NOTES AND CORRESPONDENCE

Frictional Dissipation in a Precipitating Atmosphere

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ABSTRACT

The frictional dissipation in the shear zone surrounding falling hydrometeors is estimated to be 2–4 W m^{-2} in the Tropics. A numerical model of radiative–convective equilibrium with resolved three-dimensional moist convection confirms this estimate and shows that the precipitation-related dissipation is much larger than the dissipation associated with the turbulent energy cascade from the convective scale. Equivalently, the work performed by moist convection is used primarily to lift water rather than generate kinetic energy of the convective airflow. This fact complicates attempts to use the entropy budget to derive convective velocity scales.

1. Introduction

Kinetic energy is generated in the atmosphere from the potential and internal energy made available through differential heating and is dissipated through friction. It is usually assumed that this frictional dissipation occurs as an end product of a turbulent cascade to the small scales at which molecular viscosity can act, as in other turbulent flows. This cascade is presumed to occur either in the turbulent boundary layer near the surface or in turbulent patches within the free atmosphere. We argue here that in addition to this familiar process, a substantial fraction of the dissipation in the atmosphere actually occurs in the immediate vicinity of falling hydrometeors (raindrops and ice particles), and that precipitation is the main cause of frictional dissipation in the tropical atmosphere.

Frictional dissipation is important not only for the kinetic energy budget of the atmospheric circulation but also for its entropy budget. Differential heating results in a net entropy sink for the atmosphere that must be balanced, in a steady state, by entropy production due to irreversible processes within the atmosphere. To the extent that the frictional heating associated with the dissipation of kinetic energy is the dominant irreversible process, its magnitude is constrained by the entropy budget and can therefore be estimated directly from the large-scale heat sources and sinks. In the context of tropical convection, this constraint has recently been emphasized by Rennó and Ingersoll (1996) and Emanuel and Bister (1996).

Consider the horizontally homogeneous model of the tropical atmosphere pictured in Fig. 1, thinking first of the case of a dry atmosphere heated exclusively by sensible heat exchange at the surface. The corresponding entropy flux is given by the heat flux Q divided by the surface temperature T_s. Heat is transferred upward by atmospheric motions and is then lost through radiative cooling, which must balance the heating in equilibrium. Let T_c be the average temperature at which the cooling occurs, which, given typical infrared absorber distributions in the Tropics, is a midtropospheric temperature. If kinetic energy is produced and dissipated at a rate $W > 0$, then the entropy balance reads

$$0 = Q\left(\frac{1}{T_s} - \frac{1}{T_c}\right) + \frac{W}{T_d} + \Delta S_{\rm irr}, \qquad (1)$$

where T_d is the average temperature at which the frictional heating occurs, and $\Delta S_{\rm irr}$ is the entropy production due to the other irreversible processes. As irreversible entropy sources are always positive, the frictional heating W must always be smaller than $W_{\max} = Q(T_s -$

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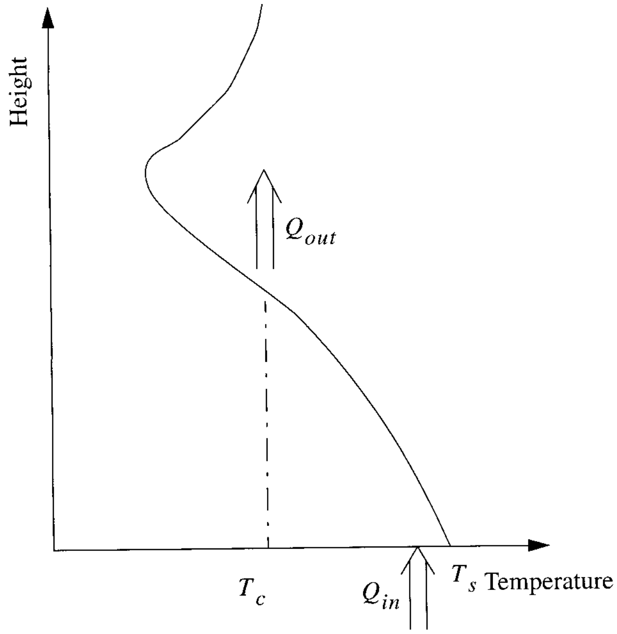

FIG. 1. Schematic representation of the atmospheric temperature profile and heat budget. The surface heat flux Q_{in} is transported by the atmospheric circulation into the troposphere where it is balanced by radiative cooling Q_{out}.

$T_c)/T_0$, where we assume for simplicity that all temperatures differ by relatively small amounts from the mean atmospheric temperature T_0. This maximum value for the production and dissipation of kinetic energy can only be realized in the absence of other entropy sources.

The appropriate treatment of the entropy budget of a moist atmosphere (Emanuel 1994) leads to exactly the same picture, in which the entropy source due to the surface heat source is equal to the total heat flux divided by the surface temperature. The only difference is that the surface heat source now includes not only the sensible heat flux but also the latent heat flux due to evaporation from the surface, the latter being the dominant term in the Tropics by nearly an order of magnitude. As long as we reinterpret Q as this total energy source at the surface, the same upper bound to the energy generation and dissipation is still in force. Irreversible processes in moist convection include—in addition to frictional heating—diffusion of heat, diffusion of water vapor, and irreversible phase changes. Typical values averaged over the Tropics are $Q \approx 100$ W m^{-2} and $(T_s - T_c)/T_0 \approx 0.1$, so that $W_{max} \approx 10$ W m^{-2}.

This constraint on frictional dissipation is of potential importance for our understanding of tropical convection. Emanuel and Bister (1996) and Rennó and Ingersoll (1996) attempt to use this entropy budget to determine the convective available potential energy over the tropical oceans. These works rely on two simplifying assumptions: 1) other entropy sources besides frictional dissipation, such as diffusion of water vapor and of heat, are negligible in the entropy balance, and 2) the frictional dissipation itself can be thought of as due to a turbulent energy cascade from the dominant energy-containing eddies of the convective turbulence to smaller scales. Given these simplifications, one can try to estimate properties of the convective eddies from knowledge of the strength of the heat source and the temperature difference between the surface and the average level at which the atmosphere is cooled radiatively. The validity of both of these assumptions can be questioned. We focus on the second assumption here, and on the distinctive manner in which dissipation of kinetic energy can occur in a precipitating atmosphere due to the sheared flow around individual raindrops and ice particles.

2. Frictional dissipation by precipitation

From a macroscopic perspective, estimating the dissipation due to falling drops is straightforward. Consider an air parcel moving with a velocity **V** whose dimensions are much larger than the individual drops. Assume also that the drops are of a fixed mass and move at a velocity $\mathbf{V} + \mathbf{V}_T$. Friction and pressure variations at the drop surface result in a drag force **F** on the drops and in an equal, but opposite, force exerted on the air parcel. This interaction performs mechanical work at a rate $\mathbf{F} \cdot (\mathbf{V} + \mathbf{V}_T)$ on the drops and $-\mathbf{F} \cdot \mathbf{V}$ on the air parcel. One can thus think of the drag as having two effects on the energetics of the system. First, mechanical energy is transferred from the falling drop to the air parcel at a rate $\mathbf{F} \cdot \mathbf{V}$. This is the work associated with the acceleration of downdrafts or the deceleration of updrafts by precipitation. Second, as the drag force acts in the opposite direction to the relative velocity of the droplets, there is also a net loss of mechanical energy, given by $\mathbf{F} \cdot \mathbf{V}_T$. Conservation of energy requires this loss to be balanced by an equivalent frictional heating.

From a microscopic perspective, there is a shear zone around each drop in which the airflow makes a transition from the velocity of the drop to the velocity of the larger-scale motion. The dissipation occurs due to the action of molecular viscosity on these shears. One can demonstrate this explicitly by integrating the equations for conservation of momentum and kinetic energy over a region containing a drop and that is large enough that the flow on the boundaries of this region asymptotes to the larger-scale air velocity.

It is a very good approximation to assume that hydrometeors in the atmosphere have reached their terminal velocity and are not accelerating. Therefore, the drag balances the gravitational force $f = -\rho_c g$, where ρ_c is the mass of the condensate per unit volume of air, and f is the drag force per unit volume. As mentioned in Emanuel and Bister (1996), the dissipation of kinetic energy associated with the flow in the vicinity of the hydrometeors is thus equal to the integral over the whole atmosphere (denoted by $\int_\Omega$),

$$W_p = \int_\Omega g\rho_c v_T, \qquad (2)$$

where $\mathbf{V}_T = -v_T\mathbf{k}$.

In statistical equilibrium, conservation of water implies that the total water flux F_w is zero at every level. The vertical water flux at any level $z = z_0$ can be decomposed into a condensate flux $F_c = \int_{z=z_0} \rho_c(w - v_T)$ and a water vapor flux $F_v = \int_{z=z_0} \rho_v w$, where ρ_v is the water vapor density and w is the vertical velocity of air. The statistical equilibrium condition $F_c + F_v = F_w = 0$ can thus be written:

$$\int_{z=z_0} \rho_c v_T = \int_{z=z_0} (\rho_c + \rho_v)w, \qquad (3)$$

which states that the downward flux of water associated with the relative velocity of condensed water is balanced by the uplift of water in all phases by the air motion. Hence the frictional dissipation W_p is equal to the potential energy imparted to the water by the atmospheric circulation

$$W_p = \int_\Omega g\rho_t w, \qquad (4)$$

where $\rho_t = \rho_c + \rho_v$.

The dissipation by precipitation can be thought as proceeding in two steps. First, water is lifted by the atmospheric circulation, increasing its potential energy. Then, during precipitation, the potential energy of condensed water is transferred to the ambient air where it is dissipated by molecular viscosity in the microscopic shear zone around the hydrometeors.[1]

To obtain an estimate of the total dissipation rate due to precipitation W_p, we can first assume it to be proportional to the precipitation rate P at the surface, which is given by the surface integral $\int_{z=0} \rho_c v_T$. The precipitation path-length H_f is defined so that the precipitation-induced dissipation is $W_p = gPH_f$. The problem of estimating W_p is thus equivalent to that of estimating H_f.

As a first approximation, one can assume that water falls immediately after condensation, without reevaporating. Then the precipitation path-length is simply the average height at which condensation occurs, h_c. For a parcel ascent, this level is given by the integral

$$h_c = -\int_0^\infty z\, dq_p = \int_0^\infty q_p\, dz, \qquad (5)$$

where $q_p(z)$ is the water vapor mixing ratio of the parcel. For an undilute saturated ascent, $q_p(z) \approx q_0 \exp(-z/h_s)$, where h_s is the e-folding height of the saturation mixing ratio, which is between 2.5 and 3 km in the Tropics. From (5), we deduce that the mean condensation altitude equals this e-folding height $h_c = h_s$, which is the estimate used in Emanuel and Bister (1996). However, the condensation level h_c increases if one takes into account the undersaturation of the subcloud layer and the entrainment of unsaturated air into the rising air parcel.

In addition, raindrops evaporate when falling through unsaturated air. Studies by Fankhauser (1988) and Ferrier et al. (1996) indicate that between one-half to two-thirds of the precipitation reevaporates before reaching the surface. If one also assumes that reevaporation occurs uniformly along the drop trajectory, this gives $H_f \approx (1.5\text{–}2)h_c$. Also, hydrometeors are lifted by convective towers after condensation, so that they fall through a larger distance before hitting the surface. Leary and Houze (1980) observe that, in convective systems in the Tropics, a significant portion of the hydrometeors is lifted into the upper troposphere after condensation within the updrafts and exported in the anvil cloud. Because of these two mechanisms, reevaporation and uplift of condensed water by the convective updrafts, the effective fallout level H_f is significantly higher than the mean condensation altitude h_c. Based on this discussion, and in the absence of direct measurement, an appropriate estimate might be $H_f \approx$ 5–10 km in the Tropics.

The ratio of the hydrometeor-related dissipative heating to the latent heat release $L_v P$, where L_v is the latent heat of vaporization, is simply

$$W_p/Q \approx gH_f/L_v = (4 \times 10^{-6})H_f. \qquad (6)$$

For $H_f \approx$ 5–10 km, this ratio is 0.02–0.04. Given the observed latent heat release averaged over the Tropics of roughly 100 W m^{-2}, the dissipation of mechanical energy in the vicinity of hydrometeors is approximately $\approx$2–4 W m^{-2}. While this is small compared to the latent heating, it is surprisingly large when compared to other estimates of kinetic energy dissipation in the atmosphere. The globally averaged rate of conversion of potential energy to kinetic energy on the large scales ($>$500 km) that are observable with the standard upper-air meteorological network and resolved in numerical models of the atmosphere, and the loss of this energy through transfer to smaller scales, mostly in the planetary boundary layer, is estimated to be only $\approx$2 W m^{-2} (Peixoto and Oort 1992). Furthermore, the bulk of these energy transfers occurs in midlatitudes where kinetic energy levels are higher than in the Tropics.

The entropy constraint (1) can be written:

$$W_{\max} = W_p + W_D + I, \qquad (7)$$

where W_D is the frictional dissipation associated with convective and boundary-layer turbulence, and $I \sim T_0 \Delta S_{\text{irr}}$ is the loss of possible mechanical work that results from the other, nonfrictional, irreversible processes in the system. Even though our estimate $W_p \approx$ 2–4 W

[1] For completeness' sake, it should be mentioned that some dissipation also occurs when precipitation hits the ground. However, the amount of energy dissipated, namely, the kinetic energy of the drops, is negligible in comparison to the dissipation occurring during the droplets' fall within the atmosphere.

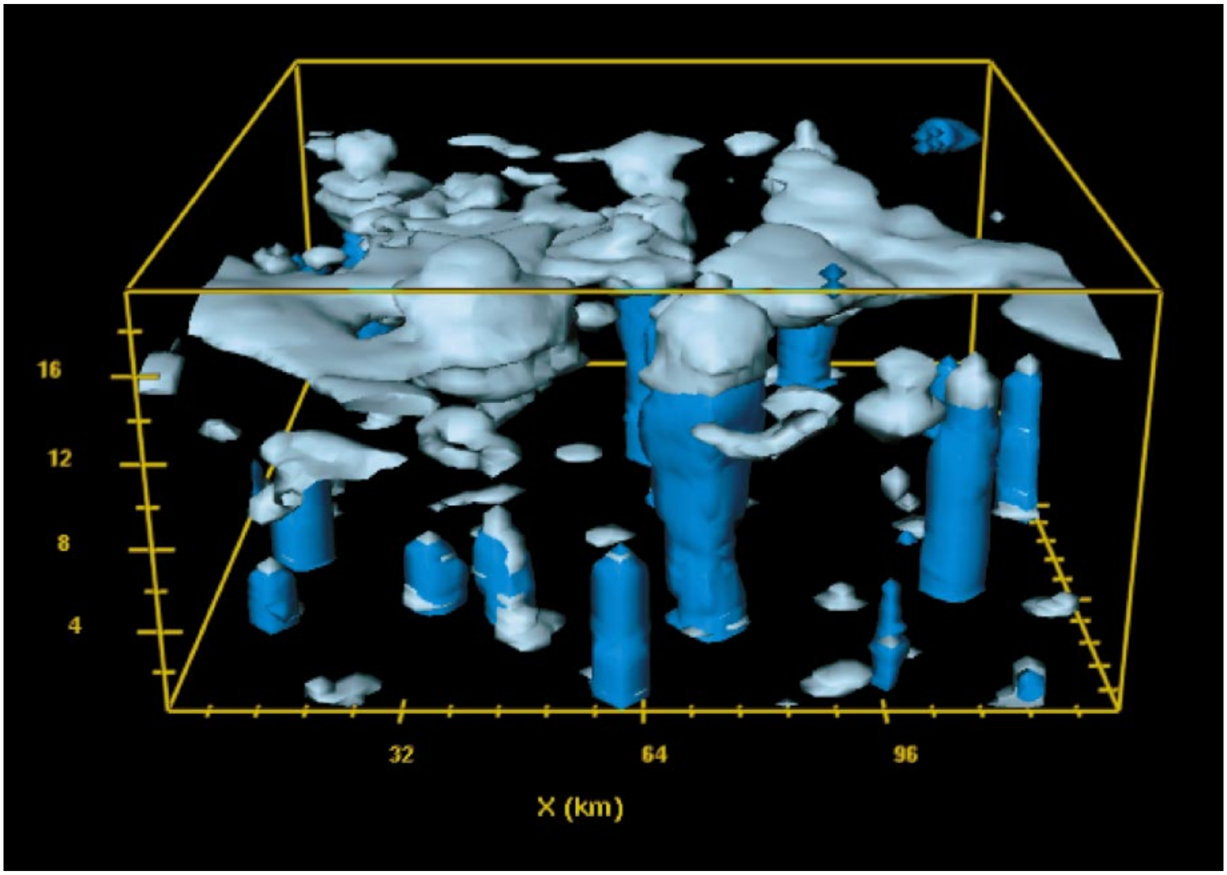

FIG. 2. Snapshot from the nonhydrostatic radiative–convective simulation showing cloud water (white) and falling precipitation (blue). Only the lower 20 km of the model atmosphere is shown.

m^{-2} is large compared to observed dissipation rate, it is still smaller than the maximum work $W_{\max} \approx 10$ W m^{-2}. On this basis, we cannot conclude that $W_D \ll W_p$. If the irreversible entropy source I is small and if this estimate of W_p is accurate, then it may still be that turbulent dissipation is larger than W_p. To investigate this we turn to numerical model of moist convection.

3. Numerical results

Numerical simulations should provide useful estimates of H_f and of the different components of the kinetic energy budget of moist convection. Studies of stochastically steady turbulent moist convection in both two and three dimensions are becoming more common as computational resources increase (e.g., Tao et al. 1987; Held et al. 1993; Sui et al. 1994; Randall et al. 1996; Tompkins and Craig 1998). In studies of deep tropical convection, these models typically have a horizontal resolution of 1–2 km. The models carry not only water vapor, but also cloud water (water drops and ice particles too small to fall) and falling hydrometeors as explicit variables. We are using a version of such a model, developed originally by Lipps and Hemler (1982), to study the radiative–convective equilibrium of the Tropics. In a horizontally periodic domain large enough to contain many convective cores, surface temperatures are prescribed, and radiative cooling destabilizes the atmosphere until convective motions develop to maintain a statistically steady state. The radiative cooling, in turn, is controlled by the predicted water vapor and cloud fields. The behavior of a two-dimensional version of this model has been documented by Held et al. (1993). We have also begun analysis of three-dimensional simulations. A scene from the statistically steady state obtained is shown in Fig. 2. This model has 64×64 grid points in the horizontal, with 2-km grid length; it has 64 unequally spaced levels in the vertical reaching into the lower stratosphere; and it equilibrates after roughly 30 days.

In this simulation, the dissipation rate associated with precipitation W_p is 3.6 W m^{-2}. Given the model's precipitation rate, the corresponding precipitation path-length is $H_f \approx 9.3$ km. This result is reasonable in the context of the discussion of section 2. The rate of dissipation from the resolved convective motions W_D is 1.4 W m^{-2}. *Therefore, the dissipation of the convective mo-*

tions is substantially smaller than the precipitation-related dissipation.

In addition, about one-third of the dissipation of the resolved motion ($\sim$0.4 W m^{-2}) is not dissipated in the troposphere but is lost to gravity waves that radiate into the stratosphere. In the model, this energy is deposited in a sponge layer used to prevent the reflection of these waves from the model top.

As the maximum work corresponding to the differential heating W_{max} is 11 W m^{-2}, and the resolved dissipation rate $W_p + W_D$ is only 5 W m^{-2}, the difference, $\approx$6 W m^{-2}, is either due to irreversible processes or dissipation by unresolved eddies. But the latter can be estimated from the subgrid-scale diffusion of heat to be less than 0.5 W m^{-2}. Irreversible phase changes and diffusion of water vapor account for the bulk of the irreversible entropy production, and result together in a reduction of the mechanical work done by the system by approximately 6 W m^{-2}. This issue will be addressed in more detail in a subsequent paper where we provide qualitative arguments for why this term is so large.

Hence, in these numerical simulations, we observe that 1) irreversible phase changes and diffusion of water vapor reduce the total work done by convection by about 50%–60%, and 2) about 70% of the frictional dissipation is due to precipitation.

This latter observation is supported by other model results, such as that described by Xu et al. (1992). Although they do not analyze the energy dissipation or entropy production in the atmosphere, one can infer a similar energy budget from the decomposition of the buoyancy flux provided by the authors.

The kinetic energy generated on the scales resolved by the model is equal to the upward buoyancy flux,

$$W_D = \int \overline{\rho} g w[\Theta'/\overline{\Theta} + (R_v/R_d - 1)\rho_v/\overline{\rho} - \rho_c/\overline{\rho}]. \tag{8}$$

Here $\overline{\Theta}$ is the horizontally averaged potential temperature, Θ' the potential temperature departure from this reference state, R_d and R_v are the gas constants for water vapor and dry air, while $\overline{\rho}$ is the horizontally averaged density. This vertical buoyancy flux is traditionally decomposed into a ''virtual temperature effect'' $\int wg[\overline{\rho}\Theta'/\overline{\Theta} + (R_v/R_d - 1)\rho_v]$ and ''condensate loading'' $\int -gw\rho_c$. Xu et al. show that these two parts of the buoyancy flux cancel to a suprising degree. For our purpose, it is more convenient to rearrange these terms as

$$W_D = W_{tot} - W_p, \tag{9}$$

where W_{tot} is the total mechanical work by the resolved eddies,

$$W_{tot} = \int wg[\overline{\rho}\Theta'/\overline{\Theta} + (R_v/R_d)\rho_v], \tag{10}$$

and W_p is the frictional dissipation due to precipitation given by (4). Since the flux of water vapor is upward, this rearrangement increases the degree of cancellation between W_p and W_{tot}. This cancellation implies that the total generation (or, equivalently, dissipation) of kinetic energy in the model's resolved scales is much smaller than the hydrometeor-related dissipation. The mechanical work produced by convective systems is primarily used to lift water to the level from which it precipitates, and only secondarily to generate kinetic energy. The potential energy imparted to the water is then lost to friction during rainfall.

4. Discussion

In moist convection, frictional dissipation occurs not only as an end result of a turbulent energy cascade from the convective scales to the smaller scale, but it also occurs in the shear zone surrounding falling hydrometeors. The amount of energy dissipated during precipitation is equal to the potential energy imparted to the water during its uplift by the atmospheric circulation. The precipitation path-length H_f is a critical parameter for moist convection. It relates the amount of precipitation-related dissipation to the precipitation rate. We find that a value $H_f \sim$ 5–10 km is a reasonable estimate for tropical convection. This is equivalent to a frictional dissipation of 2–4 W m^{-2} in the Tropics. In most studies of moist convection, the frictional dissipation is implicitly included in the buoyancy flux. However, a correct analysis of the mechanical energy and entropy budgets requires an explicit estimate of the frictional dissipation associated with falling hydrometeors.

Numerical simulations with a high-resolution cloud ensemble model are used to determine the mechanical energy budget of an atmosphere in radiative–convective equilibrium. We find that the resolved turbulent dissipation in the model is significantly smaller than the precipitation-related dissipation. Other cloud ensemble models produce similar behavior, as inferred by their buoyancy fluxes. The total frictional dissipation in the simulations is also smaller than the maximum work obtained from an entropy budget. This difference is explained by the production of entropy by irreversible phase changes and diffusion of water vapor.

We conclude that frictional dissipation in the vicinity of falling hydrometeors is the main mechanism by which mechanical energy is dissipated in the Tropics and that the use of the entropy budget to constrain the kinetic energy flowing through tropical convective motions is complicated by this fact as well as by the presence of other, even larger, entropy sources. While these effects will be less important in higher latitudes, their dominance in the Tropics is sufficient to make them important for the global atmosphere's energy generation and entropy production as well.

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REFERENCES

Emanuel, K. A., 1994: *Atmospheric Convection.* Oxford University Press, 580 pp.

——, and M. Bister, 1996: Moist convective velocity and buoyancy scales. *J. Atmos. Sci.,* **53,** 3276–3285.

Fankhauser, J. C., 1988: Estimates of thunderstorm precipitation efficiency from field measurements in CCOPE. *Mon. Wea. Rev.,* **116,** 663–684.

Ferrier, B. S., J. Simpson, and W.-K. Tao, 1996: Factors responsible for precipitation efficiencies in midlatitude and tropical squall simulations. *Mon. Wea. Rev.,* **124,** 2100–2125.

Held, I. M., R. S. Hemler, and V. Ramaswamy, 1993: Radiative–convective equilibrium with explicit two-dimensional moist convection. *J. Atmos. Sci.,* **50,** 3909–3927.

Leary, C. A., and R. A. Houze Jr., 1980: The contribution of mesoscale motions to the mass and heat fluxes of an intense tropical convective system. *J. Atmos. Sci.,* **37,** 784–796.

Lipps, F. B., and R. S. Hemler, 1982: A scale analysis of deep moist convection and some related numerical calculations. *J. Atmos. Sci.,* **39,** 2192–2210.

Peixoto, J. P., and A. H. Oort, 1992: *Physics of Climate.* American Institute of Physics, 520 pp.

Randall, D. A., K.-M. Xu, R. J. C. Somerville, and S. Iacobellis, 1996: Single-column models and cloud ensemble models as links between observation and climates models. *J. Climate,* **9,** 1683–1697.

Rennó, N. O., and A. P. Ingersoll, 1996: Natural convection as a heat engine: A theory for CAPE. *J. Atmos. Sci.,* **53,** 572–585.

Sui, C. H., K. M. Lau, W. K. Tao, and J. Simpson, 1994: The tropical water and energy cycles in a cumulus ensemble model. Part I: Equilibrium climate. *J. Atmos. Sci.,* **51,** 711–728.

Tao, W.-K., J. Simpson, and S.-T. Soong, 1987: Statistical properties of a cloud ensemble. A numerical study. *J. Atmos. Sci.,* **44,** 3175–3187.

Tompkins, A. M., and G. C. Craig, 1998: Radiative–convective equilibrium in a three-dimensional cloud-ensemble model. *Quart. J. Roy. Meteor. Soc.,* **124,** 2073–2097.

Xu, K.-M., A. Arakawa, and S. K. Krueger, 1992: The macroscopic behavior of cumulus ensembles simulated by a cumulus ensemble model. *J. Atmos. Sci.,* **49,** 2402–2420.