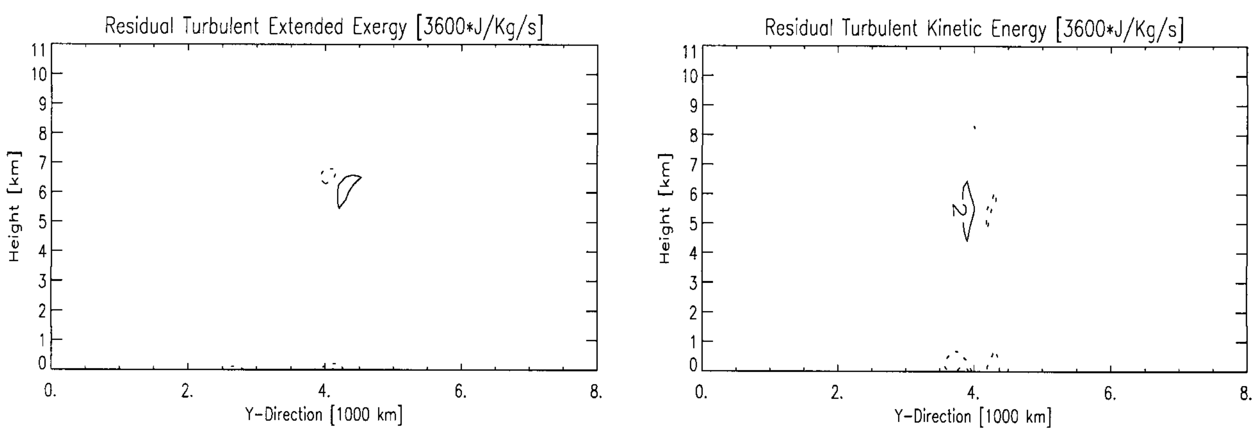

FIG. 11. The residual of the turbulent extended exergy (left) and kinetic energy (right) balances after 162 h (positive: solid contours; negative: dashed contours, CI = 2 J kg^{-1} h^{-1}).

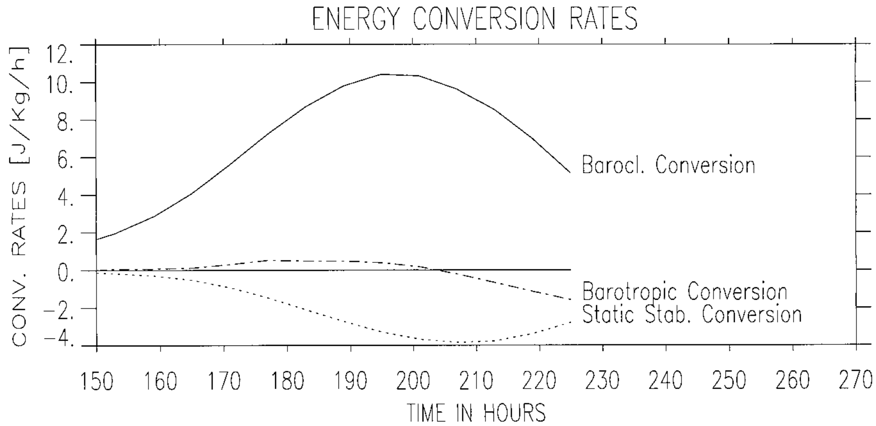

FIG. 12. Time series of the production/destruction rates of volume-average turbulent energies, starting at 150 h of development.

energy, thus reducing the energy of the cyclone. However, this is the first application of a more local available energy concept to an idealized cyclone development without making drastic simplifications to the energy equations (e.g., without the quasigeostropic assumption). It has been demonstrated that the baroclinic conversion rate has an important additional term (here called the stability-anomaly effect) compared to Lorenz-type approaches. In the dry model simulation the stability-anomaly effect is significant due to a developing tropopause fold. The effect produced a significant reduction of the baroclinic conversion and contributed significantly to the nonlinear equilibration of the baroclinic wave.

In more realistic models that include, for example, latent heat release and sensible heating, the stability-anomaly effect can be supposed to be of even greater significance. If stability anomalies were to be present before a cyclone develops, they may be responsible for the scale selection and positioning of the developing cyclone. Latent heat release in a mature cyclone certainly produces areas of pronounced stability anomalies. Since those anomalies are often persistent over several days, they could trigger, due to the stability-anomaly effect, future cyclonic developments.

This study has been performed to produced an evaluation of a dry and reversible-adiabatic reference extended energy. In such a simulation all terms in the energy evolution equations can be evaluated separately, producing a solid basis for future studies of more realistic cyclone developments in which more complex physical processes (e.g., cloud microphysics, boundary layer physics, convection) are included.

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APPENDIX

Mean and Turbulent Extended Exergy and Kinetic Energy: The General Case

The full and exact (but lengthy) derivation of the turbulent extended exergy (including the evolution equations of the following section) is performed using the exact extended exergy formulation (17) of Kucharski (1997) by applying to it the thermodynamic turbulence concept proposed by Herbert and Kucharski (1998). Here a more practical approach is presented using the extended exergy approximation (2). This means that higher-order terms in perturbations of thermodynamic variables are generally ignored, because for an arbitrary thermodynamic variable χ we have $\Delta\chi, \chi' \ll \chi$, whereas all higher-order terms involving velocity perturbations are retained. In contrast to the equations of section 3, this approach includes compressibility and diabatic and frictional effects.

In principle, all averages applied to a variable ψ, denoted as $\langle\psi\rangle$, have to be understood as density-weighted:

$$\langle\psi\rangle := \frac{\overline{\rho\psi}}{\overline{\rho}}, \qquad \text{(A1)}$$

with $\psi = \mathbf{v} = (u, v, w)$ (three-dimensional velocity vector), θ, p; and where overbar means a zonal average. The averaging of the compressible thermo-hydrodynamic equations is considerably simplified using (A1) instead of an unweighted average (e.g., Van Mieghem 1973).

Since in this study a zonal averaging is applied and, in general, the atmospheric density mainly varies in the vertical direction, $\rho \approx \rho_R(z)$, the weighted average defined in Eq. (A1) is essentially the same as the unweighted (Reynolds) average, for example, $\langle\psi\rangle \approx \overline{\psi}$.

For the deviations from the averaged variables,

$$\overline{\rho}\langle\psi''\rangle = \overline{\rho\psi''} = 0 \quad \text{with } \psi'' := \psi - \langle\psi\rangle \qquad \text{(A2)}$$

is valid.

The energies of the mean state, $\chi_m = \{e_m, k_m\}$, have to be calculated by means of the averaged variables,

$$\chi_m = \chi_m(\langle\psi\rangle), \tag{A3}$$

with $\psi = \mathbf{v}, \theta, p$. This point of view shall be highlighted by consideration of the kinetic energy k. As $k = 1/2\mathbf{v}^2$ Eq. (5) becomes

$$k_m = \frac{1}{2}\langle\mathbf{v}\rangle^2,$$

whereas for the averaged kinetic energy according to Eq. (4),

$$\langle k\rangle = \frac{1}{2}\langle\mathbf{v}\rangle^2 + \frac{1}{2}\langle\mathbf{v}''^2\rangle \tag{A4}$$

holds. It follows that there exists a positive additional term in the averaged kinetic energy $\langle k\rangle$ (which turns out to be the turbulent kinetic energy) compared to the mean kinetic energy k_m.

For the deviations of the energies $\chi = e, k$,

$$\chi := \chi_m + \chi_{\rm pert}, \tag{A5}$$

the so-called perturbation variables $\chi_{\rm pert}$, are defined. Using Eqs. (A1), (A5), and (4), the weighted average of the perturbation variables is calculated:

$$\overline{\rho\chi_{\rm pert}} = \overline{\rho\chi_t} = \overline{\rho}(\langle\chi\rangle - \chi_m) \quad \text{or} \quad \langle\chi_t\rangle = \langle\chi\rangle - \chi_m, \tag{A6}$$

where the *turbulent* variables χ_t have been introduced. Here, χ_t are defined to contain only second-order terms of the fluctuations $\mathbf{v}''$, θ'', p''.

Equation (5) applied to the extended exergy (2) gives the mean extended exergy

$$e_m \approx \frac{1}{2}\left\{\frac{g^2}{N_R^2}\left(\frac{\Delta\langle\theta\rangle}{\theta_R}\right)^2 + RT_R\frac{c_v}{c_p}\left(\frac{\Delta\langle p\rangle}{p_R}\right)^2\right\}. \tag{A7}$$

For the turbulent extended exergy the approximate expression is

$$\langle e_t\rangle \approx \frac{1}{2}\left\langle\frac{g^2}{N_R^2}\left(\frac{\theta''}{\theta_R}\right)^2 + RT_R\frac{c_v}{c_p}\left(\frac{p''}{p_R}\right)^2\right\rangle. \tag{A8}$$

The turbulent kinetic energy is given by

$$\langle k_t\rangle = \frac{1}{2}\langle\mathbf{v}''^2\rangle. \tag{A9}$$

Mean and turbulent energy evolution equations: The general case

The approximations of the turbulent kinetic and extended exergy evolution equations given in Kucharski (1997) are

$$\frac{\partial}{\partial t}(\overline{\rho}k_m) = -\nabla\cdot[\overline{\rho}\langle\mathbf{v}\rangle k_m + \langle\mathbf{v}\rangle\cdot\overline{\rho\mathbf{v}''\mathbf{v}''} + \overline{\mathbf{F}}\cdot\langle\mathbf{v}\rangle] \tag{A10}$$

$$- C(\overline{\rho}k_m, \overline{\rho k_t}) - C(\overline{\rho}k_m, \overline{\rho}e_m) - D(\overline{\rho}k_m) \tag{A11}$$

$$\frac{\partial}{\partial t}(\overline{\rho}e_m) = -\nabla\cdot\left[\overline{\rho}\langle\mathbf{v}\rangle e_m + \langle T\rangle\eta_m\frac{c_p}{\langle\theta\rangle}\overline{\rho\mathbf{v}''\theta''} + (\overline{p} - p_R)\langle\mathbf{v}\rangle\right] - C(\overline{\rho}e_m, \overline{\rho e_t}) + C(\overline{\rho}k_m, \overline{\rho}e_m) + G(\overline{\rho}e_m) \tag{A12}$$

$$\frac{\partial}{\partial t}(\overline{\rho k_t}) = -\nabla\cdot[\overline{\rho\mathbf{v}k_t} + \overline{\mathbf{F}\cdot\mathbf{v}''}] + C(\overline{\rho}k_m, \overline{\rho k_t}) + C(\overline{\rho k_t}, \overline{\rho e_t}) - D(\overline{\rho k_t}) \tag{A13}$$

$$\frac{\partial}{\partial t}(\overline{\rho e_t}) = -\nabla\cdot[\overline{\rho\mathbf{v}e_t} + \overline{p'\mathbf{v}''}] + C(\overline{\rho}e_m, \overline{\rho e_t}) - C(\overline{\rho k_t}, \overline{\rho e_t}) + G(\overline{\rho e_t}), \tag{A14}$$

with

$$C(\overline{\rho}e_m, \overline{\rho e_t}) \approx \underbrace{-\overline{\rho\mathbf{v}_H''\theta''}\cdot\frac{g^2}{\langle\theta\rangle^2}\frac{1}{N_R^2}\nabla_H\langle\theta\rangle}_{\rm I} - \underbrace{\overline{\rho w''\theta''}\frac{g}{\langle\theta\rangle}\left(\frac{N_m^2 - N_R^2}{N_R^2}\right)}_{\rm II} - \underbrace{\overline{\rho\mathbf{v}_H''\theta''}\cdot\frac{1}{\langle\theta\rangle\rho_R}\nabla_H\overline{p}}_{\rm III} \tag{A15}$$

$$C(\overline{\rho k_t}, \overline{\rho e_t}) = -\overline{\mathbf{v}''\cdot\nabla p} \approx -\overline{\mathbf{v}_H''\cdot\nabla_H p'} \approx -\nabla\cdot(\overline{p'\mathbf{v}''}) + \frac{g}{\langle\theta\rangle}\overline{\rho}\,\overline{w''\theta''} + \frac{N_m^2}{g}\overline{w''p'}$$

$$C(\overline{\rho}k_m, \overline{\rho k_t}) := -(\overline{\rho\mathbf{v}''\mathbf{v}''}\cdot\nabla)\cdot\langle\mathbf{v}\rangle := -(\overline{\rho\mathbf{v}''\mathbf{v}''}:\nabla\langle\mathbf{v}\rangle) \qquad C(\overline{\rho}k_m, \overline{\rho}e_m) := \langle\mathbf{v}\rangle\cdot\nabla\overline{p} + \overline{\rho}\langle\mathbf{v}\rangle\cdot\nabla\phi \approx \langle\mathbf{v}_H\rangle\cdot\nabla_H\overline{p}$$

$$D(\overline{\rho k_t}) := -\overline{\mathbf{F}:\nabla\mathbf{v}''} \qquad D(\overline{\rho}k_m) := -\overline{\mathbf{F}}:\nabla\langle\mathbf{v}\rangle \qquad G(\overline{\rho e_t}) \approx \overline{\left[\frac{g^2}{c_p}\frac{1}{N_R^2}\frac{\theta''}{\theta_R} + T_R\frac{R}{c_p}\frac{p'}{p_R}\right]\frac{\rho q}{\langle T\rangle}}$$

$$G(\overline{\rho}e_m) \approx \overline{\eta_m\rho q} \qquad \eta_m := \left\{\frac{g^2}{c_p}\frac{1}{N_R^2}\frac{\langle\theta\rangle - \theta_R}{\theta_R} + T_R\frac{R}{c_p}\frac{\overline{p} - p_R}{p_R}\right\}\frac{1}{\langle T\rangle}, \tag{A16}$$

where the subscript H means horizontal vector components and $p' := p - \overline{p}$. As in the main text, the notation $\mathbf{A}:\mathbf{B}$ (double scalar product) means that the tensors **A**, **B** are multiplied tensorially, and from the resulting tensor the trace has to be calculated to give the scalar $\mathbf{A}:\mathbf{B}$.

The mean Brunt–Väisälä frequency is defined as

$$N_m^2 := \frac{g}{\langle\theta\rangle}\frac{\partial\langle\theta\rangle}{\partial z}. \tag{A17}$$

The rate of heating, ρq, is

$$\rho q = -\nabla\cdot\mathbf{J}_q + J - \mathbf{F}:\nabla\mathbf{v},$$

where $\mathbf{J}_q$ is the (molecular) heat flux, J the heat supply by radiation and latent heat release, and $\mathbf{F}$ the (molecular) stress tensor.

The meaning of all reversible effects has already been discussed in section 3 of the main text. These terms occur here in the fully compressible representation, but their interpretation remains the same. Thus, we only discuss the remaining terms resulting from diabatic and irreversible processes.

- The term $D(\overline{\rho k_t}) = -\overline{\mathbf{F}:\nabla\mathbf{v}''} > 0$ is the molecular dissipation of turbulent kinetic energy.
- The term $D(\overline{\rho}k_m) = -\overline{\mathbf{F}}:\nabla\langle\mathbf{v}\rangle > 0$ is the molecular dissipation of mean kinetic energy.
- The term $G(\overline{\rho e_t})$ is the generation rate of turbulent extended exergy. The *local* deviations from the mean state in connection with the local rate of heating become relevant in producing/destroying turbulent extended exergy. For example, latent heat release in a developing cyclone is one effect characterized by this term creating additional turbulent extended exergy that could be transformed into turbulent kinetic energy. The term including the pressure deviation might be small compared to the term including the potential temperature deviation in many cases. However, in regions with deep temperature disturbances, the pressure deviations tend to increase with height due to the quasi-hydrostatic balance and may thus become relevant near the tropopause. The factor

$$\eta_t = \left[\frac{g^2}{c_p}\frac{1}{N_R^2}\frac{\theta''}{\theta_R} + T_R\frac{R}{c_p}\frac{p'}{p_R}\right]\frac{1}{\langle T\rangle}$$

  can be interpreted as a local *turbulent* Carnot factor steering the efficiency of turbulent extended exergy production given a rate of heating ρq. Depending on the rate of heating, this term might be quite efficient in producing turbulent extended exergy.
- The term $G(\overline{\rho}e_m)$ is the generation of mean extended exergy e_m. Since the first term is an order of magnitude larger than the second term, we recover the result that heating at the equator and cooling at the Pole produce mean extended exergy. The factor

$$\eta_m = \left[\frac{g^2}{c_p}\frac{1}{N_R^2}\frac{\langle\theta\rangle - \theta_R}{\theta_R} + T_R\frac{R}{c_p}\frac{\overline{p} - p_R}{p_R}\right]\frac{1}{\langle T\rangle}$$

  might be interpreted as a *mean* Carnot factor.

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