REFLECTION, ABSorption, AND TRANSMISSION OF INSOLATION
BY STRATUS CLOUD

By M. Neiburger

University of California, Los Angeles

(Manuscript received 26 July 1948)

ABSTRACT

Using Eppley pyrheliometers mounted on a blimp, observations were made of the upward and downward flux of short-wave radiation above, in, and below coastal stratus throughout the summer of 1945. These observations are summarized and evaluated in terms of the percentage of the incident radiation reflected, absorbed, and transmitted, as a function of the thickness of the cloud.

The simplified radiative transfer equations, in which the radiation is considered to be composed of upward and downward isotropic currents, are solved for general distributions of drop size and liquid-water content, and applied to the distributions found in coastal stratus. The theoretical variation with cloud thickness differs from the observed. This is taken to indicate that the simplified theory is an inadequate description because the radiation is not isotropic.

1. Introduction

The evaluation of the amount of solar radiation reflected and absorbed by clouds enters into a variety of geophysical problems. In particular, questions concerning the heat budget of the earth, the atmosphere, and the oceans require some estimate of cloud albedo for their answers. Complete solutions of the problems of forecasting maximum temperatures, thermally induced thunderstorms, and the dissipation of stratus cloud likewise must be based on an accurate evaluation of the energy available. The ablation of glaciers is another question for which this information is needed.

Previous measurements of the albedo of cloud vary from 0.5 to 0.8. The value of 0.78 is quoted in most meteorology textbooks in English, and has been used in many geophysical computations. It is based on the measurements made by Aldrich [2] from a captive balloon over stratus cloud in the San Gabriel Valley of California on 17 September 1918. The base of the stratus varied from 1000 ft to 2000 ft during the observations; the top remained at a height of 2600 ft throughout. Thus Aldrich deduced that the reflectivity of cloud was independent of thickness.

That cloud albedo should depend on thickness is an obvious consequence of the pioneer theoretical work, "Radiation through a foggy atmosphere," published by Schuster [8] in 1905. He obtained for the amount $A$ transmitted through a layer of thickness $H$ of non-absorbing cloud with a scattering coefficient $s$,

$$ A = 2A_0/(2 + sH), $$

where $A_0$ is the intensity of the incident radiation. The fraction scattered back, $1 - A/A_0$, is thus a function of $H$, and is given by

$$ A_0 = sH/(2 + sH). $$

As $H$ increases, $A_0$ increases, approaching unity for infinite cloud thickness. For an absorbing medium Schuster derived equations which are more complicated but likewise show dependence of the radiation scattered backwards on cloud thickness.

At about the same time as Aldrich's work was published there appeared an article by Luckiesh [5], giving the results of a series of measurements made from an airplane. In these measurements he found that the albedo varied from 0.36 for thin clouds to 0.78 for "very dense clouds of extensive area and great depth." Luckiesh's data have apparently been completely overlooked by meteorologists.

Meteorological applications of Schuster's theory have been carried out by Mecke [6], Dietzius [3], Albrecht [1], and Hewson [4]. Hewson presented a brief review of the previous theoretical work, and evaluated theoretically the radiation transmitted, absorbed and reflected by clouds of particular thickness, drop size, and liquid-water content. His computations gave reflectivities ranging from 1.4 per cent for very thin clouds with small liquid-water content and large drops to 93.6 per cent for thick clouds of small drops but large liquid-water content. His computed values of absorption ranged from 0.1 per cent to 6.7 per cent.

The lack of agreement between the generally accepted albedo measurement and theory led to the decision to measure the radiative flux above, in, and below the stratus cloud which characterizes the California coast in summer, and to attempt at the same time to measure the other pertinent quantities, namely drop size and liquid-water content. These measurements were carried out with a blimp (nonrigid lighter-than-
air ship) operated by the U. S. Navy, during the summer of 1945. To the excellent cooperation of the Navy Bureau of Aeronautics, and especially the personnel of the Naval Air Station at Santa Ana must be attributed the prime credit for these data.

2. Methods of measurement

Two Eppley 50-junction pyrheliometers were mounted on the blimp. The one facing upward to measure the total downward flux of short-wave radiation was mounted on the top of the balloon. The one facing downward to measure the upward-directed flux was mounted on the bottom of the cabin. It was impossible to have the latter sufficiently low so that it had a free horizon, and in fact a solid angle of about 1/24 the hemisphere was obstructed by the housing of the radar antenna. However, as has been shown elsewhere [7], the influence of the radiation from this sector is small due to the operation of the cosine law, particularly since the radiation intercepted by the housing is replaced by radiation from the same source reflected from the white paint of the housing. If it were not replaced at all, the upward flux would be in error about two per cent of its value; the replacement by radiation reflected from the white paint probably reduces this error to less than one per cent.

Readings were made at 100- or 200-ft intervals, starting near the sea surface, ascending above the cloud top, and then descending again. The positions of the base and top of the cloud were observed. At each level of observation the blimp flew horizontally for a period while the readings were being made. Two potentiometers were used, so that readings of the bottom and top pyrheliometers could be made simultaneously, and a series of five to ten readings were made at each level and averaged to reduce the influence of irregularities of cloud thickness and flight angle of the blimp.

Because the upper pyrheliometer had to be mounted above the blimp and the lower one below it, simultaneous readings did not give the upward and downward flux at the same level. The pyrheliometers were 80 ft apart. The results presented in the present paper will be based only on observations in the cloud-free air above and below the stratus, and thus will not be affected by this difference in pyrheliometer position.

Measurements of the drop size and liquid-water content were made on only a few trips near the end of the series. The methods used were developed by the Icing Research Project at the Massachusetts Institute of Technology [9]. For the drop-size measurements small slides on which a thin film of lampblack had been deposited were exposed normal to the air stream for a few seconds. The drops deposited left rings in the soot whose diameters are related to the drop diameter. The liquid-water content was measured with a capillary collector. Both measurements were corrected for the efficiency of collection.

3. Results of observations

The particular data which are of most interest are (a) the albedo, or fraction \( a_0 \) of the incident radiation reflected by the cloud, (b) the fraction \( b \) absorbed by the cloud, and (c) the fraction \( c \) transmitted through the cloud and absorbed by the earth’s surface. If \( A_0 \) is the flux of radiation incident on the cloud top, and \( B_0 \) the flux of radiation reflected upward by the cloud, the cloud albedo \( a_0 \) is given by

\[
a_0 = B_0 / A_0.
\]

If \( A_H \) is the amount of radiation passing downward from the bottom of the cloud, and \( B_H \) is the amount reflected upward by the earth’s surface and entering the cloud from below, then, neglecting absorption and scattering by the air below the cloud,

\[
c = (A_H - B_H) / A_0 = (1 - a_0) A_H / A_0 = (1 - a_0) t,
\]

where \( a_0 \) is the reflectivity of the earth’s surface and \( t = A_H / A_0 \) is the fraction transmitted by the cloud. The fraction absorbed by the cloud is

\[
b = (A_0 - A_H - B_0 + B_H) / A_0 = 1 - a_0 - c = 1 - a_0 - (1 - a_0) t.
\]

The cloud albedo \( a_0 \) is obtained by taking the ratio of the simultaneous readings of upper and lower pyrheliometers when the blimp was above the cloud. The surface albedo \( a_0 \) is obtained similarly from simultaneous readings below the cloud. If the cloud has not changed in optical thickness during the time between the measurements of \( a_0 \) and \( a_0 \), it seems safe to consider that these measurements correspond and may be used in (5) to compute \( b \). The evaluation of \( t \), however, requires the simultaneous reading of the downward flux \( A \) both above and below the cloud. Since only one blimp was used, such simultaneous readings were not made.

To get around this difficulty it was necessary to estimate the incident radiation \( A_0 \) at the time the blimp was below the cloud measuring \( A_H \). As an aid in doing this, a pyrheliometer was installed at the airways weather station on Mount Wilson. Since the stratus never enveloped the mountain, and since the distance from it to the observation points was relatively small, it was assumed that the mountain-top reading \( A_m \) was characteristic of the radiation incident on the cloud top, except for the effect of scattering by the layer of air between them. It was assumed therefore that the fractional change in \( A_0 \) was the same as that in \( A_m \) during the interval between the time the blimp was above the cloud and the time it was below.
Thus

\[ t = \frac{A_H/A_0}{\alpha_0}, \]

where \( r = A_M/A_{M'}, A_M \) is the Mount Wilson reading at the time \( A_H \) was measured, \( A_{M'} \) is the Mount Wilson reading at the nearest time when the blimp was above the cloud, when \( A_{H'} \) was measured.

There was some difficulty in timing the Mount Wilson readings, for the recording potentiometer was electrically driven, and the current frequency on the mountain was erratic. However, check points were made every three hours, and it is felt that the values used were reasonably accurate.

---

FIG. 3. Albedo of cloud as function of cloud thickness. Circles give mean values, lines standard deviations of means, for observations grouped by 250-ft intervals. Curves represent theoretical variation for various values of absorption, using observed distribution of drop size and liquid content.

---

FIG. 4. Transmission and absorption of cloud as function of thickness.

---

Flights were made on about 50 days during July, August, and September, 1945. Several ascents and descents were made in each flight. Fig. 1 shows the data obtained in a typical sounding. The values above the cloud give the albedo of the cloud \( \alpha_0 \). The values below the cloud are used to compute the albedo of the sea surface, \( \alpha_s \), and the transmission and absorption by the cloud. Approximately 200 evaluations were made of the cloud albedo and the sea-surface albedo, and about 100 of the transmission by stratus clouds ranging in thickness from 180 to 2000 ft.

In fig. 2 the observations of cloud albedo are presented as a function of geometrical cloud thickness. In general there is an increase with height, but at any
one height there is a scattering of values observed. Some of this scattering is doubtless due to errors of observation. Most of it must be attributed to variations of drop size and liquid-water content for the same thickness of cloud. The most rapid variation and the largest scattering of values occurs for clouds less than 1000 ft thick. For thicker clouds the values change less rapidly with thickness, and are less variable. In fig. 3 the observations are represented by the averages for 250-ft intervals, and the standard deviations thereof. The increase with thickness and the variability, particularly in the range 500–1000 ft, is shown clearly.

The average percentages transmitted and absorbed for the observations grouped by 250-ft intervals of thickness are shown in fig. 4. The amount transmitted decreases steadily with cloud thickness except for the thickest of the clouds observed, being affected principally by the loss due to reflection (scattering backward). For the intervals 1500–1750 ft and 1751–2000 ft the number of observations was small, so that the averages may be less representative. The standard deviations show that the mean values could very well be in error by an amount which would allow them to decrease with increasing thickness.

The percentage absorbed on the other hand shows no definite tendency to increase with cloud thickness, and the standard deviations for most of the groups are many times the average values. Quite a few of the values computed from the observations turned out to be negative, about 20 out of 75, in fact. This suggests strongly that the change in incident radiation between the times the blimp was above and below the cloud was not accurately evaluated. However, it seems plausible that the means give some indication of the correct value, and the standard deviations of the means would suggest to what extent they are reliable.

The average of all observations of the absorption is 7.0 per cent, and their standard deviation is 14.0. This gives for the standard deviation of the mean less than two per cent, so that there is a high probability that the mean absorption lies between five and nine per cent. A tabulation of the frequency of various groups of values showed that the mode was somewhat higher than the mean, with the spread toward negative values offsetting the greater frequency of high values. This departure from normal distribution reduces the significance of the standard deviation of the mean as a measure of its confidence interval.

Observations of drop size and liquid content were obtained on only a few of the flights. Fig. 5 shows the drop-size distributions found on one of the flights, and fig. 6 shows the liquid-content measurements during several ascents and descents on another flight. It should be noted that, except near the cloud base, the distribution has a single well-pronounced mode in the 13–15μ diameter group. The concentration of drop diameters around 14μ was not confined to this flight, but was fairly characteristic of all those on which drop size was measured. This size is near those most frequently found in fog and stratus cloud in other regions.

The liquid content is seen from fig. 6 to increase approximately linearly from the cloud base to a little distance below the top. This also was characteristic of all flights in which liquid content was measured, although the rate of increase varied somewhat. Averaged

![Graph showing liquid content in typical California stratus.](Flight 46, 15 September 1945.)
for all flights it was 0.40 g m\(^{-3}\) (1000 ft\(^{-1}\)) or 0.13 g m\(^{-3}\) (100 m\(^{-1}\)).

It is interesting that while the drop size remains nearly constant from base to top of the cloud, the liquid content increases. This must mean that the number of drops increases from base to top, i.e., new drops are formed as a result of continued condensation, in preference to further growth on the existing drops.

4. Theoretical variation of reflection, transmission, and absorption

The theoretical derivations referred to in the introductory remarks were concerned with taking into account correctly the processes of scattering and absorption, but regarded the cloud as homogeneous. Thus Albrecht carried out the integration allowing for the variation of the coefficient of scattering with altitude of the sun and with depth in the cloud. Hewson carried out the integration regarding the scattering coefficient constant, but taking into account the spectral distribution of the absorption coefficient and the sun’s energy. In all of the papers referred to, the drop size and liquid content were assumed constant, independent of position in the cloud.

The measurements in California stratus showed that while the drop size was practically independent of position, the liquid content increased linearly from bottom to top of the cloud. Thus it seemed desirable to regard the liquid content as variable in integrating the theoretical equations. It was thought that this variation might prove more important than the variation of the coefficient of scattering or the absorption, particularly since the latter is so small.

The differential equations for the downward flux \(A\) and the upward flux \(B\) of short-wave (sun and sky) radiation, as given by Schuster [8] and Mecke [6] are:

\[
dA = [-(\alpha - \alpha \beta (1 - \chi))A - \alpha \beta (1 - \chi)(A - B)]dz,
\]

\[
dB = [\alpha \beta (1 - \chi)A - \alpha \beta (1 - \chi)(A - B)]dz.
\]

Here \(\alpha\) is the cross-sectional area presented by the drops, per unit volume, \(\chi\) is the fraction of the incident radiation absorbed by each drop, and \(\beta\) is the fraction of the incident radiation scattered backward by the drop. For drops of uniform radius \(r\), \(\alpha = 4\pi r^2\), where \(N\) is the number of drops per unit volume.

The significance of the equation may be seen by reference to fig. 7. In a thickness \(dz\), the area of drops intercepting radiation per unit cross section is \(\alpha dz\), and this fraction of the beam \(A\) is subjected to scattering and absorption. The fraction \(\alpha \chi A dz\) which is absorbed, and the fraction of the remainder \(\alpha \beta (1 - \chi)A dz\) which is scattered backward, deplete the downward beam, but it is augmented by the fraction of \(B\) which is scattered backward, namely \(\alpha \beta (1 - \chi)B dz\). The boundary conditions are determined by the fact that at the cloud top \(A = A_0\), the amount of radiation incident from sun and sky, and at the cloud bottom the amount incident from below is related to the amount transmitted by the equation \(B_H = a_s A_H\), where \(a_s\) is the sea-surface albedo.

It is of interest to see to what extent the equations can be integrated without specifying the coefficients. In the case of \(\chi = 0\) (no absorption), for instance, the solution is easily found to be

\[
A = A_0 \frac{1 + [F(H) - F(z)](1 - a_s)}{1 + F(H)(1 - a_s)},
\]

\[
B = A_0 a_s + [F(H) - F(z)](1 - a_s),
\]

where

\[
F(z) = \int_0^z \alpha \beta \, dz.
\]

The cloud albedo is

\[
a_0 = \frac{a_s + F(H)(1 - a_s)}{1 + F(H)(1 - a_s)},
\]

and the fraction transmitted is

\[
t = [1 + F(H)(1 - a_s)]^{-1}.
\]

Thus the solution for each set of functions \(\alpha\) and \(\beta\) may be found by performing a simple quadrature, without having to solve the differential equations each time. If \(\alpha\) and \(\beta\) are constant, and \(a_s = 0\), these expressions reduce to those given by Schuster.

\[
\text{Fig. 7. Illustration of significance of terms in equations for flux of short-wave radiation through cloud.}
\]

If \(\chi \neq 0\) it turns out that it is necessary that the ratio

\[
n^2 = \chi^2 + 2\beta (1 - \chi)
\]

must be constant in order to carry out a similar procedure. If this is assumed, the solution is

\[
A = A_0 R^{-1}[(n + 1)f(n)\epsilon^{Q(H) - Q(\zeta)} - (n - 1)f(-n)\epsilon^{Q(H) - Q(\zeta)}] + (n + 1)f(n)\epsilon^{Q(H) - Q(\zeta)}]
\]

\[
B = A_0 R^{-1}[(n + 1)f(-n)\epsilon^{Q(H) - Q(\zeta)} - (n - 1)f(n)\epsilon^{Q(H) - Q(\zeta)}] + (n - 1)f(n)\epsilon^{Q(H) - Q(\zeta)}.
\]
where
\[ f(n) = [(1 + a_s) + n(1 - a_s)] \]
\[ R = (n + 1)f(n) e^{Q(n)} + (n - 1)f(-n) e^{-Q(n)} \]
and
\[ Q(z) = n \int_0^z \alpha \chi \, dz. \] (13)

The cloud albedo is given by
\[ a_\chi = R^{-1}[(n + 1)f(-n) e^{-Q(n)} + (n - 1)f(n) e^{Q(n)}] \] (14)
and the fractional transmission by the cloud is
\[ t = 4n/R. \] (15)

In practice this solution applies only if \( \chi \) and \( \beta \) are separately independent of \( z \), for it would be highly improbable that \( n^2 \) would be constant otherwise.

It can easily be shown that \( \chi = \frac{4}{7} \kappa \), where \( r \) is the drop radius and \( \kappa \) the absorption coefficient. For each wave-length of radiation, \( \chi \) depends only on \( r \); we have seen that \( r \) is roughly independent of \( z \), and so, therefore, is \( \chi \), in stratus cloud.

Albrecht [1] discusses the variation of \( \beta \) with \( z \). He shows that if \( \beta_\delta \) is the fraction of diffuse downward radiation scattered upward, and \( \beta_u \) is the fraction of an unscattered parallel beam scattered upward, the scattering coefficient for the total downward radiation is
\[ \beta = \beta_\delta - (\beta_\delta - \beta_u) \frac{A_0}{A} e^{-m(z)}, \] (16)
where \( m = \frac{1}{2} \int_0^z \alpha \beta_a \, dz \). For clouds of the drop size and liquid content observed in California stratus, the clouds must be at least 3000 ft thick in order that \( \beta \) should not differ materially from \( \beta_a \) at the bottom. For thinner clouds and at higher levels in the cloud there will be a material difference unless the sun is near 23° elevation, for which \( \beta_a = \beta_\delta \). Thus the solutions above are valid only for solar elevations near 23°.

Nevertheless they improve on those of Schuster and Mecke by allowing for variations of \( \alpha \), and should therefore give a better idea of the variation of albedo and absorption with cloud thickness than these earlier works.

5. Comparison of theory with observations

If \( a_\alpha \) is taken to be zero and \( a_\beta = s/2 \) is considered constant, (9) reduces to (2), since \( F(H) = a_\beta H = sH/2 \). In fig. 2 curves have been drawn for the albedo using \( F(H) \) corresponding to three values of \( s/2 \). The curves follow in a general way the variation of the observed values with height. The values of \( s \) used correspond, for \( 7 \mu \) radius drops and \( \beta = 0.2 \), to liquid-content values of 0.24, 0.31, and 0.39 g m\(^{-3}\).

These values correspond to the average values for stratus clouds 1000 to 2000 ft thick. Thus, as one proceeds to greater cloud thickness one might expect to shift from the left theoretical curve to the middle one, and then to the one on the right.

The curves follow, in a general way, the observed values, but give too high values both for clouds less than 500 ft thick, and for those greater than 1500 ft thick. No adjustment of the value of \( s \) would make the curve fit in both of these regions. Having a non-zero value of \( a_\alpha \) would give still higher values of the albedo for thin clouds. The question arises whether consideration of absorption will bring the theoretical curves into agreement with the observations.

In fig. 3 the curves for various values of absorption are compared. The drops were assumed to be uniformly of \( 7 \mu \) radius, so that the value of \( \chi \) for water varies from about 2 \( \times 10^{-7} \) in the violet to \( 7 \times 10^{-5} \) for the farthest infrared for which there is any significant amount of solar radiation reaching the earth. For most of the radiation (wave-length less than \( 1 \mu \) \( \chi \) is less than \( 10^{-3} \), and so the values of \( \chi \) which were used to examine the effect of absorption are \( \chi = 10^{-3} \) and \( \chi = 10^{-2} \). These are compared with the case \( \chi = 0 \), in which absorption is ignored.

The curves in fig. 3 were computed using the linear variation of liquid content which was observed, so that except for the assumption that \( \beta \) has the value 0.2 corresponding to diffuse radiation throughout the cloud, the conditions assumed in the cloud were those observed. The computations were carried out for \( a_\alpha = 0.02 \), corresponding to thin cloud and high sun, and \( a_\alpha = 0.10 \), corresponding to moderately thick cloud. It should be noted that the influence of the surface albedo on the cloud albedo decreases rapidly with cloud thickness and becomes insignificant for thicknesses greater than 1000 ft.

The curves for \( \chi = 0 \) were computed from equation (9), and those for \( \chi \neq 0 \) from equation (14). It will be seen that for \( \chi = 10^{-3} \), i.e., for most of the insolation, the computed albedo differs very little from that computed for \( \chi = 0 \), and for \( \chi = 10^{-3} \) the albedo is only a few percent less even for the thickest clouds computed. Thus the effect of absorption may practically be neglected, so far as the albedo of cloud for solar radiation is concerned.

Comparison of these curves with the observations, here averaged for 250-ft intervals of thickness, shows that the agreement is better for greater thicknesses than for the curves in fig. 2, but for thin clouds the computations now give too small values. The indication seems to be that the thin clouds are more effective scatterers of light than is suggested by the equations. Either they contain more drops than the measurements suggest, or the process is not adequately represented by the theory. It may be that the inadequacy...
of the theory is due to the assumption that the intensity is independent of direction.

The computed absorption and transmission are given by the curves in fig. 4. For \( \chi = 10^{-3} \) the absorption amounts to less than two per cent for the thickest cloud considered, but for \( \chi = 10^{-2} \) it was close to 20 per cent for clouds 2000 ft thick. This illustrates clearly the fact that the absorption is almost entirely in the far infrared, where the amount of solar radiation is small. The high variability of the values of absorption computed from the observations makes it of little point to compare those computed theoretically with them.

The transmitted radiation computed from theory decreases less rapidly with thickness for thin clouds than does the observed, but agrees quite well for the thicker clouds. For the amount absorbed at the earth's surface, the fraction reflected at the surface must be subtracted. As in the case of the cloud albedo curves, the effect of the surface albedo is small except for thin cloud. For clouds 2000 ft thick, only about 20 per cent of the radiation is available to heat the ground; for thinner clouds, a much larger portion is available.

6. Conclusions

The observations demonstrate that the albedo of cloud varies with thickness, as predicted by theory. The absorption computed from observations is small, averaging seven per cent for all observations. The theory also indicates that the absorption is small but that it should increase with cloud thickness, and that for estimates of the cloud albedo it can be neglected.

For clouds of moderate thickness the theoretical computation of the albedo agrees fairly well with the observations, but for thin cloud there is considerable deviation. It would be desirable to arrive at a theory which takes account of the anisotropy of the radiation. In the meantime, however, there is little point in attempting to integrate the equations over the spectrum, taking account of the variation of absorption with wave length.

Further information on the number and size of drops in various types of clouds is also of great importance. Both the scattering and the absorption are greatly dependent on these quantities, concerning which little information is available.

REFERENCES