A METHOD FOR QUANTITATIVE DETERMINATION OF LONG WAVES

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ABSTRACT

In an effort to find tools for the description of long waves in the upper troposphere in addition to those already available, the total field of motion aloft is divided into a nondivergent basic current and perturbations, following standard hydrodynamical methods of wave treatment. After this division the long waves appear in the form of closed perturbation centers so that latitude is added to longitude as a coordinate for defining long waves. Moreover, since the intensity of these centers is also given, the deepening and filling which they experience, as well as their displacements from day to day can be followed, much as in the case of highs and lows on the sea-level chart. Several other measurements that can be made with use of perturbation charts are also indicated.

Two examples of the suggested technique are given, and some synoptic consequences are also discussed.

1. Introduction

Previous studies (Riehl 1947; Cressman 1948) have brought out the importance of interference between short waves of high and low latitudes. Recently, supplementing the widespread attention focused on long waves in middle latitudes, it has been possible (Riehl, 1948) to demonstrate that another set of long waves frequently exists in summer in the upper troposphere above the trade-wind belt. Moreover, Namias and Clapp (1944) have shown that waves in middle latitudes, as observed at the 10,000-ft level, often break up into separate portions in higher and lower latitudes. The presence of two or three sets of long waves, reminiscent to some degree of the classical concept of a tricellular division of a hemisphere, is the rule rather than the exception. It is evident, therefore, that techniques of weather-map analysis which apply the long-wave concept developed by Rossby (1939), must be suitable for revealing the complicated structure of several trains of waves rather than the configuration of a single set, which was the initial object of long-wave analysis.

Exact determination of the long-wave pattern or patterns on weather maps is difficult to accomplish, but favorable conditions are met in the tropics, at least during summer (Riehl 1948). In higher latitudes, however, where short waves of considerable intensity coexist with the long waves, there is as yet no unique physical method available for their separation, especially if such separation is sought on daily charts. In an effort to overcome this difficulty, Allen *et al.* (1940) and Willett *et al.* (1941) have developed a statistical technique for separating the waves through the preparation of the well-known five-day mean charts. If motion and transformation of long waves is slow relative to that of short waves, the effect of the latter should be largely canceled on such charts, and the long waves should appear in sharp focus. Subsequent studies (summarized in Namias, 1947) have confirmed the usefulness of the five-day technique. Nevertheless, it is not amiss to develop additional techniques that might be useful in furnishing further descriptive features of the long waves.

2. Perturbation technique

One such technique will be described here. It consists quite simply of an attempt to determine the perturbation streamline field from weather maps. Many types of such streamlines have been drawn in theoretical studies, but this method of representation has not yet been used synoptically to any extent. We shall here follow standard hydrodynamical methods of wave treatment which divide a total velocity field into a basic current and superimposed perturbations. This is also the course chosen by Rossby (1939). The empirical approach permits elimination of some of the restrictions contained in theoretical treatments. We shall define the nondivergent part of the zonal motion as basic current, following Rossby, but here we permit the current to have an arbitrary shear in the north–south direction, in view of the findings of Staff Members of the University of Chicago (1947). We also define all residual motion as perturbation motion, an approach recently discussed also by Starr (1948).

Elimination of the fast moving short waves is most readily accomplished by recourse to high-tropospheric analysis since these waves generally lose intensity with height (Rossby 1942). The 300-mb level appears to be best suited for this purpose. In higher latitudes, this level approaches the layer in which the meridional temperature gradient reverses sign. The dynamic

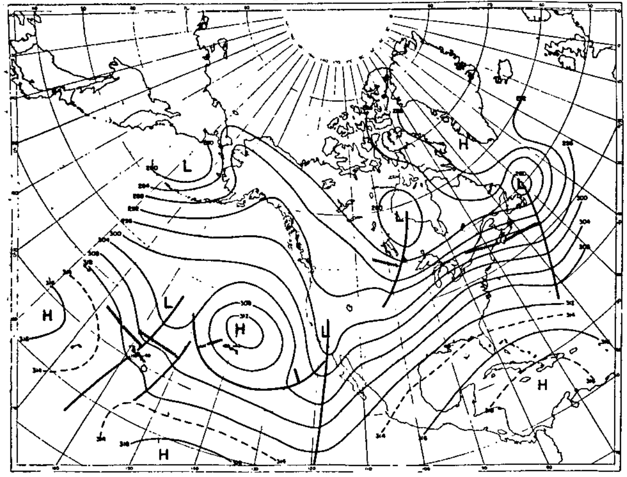

FIG. 1. Topography of the 300-mb surface, 0400 GCT 8 December 1947. Contours are given in hundreds of feet. Data have not been plotted except in the Pacific Ocean. Arrows indicate 24-hour displacement of disturbances in Pacific Ocean.

troughs and ridges which represent the long waves therefore reach their greatest strength near 300 mb. In the tropics, the 300-mb chart is also representative of the high-tropospheric stratum in which the tropical long waves are well developed. The sparsity of data at that height counters to some extent, of course, the advantages gained, and it is often necessary to employ differential analysis for the construction of 300-mb charts.[1] It is not unreasonable, however, to hope that the recent trend toward an increasing number of observations at high altitudes will continue.

Except for residual effects of the secondary disturbances, which have been identified for a number of years as rapidly moving traveling isallobaric centers, the perturbation field at 300 mb is due mainly to the long-wave trains and to effects of the primary general circulation that are produced in part by geographic factors. Although we possess certain climatic knowledge concerning the latter, at least at lower elevations, it is not possible at present to evaluate this circulation separately on given days or for five-day periods because of the nonlinearity of the equations of motion. Therefore, the method described will yield only a description of the total perturbation field, including all interference effects between different long-wave trains. It should be mentioned, however, that it is not certain whether removal of the primary circulation would be desirable from the viewpoint of physical long-wave analysis even if it could be accomplished.

3. The situation 6–9 December 1947

The following figures will show an application of the perturbation technique to the weather situation of 6–9 December 1947. This period was chosen because relatively good data were available from the scattered Pacific stations and a remarkable development occurred along the flight route from San Francisco to Hawaii. One of the charts of this series, 0400 GCT 8 December 1947, is reproduced in fig. 1. As the daily analysis still remained somewhat uncertain over the oceans, mean conditions for the four-day interval will be presented. This is permissible in the present instance since the long waves remained quasistationary. Comparison will also be made with the corresponding five-day mean chart at 700-mb. The time difference of one day between 700-mb and 300-mb mean charts should not make the comparison unrepresentative.

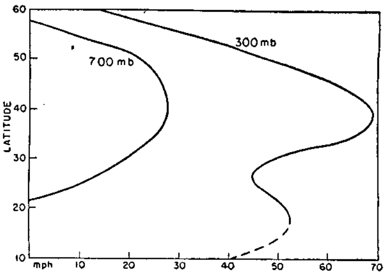

FIG. 2. Latitudinal profile of the basic westerly current at 700 mb and 300 mb, 6–9 December 1947.

The basic westerly current, computed with the customary geostrophic assumption following standard methods, has a pronounced maximum near latitude 40°N (fig. 2), where it resembles a jet stream (*cf.*, University of Chicago, 1947) and where its baroclinity is very pronounced. South of 25°N there is a renewed increase of wind speed at 300 mb, so that a secondary maximum of westerly speed and baroclinity appears near 20°N. Although the secondary peak is small, it is remarkable that it is observed at all, especially as a mean condition over several days.

A pronounced wave pattern exists at 700 mb (fig. 3), and in this instance there is little evidence of diverging and converging contours, in contrast with the daily situation shown in fig. 1. The perturbations at 300 mb

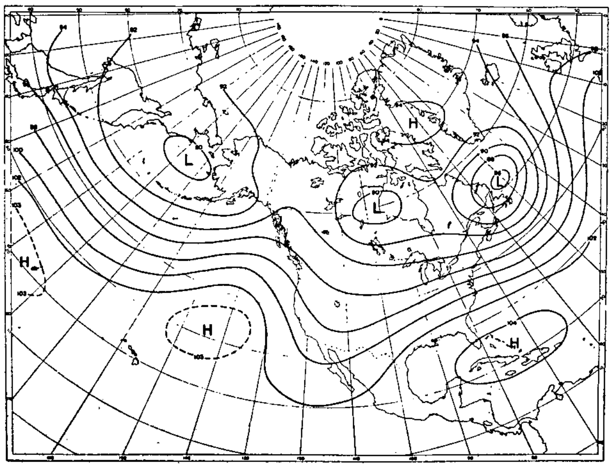

FIG. 3. 700-mb mean chart 6–10 December 1947 (contours in hundreds of feet). Data for this chart were obtained from the Extended Forecast Section, United States Weather Bureau.

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[1] H. Riehl and N. La Seur, "A study of high-tropospheric lapse rates, with application to the construction of 300-mb charts," 1949 (to be published).

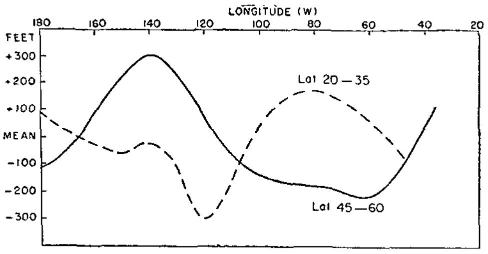

FIG. 4. Deviation of the height of the 300-mb surface (latitude belts 20°–35°N and 45°–60°N) from the average height for each of those belts, 6–9 December 1947.

can be presented in several ways. We can define the height of the 300-mb surface at any point as being composed of two components—the mean height of the latitude circle on which the point is situated, plus the deviation from this mean height. Since the former quantity is used to determine the basic zonal current, the deviation represents the perturbation at the point. If we average the deviations in the latitude belts 45°–60°N and 20°–35°N, omitting the intermediate transition zone, we obtain a plot of average deviation against longitude as shown in fig. 4. It is at once apparent from this figure that wave-like perturbations exist in both latitude belts and that these disturbances are situated out of phase. Moreover, the amplitude of both sets of waves is nearly equal, whereas one might have expected prevalence of smaller amplitudes in lower latitudes, particularly since the latitude belt 20°–35°N comprises a much larger area than the belt from 45° to 60°N.

The complete perturbation field appears in fig. 5. This chart has been obtained by subtracting the average height of each latitude circle from the 300-mb height values at each 10-degree intersection of latitude and longitude, and then drawing isolines of equal deviation. Perhaps the most interesting feature revealed by this figure are the singularities that appear in the perturbation height field. There are definite cyclonic and anticyclonic perturbation centers. Evidently we are confronted with the large-scale turbulence eddies stressed by Rossby on many occasions, most recently in a paper on the general circulation (Rossby, 1947).

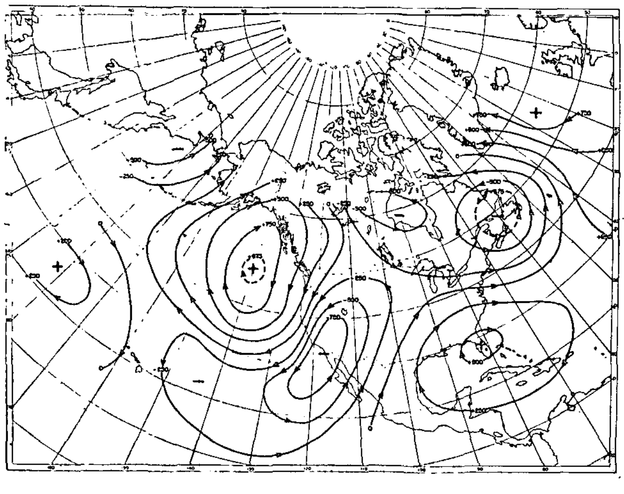

FIG. 5. Isolines of equal deviation of the height of the 300-mb surface (feet) from the average height for each latitude circle, 6–9 December 1947.

The opposition between the centers of high and low latitudes is again evident in fig. 5. In higher latitudes, the perturbation centers are situated near and north of 50°N; in lower latitudes they lie near and south of 30°N. One may conclude that two long-wave trains are defined by figs. 4 and 5. The southern train has a smaller wave length in degrees longitude, though not in miles, and the out-of-phase position of the two trains of disturbances therefore cannot be continuous around the globe, as is indeed evident near both edges of the map. Interference between the two trains is most obvious near the west coast of the United States and over the eastern Pacific Ocean.

The relative position between perturbation and jet-stream maxima is also of interest. Both sets of perturbation centers are located about 5–10 latitude degrees north of the two peaks in the westerly flow. Experience with several months of perturbation charts has demonstrated that this is not always the case, and that the disturbances experience changes in latitude relative to the associated jet streams.

As seen from the foregoing, it is an evident advantage of the perturbation chart that it adds a second coordinate—latitude—for the definition of long-wave bands. In the present case this method of charting the long waves leads to a considerably different interpretation of the weather situation from that obtained by inspection of fig. 3. This variance may be ascribed in part to differences of the synoptic picture between the 300- and 700-mb levels since the disturbances, particularly those of lower latitudes, are much weaker at 700 mb than at 300 mb. In part, however, the disagreement stems from the fact that at 700 mb the trough over the western United States and the ridge over the eastern part of the country appear as parts of a single long-wave train, whereas the great difference in latitude of these perturbation centers at 300 mb, as compared with those over the Atlantic and Pacific Oceans, suggests the definition of two wave trains. This conclusion is supported by fig. 4 and by the regular line-up of the perturbation centers relative to the. jet-stream maxima. Nevertheless, it is also possible to argue that all centers, excepting that in the lower latitudes of the central Pacific Ocean, form a continuous chain which is displaced farther from the pole over North America than over the adjacent oceans. It is not the purpose of this paper to determine which of these solutions is more proper. The objective is merely to illustrate a technique that brings out several features of the perturbation field which are rather important as long as any value is attached to the concept of the division of the total flow field into a nondivergent basic current and perturbations. Experience with daily perturbation charts covering a much larger portion of the globe than is considered here leads to the conclusion that two or three trains of disturbances can frequently be identified, which supports the literature initially quoted.

4. Perturbation indices

The perturbation chart can be used as a basis for several types of numerical calculations. Since the isolines of equal deviation are also the streamlines of the geostrophic perturbation motion, the geostrophic part of the perturbed wind field can be computed. This would not be possible if the motion associated with the disturbances were small compared to that of the basic current. Actually, however, both velocity fields have the same order of strength in the belt of westerlies, and in the tropics the perturbations predominate.

A valid objection to such calculations comes from the fact that the cyclostrophic component of motion often is very large at 300 mb. It is preferable, therefore, for many purposes to make computations with use of the perturbation heights only. The simplest index is the height deviation of the individual centers, the changes of which can be followed from day to day as are those of surface highs and lows. A second index is furnished by integration over each cyclonic and anticyclonic perturbation. Such integrals also permit comparison between the intensity of cyclonic relative to anticyclonic perturbations, if the area covered by each perturbation is taken into account. It should again be mentioned that the intensity, as defined here, is not necessarily proportional to quantities such as the kinetic energy. Further information concerning larger areas can be gained by a summation of all height deviations along each parallel without regard to sign. The resulting number can be interpreted as indicating the strength of the perturbation field. If the integration is extended over all latitudes, one obtains a single measurement of perturbation strength for the whole map. Experimentation with the indices described and other quantities is being carried out at present.

5. Synoptic development

Within the framework of the broadscale situation outlined above, a somewhat uncommon type of cyclogenesis took place in the eastern Pacific Ocean along the flight route from Honolulu to the American mainland. Between December 6 and 8 an east–west shearline (fig. 1) moved southward in that area, as indicated by a time section of radiosonde and rawin observations of the weather ship at 30°N, 140°W and the reports along the west coast of the United States. This situation is somewhat reminiscent of a beginning "surge of the trades" and transformation of a subtropical anticyclone at high levels. At the same time two troughs in the low-latitude westerly current, shown in fig. 1, advanced eastward from the central Pacific. The motion of the troughs could be determined with fair accuracy from the time sections of Honolulu, Midway Island, and Johnston Island, whereas the displacement of the east–west shearline had to be estimated south of the guardship.

On December 8, these secondary disturbances approached each other and also moved toward a region of broadscale cyclonic conditions (fig. 5). It is well known that secondary troughs moving toward an area of a mean trough will intensify (Fultz, 1945), and the previously mentioned studies on low-latitude conditions have demonstrated that superposition of troughs moving toward each other in low and high latitudes generally results in intensification of low-latitude troughs, and usually also in intensification of high-latitude troughs. Moreover, the principal eastward trough displacement took place in the lowest latitudes so that all disturbances experienced counterclockwise rotation relative to each other rather than the common clockwise rotation. This is another circumstance favorable for cyclogenesis (Riehl, 1947).

Taking all these considerations into account, deepening should be expected to occur between Honolulu and the guardship at, and following, the map time reproduced. This actually happened, and severe thunderstorms developed over a large portion of the cyclogenetic area, affecting operations of American and Australian airlines during the night of December 8–9. Wind observations taken by pilots indicated the presence of at least a strong cyclonic deformation of the trade stream near 700 mb, and several winds with west component were reported, suggesting the existence of closed cyclonic streamlines. Under these circumstances, a pronounced cyclonic center should have existed at that time at 300 mb.

6. Subsequent synoptic development

As already noted, the out-of-phase relation observed between the perturbation trains of higher and lower latitudes 6–9 December 1947—if this interpretation of fig. 5 is granted—could not have persisted around the globe since the angular wave length varied with longitude. During the following days relative motion between these trains took place so that they achieved an in-phase position over North America on December 15, which persisted through December 18. It is of interest to compare conditions during the period December 15–18 with those prevailing December 6–9.

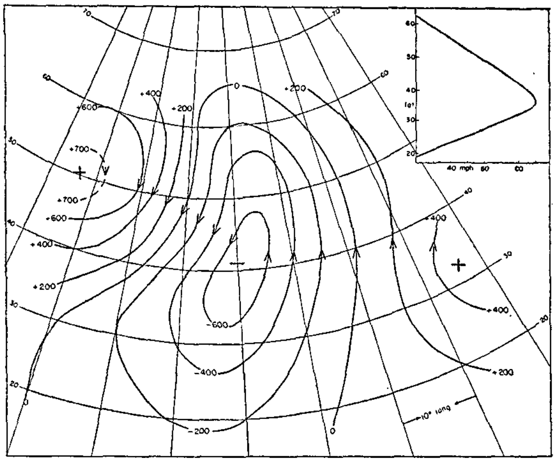

FIG. 6. Isolines of equal deviation of the height of the 300-mb surface from the average height for each latitude circle, 15–18 December 1947. Deviations are computed in a coordinate system moving eastward with the speed of a trough line. This trough line is represented by the central meridian in the figure. Insert: latitudinal profile of basic westerly current at 300 mb, 15–18 December 1947.

The profile for the basic current from December 15–18 appears in the insert of fig. 6. It is more difficult to show the perturbations for this series than for the preceding one since all major disturbances were in motion during the period. A chart computed exactly as fig. 5 would lead to a distorted picture. We may, however, avail ourselves of another technique which is standard in theoretical work, namely, to introduce a coordinate system that moves with the speed of the waves. As the speed varied with longitude and latitude on this occasion, the analysis had to be limited to a disturbance which was traveling eastward across the North American continent, moving at a rate of 7 degrees longitude per day. Although such a restriction is not desirable for many purposes, it is easily overcome if all computations are carried out on a daily basis.

In order to obtain the configuration of the North American disturbance, first the basic current was computed for each day and subtracted from the corresponding map. The resulting deviation patterns were then averaged, using the trough position on each day as common reference longitude. The composite deviation pattern which is shown in fig. 6 differs greatly from that noted in fig. 5. Only one set of perturbation centers is in evidence and the intensity of these centers gradually diminishes northward and southward. At the same time the double maximum of the westerly jet has disappeared and a single jet, stronger than that prevailing from December 6–9, is situated somewhat south of latitude 40°N. The center of the cyclonic disturbance also is observed near or slightly north of that latitude.

7. Conclusion

This paper has given an illustration of the manner in which simple theoretical methods of wave treatment can be introduced into synoptic practice. Application of the hydrodynamic concept of wave or vortex trains moving in a nondivergent zonal current to upper-air charts leads to the appearance of finite cyclonic and anticyclonic perturbation centers. Whereas previous investigations of long waves aloft have defined these disturbances primarily by their longitude, the perturbation method adds latitude as a second coordinate for their description.

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