New Third-Order Moments for the Convective Boundary Layer

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ABSTRACT

Turbulent convection is inherently a nonlocal phenomenon and a primary condition for a successful treatment of the convective boundary layer is a reliable model of nonlocality. In the dynamic equations governing the convective flux, the turbulent kinetic energy, etc., nonlocality is represented by the third-order moments (TOMs). Since the simplest form, the so-called down gradient approximation (DGA), severely underestimates the TOMs (up to an order of magnitude), a more physical model is needed. In 1994, an analytical model was presented that was derived directly from the dynamical equations for the TOMs. It considerably improved the DGA but was a bit cumbersome to use and, more importantly, it was based on the quasi-normal (QN) approximation for the fourth-order moments.

Here, we present a new analytic expression for the TOMs that is structurally simpler than the 1994 expression and avoids the QN approximation. The resulting fit to the LES data is superior to that of the 1994 model.

1. Introduction

The search for a reliable expression for the third-order moments to be used in the dynamic equations for the second-order moments such as the turbulent kinetic energy, the convective fluxes, etc., has a long history. For many years, people used the so-called down gradient approximation but large eddy simulations (LESs) have shown that said model severely underestimates the third-order moments (TOMs) (Moeng and Wyngaard 1989).

Prompted by these results, Canuto et al. (1994) undertook the task of solving directly the dynamic equations for the TOMs thus avoiding the need for phenomenological expressions. The key merit of the 1994 model was to exhibit the fact that all the TOMs are a linear combination of the gradients of all the second-order moments and not only of selected ones, as assumed in the downgradient approximation. From the performance viewpoint, the new TOMs reproduced the LES data quite satisfactorily but the predicted $\overline{w^2\theta}$ and $\overline{w\theta^2}$ were not as good as that of the other TOMs. The weakest point in the 1994 model was the use of the quasi-normal (QN) approximation for the fourth-order moments. Both these limitations have motivated us to search for new expressions for the TOMs, which are simpler and with a better physical content.

2. The new physical ingredient of the third-order moments

There are six TOMs to be considered:

$$\overline{w^3}, \quad \overline{q^2w}, \quad \overline{w^2\theta}, \quad \overline{w\theta^2}, \quad \overline{\theta^3}, \quad \overline{q^2\theta}. \tag{1}$$

Here, u, v, w, and θ are the fluctuating velocity and temperature fields and $q^2 = u^2 + v^2 + w^2$. Since the TOMs in (1) have different dimensions, we multiply the last four by appropriate variables so that all the TOMs have dimensions of a velocity cubed. Thus, we introduce the new variables x's, which have the same dimensions:

$$x_1 \equiv g\alpha\tau_V\overline{w^2\theta}, \qquad x_2 \equiv (g\alpha\tau_V)^2\overline{w\theta^2} \tag{2a}$$

$$x_3 \equiv (g\alpha\tau_V)^3\overline{\theta^3}, \qquad x_4 \equiv g\alpha\tau_V\overline{q^2\theta} \tag{2b}$$

$$x_5 \equiv \overline{wq^2}, \qquad z \equiv \overline{w^3}. \tag{2c}$$

Here, α is the volume expansion coefficient, g is the local gravity, and τ_v is a timescale that will be discussed below. The original dynamic equations for the TOM given in (1) can be found in Canuto (1992), Eqs. (37a), (38a), (39a), and (40a). These equations entail fourth-order moments that can be written in general as

$$\overline{abcd} = (\overline{ab}\,\overline{cd} + \overline{ac}\,\overline{bd} + \overline{ad}\,\overline{bc})F. \tag{3a}$$

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If the function F is taken to be unity, Eq.(3a) reduces to the QN approximation. If so, the timescale τ_v introduced before can be identified only with the dynamic timescale of turbulence $\tau = 2K/\epsilon$ (K and ϵ are the turbulent kinetic energy and its rate of dissipation). With $F = 1$ and $\tau_v = \tau$, the form of the TOMs was given in Canuto et al. (1994). The comparison with LES data was overall satisfactory but the predicted $\overline{w^2\theta}$ and $\overline{w\theta^2}$ (Figs. 10, 11 of Canuto et al. 1994) were not as good as that of the other TOMs. In addition, the expressions of the TOMs were rather cumbersome due to the explicit form of the determinant resulting from solving the algebraic equations for the TOMs.

For these reasons, we felt motivated to search for new expressions for the TOMs that are both simpler to handle and possess a better physical content. Zilitinkevich et al. (1999) tried to do so but their solution is not practical since it does not provide an expression for $\overline{w}^3$, which must be derived from outside, say from LES.

To improve the 1994 model, we were guided by the fact that with $F = 1$, the TOM can become arbitrarily large while in reality they have finite values, for example, the small value of the skewness. Thus, reducing the TOM given by the $F = 1$ case is a way to avoid unphysical results. How to directly relate this ''damping effect'' to a $F \neq 1$ is not a matter that can be carried out analytically, rather, we have used F in (3a) to formally indicate the physical motivation of our new approach. In the most successful heuristic model used to cut down the growth of the TOM, the EDQNM model (Lesieur 1992), the damping is represented by an additional timescale, which one must choose on physical grounds. Here, we suggest

$$\tau_V = \tau[1 + \lambda_0 N^2\tau^2]^{-1}, \qquad N^2 = -g\alpha\partial T/\partial z,$$
$$\tau = 2K/\epsilon. \tag{3b}$$

We shall further take

$$\lambda_0 = 0.04 \quad \text{if } N^2 > 0, \qquad \lambda_0 = 0 \quad \text{if } N^2 < 0. \tag{3c}$$

The 1994 model (Canuto et al. 1994) corresponds to $F = 1$ and/or to $\lambda_0 = 0$.

3. The new expression for the third-order moments

The new analytic expressions for the TOMs are quite simple:

$$x_1 = X_0 z - X_1 \qquad x_2 = Y_0 z - Y_1 \tag{4a}$$
$$x_3 = Z_0 x_2 - Z_1 \qquad x_4 = W_0 x_5 + c^{-1}x_2 + W_1 \tag{4b}$$
$$x_5 = \Omega_0 z - \Omega_1$$
$$z = \left(\Omega_1 - 1.2X_1 - \frac{3}{2}f_5\right)(c - 1.2X_0 + \Omega_0)^{-1}, \tag{4c}$$

where the functions X, Y, Z, W, and Ω are defined as

$$X_0 = \gamma_2\tilde{N}^2(1 - \gamma_3\tilde{N}^2)[1 - (\gamma_1 + \gamma_3)\tilde{N}^2]^{-1}$$
$$X_1 = [\gamma_0 f_0 + \gamma_1 f_1 + \gamma_2(1 - \gamma_3\tilde{N}^2)f_2] \times [1 - (\gamma_1 + \gamma_3)\tilde{N}^2]^{-1} \tag{5a}$$
$$Y_0 = 2\gamma_2\tilde{N}^2(1 - \gamma_3\tilde{N}^2)^{-1}X_0$$
$$Y_1 = 2\gamma_2(1 - \gamma_3\tilde{N}^2)^{-1}(\tilde{N}^2X_1 + \gamma_0\gamma_1^{-1}f_0 + f_1) \tag{5b}$$
$$Z_0 = \frac{3}{2}(c - 2)^{-1}\tilde{N}^2 \qquad Z_1 = \frac{3}{2}(c - 2)^{-1}f_0 \tag{5c}$$
$$\Omega_0 = \omega_0 X_0 + \omega_1 Y_0 \qquad \Omega_1 = \omega_0 X_1 + \omega_1 Y_1 + \omega_2 \tag{5d}$$
$$W_0 = \frac{1}{2c}\tilde{N}^2, \qquad W_1 = -c^{-1}f_3. \tag{5e}$$

The auxiliary functions ω's are

$$\omega_0 = \gamma_4(1 - \gamma_5\tilde{N}^2)^{-1}, \qquad \omega_1 = (2c)^{-1}\omega_0,$$
$$\omega_2 = \omega_1 f_3 + \frac{5}{4}\omega_0 f_4. \tag{6}$$

Finally, the γ's are constants that depend on the only adjustable parameter c:

$$\gamma_0 = 0.52c^{-2}(c - 2)^{-1}, \qquad \gamma_1 = 0.87c^{-2}$$
$$\gamma_2 = 0.5c^{-1} \qquad \gamma_3 = 0.60c^{-1}(c - 2)^{-1}$$
$$\gamma_4 = 2.4(3c + 5)^{-1}, \qquad \gamma_5 = 0.6c^{-1}(3c + 5)^{-1}. \tag{7}$$

Based on previous work, the suggested value is $c = 7$ but small variations are allowed. The second-order moments enter through the functions $f_{0,\ldots,5}$, which are defined as follows:

$$f_0 = (g\alpha)^3\tau_V^4 J\frac{\partial\overline{\theta^2}}{\partial z} \qquad f_1 = (g\alpha)^2\tau_V^3\left(J\frac{\partial J}{\partial z} + \frac{1}{2}\overline{w^2}\frac{\partial\overline{\theta^2}}{\partial z}\right)$$
$$f_2 = g\alpha\tau_V^2 J\frac{\partial\overline{w^2}}{\partial z} + 2g\alpha\tau_V^2\overline{w^2}\frac{\partial J}{\partial z}$$
$$f_3 = g\alpha\tau_V^2\left(\overline{w^2}\frac{\partial J}{\partial z} + J\frac{\partial K}{\partial z}\right)$$
$$f_4 = \tau_V\overline{w^2}\left(\frac{\partial\overline{w^2}}{\partial z} + \frac{\partial K}{\partial z}\right) \qquad f_5 = \tau_V\overline{w^2}\frac{\partial\overline{w^2}}{\partial z}. \tag{8}$$

All the functions f's have dimensions of velocity cubed. Finally,

$$J = \overline{w\theta}, \qquad \tilde{N}^2 \equiv \tau_V^2 N^2. \tag{9}$$

4. Test of the new TOM versus LES data

In Fig. 1 we compare the new TOMs defined in Eqs. (2) and given by Eqs. (4)–(9) versus LES data. Rather than solving the CBL dynamic equations as it was done in the 1994 paper, here we employed LES data to compute the second-order moments that appear in Eq. (8)

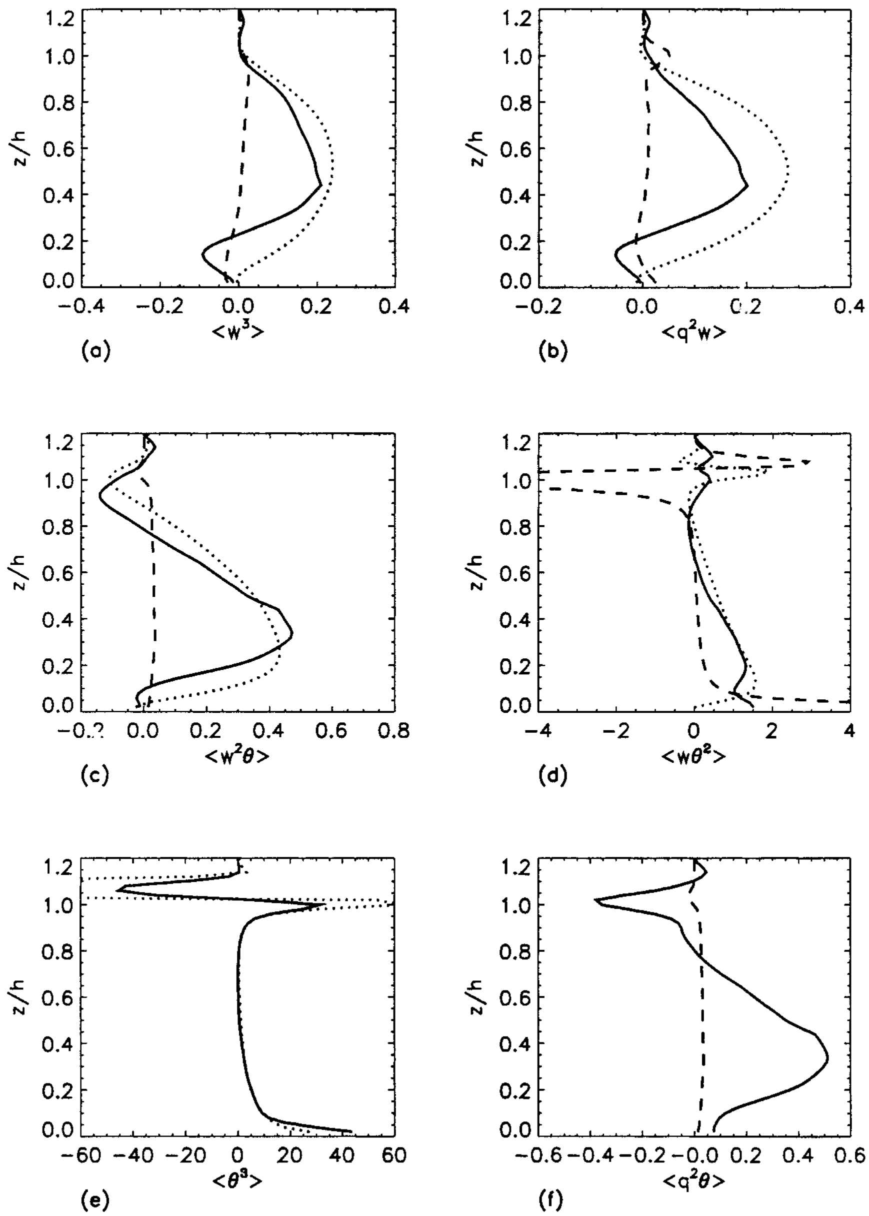

FIG. 1. Solid lines represent the third-order moments (vs z/h) defined in Eq. (1) and given explicitly in Eqs. (4)–(9) in terms of the the second-order moments. The LES data are plotted as dotted lines (Zilitinkevich et al. 1999; the LES data do not provide $\overline{q^2\theta}$). The same LES data were also used to evaluate the second-order moments appearing in Eq. (8), as well as to compute $N^2(z)$ Eq. (3b), and τ of Eq. (3b), which are needed to compute τ_v. We recall that the use of τ_v is a heuristic way to account for $F \neq 1$ in (3a), so as to avoid the quasi-normal approximation used in the 1994 model. The downgradient approximation (DGA) model corresponds to dashed lines. As well known, the DGA severely underestimates the third-order moments. All TOMs are normalized with Deardorff convective scales w_* (=2 m s^{-1}) and θ_* (=0.12 K). The value of the PBL depth h is 1010 m.

as well as $\tau = 2K/\epsilon$ and $N^2(z)$ that are needed to compute τ_v, which is given by Eq. (3b).

The better agreement with LES data with respect to the 1994 model, especially in Figs. 1c–d, is also partly due to the simpler analytical form of the TOM, which has allowed us to test slight variations around the $c =$ 7 value. Due to its rather rigid nature, the 1994 model did not allow the same freedom. However, the key reason for the better performance of the new model is of physical origin: we have abandoned the quasi-normal approximation for the fourth-order moments that we employed in the 1994 model and which corresponds to F

= 1 in (3a). This means that we have searched for a way to cut down an otherwise unphysical growth of the TOM by adopting an EDQNM-like procedure. Regrettably, at present we do not have an a priori derivation for (3b, c) which must thus be considered a heuristic expression of the $F \neq 1$ case.

5. Conclusions

The results presented in Fig. 1 satisfy the two requirements set out at the beginning, the expressions for the TOMs are simpler than those of the 1994 model and their physical content is better. As a result, the large values of $\overline{w^2\theta}$ and $\overline{w\theta^2}$ that characterized the 1994 model are no longer present and an overall better fit is obtained.

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