The Lateral Transport of Zonal Momentum Due to Kelvin Waves in a Meridional Circulation

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ABSTRACT

A modified, equatorial Kelvin wave solution is obtained in the presence of the zonal-mean meridional circulation. The modified Kelvin wave solution, which is obtained via a perturbation expansion of the linearized, primitive equations on an equatorial β plane, possesses a nonzero meridional wind component. This meridional wind component is absent when the background flow is at rest. The combination of the meridional and zonal winds induces a meridional flux of zonal momentum in the upstream direction of the background north–south flow. This flux is divergent in latitude and produces a nonzero wave-induced force even though the waves are linear, steady, and conservative. It is shown that, although such a force violates the traditional nonacceleration theorem in which the mean meridional circulation is negligible at leading order, the result is in accord with a more general nonacceleration theorem obtained from the exact generalized Lagrangian-mean theory in which the mean meridional circulation is nonzero. The meridional circulation, in effect, attempts to advect wave pseudomomentum off the equator, resulting in a nonzero acceleration in the Eulerian reference frame. The meridional flux of momentum for any equatorially trapped mode is derived from the generalized Lagrangian-mean theory. Those modes with eastward (westward) intrinsic phase velocity transport eastward (westward) zonal momentum in the upstream direction of the background meridional flow in the neighborhood of the equator. It is also shown that the vertical flux of zonal momentum is not constant with altitude in a steady vertical flow since diabatic heating/cooling is needed to sustain the vertical wind. Implications of the results for the terrestrial and Venusian atmospheres are discussed.

1. Introduction

The vanishing of the Coriolis effect at the equator supports a class of equatorially confined waves (Matsuno 1966). Those equatorial waves play important roles in the dynamics of the equatorial atmosphere. In this paper, we focus on the Kelvin wave, which is the gravest symmetric mode of the equatorial waves. The Kelvin wave propagates in the direction of the planetary rotation as an internal gravity wave, with its structure being geostrophically balanced in the meridional direction. The zonal wind and geopotential perturbations have maxima at the equator, while the meridional wind is absent in a resting background wind. The upward transport of zonal momentum by Kelvin waves is considered to play an important role in driving the quasi-biennial oscillation of the equatorial stratosphere (Baldwin et al. 2001). Kelvin waves are also observed in the Venusian atmosphere and are expected to influence the mean wind (Del Genio and Rossow 1990).

The influence of meridionally sheared zonal flows on equatorial waves has been extensively studied (Lindzen 1971; Andrews and McIntyre 1976b; Boyd 1978; Zhang and Webster 1989). Regarding the Kelvin wave, Boyd (1978) showed that (i) the shear induces a nonzero perturbation in the meridional wind and (ii) the zonal wind and geopotential perturbations are not affected significantly. The phase of the meridional wind is shifted by 90° relative to the zonal wind. Thus, it does not induce the meridional flux of zonal momentum.

Reported here for the first time is the Kelvin wave solution in the presence of a meridional circulation. In section 2, the solution is derived using an asymptotic

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series expansion, and it is shown that it has a nonzero meridional wind disturbance. Section 3 shows that the wave induces the meridional flux of zonal momentum, producing its divergence and/or convergence near the equator. Section 4 presents an alternative derivation of the momentum transport by using the generalized Lagrangian-mean theory (Andrews and McIntyre 1978b). The derivation enables us to calculate the momentum transport for any equatorially trapped mode. Section 5 discusses the implication of the results for the terrestrial and Venusian atmospheres. Section 6 states the conclusions.

2. Derivation of wave solution by perturbation method

We employ an asymptotic series expansion, similar to that of Boyd (1978), in which the advection by mean flow is treated as a small but finite perturbation in the wave equation. Adopting an equatorial β plane moving with the mean zonal wind at the equator, primitive equations are written as (Andrews et al. 1987)

$$D_t u - \beta y v + \frac{\partial \phi}{\partial x} = X, \tag{1}$$

$$D_t v + \beta y u + \frac{\partial \phi}{\partial y} = Y, \tag{2}$$

$$\frac{\partial \phi}{\partial z} = \frac{R}{H}\theta e^{-\kappa z/H}, \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w) = 0, \tag{4}$$

$$D_t \theta = Q, \tag{5}$$

with

$$D_t \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}. \tag{6}$$

Here u, v, and w are the zonal, meridional, and vertical winds, respectively; ϕ the geopotential; t the time; x the zonal distance; y the meridional distance; z the log-pressure height; H the constant log-pressure scale height; β the meridional gradient of the Coriolis parameter; R the gas constant; θ the potential temperature; κ the ratio of specific heats; ρ_0 the basic density, which is proportional to $e^{-z/H}$; X and Y the nonconservative mechanical forcing in the x and y direction, respectively; and $Q \equiv (J/c_p)e^{\kappa z/H}$ the diabatic heating term, where J is the diabatic heating per unit mass and c_p the specific heat. The distances x and y increase with the direction of the planetary rotation and northward, respectively, with y originating at the equator. Then β is always positive.

We first consider a steady, zonally symmetric basic state in which a meridional circulation exists and the wind field is independent of z. Denoting the basic state by an overbar and suffix zero, we have, from (1)–(5),

$$\overline{v}_0\frac{\partial \overline{u}_0}{\partial y} - \beta y \overline{v}_0 = \overline{X}_0, \tag{7}$$

$$\overline{v}_0\frac{\partial \overline{v}_0}{\partial y} + \beta y \overline{u}_0 + \frac{\partial \overline{\phi}_0}{\partial y} = \overline{Y}_0, \tag{8}$$

$$\frac{\partial \overline{\phi}_0}{\partial z} = \frac{R}{H}\overline{\theta}_0 e^{-\kappa z/H}, \tag{9}$$

$$\frac{\partial \overline{v}_0}{\partial y} + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 \overline{w}_0) = 0, \tag{10}$$

$$\overline{v}_0\frac{\partial \overline{\theta}_0}{\partial y} + \overline{w}_0\frac{\partial \overline{\theta}_0}{\partial z} = \overline{Q}_0. \tag{11}$$

When the basic-state absolute angular momentum depends on latitude, a forcing term $\overline{X}_0$ is required to balance the zonal momentum equation (7) when $\overline{v}_0 \neq 0$. In the present study, such a zonal forcing is attributed to the convergence of eddy momentum fluxes associated with disturbances that exist prior to introducing the Kelvin wave. We further assume that $\overline{Y}_0$ is independent of z. Then, (8) yields $\partial^2\overline{\phi}_0/\partial z\partial y = 0$ since the basic wind field is assumed to be independent of z. The results obtained under these assumptions, however, could be extended to the cases of a basic state with sufficiently small vertical shear.

We now consider a small disturbance to this basic state for each variable Z. Disturbance quantities Z' are taken to be $O(\alpha)$ or smaller when nondimensionalized, where α is a dimensionless parameter of nonlinearity and is assumed to be much smaller than 1. It is also assumed that the departure of zonal mean $\overline{Z}$ from $\overline{Z}_0$ is $O(\alpha^2)$ for each variable:

$$Z = \overline{Z}_0(y, z) + Z'(x, y, z, t) + O(\alpha^2). \tag{12}$$

The zonal mean should be a function of t, but its t dependence is assumed to be small for the discussion of the wave structure and is neglected here. Hereafter, we neglect the difference between $(\overline{u}_0, \overline{v}_0, \overline{w}_0, \overline{\theta}_0)$ and $(\overline{u}, \overline{v}, \overline{w}, \overline{\theta})$, and the latter is used throughout the rest of the paper.

The substitution of (12) into the primitive equations (1)–(5) and the use of (7)–(11) then give the following set of linear equations at $O(\alpha)$:

$$\overline{D}_t u' + \left(\frac{\partial \overline{u}}{\partial y} - \beta y\right)v' + \frac{\partial \phi'}{\partial x} = 0, \tag{13}$$

$$\overline{D}_t v' + \frac{\partial \overline{v}}{\partial y}v' + \beta y u' + \frac{\partial \phi'}{\partial y} = 0, \tag{14}$$

$$\overline{D}_t \frac{\partial \phi'}{\partial z} + \overline{w}\frac{\kappa}{H}\frac{\partial \phi'}{\partial z} + w'N^2 = 0, \tag{15}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w') = 0, \tag{16}$$

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$$\overline{D}_t \equiv \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z}; \tag{17}$$

N is the log-pressure buoyancy frequency, which is defined by $N^2 \equiv H^{-1}\mathrm{Re}^{-\kappa z/H}\partial\overline{\theta}/\partial z$; terms of $O(\alpha^2)$ have been neglected; and the nonconservative terms (X', Y', Q') are assumed to be absent for simplicity. Note that the advection of disturbance quantities by the meridional circulation is not neglected in the present study, in contrast to the conventional linear wave theories that assume the meridional circulation to be $O(\alpha^2)$ or smaller.

For simplicity, we assume the following basic meridional and vertical winds satisfying (10):

$$\overline{v} = V_a + V_s y, \tag{18}$$

$$\overline{w} = HV_s. \tag{19}$$

The parameter V_a represents a cross-equatorial northward flow ($V_a > 0$) or southward flow ($V_a < 0$). The parameter V_s represents a simple equatorially symmetric flow with horizontal divergence ($V_s > 0$) or convergence ($V_s < 0$), with the former (latter) representing, for example, the upper (lower) branch of a symmetric Hadley circulation. The basic zonal wind is also taken into account after Boyd (1978):

$$\overline{u} = U_c + U_a y + U_s y^2, \tag{20}$$

where U_c represents the constant component, U_a the linear shear across the equator, and U_s the quadratic shear. We may assume, without loss of generality, that $U_c = 0$ since the equatorial β plane moving with the mean zonal wind at the equator is adopted.

Since the mean winds $\overline{u}$, $\overline{v}$, and $\overline{w}$ are independent of height, the linearized equations can be solved by using the conventional separation of variables. We seek a wave solution with variables depending on x, z, and t as $\exp[ikx + (im + 1/2H)z - i\omega t]$, where k and m are the zonal and vertical wavenumber, respectively, and ω is the frequency. The frequency ω is set to be positive, so m is negative when the phase velocity is downward, or the group velocity is upward. From (13)–(16), horizontal structure equations are obtained after separating out the dependence on x, z, and t as

$$-i(\omega + \Delta\omega)\hat{u} + \left(\frac{\partial\overline{u}}{\partial y} - \beta y\right)\hat{v} + ik\hat{\phi} = 0, \tag{21}$$

$$-i(\omega + \Delta\omega)\hat{v} + \frac{\partial\overline{v}}{\partial y}\hat{v} + \beta y\hat{u} + \frac{\partial\hat{\phi}}{\partial y} = 0, \tag{22}$$

$$-i\left(\omega + \Delta\omega + i\overline{w}\frac{\kappa}{H}\right)\epsilon\hat{\phi} + ik\hat{u} + \frac{\partial\hat{v}}{\partial y} = 0, \tag{23}$$

where $\hat{u}$, $\hat{v}$, and $\hat{\phi}$ represent the complex amplitudes of u', v', and ϕ', respectively;

$$\epsilon = \frac{m^2 + 1/4H^2}{N^2} \tag{24}$$

is the separation constant; and $\Delta\omega(y)$ is a y-derivative operator representing the Doppler shift due to the latitude-dependent zonal wind and the meridional circulation given by

$$\Delta\omega \equiv -k\overline{u} - \left(m - \frac{i}{2H}\right)\overline{w} + i\overline{v}\frac{\partial}{\partial y}. \tag{25}$$

For a given combination of ω and k, ϵ is obtained as an eigenvalue.

If neither the meridional shear nor the meridional circulation exists in the basic state, that is, $\Delta\omega = 0$, the Kelvin wave solution is derived as

$$\hat{u}_0 = \tilde{u}\exp\left(-\frac{y^2}{2L^2}\right), \tag{26}$$

$$\hat{\phi}_0 = \frac{\omega}{k}\hat{u}_0, \tag{27}$$

$$\hat{v}_0 = 0, \tag{28}$$

$$\epsilon_0 = \frac{k^2}{\omega^2}, \tag{29}$$

where $\tilde{u}$ is the zonal wind amplitude at the equator and $L = (\omega/k\beta)^{1/2}$ is the equatorial deformation radius. We now define a perturbation parameter δ as

$$\delta \equiv \left|\frac{\Delta\omega_L}{\omega}\right|, \tag{30}$$

where $\Delta\omega_L$ is $\Delta\omega$ at $y = L$ with $\partial/\partial y$ substituted by L^{-1}. If δ is much smaller than 1, (21)–(23) can be solved asymptotically with the zeroth order solution being (26)–(28):

$$\hat{Z} = \hat{Z}_0 + \hat{Z}_1 + O(\delta^2\alpha), \tag{31}$$

where $\hat{Z}_1$ is $O(\delta\alpha)$. Hereafter the corresponding real representations of $\hat{Z}_0$ and $\hat{Z}_1$ are written as Z'_0 and Z'_1, respectively.

Note that, although $|\overline{u}|$ and $|\overline{v}|$ increase with $|y|$ in the present formulation, the scaling with δ is valid, since the solution decays exponentially with $|y|$ (Boyd 1978). Note also that the terms of $O(\delta\alpha)$ are retained here, while terms of $O(\alpha^2)$ have been neglected when deriving the linearized equations. Then, the use of (21)–(23) formally requires $\delta > \alpha$. However, because $O(\alpha^2)$ terms involve either zonal mean or wavenumber $2k$ disturbances and thus are not relevant to the structure of the wavenumber k disturbance of interest here, the results of this study should be appropriate even when $\delta < \alpha$. For the Kelvin wave, α should be defined as

$$\alpha \equiv \frac{\tilde{u}k}{\omega}, \tag{32}$$

which is the ratio of the nonlinear advection ($u'\partial Z'/\partial x$) to the time rate of change ($\partial Z'/\partial t$).

Substituting (31) into (21)–(23), we obtain the first-order perturbation equations as in Boyd (1978):

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$$-i\omega\hat{u}_1 - \beta y\hat{v}_1 + ik\hat{\phi}_1 = i\Delta\omega\hat{u}_0 - \frac{\partial\overline{u}}{\partial y}\hat{v}_0, \tag{33}$$

$$-i\omega\hat{v}_1 + \beta y\hat{u}_1 + \frac{\partial\hat{\phi}_1}{\partial y} = i\Delta\omega\hat{v}_0 - \frac{\partial\overline{v}}{\partial y}\hat{v}_0, \tag{34}$$

$$-i\epsilon_0\omega\hat{\phi}_1 + ik\hat{u}_1 + \frac{\partial\hat{v}_1}{\partial y} = i\epsilon_0\left(\Delta\omega + i\overline{w}\frac{\kappa}{H}\right)\hat{\phi}_0 + i\epsilon_1\omega\hat{\phi}_0, \tag{35}$$

where terms of $O(\delta^2\alpha)$ have been neglected. Here, the modifications of the eigenvalue and the vertical wavenumber are also taken into account as $\epsilon = \epsilon_0 + \epsilon_1$ and $m = m_0 + m_1$ since a change in the coefficients of the equations requires a change in the eigenvalue. The $\Delta\omega$ in (33)–(35) is regarded as (25) with m_0 being inserted instead of m. Following Boyd (1978), the first-order solution in terms of δ is obtained as polynomials of y multiplied with $\exp(-y^2/2L^2)$. Substituting (26)–(29) into (33)–(35) and solving them by matching powers of y, one finds

$$\hat{u}_1 = -\frac{k^2}{\beta}\left(U_a y + \frac{U_s}{2}y^2\right)\hat{u}_0 + \left[\frac{m_0H - i/2}{\omega}V_s - i\frac{k^2V_a}{\omega}y - \frac{2\beta m_0H + i(2k\omega - \beta)}{4\omega^2}kV_sy^2\right]\hat{u}_0, \tag{36}$$

$$\hat{\phi}_1 = -\frac{k^2}{\beta}\left(U_a y + \frac{U_s}{2}y^2\right)\hat{\phi}_0 + \left[-i\frac{k^2V_a}{\omega}y - \frac{2\beta m_0H + i(2k\omega - \beta)}{4\omega^2}kV_sy^2\right]\hat{\phi}_0, \tag{37}$$

$$\hat{v}_1 = i\frac{k}{\beta}(U_a + U_sy)\hat{u}_0 - \frac{k}{\omega}(V_a + V_sy)\hat{u}_0, \tag{38}$$

$$\epsilon_1 = \frac{U_s}{\beta}\epsilon_0 + \frac{2m_0H - i\kappa}{\omega}V_s\epsilon_0. \tag{39}$$

The first and second terms on the rhs of (36)–(39) originate in the imposed zonal flow $\overline{u}$ and the meridional circulation $(\overline{v}, \overline{w})$, respectively. The solution is arbitrary up to a constant multiplied by the Kelvin wave solution (26)–(28), and the terms that are constants multiplied by $\exp(-y^2/2L^2)$ in $\hat{u}_1$ and $\hat{\phi}_1$ (denoted with $\hat{u}_{1,0}$ and $\hat{\phi}_{1,0}$ here) must satisfy the condition that

$$\hat{u}_{1,0} = \frac{k}{\omega}\hat{\phi}_{1,0} + \frac{m_0H - i/2}{\omega}V_s\hat{u}_0. \tag{40}$$

The solution above is obtained by taking $\hat{\phi}_{1,0} = 0$.

Equations (36) and (37) imply

$$\hat{u}_1 - \frac{m_0H - i/2}{\omega}V_s\hat{u}_0 = \frac{k}{\omega}\hat{\phi}_1. \tag{41}$$

When the meridional circulation is independent of y, that is, $V_s = 0$, (41) reduces to $\hat{\phi}_1 = (\omega/k)\hat{u}_1$, which is identical to (27) except that the zeroth-order fields have been replaced by corresponding first-order quantities. In this case, the sum of the first and third terms on the lhs of (33) is zero, and the sum of the first and second terms on the lhs of (35) is also zero. Then, noting that $\hat{v}_0 = 0$ in (33) and (34), we find that (i) the advection of wave fields by mean winds in Eqs. (33) and (35) are entirely balanced by nonzero $\hat{v}_1$ and ϵ_1 and (ii) the role of $\hat{\phi}_1$ and $\hat{u}_1$ is to balance the $\hat{v}_1$ term in the meridional momentum equation (34). The same observations were reported by Boyd (1978), who derived solutions without the meridional circulation.

Note that the derived ϵ is complex when $V_s \neq 0$. In this case, the vertical wavenumber m is also complex and its imaginary part is given by

$$m_i \simeq -\frac{\kappa V_s}{2m_0\omega}\left(m_0^2 + \frac{1}{4H^2}\right) \tag{42}$$

at leading order in δ from (24) and (39). This means that the wave amplitude varies with height. The imaginary part is attributed to the second term on the lhs of (15), which represents the influence of the mean vertical wind on the temperature disturbance. Since by (11) the background temperature does not change with time, thereby excluding adiabatic vertical displacement of surfaces of constant background potential temperature, the resulting mean vertical motion across these surfaces causes the magnitude of the disturbance potential temperature to change relative to the (assumed steady) background potential temperature. Evidently the breakdown of nonacceleration conditions in this example is attributable, not to the disturbance heating, but to mean heating that causes the wave amplitude to behave, as it were, in a nonconservative manner. Note also that the rhs of (42) is proportional to κ, and not H, indicating that the magnitude of the effect depends on the diatomic nature of the gas and not on compressibility per se.

3. Meridional transport of zonal momentum

Let us now consider the Eliassen–Palm flux (EP flux) associated with the Kelvin wave in the meridional circulation. The y component of the EP flux, $F^{(y)}$, is equal to $-\rho_0\overline{u'v'}$ since $\partial\overline{u}/\partial z = 0$. For the Kelvin wave, it is derived at leading order in δ as follows:

$$F^{(y)} \simeq -\rho_0\overline{u_0'v_1'} = \rho_0(V_a + V_sy)\frac{\overline{u_0'^2}}{c}, \tag{43}$$

where $c \equiv \omega/k$. Here, nonzero $F^{(y)}$ arises in the presence of the meridional circulation since the phase of $\hat{v}_1$ is 0° or 180° with respect to that of $\hat{u}_0$. On the other hand, nonzero $F^{(y)}$ is not induced by the basic zonal wind

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since the phase difference between $\hat{v}_1$ and $\hat{u}_0$ is $\pm 90°$ in this case. The direction of zonal momentum transport is upstream of the north–south flow, and the flux is equal to $-F^{(y)} \sim -\rho_0 \overline{v} c \exp(-y^2/L^2)/2$ when the wave has a saturation amplitude (limit of convective instability) $\tilde{u} = c$. Note, however, that the validity of the result is uncertain for a wave close to the saturation amplitude since $\alpha \sim 1$ in that case.

The z component of the EP flux is given by

$$F^{(z)} \simeq -\rho_0 \overline{u'w'} + \rho_0 \beta y \frac{\overline{v'\theta'}}{\overline{\theta}_z}, \tag{44}$$

with subscripts denoting partial derivative. The first term on the rhs of (44) is zeroth order in δ, while the second term is first order. Since both of the two terms depend on z as $\exp(-2m_i z)$, the divergence of $F^{(z)}$ is written, at leading order in δ, as

$$\frac{\partial F^{(z)}}{\partial z} \simeq 2m_i \rho_0 \overline{u_0' w_0'} \simeq \frac{\rho_0 \kappa c V_s}{N^2}\left(m_0^2 + \frac{1}{4H^2}\right)\overline{u_0'^2}. \tag{45}$$

From (43) and (45), the EP flux divergence $\nabla \cdot \mathbf{F}$ is given at leading order in δ by

$$\frac{1}{\rho_0}\nabla \cdot \mathbf{F} \simeq \frac{\partial}{\partial y}\left[(V_a + V_s y)\frac{\overline{u_0'^2}}{c}\right] + \frac{\kappa c V_s}{N^2}\left(m_0^2 + \frac{1}{4H^2}\right)\overline{u_0'^2}, \tag{46}$$

implying that $\nabla \cdot \mathbf{F}$ is not zero in the presence of a meridional circulation. This result, at first sight, appears inconsistent with the nonacceleration theorem, which states that $\nabla \cdot \mathbf{F}$ is $O(\alpha^3)$ or smaller when the disturbance is steady and dissipation is absent (e.g., Andrews and McIntyre 1976a). However, the theorem was derived by neglecting $\overline{v}$ and $\overline{w}$, assuming that they are of $O(\alpha^2)$ or smaller. It will be shown in section 4 that the occurrence of nonzero EP flux divergence for steady, conservative waves in a mean meridional circulation is, in fact, consistent with the generalized Lagrangian-mean theory.

It should be noted that the eddy-flux component of the residual mean circulation due to the Kelvin wave does not contribute to the zonal momentum balance at $O(\delta)$. The transformed Eulerian-mean zonal momentum equation is written as (Andrews et al. 1987)

$$\frac{\partial \overline{u}}{\partial t} + \left(\frac{\partial \overline{u}}{\partial y} - f\right)\overline{v}^* + \frac{\partial \overline{u}}{\partial z}\overline{w}^* = \frac{1}{\rho_0}\nabla \cdot \mathbf{F} + \overline{X}, \tag{47}$$

where f is the Coriolis parameter and $(\overline{v}^*, \overline{w}^*)$ is the residual mean meridional circulation, defined as

$$\overline{v}^* \equiv \overline{v} - \frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 \frac{\overline{v'\theta'}}{\overline{\theta}_z}\right), \tag{48}$$

$$\overline{w}^* \equiv \overline{w} + \frac{\partial}{\partial y}\left(\frac{\overline{v'\theta'}}{\overline{\theta}_z}\right). \tag{49}$$

Here the second terms on the rhs of (48) and (49) are the eddy-flux terms. Although the eddy flux $\overline{v'\theta'}$ is order δ (not zero), the second term on the rhs of (48) is $O(\delta^2)$ since the vertical divergence in this term introduces another order of δ. The second term on the rhs of (49) is order δ, but the third term on the lhs of (47) is zero since $\overline{u}_z = 0$. Therefore, to first order in δ, all the eddy terms are dropped from the lhs of (47): the Kelvin wave contributes to the zonal momentum balance only through the EP flux divergence. The same conclusions would apply if $\overline{u}_z$ were order δ. In section 5, we examine the momentum transport by Kelvin waves with a few examples.

4. Lagrangian derivation and generalization to other equatorial wave modes

In this section we show an alternative derivation of the momentum transport by using the generalized Lagrangian-mean (GLM) theory (Andrews and McIntyre 1978b). The derivation will provide a physical insight into the transport, demonstrating that a nonzero forcing of the Eulerian mean flow is required in the presence of a meridional circulation even when the conditions of the ''nonacceleration theorem'' apply, namely, steady and conservative waves (Andrews and McIntyre 1978b; Dunkerton 1980). We show, furthermore, that this type of zonal wind forcing occurs for any equatorially trapped mode and are able to anticipate the direction of the momentum transport near the equator.

a. GLM derivation and nonacceleration theorem

Andrews and McIntyre (1978b) derived exact equations that govern Lagrangian-mean motions of fluids. When the mean is taken along circles of constant latitude, the equation governing the Lagrangian-mean zonal wind $\overline{u}^L$ is written in the Cartesian coordinate on the equatorial β plane as

$$\overline{D}_t^L \overline{u}^L - \overline{(\beta y v)}^L - \overline{X}^L = \overline{D}_t^L A + \overline{\xi_x X^l} + \overline{\eta_x Y^l} + \overline{(p^l)_x q}. \tag{50}$$

Here,

$$\overline{D}_t^L \equiv \frac{\partial}{\partial t} + \overline{u}^L\frac{\partial}{\partial x} + \overline{v}^L\frac{\partial}{\partial y} + \overline{w}^L\frac{\partial}{\partial z^*}, \tag{51}$$

$$A \equiv -(\overline{\xi_x u^l + \eta_x v^l + \beta y \xi \eta_x}), \tag{52}$$

$$q \equiv \frac{1}{\rho(\overline{\theta}^L, p^\xi)} - \frac{1}{\rho(\theta^\xi, p^\xi)}; \tag{53}$$

p is the pressure; the Lagrangian-mean operator $\overline{(\ )}^L$ indicates the zonal average taken with respect to the displaced positions $\mathbf{x} + \boldsymbol{\xi}$ with $\boldsymbol{\xi}(\mathbf{x}, t)$ being the disturbance-associated particle displacement field; ξ, η, and ζ are the x, y, and z components of $\boldsymbol{\xi}$, respectively; ρ is the density; z^* is the geometric height; and the suffix x represents the partial derivative with respect to it. The

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notation follows Andrews and McIntyre (1978b), where a superscript ξ indicates a quantity evaluated at the displaced position; for example, $u^{\xi}(\mathbf{x}, t) = u(\mathbf{x} + \boldsymbol{\xi}, t)$. A superscript l indicates the difference between a quantity evaluated at the displaced position and the Lagrangian-mean of that quantity at the reference position; for example, $u^l = u^{\xi} - \overline{u}^L$.

By introducing the scaling with α and δ, the parcel displacement can be eliminated by using the leading-order approximation $(u^l, v^l) \simeq -i\omega(\xi, \eta) \simeq (u'_0, v'_0)$. Therefore, (52) can be approximated as follows:

$$A = \frac{1}{c}\left(\overline{u_0'^2 + v_0'^2 + i\beta y \frac{u'_0 v'_0}{\omega}}\right). \tag{54}$$

It follows that

$$\overline{D}_t^L A = \frac{1}{\tilde{\rho}}\left[\frac{\partial(\tilde{\rho}A)}{\partial t} + \frac{\partial(\tilde{\rho}\overline{v}^L A)}{\partial y} + \frac{\partial(\tilde{\rho}\overline{w}^L A)}{\partial z^*}\right] \tag{55}$$

by using the continuity equation, where $\tilde{\rho}$ is the Lagrangian-mean density, which can be approximated by $\rho_0(z)$. The first and third terms of the rhs can be dropped so that

$$\overline{D}_t^L A \simeq \frac{\partial(\overline{v}A)}{\partial y} \tag{56}$$

at $O(\delta^1\alpha^2)$ if the disturbance is steady and $\overline{w}$ is independent of z. If we suppose a Kelvin wave, $v'_0 = 0$, and thus from (54) and (56),

$$\overline{D}_t^L A \simeq \frac{\partial}{\partial y}\left(\overline{v}\frac{\overline{u_0'^2}}{c}\right). \tag{57}$$

The remaining terms on the rhs of (50) for a Kelvin wave are, for the current scaling with the assumption that $X' = Y' = Q' = 0$ and $\partial\overline{w}/\partial z = 0$, given by

$$\overline{(p^l)_x q} \simeq \frac{\kappa c\overline{w}}{HN^2}\left(m_0^2 + \frac{1}{4H^2}\right)\overline{u_0'^2}, \tag{58}$$

$$\overline{\xi_x X^l} \simeq 0, \tag{59}$$

$$\overline{\eta_x Y^l} \simeq 0. \tag{60}$$

The derivation of (58) and (59) is given in the appendix, and (60) is based on the assumption of $\overline{Y}_z = 0$ in the present study, which yields $Y^l \simeq \eta\overline{Y}_y$.

We further note that for steady disturbances the Coriolis term in (50) reduces to

$$\overline{(\beta y v)}^L \simeq \beta y\overline{v}^L \tag{61}$$

at second order in α (Andrews and McIntyre 1978b). Then, from (50) and (57)–(61), we have

$$\overline{D}_t^L\overline{u}^L - \beta y\overline{v}^L - \overline{X}^L \simeq \frac{\partial}{\partial y}\left(\overline{v}\frac{\overline{u_0'^2}}{c}\right) + \frac{\kappa c\overline{w}}{HN^2}\left(m_0^2 + \frac{1}{4H^2}\right)\overline{u_0'^2}. \tag{62}$$

The rhs of (62) is identical to $\rho_0^{-1}\boldsymbol{\nabla}\cdot\mathbf{F}$ due to the Kelvin wave derived in the previous section. Note, however, that (62) is derived without using the solution of the perturbed wave structure, or only by using the zeroth-order solution. The transformed Eulerian and Lagrangian-mean meridional circulations are equivalent for linear, steady, and conservative waves. Furthermore, the difference between $\overline{u}^L$ and $\overline{u}$ (Stokes correction) is first order in δ for Kelvin waves in zero vertical shear, although it is order unity for other equatorial waves. Therefore, (62) yields the same result as (47) at $O(\delta^1\alpha^2)$.

The derivation here suggests that the momentum transport obtained earlier is consistent with the generalized Lagrangian-mean theory. The acceleration due to the latitudinal EP flux divergence arises because $\overline{v}^L A$ is generally not constant since the meridional distribution of A is determined (almost) independently of $\overline{v}^L$ to form an equatorially trapped mode. This result does not contradict the so-called nonacceleration theorem, valid for nonlinear, steady and conservative waves (Andrews and McIntyre 1978b; Dunkerton 1980), which states, in effect, that Kelvin's circulation around a material circuit is conserved following the Lagrangian mean motion. In our case, the eigenmode (Kelvin or other equatorial wave, see below) is locked to the geographical equator, while a nonzero meridional circulation at leading order advects the wave's pseudomomentum in latitude: toward, across, or away from the equator. Note that the zonal forcing in (57) has the form of a latitudinal derivative, implying that the latitudinal integral of the force is identically zero since the equatorial-wave amplitude vanishes at large y. This constraint is consistent with angular momentum conservation: the force imparted to the flow by the wave at any altitude (e.g., where the wave enters the domain) is exactly balanced by an equal and opposite force at any other altitude (e.g., where the wave leaves the domain) because the wave is steady and conservative.

The acceleration due to the vertical EP flux divergence arises when $V_s \neq 0$; that is, $\overline{w} \neq 0$. From the point of view of the GLM theory, the acceleration is attributed to the diabatic effect q (or $\partial\overline{Q}/\partial z$, see appendix), which is needed to maintain a constant $\overline{w}$ through (11). In this case, the wave cannot be regarded as a conservative wave even when $Q' = 0$.

b. *Generalization to other equatorially trapped modes*

Equation (56) is not restricted to the Kelvin wave. For any equatorially trapped mode, the meridional structure of u'_0 and v'_0 in the neighborhood of the equator can be approximated as $\hat{u}_0 = a$ and $\hat{v}_0 = by$ (for the symmetric modes) or $\hat{u}_0 = by$ and $\hat{v}_0 = a$ (for the antisymmetric modes), where a and b are constants, and a is nonzero. Substituting these into (54), we obtain

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$$A = \frac{1}{2c}\left[a^2 + \left(b^2 \pm \frac{\beta ab}{\omega}\right)y^2\right], \quad (63)$$

for both cases. Since the first term on the rhs is dominant if y is sufficiently small, we can conclude that the lateral momentum transport $(-\overline{v}^L A)$ is directed against (in) the direction of $\overline{v}^L$ when $c > 0$ $(c < 0)$ in the neighborhood of the equator. If $\overline{v}^L \simeq V_s y$, for instance, then

$$\overline{D}_t^L A \simeq \frac{V_s a^2}{2c}. \quad (64)$$

Thus, the direction of the zonal acceleration due to the lateral convergence of the momentum flux is determined by the sign of V_s/c.

5. Applications to planetary atmospheres

a. Brewer–Dobson circulation at solstices

The theory developed above is applied to the cross-equatorial Brewer–Dobson circulation at solstices in the terrestrial stratosphere. The flow velocity is estimated to be 1–2 m s^{-1} in the upper stratosphere (Eluszkiewicz et al. 1996). The circulation assumed here is $\overline{v} = V_a = 2$ m s^{-1} and $\overline{w} = 0$, representing the northern winter condition.

Among various Kelvin waves observed (e.g., Andrews et al. 1987), let us consider a wave with $\omega/k = 30$ m s^{-1}, $\tilde{u} = 6$ m s^{-1}, and the zonal wavenumber of 1. It has the intrinsic period of 16 d (d = Earth days), the vertical wavelength of 9.4 km for $N = 0.02$ s^{-1} and $H = 6.5$ km, and the equatorial deformation radius of 1100 km corresponding to the latitudinal width of 10°. In this case, the amplitude parameter (32) is $\alpha = 0.20$, and the perturbation parameter (30) is $\delta = 0.37$.

Figure 1 shows the Kelvin wave solution in the meridional circulation calculated from (26)–(28) and (36)–(38). The modifications in the zonal wind $\hat{u}$ and the geopotential $\hat{\phi}$ occur only in their imaginary parts and are negligibly small. The meridional wind $\hat{v}$ has only a real part, and its latitudinal structure is the same as those of $\hat{u}$ and $\hat{\phi}$, with the maximum amplitude being 0.4 m s^{-1}. The modification of ϵ is zero according to (39).

Since the phase of v' is 180° with respect to u', the combination of v' and u' transports zonal momentum southward. Figure 1 also shows the momentum flux per unit mass, $-\rho_0^{-1}F^{(y)}$, and the EP flux divergence per unit mass, $\rho_0^{-1}\partial F^{(y)}/\partial y$, calculated by (43) as functions of latitude. The momentum transport is confined within the wave's e-folding width and peaks at the equator with a maximum of 1.2 m^2 s^{-2}. The EP flux divergence has a maximum at $y \sim -0.7L$ and a minimum at $y \sim 0.7L$, with peak magnitudes of 0.08 m s^{-1} d^{-1}. If we naively extend the estimate to a Kelvin wave with a saturation amplitude, the peak magnitude of the divergence is estimated to be 2.0 m s^{-1} d^{-1}, which is comparable to the Coriolis force acting on $\overline{v}$ at $y = 0.7L$ of $0.7L\beta\overline{v} \sim$ 3.2 m s^{-1} d^{-1}. However, since α is equal to 1 in this case, it is uncertain to what extent this estimate is valid.

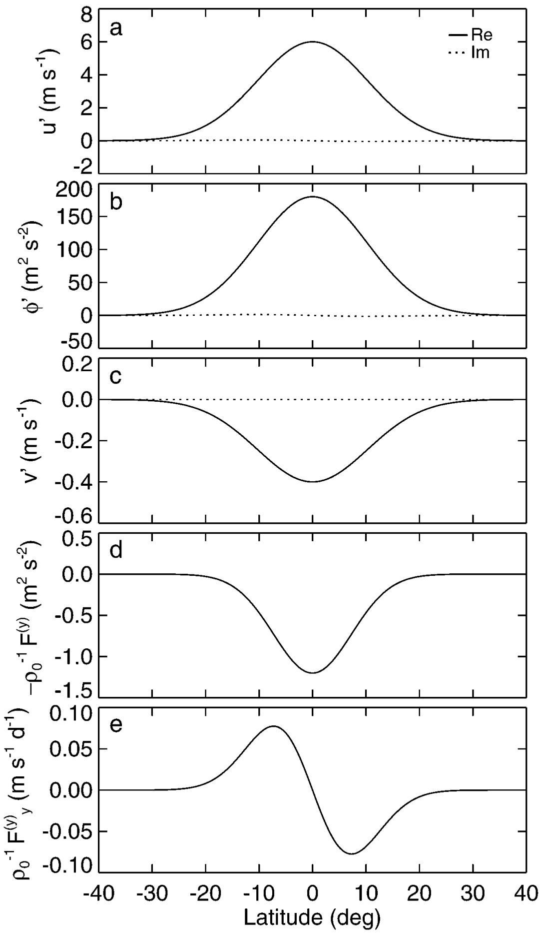

FIG. 1. Latitudinal structure of a Kelvin wave with the wavenumber of 1, the phase velocity of 30 m s^{-1}, and the zonal wind amplitude of 6 m s^{-1}, in a cross-equatorial northward flow of $\overline{v} = 2$ m s^{-1}: (a) zonal wind u', (b) geopotential ϕ', (c) meridional wind v', (d) minus the meridional EP flux per unit mass $-\rho_0^{-1}F^{(y)}$, and (e) EP flux divergence per unit mass $\rho_0^{-1}\partial F^{(y)}/\partial y$. Solid and dotted curves in (a)–(c) represent real and imaginary parts, respectively.

b. Brewer–Dobson circulation at equinoxes

Next, the theory is applied to the Brewer–Dobson circulation at equinoxes in the stratosphere, which is symmetric about the equator. The poleward flow velocity increases with latitude, and at 30° latitude it reaches 1–2 m s^{-1} in the upper stratosphere (Eluszkiewicz et al. 1996). Then, the meridional circulation is taken as $\overline{v} = V_s y$ and $\overline{w} = HV_s$ with $V_s = 3.6 \times 10^{-7}$ s^{-1} and $H = 6.5$ km, giving $\overline{v} \sim 1.2$ m s^{-1} at 30° latitude and $\overline{w} \sim 2.3 \times 10^{-3}$ m s^{-1}. The wave parameters adopted are the same as those in section 5a. In this case, the perturbation parameter is $\delta = 0.24$.

Figure 2 shows the Kelvin wave solution in the meridional circulation calculated from (26)–(28) and (36)–(38).

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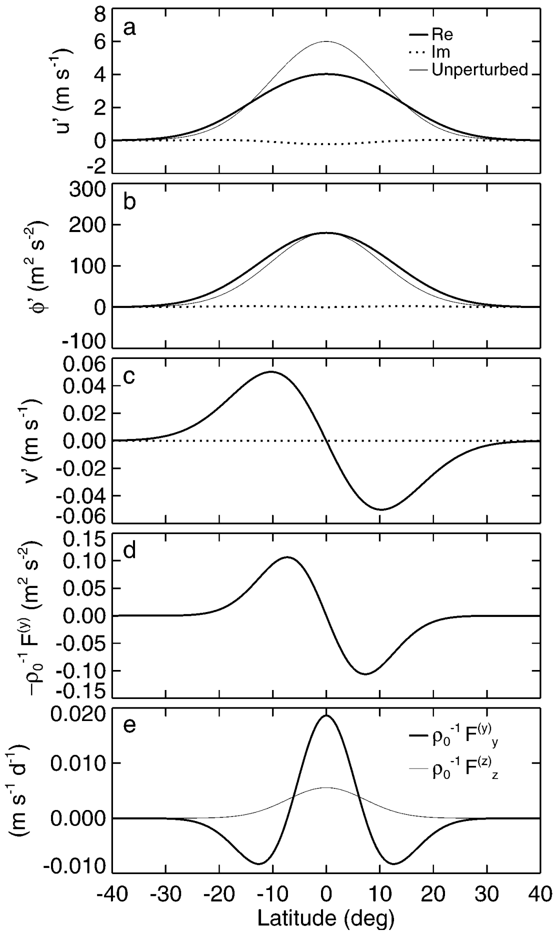

FIG. 2. Similar to Fig. 1 but for a wave in a meridional circulation that is symmetric about the equator. The flow velocity is given by $\overline{v} = (3.6 \times 10^{-7}\ \mathrm{s}^{-1}) \times y$ and $\overline{w} = (3.6 \times 10^{-7}\ \mathrm{s}^{-1}) \times H$. Thin solid curves in (a) and (b) are the solution in an atmosphere at rest. The divergence of the vertical EP flux is also plotted for comparison by a thin curve in (e).

The meridional wind $\hat{v}$ has only a real part and its amplitude peaks at $y \sim \pm L$ with the maximum of 0.05 m s^{-1}. The eigenvalue ϵ is decreased by 47% compared to ϵ_0, resulting in a decrease of the vertical wavenumber by 28%. Since the phase of v' is 180° with respect to u' in the Northern Hemisphere and 0° in the Southern, the combination of v' and u' transports zonal momentum equatorward. The divergence of the resultant lateral EP flux has a maximum of 0.019 m s^{-1} d^{-1} at the equator and minima at $y \sim \pm 1.2L$. The divergence of the vertical EP flux is also plotted for comparison in the bottom panel, indicating that its contribution to the zonal acceleration is relatively minor when compared to the lateral EP flux in this case.

If we naively extend the estimate to a Kelvin wave with a saturation amplitude, the maximum value at the equator is estimated to be 0.47 m s^{-1} d^{-1}, which is comparable to the Coriolis force acting on $\overline{v}$ at $y = 0.7L$ of $0.7L\beta\overline{v} \sim 0.46$ m s^{-1} d^{-1}.

c. Implication for the Venusian stratosphere

The proposed mechanism is expected to play an important role in the Venusian stratosphere, where Kelvin waves with the wavenumber of 1, $\omega/k \sim 15$ m s^{-1}, and $\tilde{u} \sim 10$ m s^{-1} have been observed in cloud motions (Del Genio and Rossow 1990; Belton et al. 1991). Although the rotation of Venus is westward and slow with the period of 243 d, westward wind speed increases with height, and the stratosphere near the cloud top circles around the planet with the period of 4.7 d (Schubert 1983). This wind system, called the superrotation, allows Kelvin waves to exist. The poleward flow associated with the Hadley circulation is also observed at the cloud top, with $\overline{v}$ being 1–10 m s^{-1} in the midlatitude according to cloud tracking (Rossow et al. 1990). Then, adopting the wave parameters above and $V_S = 10^{-6}$ s^{-1}, which gives $\overline{v} = 3.2$ m s^{-1} at 30°, the equatorward momentum flux at $y = \pm 0.7L$ (±11.5°) and its convergence at the equator per unit mass are calculated to be 2.5 m^2 s^{-2} and 0.3 m s^{-1} d^{-1}, respectively, by using (43). However, since $\delta = 3.5$ in this case, the quantitative validity of the result is questionable. Nevertheless, we can speculate that the result may be correct at least qualitatively, if the underlying physics is robust.

Such a momentum transport may partly be contributing to the maintenance of the superrotation, since zonal momentum is accumulated at high altitudes if the Hadley circulation transports zonal momentum upward in the equatorial region and poleward at high altitudes, and some eddies transport it equatorward (Gierasch 1975). The estimated EP flux divergence is comparable to the equatorial acceleration required to maintain the superrotation, which is estimated to be ~0.5 m s^{-1} d^{-1} (Newman and Leovy 1992). Further investigations are needed to evaluate the contribution of the present mechanism.

6. Conclusions

The solution of the equatorial Kelvin wave in the meridional circulation was obtained for the first time, using a perturbation theory. In this study, the meridional circulation is supposed to exist in the basic state and is not neglected in the linearized wave equations, in contrast to the conventional linear wave theories that assume meridional circulation to be second order with respect to the wave amplitude. The wave solution was obtained analytically after separating the vertical dependence from the horizontal dependence under the assumption that the mean flow is independent of height. The result shows that the wave is modified to have a nonzero meridional wind disturbance, which is absent when the background atmosphere is at rest. The com-

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bination of the meridional and zonal winds induces a meridional flux of zonal momentum in the upstream direction of the background north–south flow. Consequently, the EP flux divergence can be nonzero in the meridional circulation even when the wave transience and dissipation are absent. When a Kelvin wave is present in a cross-equatorial circulation, the wave transports momentum from the winter hemisphere to the summer hemisphere assuming the flow is from summer to winter hemisphere. In the upper branch of the Hadley circulation, the wave transports zonal momentum equatorward. It was shown that the momentum transport can be derived using the generalized Lagrangian-mean theory (Andrews and McIntyre 1978b). Via this approach the zonal wind forcing for any equatorially trapped mode can be calculated: the momentum transport is directed against (in) the direction of the north–south flow when the zonal phase velocity is positive (negative) in the neighborhood of the equator. When a steady vertical flow exists, the divergence of the vertical momentum flux is also nonzero because the steady vertical flow requires a mean diabatic heating, which induces the change of the amplitude of the temperature disturbance relative to the background in the course of the vertical advection.

The solution was applied to the terrestrial stratosphere for the condition of the Brewer–Dobson circulation at solstices and equinoxes. When a small-amplitude wave is assumed to ensure the validity of the linear theory, the resultant EP flux divergence seems to be too small to affect the momentum balance of the equatorial stratosphere. However, the divergence becomes large enough when a saturation amplitude is given, although it is uncertain to what extent such estimates are valid. The solution was applied also to the Venusian stratosphere, where the upper branch of the Hadley circulation exists. It was suggested that the equatorward transport of momentum by Kelvin waves may be significant, although the meridional circulation was found to be too strong to apply the analytical solution obtained by the perturbation method.

In the employed analytical technique, the vertical shear of the background wind is assumed to be zero. In the real atmosphere, however, the vertical variation in mean winds is often not negligible within the depth corresponding to the typical vertical wavelength of the Kelvin wave. Nevertheless, this simple approach is suitable for understanding the basic physics underlying the process and for providing a simple formula of momentum flux, with a more realistic approach being left for future research.

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APPENDIX

Derivation of Eqs. (58) and (59)

The Lagrangian disturbance pressure p^l is related to the disturbance vertical velocity in pressure coordinate, ω', as $\overline{D}_t p^l \simeq \omega'$ (Andrews and McIntyre 1978a). This expression is obtained from the exact relation $\overline{D}_t^L p^l = \omega^l$ (Andrews and McIntyre 1978b) after linearizing and noting that (consistent with the assumptions of our paper) the Stokes correction $\overline{u}^S$ (difference between $\overline{u}^L$ and $\overline{u}$) and correction to ω' (difference between ω^l and ω') are zero. Then, assuming that disturbance fields depend on x and t as $\exp(ikx - i\omega t)$, $(p^l)_x$ is rewritten in terms of the log-pressure disturbance vertical velocity $w'(=-\overline{p}^{-1}H\omega')$ as

$$(p^l)_x \simeq \frac{k\overline{p}}{\omega H} w'. \tag{A1}$$

As for q, it is not zero due to the nonzero diabatic heating in the presence of a steady vertical flow, even when $Q' = 0$. We first note

$$q \simeq -\frac{\partial}{\partial\theta}\left[\frac{1}{\rho(\theta, p)}\right]\theta^l \simeq -\frac{\theta^l}{\overline{p}\,\overline{\theta}}. \tag{A2}$$

Using $\overline{D}_t^L \theta^l = Q^l$ and $\partial Q^l/\partial t \simeq \overline{Q}_z w'$ with z being the log-pressure height, θ^l is related to w' as

$$\theta^l \simeq \frac{i}{\omega} Q^l \simeq -\frac{1}{\omega^2}\frac{\partial\overline{Q}}{\partial z} w'. \tag{A3}$$

Since the basic state temperature does not depend on y (section 2) and $\overline{w}$ and N do not depend on z in the present study, (11) and the definition of N yields

$$\frac{\partial\overline{Q}}{\partial z} = \frac{\partial^2\overline{\theta}}{\partial z^2}\overline{w} = \frac{\kappa N^2\overline{\theta}\,\overline{p}\,\overline{w}}{\overline{p}}. \tag{A4}$$

Substituting (A3) and (A4) into (A2), we have

$$q \simeq \frac{\kappa N^2 \overline{w}}{\omega^2 \overline{p}} w'. \tag{A5}$$

From (A1) and (A5), we obtain

$$\overline{(p^l)_x q} \simeq \frac{\kappa k N^2 \overline{w}}{H\omega^3}\overline{w'^2}. \tag{A6}$$

If we suppose a Kelvin wave, w' is related to u' through (15), (26), and (27). Substituting these into (A6), we get a relation at first order in δ:

$$\overline{(p^l)_x q} \simeq \frac{\kappa c \overline{w}}{HN^2}\left(m_0^2 + \frac{1}{4H^2}\right)\overline{u_0'^2}. \tag{A7}$$

This result is consistent with the second term on the rhs of (46) so that the Lagrangian and Eulerian derivations agree.

The X^l that appears in $\overline{\xi_x X^l}$ is approximated by

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$$X^l \simeq \eta\frac{\partial\overline{X}}{\partial y} + \zeta\frac{\partial\overline{X}}{\partial z}. \tag{A8}$$

The first term on the rhs is second order in δ since both η and $\overline{X}$ are first order. Since the mean winds do not depend on the altitude in the present study, (7) gives $\partial\overline{X}/\partial z = 0$. Therefore, X^l is second order in δ, and $\overline{\xi_x X^l}$ is also second order.

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