PRELIMINARY REPORT ON TEMPERATURE MEASUREMENT BY SONIC MEANS

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ABSTRACT

Following a brief review of the assumptions involved in the usual (Laplacian) expression for the speed of sound waves, an instrument, the sonic thermometer, is described which utilizes this relationship to measure the air temperature. The advantages of the sonic thermometer are then discussed, the main advantages being the absence of radiational errors and extremely low lag—a result of the fact that the measured variable, the speed of sound, is independent of the properties of the measuring elements.

During the past year a program of investigation into the practicability of determining free-air temperatures by measurements of sound velocity has been carried out in the experimental meteorological laboratory at the University of Chicago under contract with the Signal Corps Engineering Laboratories. The ultimate objective of this investigation has been to develop practical equipment which can be utilized in routine balloon soundings from the ground to altitudes of about 30 km. Measurements should be accurate within one degree centigrade and should be rendered as independent of other physical quantities as possible.

1. Basic theory of the measurement

It is shown in standard works on acoustics or physics that the velocity of an adiabatic compressional wave in a homogeneous gaseous medium may be expressed by the relation

$$c^2 = \rho^{-1}\gamma p, \tag{1}$$

where c is the speed of wave propagation, $\gamma = c_p/c_v$ is the ratio of the specific heats of the gas, p is the static pressure, and ρ is the density of the gas. In place of the density the equivalent expression from the equation of state, $pm = \rho RT$, may be substituted (where m is the molecular weight, R is the universal gas constant, and T is the absolute temperature), giving an expression which is independent of static pressure:

$$c = (\gamma RT/m)^{\frac{1}{2}}, \tag{2}$$

thus eliminating from primary consideration this widely varying quantity. On substitution of numerical values for the constants γ, R, and m (for dry air), this equation reduces to

$$c = 20.067T^{\frac{1}{2}}, \tag{3}$$

where c is expressed in m sec^{-1}, yielding a simple direct relationship between the acoustical wave velocity and the prevailing temperature.

Having derived this fundamental relationship between the desired quantities, it next becomes necessary to consider the dependence of the relationship upon those other physical variable phenomena which are encountered in the atmosphere. In this connection it is necessary to consider some of the tacit assumptions underlying equation (1). In the derivation of that equation it is assumed that the incremental pressure due to the sound wave is very small compared to the static pressure of the medium. This condition is met in the atmosphere near the surface of the earth even with quite loud sounds; however, at heights of 30 km the sound pressures from intense sources become an appreciable fraction of the ambient pressure (10 mb), and the velocity becomes a function of the ambient pressure. Thus, the sound impulse cannot be transmitted at excessively high intensity if the desirable simplicity of the foregoing relations is to be retained. Fortunately this imposes no serious practical limitations on the present investigation, since the pulse amplitudes employed are small even in comparison to the pressure at 30 km.

A second limiting assumption is made in treating the quantity γ as constant for a given gas or mixture, independent of other physical variables. It has been established by experiment that a velocity dispersion with respect to the sound frequency exists for ultrasonic waves of very high frequency, the velocity showing an increase with frequency. An explanation for this dispersion has been put forward [1; 2], based upon the assumption that owing to the great accelerations of the molecules produced by high-frequency sound waves, energy imparted to the molecules can-

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not be distributed among the various vibrational degrees of freedom in accordance with equilibrium theory. It appears that a small but finite time ("relaxation time") is required for a molecule to adjust itself to a change in total energy imposed from outside the molecule; if the sonic pressure changes act in too rapid succession, the molecule cannot achieve equilibrium. Since γ depends upon the specific heats, c_p and c_v, and these in turn depend upon the energy distribution among the various translational, vibrational, and rotational degrees of freedom of the molecules, it follows that γ cannot be regarded as a constant, but is a frequency-dependent variable. Practical considerations show, however, that dispersion can be neglected when the sound frequencies are less than about 5×10^5 cycles $\sec^{-1}$.

A third restriction imposes greater limitations and requires an empirical correction factor. It is obvious that the quantities γ and m must be considered as functions of chemical composition of a gas or mixture. Observational evidence obtained to date indicates that the composition of the atmosphere remains substantially constant up to heights in excess of 20 km, with the exception of water vapor and carbon dioxide.

The small carbon-dioxide content of the normal atmosphere may be neglected in this study; however, the water-vapor variation cannot be neglected without causing errors in excess of one degree centigrade since it is found that the speed of sound in pure water vapor is considerably higher than in dry air. On the assumption that the speed of sound may be regarded as a linear function of the relative concentration of the components of a mixture, it is possible to compute a correction factor based on the vapor pressure of water. This has been done by Ishii [1] who determined this factor both theoretically and experimentally. His correction may be expressed as a multiplier under the radical in equation (3), resulting in the complete practical equation:

$$c = 20.067[T(1 + 0.3192e/p)]^{\frac{1}{2}} \qquad (4)$$

(with e, the vapor pressure, and p expressed in the same units).

The similarity between the expression in brackets in (4) and the common meteorological quantity known as "virtual temperature" leads to the name "sonic virtual temperature," which we have assigned to the quantity $T(1 + 0.3192e/p)$. If one makes a measurement of the sonic velocity and applies equation (3) directly without moisture corrections, it is the sonic virtual temperature which one obtains. At a temperature of 30C and of 30 per cent relative humidity the sonic virtual temperature is approximately 31C. As in the case of ordinary virtual temperature, the correction becomes small at higher altitudes in the atmosphere and may be neglected above about 15,000 ft.

2. Experimental and practical considerations

In order to utilize the theoretical relationship already derived, it is necessary to *(1)* generate sound waves by electrical or mechanical means, *(2)* transmit the sound through a region of the atmosphere, *(3)* receive the sound and convert it to electrical impulses, *(4)* express the time of travel of the sound wave in terms of some electrical parameter, and *(5)* provide for radio telemetering.

A considerable number of experimental systems have been tested in the laboratory. Of these, one particular system has shown itself to be more accurate and reliable than the remainder, although it is somewhat complex electrically. It has been found that the more simple and obvious methods fail to yield satisfactory results because of the inevitable deviations of actual acoustical and electronic components from their ideal theoretical prototypes.

In the system now under discussion, which has been named the "pulse-feedback system," a short pulse of high-frequency sound is generated by applying a pulse of voltage to a piezoelectric crystal. Due to the oscillatory characteristics of such a crystal, the sound is emitted in the form of a damped train of sinusoidal waves. This wave train is allowed to propagate through a fixed distance, after which it is received by another piezoelectric crystal whose characteristics are as similar as possible to those of the transmitter. Due to the reversibility of the piezoelectric effect, a small voltage wave train is produced by the receiving crystal. This voltage is then amplified by means of vacuum tubes to a usable level. By the use of a detector and wave filter, the individual sinusoidal oscillations may be stripped from the voltage wave, and a new voltage wave having the shape of the envelope of the original wave train is produced. The leading edge of this envelope is then sharpened by suitable clipper amplifiers and peaking circuits so that a sharp pulse or "spike" of voltage is produced. This spike serves to "trigger" the pulse generator so as to cause a new pulse to actuate the transmitting crystal. Thus each pulse sent out by the transmitter gives rise to the next, after it has traversed the fixed distance. The time interval between pulses is determined by the time of travel of the sound pulse, which is a function of the (fixed) distance and the speed of sound, and by the time required for the electrical impulses to pass through the vacuum tubes and circuits, which should for best results be constant, but which varies very slightly in practice. With a path length of about 16 inches the travel time varies between about 1000 and 1200 microseconds depending upon the air tempera-

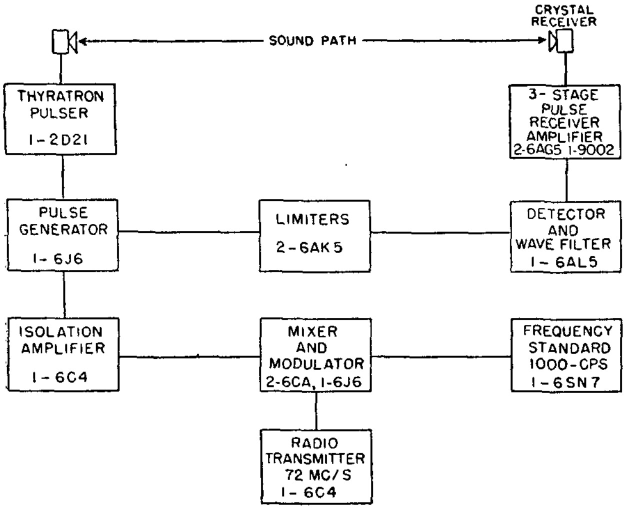

Fig. 1. Block diagram of laboratory model of pulse-feedback sonic thermometer.

ture, while the electrical time delay has been calculated as 33.7 microseconds for strong impulses and slightly greater for weak signals.

Since the pulses are generated successively and continually, they may be said to possess a frequency or repetition rate which equals numerically the reciprocal of the sum of the travel time plus the electronic delay time. As will be shown in the following paragraph, this frequency is a measure of the velocity and hence of the temperature. If these pulses are allowed to modulate a radio transmitter, the desired data may be easily telemetered to the ground and measurement of the frequency made at the ground station. The use of a frequency as the recorded intelligence facilitates the measurement, since standard techniques are available for frequency measurement and recording.

Fig. 1 illustrates a block diagram of the system while fig. 2 is a photograph of the experimental apparatus as used for measurements in the laboratory.

The relationship between frequency, distance, sonic velocity, and circuit delay time may be expressed as

$$f^{-1} = c^{-1}d + t_0,$$

where f is the frequency of pulse repetition, d is the fixed distance traversed by the sound, c is the speed of sound, and t_0 is the circuit delay time. Thus, with d and t_0 measured, a direct relationship between f and c is obtained and a calibration chart may be drawn.

Simulated ascents have been made with the laboratory model. Operation has been obtained up to pressure altitudes of 88,000 ft, and it is expected that when a stronger source of voltage pulses to actuate the transmitting crystal has been constructed, this range may be extended to 100,000 ft or higher. Failure of operation at low pressures occurs because at low pressures the acoustical energy is transferred from the crystal transmitter to the air and from the air to the receiving crystal with very low efficiency so that the received signal amplitude is very small as compared to that obtained under surface conditions. Additional amplification of the received signal does not overcome the difficulty, since stray acoustical noise and circuit and tube noises are amplified along with the desired pulse. The operation of the system depends on the signal to noise ratio; once the noise has been reduced to as low a level as the state of the art permits, operation at lower pressures depends on increasing the strength of the transmitted pulse. Experimental data indicate that in order to obtain operation at a pressure of 10 mb the sound intensity must be 200 times greater than the minimum necessary for operation at 1000 mb.

3. Meteorological significance

Having shown that it is possible to automatically and remotely measure the velocity of sound at various levels throughout the free atmosphere, it becomes desirable to indicate those special advantages which accrue from the use of this technique as contrasted with other methods of temperature measurement now employed. The foremost of these advantages, from the viewpoint of higher-atmosphere sounding, arises from the almost complete indifference to radiation effects resulting from the sonic technique. All existing thermometer devices with which the writers are acquainted make use of some piece of matter as a measuring element, with the assumption that this piece of matter is in thermal equilibrium with the atmosphere in its vicinity. The actual measurement involves a determination of the numerical value of some physical variable quantity *within* this piece of matter, with the supposition that a unique correspondence exists between this variable (electrical resistance, volume, density, thermal e.m.f., *etc.*) and the "temperature" of the surrounding air. Because of the fact that thermometers of these types have been most useful practically, physicists and engineers have defined "temperature"

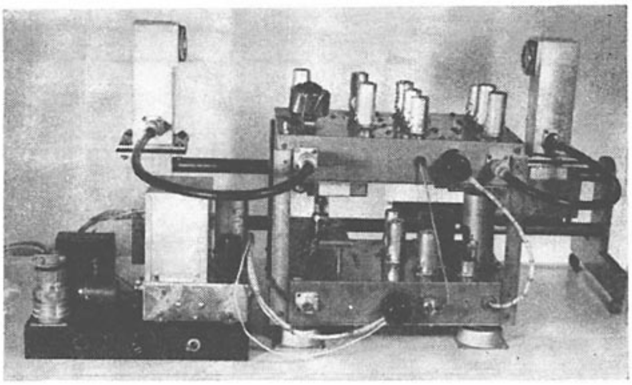

Fig. 2. Photograph of laboratory model of pulse-feedback sonic thermometer.

in terms of these physical variables *inside* the piece of matter used in the thermometer.

Experience indicates that for a very large number of applications these techniques and definitions are satisfactory. For the case of atmospheric temperature in particular, however, care must be exercised, since the piece of matter may be acted upon by two principal physical variables. The first of these is the air temperature, which for purposes of this paper will be defined as being directly proportional to the mean translational kinetic energy for a large number of air molecules. In the absence of any other physical forces or energies, experience and thermodynamic considerations indicate that the molecules composing the thermometer will, in time, acquire energy from the air until the net transfer of energy between air and thermometer approaches zero. Under these conditions, the internal properties of the thermometer may be considered as uniquely determined by the energy of the surrounding air, *i.e.*, the air temperature. However, as the instrument ascends higher into the atmosphere, the number of molecules which can exchange energy with the thermometer decreases, while another physical factor, namely radiant energy from the sun, becomes more prominent. This radiation is capable of supplying energy to the thermometer quite independently of the effect of air molecules. In the limiting case, the effect of air molecules approaches zero and the "thermometer" becomes a pyrheliometer or bolometer and measures "temperature" at which energy gained by radiation equals that lost by radiation. Thus it is clear that two conflicting definitions of temperature at a point in the atmosphere can exist and that ordinary thermometers cannot clearly distinguish between them. Practical devices such as radiation shields and highly reflective coatings serve to reduce this ambiguity but cannot entirely eliminate it.

In contrast to the conventional thermometer, the sonic thermometer yields a measurement based upon the very property which is used to define molecular temperature, namely the molecular energy of the air. Since it is unnecessary to utilize an intermediate step involving the internal properties of a piece of matter, the ambiguity in the meaning of "temperature" is reduced. Thus the introduction of sonic thermometers would tend to reduce the confusion which confronts physicists of the upper atmosphere who attempt to abstract information from ordinary radiosonde data at high levels.

The second major advantage may be considered as a corollary of the first. Since no piece of matter acts as an intermediary in the measurement, the time lag associated with ordinary thermometers is not inherent in the sonic technique. No transfer of "heat energy" from air to thermometer, with the usual exponential relationship to time, is required. A time lag does exist, but the magnitude of this lag is of the order of 0.001 sec; that is, the time required for one pulse to travel the fixed distance. Thus, the sonic technique is highly suited to measurements of temperature where the temperature varies widely in short time intervals. This feature makes the technique invaluable for micrometeorological studies of temperature variation.

A third advantage obtainable by the sonic technique involves the property of space integration which is inherent in the method. Because of the high velocity of sound waves it is possible to lengthen the distance through which the sound travels to fairly large values, of the order of hundreds of meters. If the atmosphere through which the sound passes is not homogeneous in temperature, the temperature indicated by the sonic method represents an average over the travel distance. The integration cannot be considered as absolutely independent of time, of course, since the velocity of sound cannot be infinite; however, the time interval over which the integration is made is so small in comparison with the rapidity of local temperature fluctuations in the atmosphere that for practical purposes the measurement may be regarded as a space integration made at a particular instant of time. Such space integrations may be of value in determining representative surface temperatures of air masses by elimination of the local effects which affect "spot" temperature measurements at weather stations and by at least a partial averaging of such variations in temperature as are due to local ground heating and/or turbulent motion along the path.

The only disadvantage at present involved in the use of a sonic thermometer appears to lie in the relative complexity of the apparatus required; this may be regarded as primarily an economic rather than a scientific objection, and in any case may be minimized by future research.

In summary, it may be safely concluded that the sonic technique of temperature measurement constitutes a major improvement over other procedures because of (*a*) its indifference toward radiation, (*b*) its measurement of a property of the air directly related to air temperature, without the use of intermediate matter which must establish thermal equilibrium with the air, (*c*) its inherent freedom from the exponential time lag associated with ordinary thermometers, and (*d*) its utility in connection with space integrations of air temperature to obtain more representative values for synoptic studies.

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