Bimodality of the Planetary-Scale Atmospheric Wave Amplitude Index

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ABSTRACT

The evidence for multiple flow regimes in the planetary-scale atmospheric wave amplitude index (WAI) is studied using the 56 winters from the NCEP reanalysis data. The regimes are identified by bimodality in the probability density estimates. Both the probability density of the WAI alone and the probability density in the two-dimensional space spanned by the WAI and its temporal rate of change are examined. The latter procedure allows us to exploit the quasi stationarity of the regimes and increase the statistical significance. The statistical significance of bimodality in the probability densities is tested by a Monte Carlo approach using surrogate time series that preserve the full autocorrelation spectrum of the original WAI. By using a longer dataset and including the rate of change, some of the questions raised in previous studies about the robustness and statistical significance of the bimodality of the WAI are resolved.

Statistically significant bimodality is found in the WAI based on the 500-hPa height. The probability density of the WAI shows considerable low-frequency variability on decadal scales. However, the bimodality is reproduced in all decadal subperiods although without statistical significance. The last decade has been dominated by a strong (disturbed) regime while a weak (zonal) regime dominates the previous decades. This recent change toward the disturbed regime is statistically significant. Imprints of the regimes are found at other tropospheric levels including the sea level. In particular, the regimes are found with statistical significance in the WAI based on the sea level pressure for the subperiod 1979–2003. Systematically varying the upper and lower boundaries of the latitudinal interval over which the geopotential height is averaged shows that the bimodality of the WAI is rather sensitive to these parameters, but also that statistically significant bimodality is found for a range of intervals with the lower boundary at 45°–50°N.

1. Introduction

An important question in the study of atmospheric low-frequency variability in the extratropics is whether multiple atmospheric regimes exist. The answer to this question may have far-reaching consequences for our understanding of the climate system and for the detection and interpretation of climate change, as pointed out repeatedly in the literature (Wallace et al. 1991; Palmer 1999).

The most direct method to infer multiple atmospheric regimes from observations is to demonstrate multimodality in a probability density function. As the probability density function can only be reliably estimated in one and two dimensions, other methods such as cluster analysis have also been used. Ghil and Robertson (2002) present a recent review including research based on both probability density functions and cluster analysis. A drawback of cluster analysis methods such as the "k-means" algorithms is that the number of clusters have to be specified prior to the analysis and appropriate statistical tests need to be designed to determine the most probable number of clusters. The design of these tests is far from trivial. Often the test statistic is chosen as a classifiability index that measures the convergence properties of the clustering algorithm (Michelangeli et al. 1995) and the null hypothesis is a multinormal distribution. In comparison, statistical tests for multimodality in probability density functions are directly meaningful and the null hypothesis can readily be extended beyond that of normality (Silverman 1986). Note that, even if bimodality in a probability density function has been established with statistical rigor, it only indicates an instability in the atmospheric system but not necessarily the existence of multiple atmospheric regimes (Hansen and Sutera 1995).

Although there has been a large recent interest in the

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subject, the existence of multiple regimes in the troposphere is not universally accepted in the scientific community. The basic problem lies in the relatively short atmospheric records and the consequent weak statistical significance of the results. A further problem when analyzing data with high sampling rate is the serial correlations and the resulting reduced number of independent points. The claims of Corti et al. (1999) and Monahan et al. (2001) of multiple regimes in the tropospheric circulation have been criticized on these grounds (Hsu and Zwiers 2001; Christiansen 2002). Very recently, Stephenson et al. (2004) reported a thorough statistical study of the space spanned by the two leading EOFs of the monthly mean 500-hPa geopotential heights and found that the unimodal hypothesis cannot be rejected. Multimodality might be easier to detect in the stratosphere due to the dominance there of larger spatial scales. Recently, Christiansen (2003) reported two statistically significant circulation regimes in the interannual stratospheric variability and a regime shift in the late 1970s. Perlwitz and Graf (2001) found a convincing bimodality in the distribution of anomaly correlations of the stratospheric zonal mean wind. We will comment on the latter result in the discussion.

An early and much cited observation of bimodality in the extratropical atmospheric variability was presented in papers by Hansen and Sutera (Sutera 1986; Hansen 1986; Hansen and Sutera 1986), who studied a planetary-scale atmospheric wave amplitude index (WAI). It is a particular strength of these studies that they are based on physical theory developed before the observational study. The theory suggests, based on a simple barotropic model, that multiple steady states in the zonal wind and the wave amplitude develop due to an orographically induced instability. This theory, put forward by Charney and Devore (1979), and some extensions, are discussed in the original papers by Sutera (1986) and Hansen (1986).

The first study of the WAI (Sutera 1986) only included data from 4 winters, 1981–84, while Hansen and Sutera (1986) extended the study to 16 winters, 1965–80. The bimodality based on the 16 winters was claimed to be statistically significant based on a Monte Carlo test where 100 realizations were created by drawing random numbers from an appropriate unimodal distribution and only 9% of the realizations showed a bimodality comparable to that of the WAI. Serial correlations were treated by reducing the number of random numbers drawn with a factor of 4.5 compared to the number of winter days in the 16 years. This procedure was criticized by Nitsche et al. (1994) who pointed out that it is inconsistent if the smoothing parameter of the density estimate is not changed accordingly. They also noted that the density estimates are highly sensitive to the precise way that the WAI is calculated—in particular to the time filtering and to the latitudinal boundaries. Furthermore, Nitsche et al. (1994) did not find bimodality in the longest period considered (1946–91). With an improved statistical test Hansen and Sutera (1995) found that the bimodality was statistically significant in a 42-yr (1947–88) dataset but not in the dataset considered in their original work (Hansen and Sutera 1986). However, the test statistic that they used to measure the severity of the bimodality was the separation between the modes and not the depth, which seems more natural and would be in line with the discussion in Silverman (1986). The sensitivity of the results reported by Nitsche et al. (1994) seems supported by the changes of the details of the algorithm used in the papers by Hansen and Sutera (Sutera 1986; Hansen 1986; Hansen and Sutera 1986, 1995). The advent of longer time series has not led to increased significance. In Cerlini et al. (1999) a probability density estimate of the WAI based on 44 years (1950–94) is shown (their Fig. 4a) and it is obvious that the bimodality is very weak and almost nonexistent, although no attempt to calculate the statistical significance is presented.

In this paper we investigate the bimodality in the WAI calculated from the longest existing datasets of the 500-hPa geopotential height. The 56 years covered by these data are significantly longer than those used in previous studies. Our analysis differs from previous analyses in two ways. First, the Monte Carlo approach is based on surrogate data that preserve all the temporal characteristics of the original WAI. Second, we introduce the rate of change of the WAI and show that, when only the slow parts of the index are considered, the bimodality becomes much clearer and its statistical significance increases drastically. The increased statistical significance allow us to study, at least cursorily, the low-frequency changes in the probability distribution of the WAI.

2. Data and methodology

a. Data and the WAI

We have mainly used daily values of the geopotential heights and sea level pressure for the period 1948–2003 from the National Centers for Environmental Prediction–National Center for Atmospheric Research (NCEP–NCAR) reanalysis (Kalnay et al. 1996). These data are defined on a grid with 144 longitudes and 73 latitudes. For comparison we have also used daily 500-hPa Northern Hemisphere geopotential heights for the period 1945–2003 from the Hadley Centre at the U.K. Meteorological Office (UKMO). The data were provided by the British Atmosphere Data Centre at a 5° × 10° latitude/longitude grid. Data gaps, which exist in the UKMO dataset in the extratropical winter months before 1968, have been filled by simple interpolation. The older parts of UKMO dataset are not necessarily homogeneous as they are patched together from several different sources and analyzed by different methods. However, since 1976 the dataset is based on the UKMO operational analyses (Parker 1980; D. E. Parker 2004, personal communication).

In Hansen and Sutera (1995) the wave amplitude index (WAI) was calculated as follows:

1) The daily 500-hPa heights are averaged over latitudes between 45 and 70°N;
2) the average heights are Fourier decomposed in longitude and the index is computed as the root-mean-square of the amplitudes of waves with wave numbers 2, 3, and 4;
3) the annual harmonic and periods less than 5 days are removed by Fourier filtering; and
4) the 90 winter days, December, January, and February, are extracted from each year.

We will follow this prescription but allow for variations of the constants in steps 1 and 3.

We calculate probability density functions by the kernel density estimate procedure with a Gaussian kernel using the algorithm based on the fast Fourier transform (Silverman 1986). The kernel density algorithm contains a smoothing parameter, h, which is analogous to the bin width in the histogram estimate. The smoothing parameter can be determined objectively by a least squares cross-validation procedure. We choose the optimal smoothing parameter near the center of the broad minimum in the score function, which measures the integrated error between the probability estimate and the true probability density. The structure of the probability densities of the WAI is only weakly sensitive to the choice of h. Also, we find that the optimal smoothing parameter depends only little on the sample size n (as $n^{-0.2}$ according to Silverman 1986) and we are therefore justified in using the same value of h for both the whole period and for shorter subperiods.

b. *Surrogate data and the Monte Carlo test for bimodality*

When bimodality has been observed in a probability density, P_0, with a smoothing parameter, h_0, it is necessary to test if the bimodality is a chance occurrence due to sampling variability or if it is due to the underlying physics. As in Christiansen (2003) we make Monte Carlo tests following Silverman (1986). First we fit the original data to a unimodal distribution, P_u, by increasing the smoothing parameter from h_0 until the distribution just becomes unimodal. We proceed by first constructing 1000 surrogate time series following this unimodal distribution, then calculating their probability densities with the smoothing parameter h_0, and finally counting how many of these probability densities have a more severe bimodality than P_0. This test is quite robust to changes in the smoothing parameter h_0. The reason is that the test compares probability densities of the original and the surrogate data estimated with the same value of h_0 so that an increase of h_0 will result in a decrease of the severity of the bimodality in both the original data and the surrogate data. Strictly speaking we test unimodality against multimodality and when unimodality is rejected we should proceed and test bimodality against higher orders of multimodality. However, the distributions of the WAI are clearly at most bimodal and we may safely omit the last step.

The surrogate time series should be constructed to mimic the temporal characteristics of the original WAI. Thus, the surrogate time series should have the following properties: 1) same length, same autocorrelation spectrum, and same seasonal nonstationarity (cyclostationarity) as the original WAI and 2) they should be drawn from the unimodal test distribution P_u. In Christiansen (2003) we worked with annual data that were serially uncorrelated and the surrogate time series could therefore be constructed simply by drawing the numbers independently one by one from the unimodal distribution P_u. In the present case where the daily data are serially correlated we proceed as follows: The first requirement is fulfilled by randomizing the phases of the Fourier transform of the original WAI (as it appears after step 3 but before the winter months have been extracted) and scaling the new time series with the annual periodic envelope of the standard deviation of the original WAI. This procedure was used in Christiansen (2001) and Thejll et al. (2003) when testing the significance of correlations. The resulting time series are drawn from an underlying distribution, P_t (approximately Gaussian), which can be estimated as the average over many realizations. The second requirement is now fulfilled by transforming from $P_t(x)$ to $P_u(y)$ using $\int_{-\infty}^{x} P_t(z)\,dz = \int_{-\infty}^{y} P_u(z)\,dz$.

Our surrogate time series differ from those used in Hansen and Sutera (1995) by preserving the whole autocorrelation spectrum and not only the coefficient at lag one. This is necessary because the WAI is poorly described by an autoregressive process of order one. We find that the autocorrelation coefficient at lag one of the filtered WAI (following the precise prescription of the preceding section) is 0.90 and that the integral correlation time is close to 8 days, which agrees with the

7.5 days found in Hansen and Sutera (1995). However, the residuals are not white but have a considerable autocorrelation coefficient, 0.57 at lag one, and the procedure used in Hansen and Sutera (1995) may therefore be biased. Note that preserving the whole autocorrelation spectrum, and consequently the whole power spectrum, results in a conservative test as it ensures that the degrees of freedom are not overestimated.

We use three different measures of the severity of the bimodality in a time series. The first measure, D, is based directly on the probability density of the time series and is the depth between the minimum and the lowest neighboring maximum as defined more precisely in Christiansen (2003). The last two measures involve the temporal rate of change of the time series. We have calculated this velocity both as a centered difference and with a three-point Lagrangian interpolation and found that the difference is marginal. The second measure, D_{slow}, is then the depth between the minimum and the lowest neighboring maximum in the probability density of the slowest half of the time series. The third measure, D_{2D}, is based on the two-dimensional probability density distributions in the space spanned by the time series and its velocity and is defined as the two-dimensional analog to the depth described above. More precisely, D_{2D} is the difference in height between the secondary maximum and the saddle point between the two maxima.[1] The first measure, D, is calculated from the low-pass filtered WAI (cutoff at 5 days, exactly as in Hansen and Sutera 1995) while the two others, D_{slow} and D_{2D}, are calculated from the unfiltered WAI. The last two measures exploit the quasi stationarity of the regimes, which in the nonlinear paradigm can be considered as attractor basins (Wallace et al. 1991) or connected to unstable fixed points with only few unstable directions.

3. Low-frequency variability and robustness

The probability density of the WAI calculated with the original algorithm of Hansen and Sutera (1995) given in section 2a is shown in Fig. 1a. The NCEP data for the full period 1948–2003 have been used. The full curve is the kernel density estimate with a smoothing

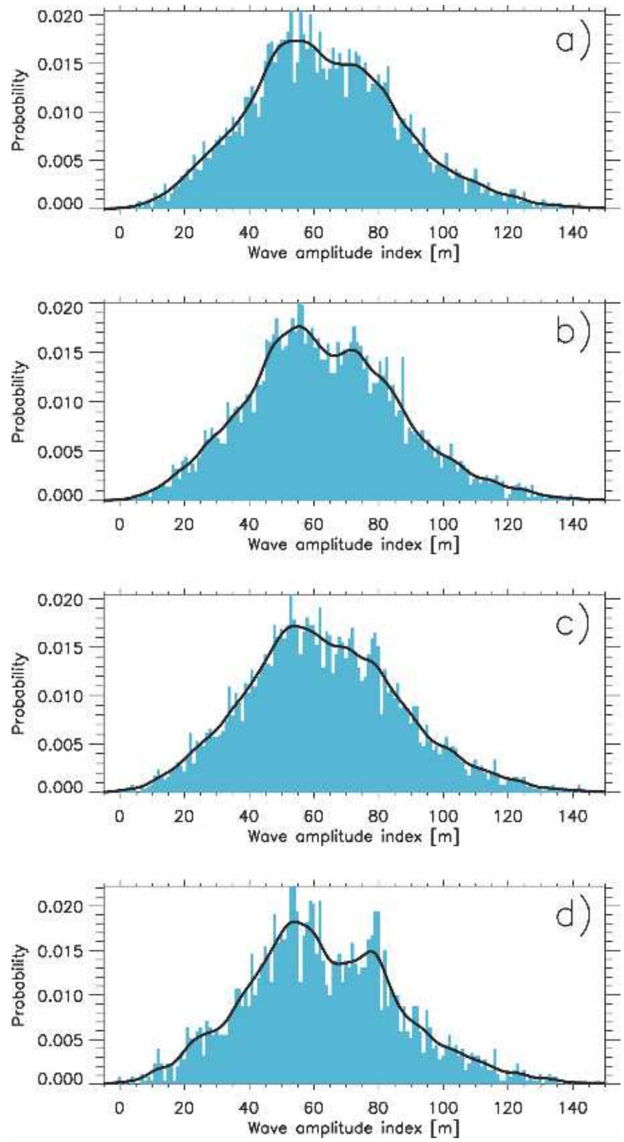

FIG. 1. Probability density of the WAI for the winter days in the period 1948–2003 calculated as a kernel density estimator with a smoothing parameter of $h = 3$ m. Also shown are the histogram estimates calculated with a bin width of 1 m: (a) variability faster than 5 days has been filtered out, (b) variability faster than 4 days has been filtered out, (c) all high-frequency variability has been retained, and (d) only the slowest half of the WAI has been included. Data are from the NCEP reanalysis.

parameter $h = 3.0$ m. The histogram estimate with a bin width of 1 m is also shown. In contrast with Hansen and Sutera (1995) the distribution is unimodal although it is highly skewed with the maximum at 53 m and a bump near 70 m. The detailed shape of the probability distribution is very sensitive to the low-pass filtering (step 3). In Fig. 1b the probability density is shown where the only difference from the algorithm in Hansen and

[1] The depth is calculated by simulated draining. Consider the set $P > s$ for a scalar s, where P is the two-dimensional probability density. If s_1 is the height of the largest peak [$s_1 = \max(P)$] then the set is empty for $s > s_1$. When s decreases the set is a growing connected island until another island appears at $s = s_2$ (the height of the second peak). When s decreases further the two separated islands grow until they merge for $s = s_m$. The depth is then $D_{\mathrm{2D}} = s_2 - s_m$.

Sutera (1995) is that the low-pass cutoff is 4 days instead of 5 days. Now, the distribution is bimodal with the peaks near 55 and 73 m as in Hansen and Sutera (1986). The bimodality is statistically significant at the 93% level according to a Monte Carlo test performed as described in section 2b. However, the bimodality may be a coincidence as it disappears if the cutoff frequency is changed further or if all the high frequencies are retained (Fig. 1c). We do not find bimodality when the cutoff period is 2, 3, 5, 6 days or longer. To investigate the influence of the different frequencies we stratify the WAI according to its velocity as described in section 2b and calculate the probability density of only the slowest part of the trajectory. In Fig. 1d the probability densities are shown including only the slowest half of the days but without any low-pass filtering. The bimodality is now much more pronounced with a deeper minimum between the maxima. As in Fig. 1b the left maximum is higher and broader than the right maximum. As mentioned in section 2, the results are not sensitive to the precise choice of h. The probability density of the slowest half of the days shows statistically

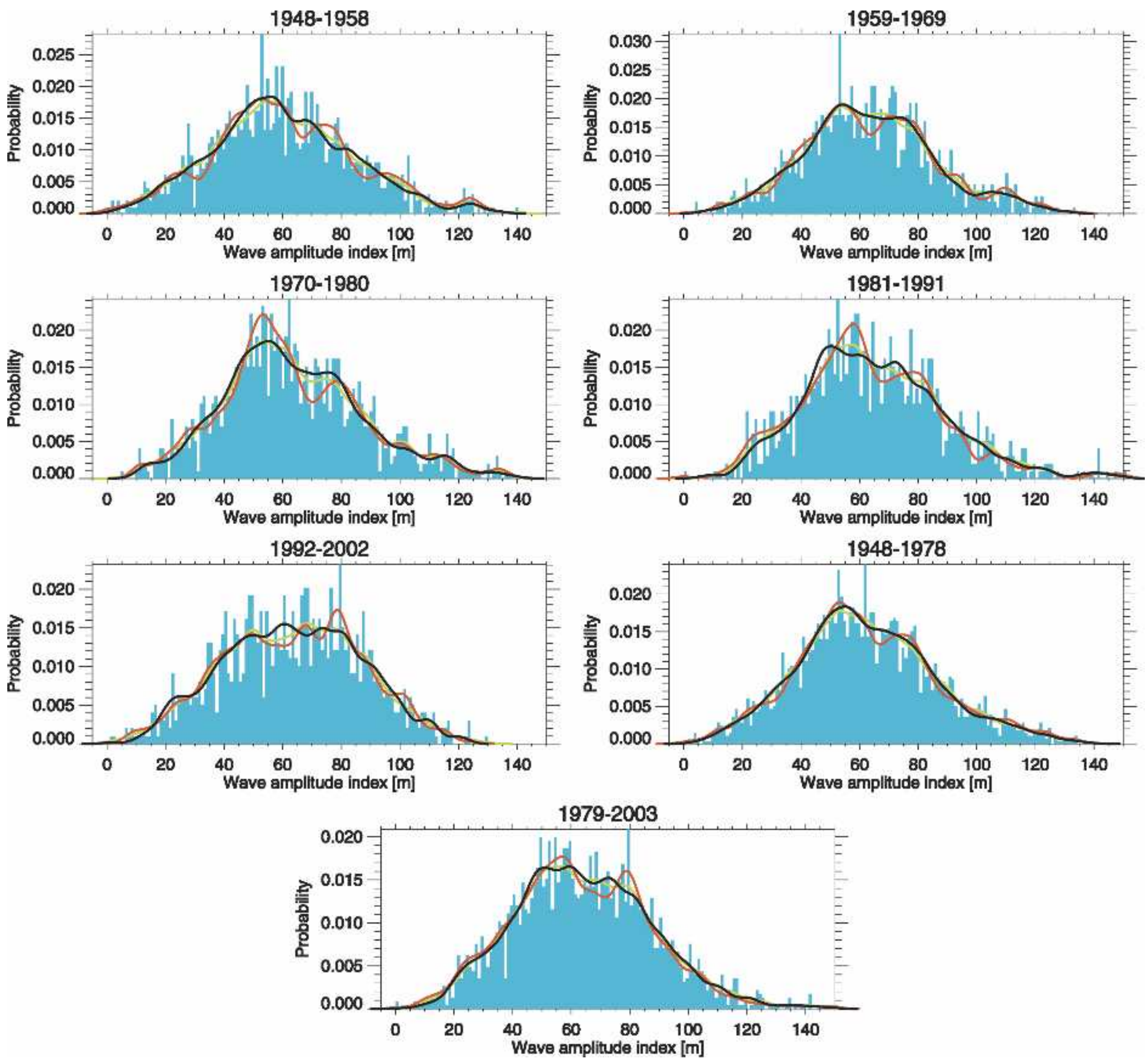

FIG. 2. Probability density of the WAI for the winter days in different periods calculated as a kernel density estimator with a smoothing parameter of $h = 3$ m. The red curve is based on the slowest half of the data, the yellow curve is based on unfiltered data, and the black curve is based on data where variability faster than 5 days has been filtered out. Also shown are the histogram estimates of the unfiltered WAI calculated with a bin width of 1 m. Data from the NCEP reanalysis.

significant bimodality for h between 2.5 and 5 m. For h between 2.5 and 1 m additional peaks are present but the distribution is dominated by the two original peaks.

Figure 2 shows the probability densities in five consecutive 11-yr subperiods and in the pre- and postsatellite periods 1948–78 and 1979–2003. We show both the probability density based on the unfiltered data, based on data low-passed filtered with the cutoff at 5 days, and based on the slowest half of the data. Only the latter method results in bimodality in all subperiods, although the bimodality is not necessarily statistically significant as we will see below. We note that the bimodality is least clear in the recent period, 1992–2002. For both the unfiltered data and the low-pass filtered data the bimodality is weak or absent in all periods, although the distributions are considerably skewed.

More details about the influence of the different time scales are shown in the two-dimensional probability density distributions in the space spanned by the WAI (unfiltered) and its velocity in Fig. 3 for the full period and for the five decadal subperiods. It is clear that the bimodality is concentrated around the region with low velocity and the bimodality is typically absent when the velocity is numerically larger than 6–7 $\mathrm{m\,s^{-1}}$. The bimodality is observed in both the full period and in all the subperiods including the recent period 1992–2002.

The results of the Monte Carlo test are shown in Table 1. The significance is given for the full period, the five decadal subperiods, and the two periods 1948–78 and 1979–2003. The probability densities for the two latter periods are shown in Fig. 7. The table reports the significance for the three measures of the bimodality D, D_{slow}, and D_{2D}. While no significant bimodality is found in the whole period or any of the subperiods when the measure D is used, this changes drastically when we focus on the slow parts of the WAI using the measures D_{slow} or D_{2D}. The bimodality is now statistically significant at least at the 95% level in the postsatellite period 1979–2003 and in the total period. Weaker significance, about the 85% level, is found for the subperiods 1959–69 and 1970–80, while the bimodality in the other decadal subperiods and the presatellite period 1948–78 could easily be chance occurrences. Although the bimodality in the subperiods is not statistically significant the reproducibility of the peaks and their positions in the subperiods shown in Figs. 2 and 3 add to the credibility of the bimodality.

The probability distribution (Fig. 3) shows an interesting low-frequency variability expressed in the relative strength of the two regimes. In the three subperiods 1948–58, 1970–80, and 1981–91, the weak (zonal) regime is stronger than the strong (disturbed) regime. This is also reflected in the probability distribution of the full period. In the subperiod 1959–69 the strengths of the regimes are comparable but the largest anomaly is found in the recent period, 1992–2002, where the disturbed regime is much stronger than the zonal regime. To test if the changes are statistically significant we perform a Monte Carlo test where, as above, the surrogate data have the same length, same autocorrelation spectrum, and same seasonal nonstationarity (cyclo-stationarity) as the original WAI. Now we force the surrogate data to have the same probability distribution, P_0, as the original WAI. As the measure of the difference between the whole period and the subperiod we choose $\Delta = \int (P_{\mathrm{all}} - P_{\mathrm{sub}})^2\, dx$, where P_{all} is the probability density of the whole period and P_{sub} is the probability density of the subperiod. Considering all the data, the slowest half, and the two-dimensional space spanned by the WAI and its velocity gives us three different measures Δ, Δ_{slow}, and Δ_{2D}. As with the depths the first measure, Δ, is calculated from the low-pass filtered WAI (cutoff at 5 days) while the two others, Δ_{slow} and Δ_{2D}, are calculated from the unfiltered WAI. The measures calculated for the original WAI are compared to the measures calculated for the surrogate time series and the results are shown in Table 2. We find that the probability distribution of the subperiod 1992–2002 is significantly different from the probability distribution of the whole period at least at the 90% level regardless of the measure. The probability distributions of the other subperiods do not differ significantly from the probability distribution of the whole period.

The low-frequency variability is also apparent in Fig. 4a where the probability density has been calculated for each winter and plotted as function of time. In particular we note the dominance of the weak regime in the middle of the 1970s and the dominance of the strong regime since 1999. It is interesting that the bimodality is very clear in the beginning of the 1980s, the period analyzed in the original papers of Sutera and Hansen. For comparison Fig. 4b show a similar plot based on the UKMO data. The agreement between the two datasets is good. The few differences are mainly found in the early period where data are sparse and the source of the UKMO dataset underwent several changes.

An important parameter for calculating the WAI is the latitudinal interval over which the geopotential height is averaged. This interval was rather broad, 25°–80°N, in the early papers (Hansen 1986; Sutera 1986) and narrowed, to 45°–70°N, in the later papers (Hansen and Sutera 1986, 1995). Nitsche et al. (1994) considered both intervals and found considerable differences in the results. Here we have systematically studied all intervals $[\phi_1, \phi_2]$, with $\phi_1 = 25°, 30° \cdots, 85°$ and $\phi_2 = \phi_1 + 5°$,

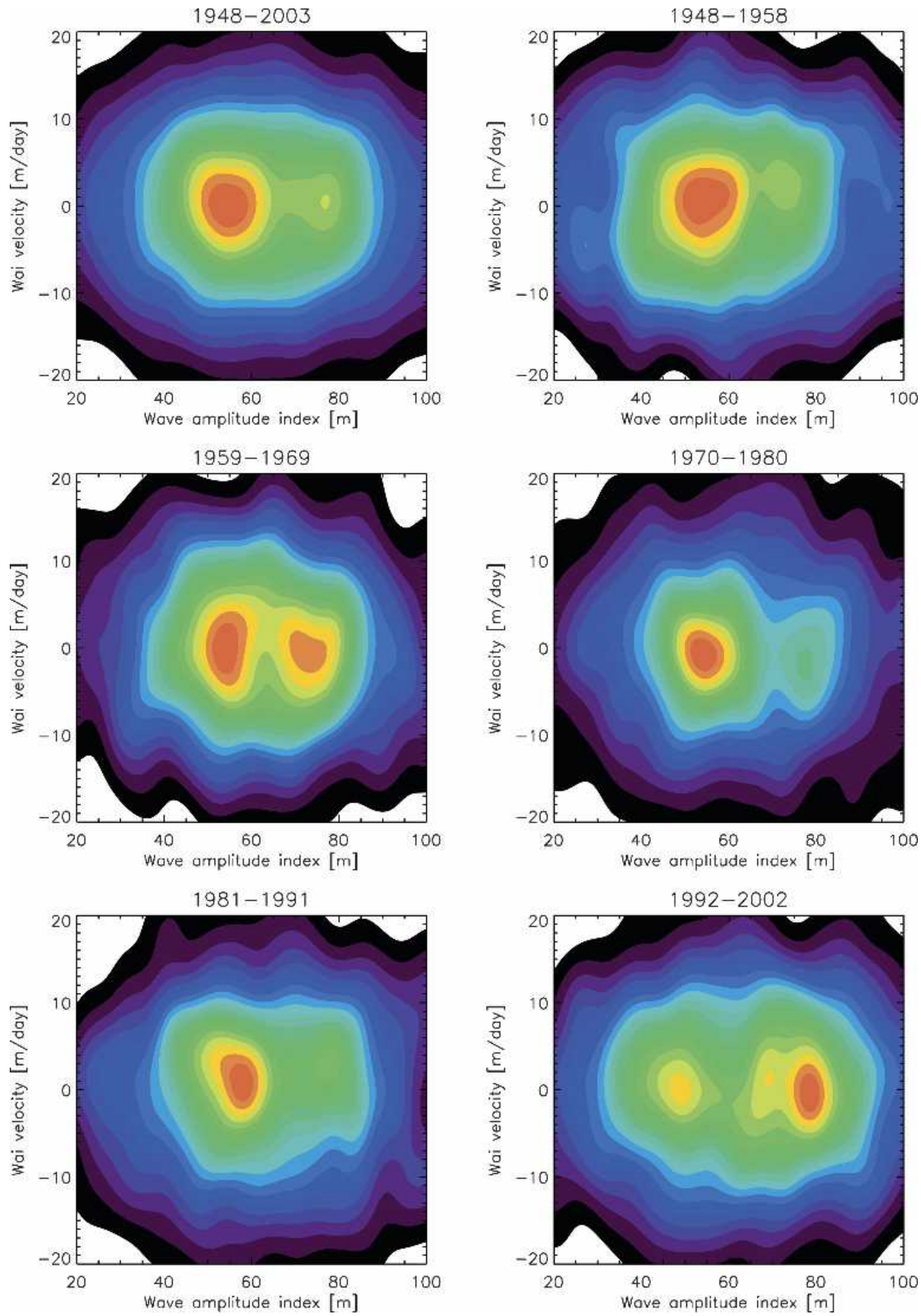

FIG. 3. Probability density in the space spanned by the WAI and its rate of change. The upper left panel includes winter days from the total period. The rest of the panels include data from five consecutive 11-yr periods (h = 3.5 m). Based on the unfiltered WAI.

TABLE 1. Statistical significance of the bimodality in the WAI for the full period and the subperiods. The significance is obtained with a Monte Carlo approach with 1000 realizations. The surrogate data are constructed to mimic the full autocorrelation spectrum of the original WAI and are drawn from a unimodal distribution. Three different statistics measuring the severity of the bimodality is used (see text). The table shows in percent the number of realizations with measure larger than that found with the original WAI; NB means that no bimodality was found in the original WAI.

	Period							
Statistic	1948–2003	1948–78	1979–2003	1948–58	1959–69	1970–80	1981–91	1992–2002
D	NB	NB	58	81	61	75	NB	33
D_{slow}	2	8	2	27	10	11	55	59
D_{2D}	5	25	4	44	18	15	34	31

$\phi_1 + 10° \cdots$, 85°N. For each interval we have calculated the WAI, the probability density of its slowest half, and the two-dimensional probability density in the space spanned by the WAI and its velocity. The results are shown in Fig. 5 where an open circle indicates bimodality and a filled circle indicates bimodality, which has been found to be statistically significant at least at the 95% level. In Fig. 5a we have used D_{slow} and in Fig. 5b D_{2D} to measure the severity of the bimodality. While the one-dimensional measure, D_{slow}, in general reports bimodality in latitude intervals at lower latitudes than the two-dimensional measure, D_{2D}, the two methods show a consistent picture of statistically significant bimodality in intervals with ϕ_1 between 45° and 50°N. In previous work the latitudinal average of the geopotential heights (step 1) was calculated with equal weighting. To test the consequence of this choice we have reproduced the distributions of Fig. 2 with the geopotential heights weighted with the cosine of the latitudes. We find that the influence of the weighting is negligible.

We briefly comment on the horizontal structure of the regimes. The mean of the 500-hPa geopotential height over all days in the disturbed regime (WAI $>$ 66 m) and the zonal regime (WAI $<$ 66 m) is shown in Fig. 6 together with the difference of the two patterns. The difference is dominated by an enhanced west Atlantic trough accompanied by east Pacific and east Atlantic ridges. Compared to Hansen and Sutera (1986) this pattern is even more localized in the Western Hemisphere. Almost the same pattern appears if the WAI is correlated with the geopotential height. Here we note that the same pattern also appears as a "teleconnection" pattern if the geopotential height in the center of the west Atlantic trough (55°N, 72°W) is correlated with all other 500-hPa geopotential heights. Alternatively, this pattern can be seen (as in Fig. 6d) in the difference between the average geopotential height when the anomalous height at the center is negative and when it is positive. Note, that the amplitudes in Figs. 6c and 6d are comparable. While Cerlini et al. (1999) found that the WAI is statistically connected to both the North Atlantic Oscillation and the Pacific–North American pattern the observation above lends some credibility to the physical reality of the WAI regimes.

4. Vertical structure

In the existing literature the WAI has exclusively been defined from the 500-hPa geopotential heights, while other levels have not been considered. This is quite natural when the roots of the WAI in the Charney–Devore theory are recalled. However, the study of other levels may add information of the consistency and robustness of the bimodality. Also, if the existence of bimodality can be established in the sea level pressure, longer datasets could be exploited. Figure 7 displays the two-dimensional probability density for the WAI calculated from the geopotential heights at

TABLE 2. Statistical significance of the difference between the probability densities of the whole period and the subperiods. The significance is obtained with a Monte Carlo approach with 1000 realizations. The surrogate data are constructed to mimic the full autocorrelation spectrum of the original WAI and are drawn from the same distribution as the original WAI. Three different statistics measuring the difference of the probability densities are used (see text). The table shows in percent the number of realizations with measure larger than that found with the original WAI.

	Period						
Statistic	1948–78	1979–2003	1948–58	1959–69	1970–80	1981–91	1992–2002
Δ	46	48	26	46	83	95	5
Δ_{slow}	70	91	66	79	47	91	7
Δ_{2D}	54	53	43	34	34	85	8

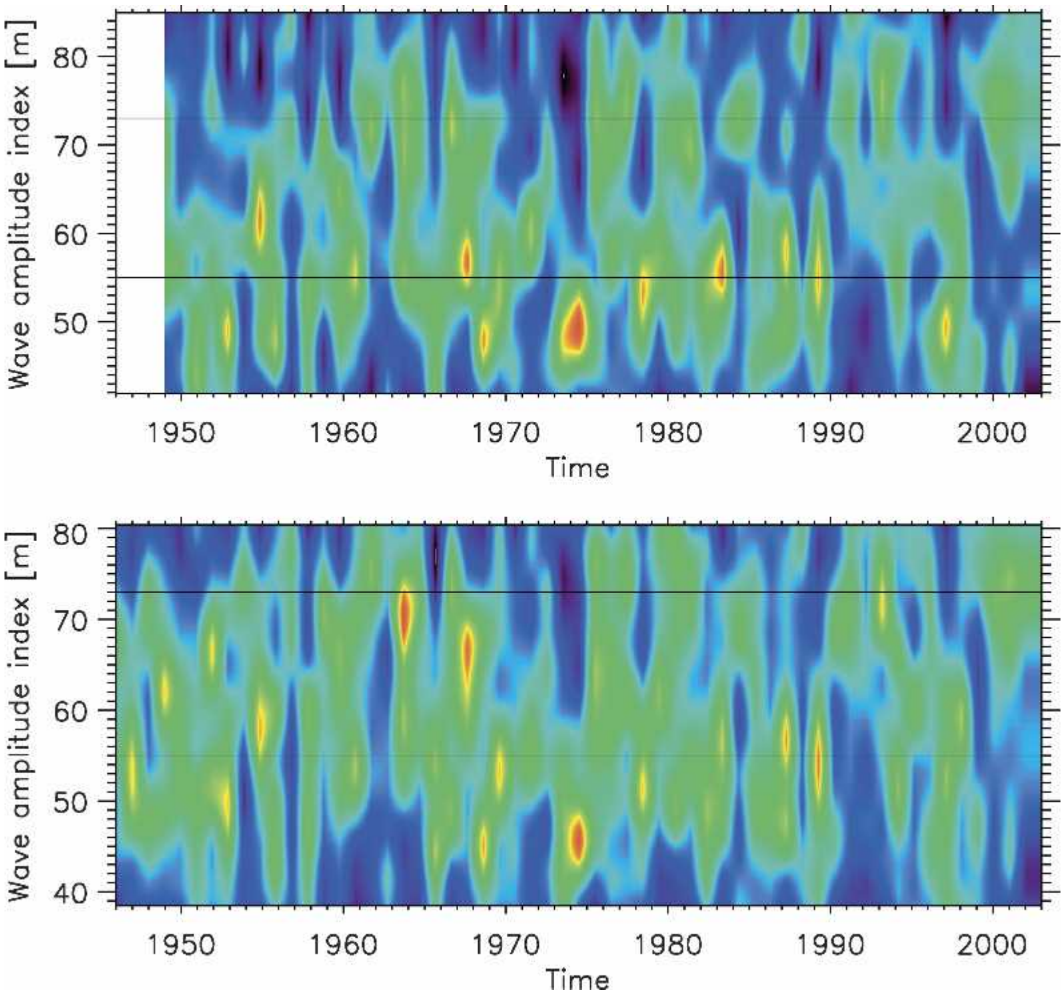

FIG. 4. The distribution of the WAI as a function of time. The distribution has been calculated for each winter with $h = 3$ m. The winters are denoted by the year that includes Jan. The horizontal lines indicate the positions of the maxima in Fig. 1a. Data are from (top) NCEP and (bottom) UKMO.

100, 200, 500, 850 hPa and from the sea level pressure. The probability densities are shown both for the full period and for the two subperiods 1948–78 and 1979–2003. For the full period weak bimodality is found at 100, 500 hPa, and at the sea level. At 200 and 850 hPa the distributions are somewhat skewed but the disturbed regime is not separated from the zonal regime. The results are consistent in that the disturbed regime is weaker than the zonal regime at all levels. In the late subperiod the bimodality is more pronounced and present at all levels except at 200 hPa. In particular a clear bimodality is seen in the sea level pressure. In the early period bimodality is only seen at 500 hPa. The bimodality is found to be statistically significant at the 95% level only at 500 hPa and at the sea level. At 500 hPa this holds for the late subperiod and for the full period while at the sea level only the late subperiod shows statistically significant bimodality at the 95% level.

For consistency we need to investigate if the strong and weak regimes at the 500-hPa level correspond to the strong and weak regimes at the sea level. This is so much more important as the WAI at the sea level and the WAI at the 500-hPa level are only weakly linearly correlated with correlations coefficients of 0.17 for the whole period and 0.12 and 0.22 for the early and late subperiod, respectively. Focusing on the late subperiod, 1979–2003, where significant bimodality is found at both levels, Fig. 8 shows the probability density in the space spanned by the two WAIs. When all data are included (Fig. 8a) maximum probability is found along the diagonal indicating a strong statistical connection between the WAI at 500 hPa and the WAI at the sea level. When including only the slowest half of the tra-

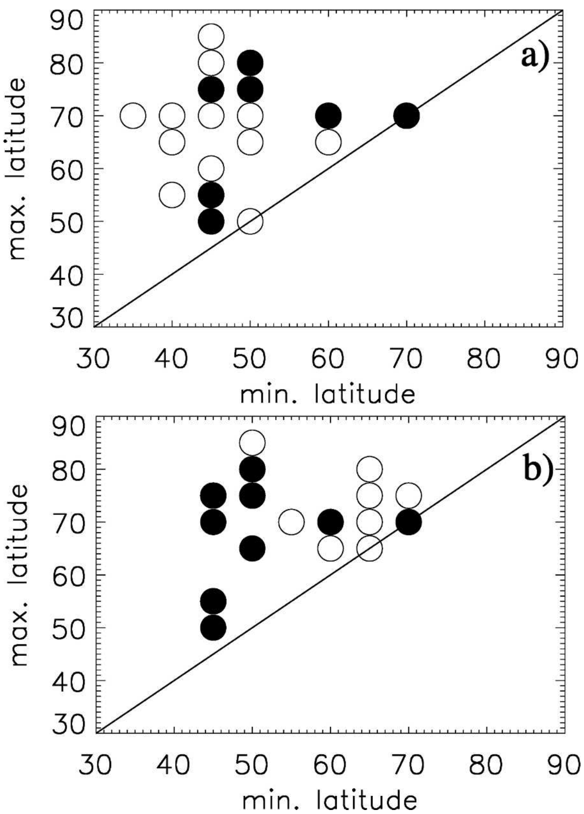

FIG. 5. The sensitivity of the bimodality of the WAI to the minimum and maximum latitudes over which the 500-hPa geopotential heights have been integrated: (a) D_{slow} and (b) D_{2D} have been used to measure the severity of the bimodality. Circles indicate latitudinal intervals where bimodality was observed in the distribution, filled circles where the bimodality is statistically significant. Data are from the NCEP reanalysis.

jectory in the analysis (Fig. 8b) the two regimes are clearly separated on the diagonal of the probability density. We therefore conclude that the regimes at the two levels are manifestations of the same physical phenomenon.

5. Discussion

The present study has been largely motivated by the diverse and conflicting results on the bimodality of the WAI in the previous literature and the resulting lack of consistency and reduced credibility. As the existence of multiple regimes in the atmospheric circulation has important consequences for our basic understanding of the climate and climate change, this situation is precarious. While a recent study (Stephenson et al. 2004) has shown that normality cannot be rejected in the space spanned by the two leading EOFs of the extratropical 500-hPa heights the WAI offers a physically sound alternative because the bimodality in this time series was predicted by theory before the original analyses by Hansen and Sutera (1986).

In this paper we have studied the bimodality in the WAI using the NCEP reanalysis data. Our study differs from previous studies mainly by stratifying the WAI according to its rate of change and by using an elaborated statistical test for bimodality where the surrogate data reproduce the whole autocorrelation spectrum of the original WAI. The former helps to enhance the statistical significance of the bimodality because it exploits the quasi stationarity of the potential regimes. The latter is important because we find that the WAI is not well modeled by an autoregressive process of order one. Our study also differs from previous studies by the length of the dataset, by systematically investigating the influence of the chosen latitudinal interval, and by incorporating vertical levels other than the 500-hPa height. The enhanced statistical significance and the length of the dataset allow us investigate the low frequency variability of the WAI.

We have shown that the bimodality in the WAI is very sensitive to the low-pass filtering. We find statistically significant bimodality only with a cutoff period of 4 days, while the distributions are unimodal for other cutoffs including the 5 days used in Hansen and Sutera (1995). This high sensitivity is probably responsible for the conflicting results of previous studies. Exploiting the quasi stationarity of the potential regimes by stratifying the WAI according to its velocity gives a consistent picture of the bimodality, which now is clearly visible both in the full period and in all decadal subperiods, although it is only statistically significant to the 95% level in the total period and the postsatellite period 1979–2003.

The probability distribution of the WAI shows a strong low-frequency variability. In particular the last decade stands out with a strong affinity for the disturbed regime. This recent change is statistically significant at least at the 90% level. The sign of the change is consistent with the concurrent change in the North Atlantic Oscillation toward its positive phase as the WAI is positively correlated with the North Atlantic Oscillation (Cerlini et al. 1999). We do not find any statistically significant difference between the periods 1948–78 and 1979–2003. This is worth noting, as a stratospheric regime change in the late 1970s toward the more zonal regime in the Northern Hemisphere has been reported (Christiansen 2003).

By systematically varying the latitudinal interval included in the definition of the WAI we find that although the WAI is sensitive to this parameter, statistically significant bimodality is present for a range of

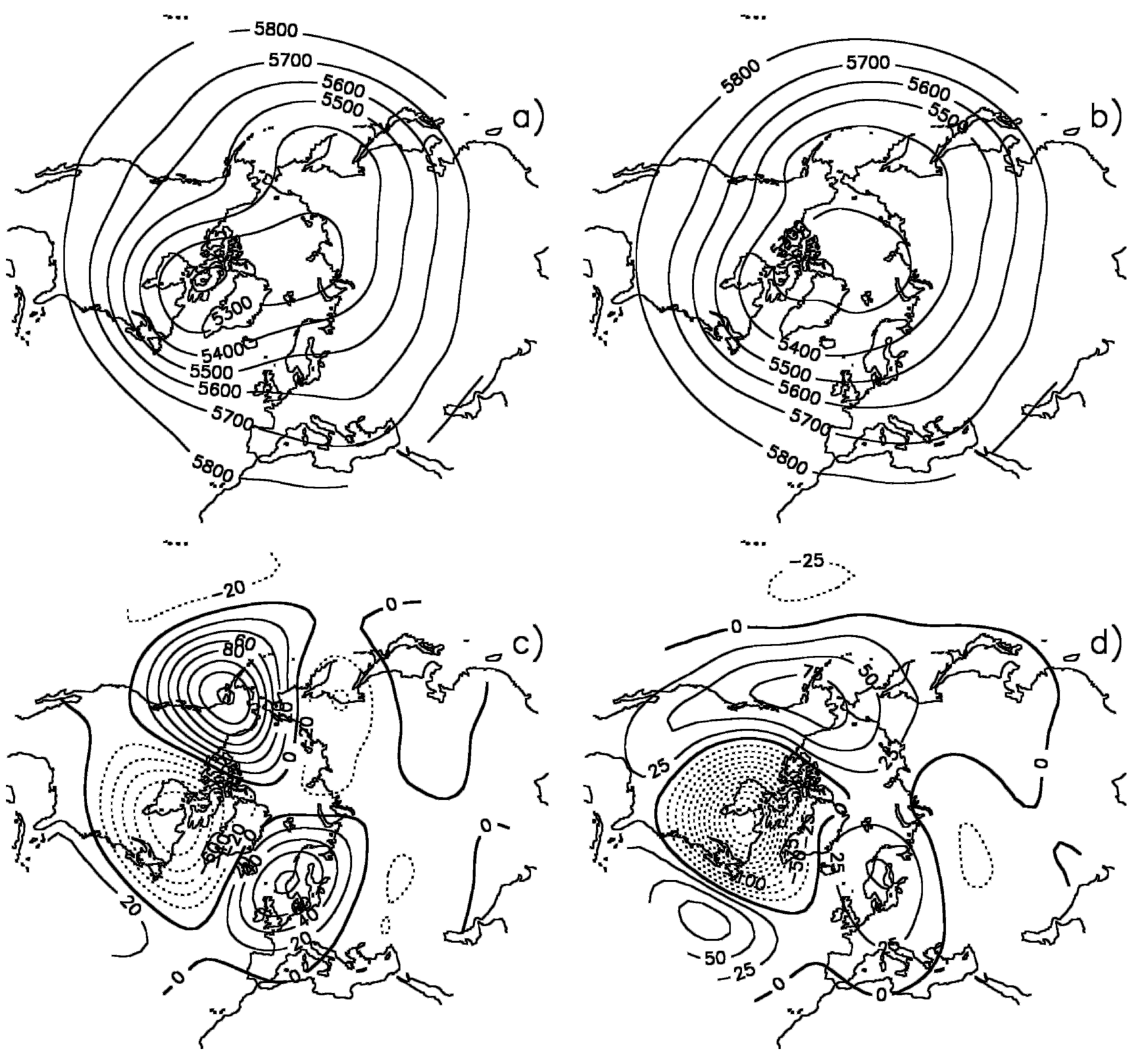

FIG. 6. The average of the 500-hPa geopotential heights (m) over (a) high index days (WAI > 66 m) and (b) low index days (WAI < 66 m), and (c) the difference. (d) The average height difference between days with negative and positive anomalous height at 55°N, 72°W.

intervals with the lower boundary near 45°–50°N. Studying the WAI at other vertical levels than 500 hPa shows a consistent picture, in particular in the period 1979–2003 when bimodality is seen at almost all tropospheric levels. At sea level the bimodality is statistically significant in the period 1979–2003 at the 95% level.

We think that the evidence presented in this paper, including the strict testing of statistical significance, the reproducibility in different periods, robustness to changes in the latitudinal interval, and consistency between different vertical layers make a strong case for the existence of bimodality in the WAI.

Applying different kinds of clustering algorithms to the Northern Hemisphere tropospheric geopotential height often results in three clusters (Ghil and Robertson 2002). The relation between the two WAI regimes and the regimes found in other studies has previously been discussed briefly by Smyth et al. (1999) and Cerlini et al. (1999). As mentioned by these authors the disturbed WAI regime (Fig. 6c) resembles regime A of Smyth et al. (1999) found by mixture model clustering and regime A of Cheng and Wallace (1993) found by hierarchical clustering. We also note a connection between cluster G in Smyth et al. (1999) and the undisturbed WAI regime. However, a full reconciliation of the WAI regimes and the regimes identified with clustering algorithms is not possible at present. A complicating factor is that at least some clustering methods will identify multiple regimes in distributions that are skewed but otherwise smooth and without bumps or shoulders. As an extreme example of unwanted behavior we mention nonlinear principal component analy-

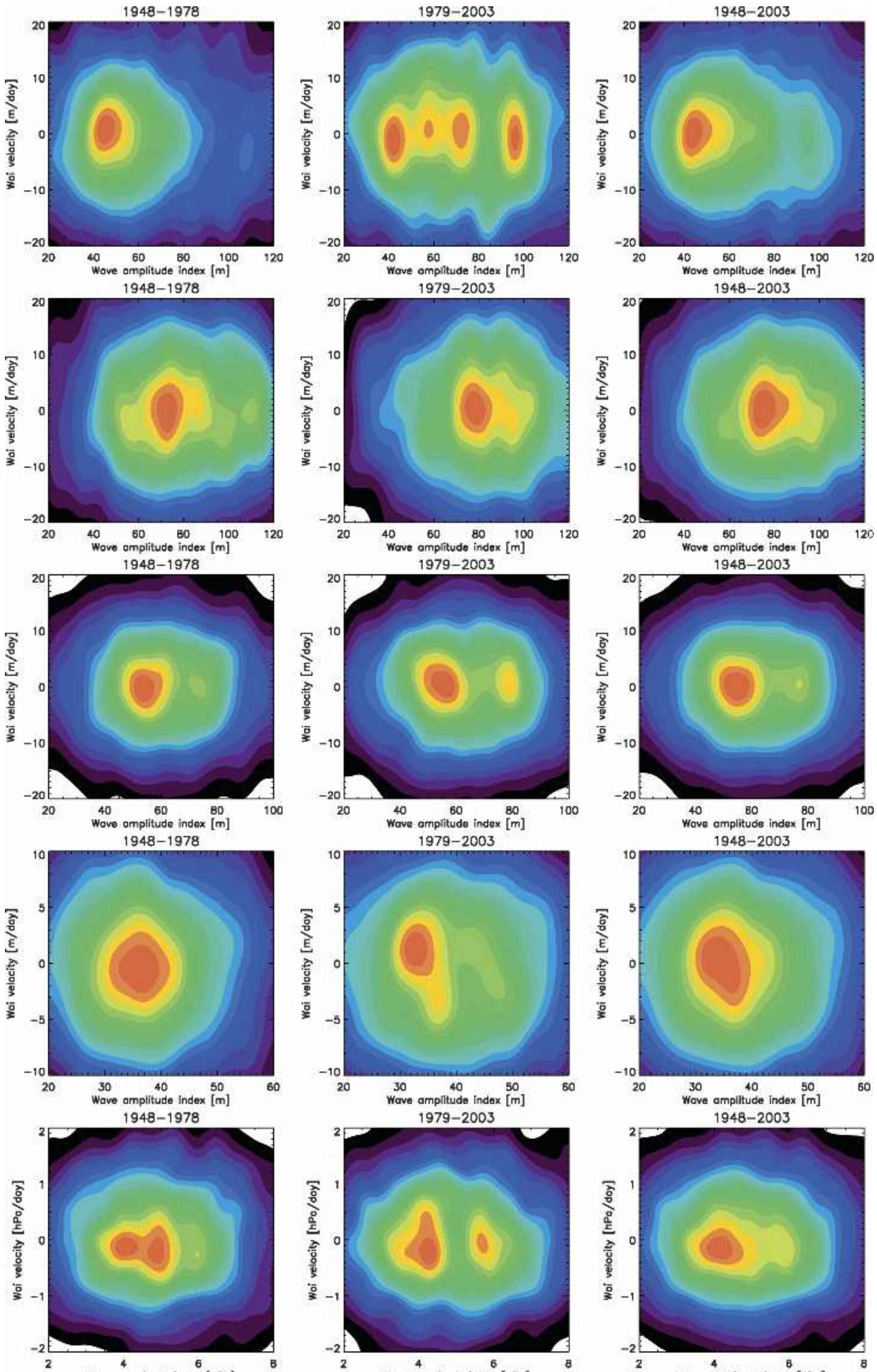

FIG. 7. The WAI for different vertical levels. The probability density is shown in the space spanned by the WAI and its rate of change. The WAI is calculated from the geopotential height at (first row) 100, (second) 200, (third) 500, and (fourth) 850 hPa. (last row) The WAI is calculated from the sea level pressure. The distributions are calculated from the periods (left) 1948–78, (middle) 1979–2003, and (right) 1948–2003. The values of *h* are 4 m at 100, 200 hPa; 3.5 m at 500 hPa; 2.25 m at 850 hPa; and 0.25 hPa at sea level. Data from the NCEP reanalysis.

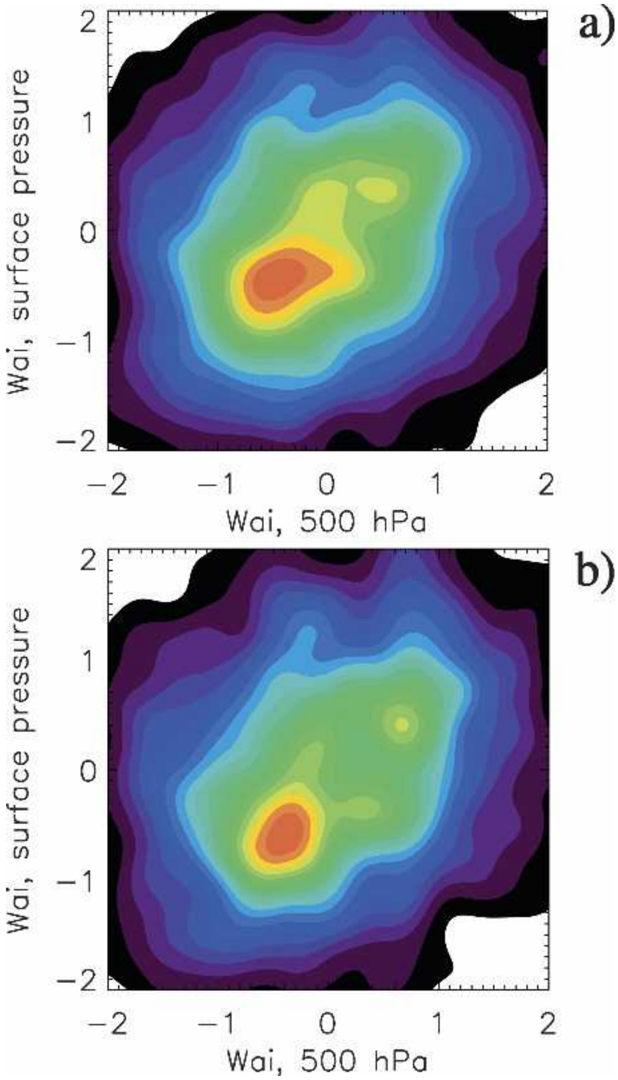

FIG. 8. The two-dimensional probability density in the space spanned by the WAI at 500 hPa and the WAI in the sea level pressure. The time series have been normalized and $h = 0.2$. (top) All data from 1979–2003 have been used; (bottom) only data from the slowest half have been used.

sis (Monahan et al. 2001), which often reports two regimes for data that are perfectly Gaussian distributed (Christiansen 2004, manuscript submitted to *J. Climate*).

We close the paper by a comment on anomaly correlations, as promised in the introduction. The probability distribution of anomaly correlations has occasionally been used in the search for regimes and nonlinearities in the atmosphere, and bimodality in this distribution has been interpreted as evidence for multiple atmospheric regimes. This interpretation was indicated in Wallace et al. (1991) and later used to argue for multiple regimes in the stratosphere (Perlwitz and Graf 2001). However, bimodality in the density distribution of anomaly correlations is not evidence for a system with multiple atmospheric regimes but only for a system with few degrees of freedom. This can be seen in the following way. Let $\Sigma_{i=1}^{N} a_i(t)\xi_i(x)$ be the principal component expansion of the field $\phi(x, t)$ of interest. We have the normalizations $[\xi_j(x)\xi_k(x)] = \delta_{jk}$, $\overline{a_j(t)a_k(t)} = \lambda_j^2\delta_{jk}$, where the brackets and the overbar indicates spatial and temporal averaging, λ_j^2 is the explained variance of the jth mode, and the modes are sorted with descending λ_j. The anomaly correlations are then

$$R_{kl} = \frac{[\phi(x, t_k)\phi(x, t_l)]}{\sqrt{[\phi(x, t_k)\phi(x, t_k)][\phi(x, t_l)\phi(x, t_l)]}} = \frac{a(t_k) \cdot a(t_l)}{|a(t_k)|\,|a(t_l)|},$$

where $a = (a_1, a_2, \ldots, a_N)$. The right-hand side is recognized as the cosine to the angle between $a(t_k)$ and $a(t_l)$. For a one-dimensional system ($N = 1$) the R_{kl} reduces to the product of the signs of $a_1(t_k)$ and $a_1(t_l)$, and the distribution of R_{kl} is clearly bimodal with sharp peaks at -1 and 1 regardless of the distribution of a_1. If the a_i are normally distributed then the distribution of R_{kl} is also bimodal for two- and three-dimensional systems regardless of the values of the λ. In general, we find that the distribution is bimodal if $3\lambda_1 - \Sigma_{i=1}^{N}\lambda_i > 0$. This is indeed the case in the stratosphere, which is dominated by a single mode explaining about half of the variance (Perlwitz and Graf 2001).

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