# Reply

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## 1. Introduction

Recently, Thuburn et al. (2002b) and Kasahara (2003a,b) analyzed the normal modes of a compressible, stratified, nonhydrostatic tangent-plane atmospheric model, bounded by top and bottom rigid boundaries. When the Coriolis terms arising from both the vertical and horizontal components of the earth's angular velocity $\Omega$ are included, in addition to usual kinds of the acoustic and inertio–gravity modes, there emerges different normal modes that are called ''new modes'' by Thuburn et al. (2002b), because they are new in the sense that no corresponding modes exist when the horizontal component of the Coriolis vector is ignored, and are called ''boundary-induced inertial (BII) modes'' by Kasahara (2003a), because without boundaries these modes do not appear. Durran and Bretherton (2004) offer their physical interpretation to these ''unique'' modes. For the purpose of referencing, here we continue to use the terminology of BII modes for these unique modes, although we wish that a better terminology will be found.

Durran and Bretherton (2004) first present a review of the BII modes that appear in the homogeneous rotating incompressible model bounded by horizontal boundaries in the vertical. Then they show that when the rotation vector is tilted with respect to the vertical, there are two possible ways of superposing two plane waves, which have different vertical wavenumbers, but the same horizontal wavenumber and frequency, at imposed boundaries to satisfy rigid conditions. The BII modes are explained by one of the two ways that the plane propagating inertial waves are superposed. Thus, it is clear that the BII modes are not in the class of edge waves, such as a Kelvin or Lamb wave in which the wave amplitude decays exponentially away from the boundary. Durran and Bretherton further extended their physical consideration of the two different kinds of normal modes to the case of a Boussinesq model treated by Kasahara (2003a), and demonstrated again that the BII modes appear as the result of a unique superposition of the two inertio–gravity waves to satisfy the rigid condition at horizontal boundaries when the Coriolis vector is tilted against the vertical.

In this reply, we will complement the analysis of BII modes by Durran and Bretherton (2004) by referring to the past studies that examined the normal mode problem of this general Boussinesq model to place the emergence of BII modes in a historical context, and call attention to the fact that both the inertio–gravity and BII modes are essential parts in understanding the inertial motions in the atmospheres and oceans. Further comments are added on the likelihood of having similar modes as the BII modes in the spherical version, but our understanding on this question is far from complete.

## 2. Normal modes of a Boussinesq model

The perturbed vertical velocity $w(x, y, z, t)$ of the linearized Boussinesq model, given in section 3 of Kasahara (2003a), is expressed by

$$w = W(z)\exp[i(mx + ny - \sigma t)], \qquad (1)$$

where $W(z)$ is the solution of the vertical structure equation

$$\frac{d^2W}{dz^2} + \frac{2if_Vf_Hn}{f_V^2 - \sigma^2}\frac{dW}{dz} + \left[\frac{(\sigma^2 - N^2)(m^2 + n^2) - n^2 f_H^2}{f_V^2 - \sigma^2}\right]W = 0, \qquad (2)$$

in which $N$ denotes the Brunt–Väisälä frequency, treated as constant.

The solutions $W(z)$ are considered to satisfy the following rigid top and bottom boundary conditions:

$$W(z = z_T) = 0 \quad \text{and} \quad W(z = 0) = 0. \qquad (3)$$

Note that, if $N = 0$, Eq. (2) is reduced to the incompressible and homogeneous case.

When the above boundary conditions are ignored, the perturbed vertical velocity can be expressed by the plane wave solutions

$$w = \exp[i(mx + ny + lz - \sigma t)] \qquad (4)$$

that yield the following quadratic dispersion equation (Phillips 1966):

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$$\sigma^2 = \frac{N^2(m^2 + n^2) + (lf_V + nf_H)^2}{m^2 + n^2 + l^2}. \qquad (5)$$

Here, it must be assumed that $\sigma^2 \neq f_V^2$, as we will see the significance of this condition later.

The dispersion equation (5) has one unique property—that the frequency $\sigma$ depends on the sign of $l/n$, if $f_H \neq 0$. Therefore, upward and downward propagating plane wave solutions of the form $\exp[i(mx + ny \pm lz)]$ cannot be combined to satisfy the boundary conditions (3).

To satisfy the boundary conditions (3), the solutions of the vertical structure equation (2) must be given in the form

$$W(z) = \sin(kz)\exp(i\Gamma_2 z), \qquad (6)$$

where

$$\Gamma_2 = -\frac{f_H f_V n}{f_V^2 - \sigma^2}, \qquad (7)$$

and the parameter $\Gamma_2$ is equal to minus one-half of the coefficient of $dW/dz$ of Eq. (2), multiplied by $i = \sqrt{-1}$.

In other words, if $f_H \neq 0$ and the boundary conditions (3) are to be satisfied, the vertical wavenumber $l$ in (4) must take the form of $\Gamma_2 \pm k$. As shown by Eckart (1960, p. 133), the eigenfunctions $w$ are expressed by the superposition of two modified plane waves in the form

$$w = \{\exp[i(k + \Gamma_2)z] - \exp[-i(k - \Gamma_2)z]\} \times \exp[i(mx + ny - \sigma t)]. \qquad (8)$$

The two terms in Eq. (8) are interpreted as reflected and incident waves, but the angles of incidence and reflection are not equal, as also noted by Durran and Bretherton (2004).

Since the vertical wavenumber $l$ must be modified by $l = \Gamma_2 \pm k$, and the dispersion equation (5) is also subjected to this modification, the substitution of $l = \Gamma_2 \pm k$ into (5) leads to a modified dispersion equation in the form

$$\begin{aligned}(m^2 + n^2 + k^2)\sigma^4 &- [(m^2 + n^2)N^2 + (m^2 + n^2 + k^2)f_V^2 + n^2 f_H^2 \\ &+ k^2 f_V^2]\sigma^2 \\ &+ [(m^2 + n^2)N^2 + k^2 f_V^2]f_V^2 = 0,\end{aligned} \qquad (9)$$

which is a quadratic equation of $\sigma^2$.

Because ordinarily $N^2 \gg \Omega^2$, we can obtain fairly accurate solutions of (9) by approximation. For the high-frequency approximate solutions of (9), we have

$$\sigma_g^2 \doteq \frac{N^2(m^2 + n^2) + n^2 f_H^2 + k^2 f_V^2}{m^2 + n^2 + k^2}. \qquad (10)$$

The above approximate solutions are derived as follows. Let us add the term $n^2 f_H^2 f_V^2$ to (9). Then, the resulting equation can be divided by $(\sigma^2 - f_V^2)$ and the quotient gives (10). The form of (10) of $\sigma_g^2$ is more accurate than (3.17) of Kasahara (2003a), which was derived by neglecting the last bracketed term of (9). Note that the form (10) is very similar to that of the inertio–gravity waves for the unbounded case given by (5).

For the low-frequency approximate solutions of (9), we expect that another $\sigma^2$ is close to $f_V^2$, and obtain

$$\sigma_I^2 \doteq f_V^2\left[1 - \frac{n^2 f_H^2}{(m^2 + n^2)(N^2 - f_V^2) + n^2 f_H^2}\right]. \qquad (11)$$

Because these modes have frequencies very close to the Coriolis frequency $\pm f_V$, and appear only when the boundary conditions (3) are imposed, Kasahara (2003a) refers to these modes as the boundary-induced inertial (BII) modes.

The same form of the $\sigma_I^2$ of (11) within the approximation of $N^2 \gg \Omega^2$ is found also by Thuburn et al. (2002b), who refer to these modes as ''new modes,'' because it appears that the modes of this kind have never been mentioned in the past. There is a good reason why both Thuburn et al. (2002b) and Kasahara (2003a) felt that this kind of mode is rather unique. In fact, this question came up in an attempt to classify the normal modes of a nonhydrostatic, compressible, stratified tangent-plane atmospheric model, including both the effects of $f_V$ and $f_H$. If the model space is unbounded, it has been shown by Phillips (1990) that the dispersion equation for $\sigma$ is a fifth-degree polynomial whose four roots are identified as two pairs of acoustic and inertio–gravity modes. The remaining root corresponds to the steady-state solution $\sigma = 0$ that can be interpreted to correspond to a Rossby mode in the event that the meridional derivative of $f_V$ is considered. However, when the boundary conditions (3) are included, the dispersion equation for $\sigma$ becomes a sixth-degree polynomial, even though there are only five time-dependent equations in the system. Among the six roots, four roots correspond to two pairs of acoustic and inertio–gravity modes, and the remaining two roots essentially correspond to the $\sigma_I^2$ of (11) and Eq. (5.1) of Thuburn et al. (2002b). It is important to recognize that the BII modes are little affected by the compressibility of fluid.

It may be worthwhile to mention here some of the past studies on the normal modes of the Boussinesq model relevant to this note. Kamenkovich and Kulakov (1977) (see also Miropol'sky 2001) investigated the role of $f_H$ on the normal modes of the Boussinesq model identical to the one considered here. They adopted a free surface condition at the model top, but the rigid condition at the bottom. Because of the use of free surface condition, the dispersion equation for $\sigma$ becomes transcendental. Nevertheless, by neglecting some small terms, they are able to identify the presence of two different kinds of internal modes which are well ''separated'' if $N^2 \gg \Omega^2$. In particular, when the rigid conditions are used at both the top and bottom boundaries, two different kinds of $\sigma^2$ are approximated by

$$\sigma_g^2 \doteq N^2\left[1 + \frac{n^2 f_H^2}{(m^2 + n^2)N^2} + O\left(\frac{\Omega^4}{N^4}\right)\right] \quad \text{and} \tag{12}$$

$$\sigma_I^2 \doteq f_V^2\left[1 - \frac{n^2 f_H^2}{(m^2 + n^2)N^2} + O\left(\frac{\Omega^4}{N^4}\right)\right]. \tag{13}$$

Kamenkovich and Kulakov (1977) further discussed approximate solutions for the case of $N^2 \ll \Omega^2$; namely, the effect of buoyancy is much weaker than that of rotation. In this case, $\sigma_g^2$ corresponds to low-frequency gravity modes and $\sigma_I^2$ to high-frequency inertial modes. However, their approximate solutions are valid only when $N \neq 0$ and not applicable to the case of $N = 0$, which reduces the Boussinesq model to the incompressible and homogeneous model. It is important to note that two kinds of $\sigma^2$ are present even in the incompressible and homogeneous model as shown by Kasahara (2003a) and Durran and Bretherton (2004).

Actually, we were unaware until recently that Stern (1975, section 9.9) even earlier treated the normal modes of the same Boussinesq model with boundary conditions (3) and presented two kinds of $\sigma^2$, as we have been discussing. In fact, Stern (1975) gives an approximate expression of $\sigma_I^2$ in the same way as (13).

In the case of $f_V \neq 0$ and $f_H = 0$, Eq. (9) can be divided by $(\sigma^2 - f_V^2)$, which should not vanish, and we have only one kind of root in the form

$$\sigma^2 = \frac{(m^2 + n^2)N^2 + k^2 f_V^2}{m^2 + n^2 + k^2}, \tag{14}$$

which gives the frequency of "traditional" inertio–gravity waves as shown in many textbooks.

Thus, the presence of two kinds of $\sigma^2$ in the general Boussinesq model with the rigid top and bottom horizontal surfaces has been known in the past. The significance of two different kinds of $\sigma^2$, however, appears not to be brought into close scrutiny until recently. It is clear that the degeneracy of $(\sigma^2 - f_V^2) = 0$ is saved by the presence of $f_H$ terms in (9).

Durran and Bretherton (2004), in their section 4, offer an illustrative discussion on why the two kinds of wave modes, one characterized by $|\sigma_g|$ of (10) or (12) and the other by $|\sigma_I|$ of (11) or (13), appear when the wave motions are constrained by the presence of boundary conditions. They show that these two kinds of normal modes emerge as the result of a unique superposition of two vertically propagating inertio–gravity waves that satisfy the boundary conditions when the earth's angular velocity vector is not normal to the boundaries.

Since the main distinction between the two forms of the eigenfunctions (4) and (8) is the presence of the factor $\exp(i\Gamma_2 z)$, it is instructive to estimate the magnitude of $\Gamma_2$, that has the dimension of vertical wavenumber, for the two kinds of modes, $\sigma_I^2$ and $\sigma_g^2$.

For $\sigma_I^2$, the magnitude of $\Gamma_2$, as given by (7), may be estimated by using (13) as

$$|\Gamma_2| = \frac{(m^2 + n^2)N^2}{n f_H f_V} = \frac{2\pi}{D}, \tag{15}$$

where we introduce the vertical scale $D$ to represent the vertical wavenumber $\Gamma_2$. By assuming that $m \sim n = 2\pi/L$, where $L$ denotes the horizontal scale of motion, we obtain the aspect ratio $D/L$ that can be expressed as

$$\frac{D}{L} = \frac{f_H f_V}{2N^2}. \tag{16}$$

By choosing that $f_H \sim f_V = 10^{-4}\ \mathrm{s}^{-1}$, the value of $D/L$ becomes $0.5 \times 10^{-4}$ for $N = 10^{-2}\ \mathrm{s}^{-1}$ and $0.5 \times 10^{-2}$ for $N = 10^{-3}\ \mathrm{s}^{-1}$. Thus, in this range of $N$ that is appropriate for the oceans and the atmosphere, the value of $D$ ranges from 5 to 500 m for $L = 100$ km. This means that the vertical structure of the BII modes has a rather high variability modulated by the $\sin(kz)$ term. The same observation was made by Thuburn et al. (2002b, Fig. 7) who illustrated the eigenstructures of the zonal wind component and the pressure which are strongly tilted with a very high vertical variability of "typically a few meters to a few hundred meters" in vertical wavelength. Also, they suggested that "these modes might justifiably be called a kind of inertial mode."

In contrast, for $\sigma_g^2$, the magnitude of $\Gamma_2$ becomes extremely small. A similar analysis shows that the aspect ratio $D/L$ in this case is estimated by

$$\frac{D}{L} = \frac{N^2}{f_H f_V}, \tag{17}$$

using the definition (7) of $\Gamma_2$ and the approximation of $\sigma_g^2$ by (12). Thus, in the range of $N$ from $10^{-2}$ to $10^{-3}\ \mathrm{s}^{-1}$, the value of $D$ ranges from $10^9$ m to $10^7$ m for $L = 100$ km. Therefore, the factor $\exp(i\Gamma_2 z)$ is practically unity and can be neglected for the $\sigma_g$ modes.

It has been known that the oscillations of ocean currents with the periods close to the Coriolis frequency $f_V$ are ubiquitous, though intermittent, at all depths ranging from subtropical to polar latitudes (e.g., Webster 1968). Yet, the observational characters of such near-inertial-period oscillations in the seas are rather elusive. It seems useful to quote here some of the observational features of such oscillations from Munk and Phillips (1968) who investigated the mechanics of the oscillations. "Typical velocities are a few centimeters per second. The oscillations are intermittent, but, when they do occur, the inertial frequency is quite prominent. . . . Another point of agreement is that, whenever simultaneous measurements are made at nearby locations, the records appear remarkably dissimilar, apart from having the same prominent frequency, whether the separation is east–west, north–south, or up–down."

The question of the resemblance between the BII modes and the near-inertial oscillations in the seas will be further discussed in section 4.

## 3. Do the BII modes exist in the spherical version?

Referring to the results of the normal mode study by Thuburn et al. (2002a) on a compressible, stratified, nonhydrostatic, deep atmospheric model in spherical geometry, including the $\cos\phi$ Coriolis terms, where $\phi$ is latitude, Durran and Bretherton (2004) brought out the observation that the BII modes are not present in the spherical counterpart. Here, we reiterate the same observation through our recent attempt to solve the same problem using a different numerical approach, though the posed question is still unanswered satisfactorily.

Unlike the normal mode problem of shallow hydrostatic primitive equations on the sphere that can be solved by the separation of variables in the horizontal and vertical directions, the problem of the deep model is nonseparable and notoriously complex. Nevertheless, normal mode solutions can be obtained numerically by setting up an eigenvalue–eigenfunction matrix problem. Thuburn et al. (2002a) adopted a finite-difference method in a two-dimensional grid in the meridional and radial directions, while harmonic solutions are assumed in longitude. Kasahara (2004) adopted a spectral approach in the horizontal direction using a spherical harmonic expansion. A finite-difference scheme is used to discretize the variables in the radial direction.

Our solutions are obtained using the terrestrial conditions similar to those adopted by Thuburn et al. (2002a). Therefore, despite differences in numerical methods, our results are in agreement with theirs in many respects. For example, one of the most significant results is that species of normal modes of the deep nonhydrostatic model are found to be identical to those of the shallow nonhydrostatic model in which the $\cos\phi$ Coriolis terms are neglected and the model atmosphere is assumed to be shallow (Kasahara and Qian 2000). Namely, there are three kinds of internal modes: inertio–gravity, rotational (Rossby), and acoustic modes. Also, there are two kinds of external modes. In the shallow model, one kind of external mode is referred to as the Lamb waves—horizontal motions with properties close to the inertio–gravity modes for large-scale motions and that asymptote to the acoustic modes for small-scale motions. The other kind of external mode belongs to the rotational modes. Although some differences are found in the eigenstructures of normal modes between the deep and shallow models, such as those of the external modes and large-scale internal acoustic modes, no additional modes similar to the BII modes are found in the deep model.

Unfortunately, as noted by Thuburn et al. (2002b), it is rather difficult to detect something like the BII modes, even though they existed, by using finite-difference or spectral schemes in the vertical, unless perhaps the grid resolution is extremely high. This is due to a high variability of the BII eigenfunctions in the vertical as shown in section 2. Similarly, unless a very high resolution is used in the discretization of grid or spectral space in the meridional direction, it may be difficult to obtain numerical solutions for phenomena unique to the equatorial area, such as zonally symmetric trapped low-frequency inertial oscillations, investigated by Stern (1963) and further discussed by Bretherton (1964) as reflections of standing inertial waves between the two concentric thin spherical surfaces.

Considering the viewpoint that the BII modes are recovered from saving the degeneracy of $(\sigma^2 - f_V^2) = 0$ in the tangent-plane model, in which $f_V$ is treated as constant, the possibility of having something similar to the BII modes in the spherical model, in which $f_V$ depends on latitude, is also questioned by Thuburn et al. (2002a). In fact, concerning the role of the $\cos\phi$ Coriolis parameter on free oscillations of an isothermal atmosphere on plane level surfaces, Eckart (1960, 135–136) already anticipated the presence of solutions that correspond to a kind of BII modes. Eckart then went on to make another conjecture that, ‘‘This has been established only for plane level surfaces. If the level surfaces are spheres, [$2\Omega \sin\phi$] will depend on latitude. It may be argued that the effects in question will then occur only in a relatively narrow band of latitudes and that this band will be different for each value of [frequency]. At any given latitude, those values of [frequency] for which these effects are large may be relatively unimportant compared to the much larger range of frequencies for which the effects are small.’’

The questions of the singularities at critical latitudes in the deep spherical model and on what role the $\cos\phi$ Coriolis terms play to provide valid solutions within the parametric limit of modeling, as examined by Miles (1974), starting from the full equations of motion for a rotating, stratified, compressible fluid, are beyond the scope of our present discussion.

## 4. Conclusions

As seen from the case of spherical geometry, the presence of $\cos\phi$ Coriolis terms prevents the separation of basic dynamical equations into the two systems of horizontal and radial structure equations. This nonseparability of dynamical systems makes understanding of the role of $\cos\phi$ Coriolis terms extremely difficult. Thus, the case of tangent-plane geometry, as we have been considering, offers a rare opportunity to have a glimpse of the complex nature of solutions in nonseparable dynamical systems.

In an unbounded domain in Cartesian coordinates, the eigenfunctions of constant coefficient systems can be represented by plain harmonics in all directions as shown by (4), regardless of the presence of the $\cos\phi$ Coriolis terms. In this case, the $\cos\phi$ Coriolis terms simply augment the role of $\sin\phi$ Coriolis terms, and the degeneracy of $(\sigma^2 - f_V^2) = 0$ still occurs. However, if the vertical motion is required to satisfy the rigid boundary conditions (3), the eigenfunctions must be represented by the superposition of incident and reflected waves, which are modified plane waves in the form of (8). Because of the presence of $\Gamma_2$, the angles of wave incidence and reflection at a horizontal boundary surface are different, causing the resulting propagation vector to be inclined relative to the horizontal surface (see Fig. 25 of Eckart 1960). The slope of inclination depends on the magnitude of $\Gamma_2$. The slope of inclination is small (large) when $|\Gamma_2|$ is small (large). Therefore, for the BII modes, the slope of the mean propagation vector can be very large, while for the inertio–gravity modes the slope is practically negligible.

It is rather surprising to realize that the characteristics of the BII modes fit the description of the observational features of ubiquitous near-inertial-period oscillations in the oceans. The BII modes are also ubiquitous in the sense that they are present in a variety of rotating fluid models on tangent planes, as long as the $\cos\phi$ Coriolis terms are accompanied with the $\sin\phi$ Coriolis terms and the vertical motions are bounded by rigid surface so that wave reflection can occur. Above all, the frequencies of the BII modes are very close to the inertial frequency $f_V$, and they are rather insensitive to the thermal stratification of fluid, as well as the horizontal scale $L$ of motions. However, their vertical modal structures are very sensitive to the thermal stability $N$ and, to a lesser extent, to the horizontal scale $L$. For example, the vertical scale of modal structure is on the order of 5 m in the thermocline ($N = 10^{-2}$ s$^{-1}$) and it becomes on the order of 500 m below the thermocline ($N = 10^{-3}$ s$^{-1}$) for $L = 100$ km. Complexities in the horizontal coherence of modal structure of the BII modes are expected, as in the observations, due to the slanted nature of their wave fronts. In short, the possibility of explaining the observed features of near-inertial oscillations from the standpoint of adjustment involving the traditional inertio–gravity and BII modes seems to deserve further study.

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