Direct Numerical Simulation of the Plumb–McEwan Laboratory Analog of the QBO

NILS P. WEDI

European Centre for Medium-Range Weather Forecasts, Reading, United Kingdom

PIOTR K. SMOLARKIEWICZ

National Center for Atmospheric Research, Boulder, Colorado

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ABSTRACT

The laboratory experiment of Plumb and McEwan demonstrates the principal mechanism of periodically reversing winds observed in the stratosphere—the quasi-biennial oscillation (QBO). However, despite numerous studies, some aspects of the QBO and the connection to its laboratory analog remain unclear. Incorporating the rapidly undulating boundaries of the laboratory experiment into the numerical algorithm—via time-dependent curvilinear coordinates—allows for the reproduction of the experimental setup, while minimizing numerical uncertainties. Results are presented of the first direct numerical simulation of the phenomena that lead to the zonal-mean flow reversal in the laboratory analog. The aim of this research is to narrow the widening gap between the theoretical understanding of laboratory-scale, internal-gravity wave processes and the complexity of global-scale circulations. A detailed study is presented on the parametric and numerical sensitivities of the oscillation. The results confirm a number of sensitivities, addressed in earlier studies. The analogy of radiative damping in the atmosphere and the role of molecular viscosity in the zonally varying laboratory flow are discussed, emphasizing the dominant role of wave–wave and wave–mean flow interactions in the latter, and in particular the retroaction of the induced mean flow on the waves. The findings elevate the importance of the laboratory setup for its fundamental similarity to the atmosphere. Implications are discussed for the theory and numerical realizability of equatorial zonal-mean zonal flow oscillations. The study corroborates the dependence of global-scale motions on small-scale wave-driven fluctuations, while being independent of parameterized or approximated means of forcing and wave dissipation.

1. Introduction

Irreversible processes are known to lead to the self-organization of coherent spatiotemporal structures (Kondepudi and Prigogine 1998, p. 427), which Prigogine called “dissipative structures,” to emphasize the constructive role of irreversibility resulting from dissipative processes. Small-scale fluctuations hereby play a decisive part while driving a system away from thermodynamic equilibrium (Kondepudi and Prigogine 1998, chapters 18 and 19), or in a fluid dynamical system away from solid-body rotation (McIntyre 2003). The notion of an emerging self-organization arises in a variety of disciplines, such as astrophysics, biology, human sciences, and economics (Kondepudi and Prigogine 1998, p. XVI). The ideas are closely related to the ultimate limitations of underlying “balanced” flow, slow-manifold, and potential vorticity inversion concepts in atmospheric studies (Ford et al. 2000).

In geophysical fluid dynamics, irreversibility manifests itself through the transfer of energy and momentum from smaller-scale fluctuations toward an emerging mean flow through nonlinearity and dissipation. The dynamical coupling of wind and ocean waves through wave-induced airflow represents one example of such a complex feedback resulting in coherent spatiotemporal structures (Hristov et al. 2003). In the atmosphere a prominent example is the quasi-biennial oscillation, and economics (Kondepudi and Prigogine 1998, p. XVI). The ideas are closely related to the ultimate limitations of underlying “balanced” flow, slow-manifold, and potential vorticity inversion concepts in atmospheric studies (Ford et al. 2000).

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1 Interestingly, the wind–wave coupling over open oceans is in remarkable agreement with “critical layer” theory (Hristov et al. 2003) referred to in QBO studies.
oscillation (QBO). The wave-driven QBO represents the dominant variability in the earth's equatorial stratosphere, exhibiting quasi-regular zonal-mean wind reversals with an average period of approximately 28 months.

The principal interaction of waves with a mean flow may be illustrated in a viscous, nonrotating Boussinesq fluid (Plumb 1977) by the momentum equations averaged in a horizontally periodic domain as

\[
\frac{\partial U}{\partial t} - \nu \frac{\partial^2 U}{\partial z^2} = \sum_i \frac{\partial F_i}{\partial z},
\]

where \( U_i \) denotes the horizontally averaged (mean) flow, \( \nu \) denotes the kinematic viscosity, and \( F_i := \bar{u} \bar{w} \) expresses a contribution to the averaged nonlinear momentum flux. Most atmospheric research of the QBO is devoted to finding the precise physical mechanisms of the atmospheric QBO (Baldwin et al. 2001).

In the laboratory, the averaged momentum flux has been attributed almost exclusively to viscous internal wave dissipation (Plumb 1977; Plumb and McEwan 1978; McIntyre 2003), in analogy to equatorial Kelvin and Rossby–gravity wave attenuation by infrared cooling and, hence, consistent with the theory of Holton and Lindzen (1972). The latter theory motivated the laboratory experiment and its corresponding conceptual synthesis (Plumb 1977; Plumb and McEwan 1978) that established the fundamental picture of the QBO as forced by large-scale upward-propagating waves, with the amplitude and the rate of downward propagation determined by the waves' phase speeds and intensity, respectively (Lindzen 1987).

To further the understanding, a number of 1D and 2D mechanistic models (Holton and Lindzen 1972; Plumb and Bell 1982; Saravanan 1990) have been employed. In these models the zonally averaged momentum Eq. (1) is solved together with linearized equations for the waves' properties, using simplifying assumptions such as the slow variation in space and time of wave amplitude, zonal-mean wind, density, and damping compared to the individual waves' phase speed. Equation (1) has also been extended to account for the mean density change in the atmosphere (Plumb 1977), a mean upward vertical motion, and a continuous forcing spectrum of waves (Saravanan 1990; Dunkerton 1991). While constraint by the simplifications, mechanistic models have added substantially to the understanding of QBO-like oscillations.

Notwithstanding, the two-wave model with oppositely traveling Kelvin and Rossby–gravity waves (Holton and Lindzen 1972; Plumb 1977) is incomplete. Observations showed that there must be an additional easterly gravity wave mode required to account for the easterly acceleration of the QBO (Lindzen and Tsay 1975). Furthermore, tropical upwelling, a climatic mean upward motion of the tropical atmosphere, was shown
to necessitate contributions to the mean flow momentum budget of other wave types. In particular, Dunkerton (1997a) argued that shorter-scale gravity waves contribute up to 70% to the forcing, thus indicating a considerable uncertainty about the precise origin and the nature of the waves responsible for driving the QBO. Consistently, high-resolution 3D simulations demonstrated insufficient provision of wave momentum flux by the classical two-wave model, given realistic amplitudes of Kelvin and Rossby–gravity waves (Takahashi and Boville 1992). By relaxing the simplifying assumptions characteristic of 1D and 2D mechanistic models, Takahashi and Boville found wave–wave interactions of Kelvin and Rossby–gravity waves, and subsequent modifications of the background wind, to be important for the simulated QBO period.

An earlier explanation of the QBO suggested that the stratospheric mean flow oscillation is driven by critical-layer\(^3\) attenuation of a spectrum of gravity waves (Lindzen and Holton 1968). Similarly, Dunkerton (1981b) and McIntyre (1994) considered wave transience and wave breaking as chronologically more important primary causes of the zonal-mean flow oscillation in the atmosphere. Hence, the laboratory experiment of Plumb and McEwan has also been criticized for its apparent fundamental difference to the QBO (Dunkerton 1981b).

In large-scale numerical simulations, Horinouchi and Yoden (1998) found a critical-layer mechanism and contributing waves consistent with Lindzen and Holton (1968) to be responsible for QBO-like oscillations. Notably, the latter study used an “aquaplanet” idealization with parameterized convection, exciting a spectrum of shorter-scale gravity waves. The intercomparison study of several general circulation models (GCMs) (Horinouchi et al. 2000) corroborates a correlation between the simulated QBO and convective processes, while finding equatorial Kelvin and Rossby–gravity waves to be unimportant relative to other types of wave forcing. Moreover, the European Centre for Medium-Range Weather Forecasts (ECMWF) 15-yr Re-Analysis (ERA-15; Gibson et al. 1999) shows insufficient spectral power of equatorially trapped planetary waves (Tindall 2003). Consistently, successful numerical modeling of a realistic 3D atmospheric QBO typically required the parameterization of nonorographic vertically propagating gravity waves (Scaife et al. 2000; Giorgetta et al. 2002).

In this paper we aim to narrow the widening gap between the theoretical understanding of laboratory-scale internal-gravity wave processes and the growing complexity of global-scale climate models (Held 2005). Hence, we perform a quantitative analysis of the direct numerical simulation (DNS) of the Plumb and McEwan laboratory experiment. By DNS we mean integrating the Navier–Stokes equations under the Boussinesq approximation for salty water without any parameterizations. Our simulations resolve the fluid motion up to the Kolmogorov length scale \(\eta = (\nu/\ell)^{1/4}\), where \(\nu\) is the kinematic viscosity and \(\ell\) denotes the kinetic energy dissipation rate. Typical grid sizes in our simulations are \(\mathcal{O}(\eta)\); see Moin and Mahesh (1998) for a review of DNS. The accuracy of our approach removes the uncertainty with respect to the approximate nature of either mechanistic or global circulation models, which arises from the lack of knowledge of the required wave forcing and dissipation together with the approximate numerical realization of these mechanisms.

In our simulations we observe wave–wave interactions\(^3\) of waves with equal horizontal wavelength, a resulting mean flow acceleration and subsequent mean flow wave retroaction (Galmiche et al. 2000) that emerge as the chronologically more important primary cause of the zonal-mean flow oscillation in the laboratory analog. Subsequent wave instabilities, and later critical-layer momentum flux absorption, as well as viscous dissipation are important but chronologically secondary. The mechanism we observe in our experiments is initiated by localized, nonlinearly induced shear instability, which cannot be deduced when wave–wave interaction and associated mean flow wave retroaction are neglected. Thus, our results suggest that the critical-layer mechanism—contrary to common belief (Alexander and Holton 2004; Dunkerton 1997a; McLandress and Scinocca 2005)—does not require a “separate process” to initiate shear zones at high levels, because nonlinear wave–wave interactions of the same waves can catalyze the onset of their own shear zones aloft (Galmiche et al. 2000).

The next section describes our virtual (computational) laboratory of the QBO analog, summarizes the numerical framework, and presents the principal result, that is, the numerical reproducibility of the laboratory experiment. Section 3 reports on the parametric and numerical sensitivities in our simulations of QBO-like

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\(^2\) Following Andrews et al. (1987) and McIntyre (2003), we refer to a “critical layer” as not necessarily a zonally continuous height band, in which a group of waves are attenuated by a variety of dissipative processes. A critical surface or level, first elucidated by Bretherton (1966) and Booker and Bretherton (1967), represents an upper asymptotic limit to the critical layer.

\(^3\) Wave–wave interaction in the context of this paper is not to be confused with nonlinear triadic (or resonant) wave interaction; cf. section 4b.
oscillations, verifying a number of sensitivities found by other authors, while adding detail not easily deduced from previous laboratory or numerical studies. Section 4 presents a revised conceptual model of the zonal-mean zonal flow oscillation in the laboratory experiment, together with a supporting theoretical analysis. In section 5, we discuss our observations in the context of earlier studies, implications for the atmospheric QBO, and for its realizability in numerical weather prediction and climate models.

2. Virtual laboratory

a. The simulation

The laboratory experiment of Plumb and McEwan was conducted in a transparent cylindrical annulus (radii \( a = 0.183 \) m and \( b = 0.3 \) m) filled with density-stratified salty water to a height of \( z_{ab} = 0.43 \) m. The lower boundary consisted of a thin rubber membrane oscillating with a constant frequency of \( \omega_0 = 0.43 \) s\(^{-1}\) and an amplitude of \( \varepsilon = 0.008 \) m (Plumb and McEwan 1978). In the analogous numerical simulation we assume an initially stagnant fluid, forced by an oscillating lower boundary. The cylindrical laboratory tank is represented by a zonally periodic, rectangular computational domain composed of 639 \( \times \) 188 grid intervals with \( L_x = 2\pi(a + b)/2, L_y = b - a, L_z = 0.43 \) m. The impermeable lower and lateral boundaries are no slip. At the upper boundary a free-slip rigid lid is assumed. The lower surface is prescribed as a linear combination of elementary shape functions,

\[
z(x, y, t) = \sum_{n=0}^{N-1} e_n \xi_n(x) \sin \left( \frac{\pi}{L_y} y \right) \sin(\omega_0 t + \phi_n),
\]

(2)

with individual amplitudes \( e_n \) and zonal profiles,

\[
\xi_n = \begin{cases} \cos^2(\pi \eta /2L_n) & \text{if } \|r_n/L_n\| \leq 1 \\ 0 & \text{otherwise} \end{cases},
\]

(3)

where \( L_n = L_x/\mathcal{N} \) and \( r_n = x + (\eta - 0.5)L_n \). In general, (2)–(3) describe a subdivision of the periodic domain into \( \mathcal{N} \) independently oscillating chambers. For \( \mathcal{N} = 16 \), \( e_n = 0.008 \) m \( \forall \eta \), and \( \phi_n = \pi \) for even \( \eta \) (\( \phi_n = 0 \) for odd \( \eta \)), the equation collapses into the symmetric solution of the 2D wave equation (Fig. 1), employed in the original Plumb and McEwan laboratory experiment.

For the initial condition a static state was assumed with buoyancy frequency \( N = 1.57 \) s\(^{-1}\), as specified in the original movie of the laboratory experiment. The integration time was several hours with a time step of \( dt = 0.05 \) s. The zonal-mean zonal flow oscillation was seeded with a zonal background flow \( u_e \) in the near-membrane layers, using \( u_e = u_0[1 - 0.5\delta(1 + \tanh(z - d_0)/\gamma)] \) with \( u_0 = 0.02 \) m s\(^{-1}\), \( \delta = 0.9999999 \), \( d_0 = 0.06 \) m, and \( \gamma = \Delta z \).

In section 3, we motivate the above-described principal setup, while discussing various deviations and the associated solution sensitivities.

b. The numerical model

The Boussinesq equations of motion—an accurate approximation for salty water (Gill 1982)—for a non-rotating, density-stratified, viscous fluid are cast in a time-dependent curvilinear framework (Prusa and Smolarkiewicz 2003; Smolarkiewicz and Prusa 2005):

\[
\frac{\partial (\rho^* \varpi^k)}{\partial \xi^k} = 0,
\]

\[
\frac{d\varpi^j}{dt} = -G^j_k \frac{\partial \varpi^j}{\partial \xi^k} - \frac{\rho^*}{\rho_0} \delta^j_s + \varpi^j,
\]

\[
\frac{d\varpi^i}{dt} = -\varpi^j \frac{\partial \rho^*}{\partial \xi^j} + \mathcal{N},
\]

(4)

Here, \( \rho^* := \rho_0 \mathcal{G} \), with \( \mathcal{G} \) denoting the Jacobian of the transformation between physical \((t, x, y, z)\) and computational \((\xi, \tau, \eta, \zeta)\) space. Indices \( j, k = 1, 2, 3 \) correspond to the \( \xi, \tau, \eta, \zeta \) components, respectively; summation is implied by repeated indices, unless stated otherwise. The total derivative is \( d/dt := \partial/\partial t + \varpi^j (\partial/\partial \xi^j) \), where \( \varpi^j := \varpi^j - \partial \varpi^j / \partial t \). The components of physical velocity \( \varpi^j \) are related via \( \varpi^j = G^j_i \varpi^i \), where \( G^j_k = (\partial \xi^j / \partial \xi^k) \) are transformation coefficients; \( \rho^* \) and

![Fig. 1. Shape of the oscillating membrane as described in Eqs. (2)–(3).](image)
\( \pi' \) denote, respectively, density and normalized-pressure perturbations with respect to the static ambient state characterized by the linearly stratified profile \( \rho_0 \). \( g \) symbolizes the gravitational acceleration; \( \rho_0 \) is a constant reference density; and \( \delta' \) is the Kronecker delta. The density diffusivity \( \varphi' \sim \nabla \cdot \kappa \nabla \rho' \) and the momentum dissipation \( \psi' \sim \nabla \cdot \nu (\nabla v + \nabla v') \) are detailed in Smolarkiewicz and Prusa (2004). Here, we specify a kinematic viscosity \( \nu = 1.004 \times 10^{-6} \text{ m}^2 \text{s}^{-1} \) and a diffusivity of salt in water of \( \kappa = 1.5 \times 10^{-9} \text{ m}^2 \text{s}^{-1} \).

We employ the generalized Gal-Chen coordinate transformation,

\[
\begin{align*}
\tilde{t} &= t, \quad \tilde{x} = x, \quad \tilde{y} = y, \\
\tilde{z} &= H_0 \frac{z - z_s(x, y, t)}{H(x, y, t) - z_s(x, y, t)},
\end{align*}
\]

whose theoretical development and efficient numerical implementation were discussed thoroughly in Wedi and Smolarkiewicz (2004). The transformation (5) allows for time-dependent upper, \( H(x, y, t) \), and lower, \( z_s(x, y, t) \), boundary forcing without small-amplitude approximations.

The governing Eqs. (4) are discretized in the transformed space using a second-order-accurate, optionally semi-Lagrangian or Eulerian, nonoscillatory forward-in-time (NFT) approach, broadly documented in the literature (cf. Smolarkiewicz and Prusa 2002; Smolarkiewicz 2006 for a recent review). Here, we employ the flux-form Eulerian, semi-implicit version of the algorithm, unless specified otherwise. All prognostic equations in (4) are integrated using the trapezoidal rule, solving the viscous and diffusive terms are computed explicitly to first-order accuracy; see section 3.5.4 in Smolarkiewicz and Margolin (1998). Together with the curvilinearity of the coordinates, this leads to a complicated elliptic problem for pressure (see appendix A in Prusa and Smolarkiewicz 2003 for the complete description) solved iteratively using a preconditioned nonsymmetric Krylov-subspace method (Smolarkiewicz and Margolin 2000).

c. The result

Figure 2 juxtaposes the time–height cross sections of the zonal-mean zonal flow from the laboratory experiment (adapted from Fig. 10 of Plumb and McEwan 1978) and from our numerical simulation (representative for the center of the annulus). The overall agreement of the solution structure is apparent. Deduced from Fig. 2b, the oscillation period is \( T = 92 \text{ min} \), the maximum amplitude is \( A_0 = 0.0085 \text{ m s}^{-1} \), and the vertical extent is 21–28 cm. The latter is defined by the lowest nonzero contour line (2 mm s\(^{-1}\)) of the zonal-mean zonal flow in Fig. 10 of Plumb and McEwan (1978). In the introduction to their paper, Plumb and McEwan state an observed oscillation period of typically 1 h. However, measuring the oscillation period from Fig. 2a, one obtains \( T = 78 \text{ min} \). For the same case, our own measurements taken directly from the original movie (using the changing direction of the neutrally buoyant spheres as an indicator) give an average value of \( T = 85.0 \text{ min} \), with individual periods ranging from 70 to 100 min. A mean flow oscillation is observed throughout the annulus (moving spheres on the water surface also indicate a change in direction) with increasing magnitude in the lower half of the water tank. Plumb and McEwan (1978) state a maximum observed mean flow velocity of \( A_0 = 0.0085 \text{ m s}^{-1} \).

The close comparability of the experimental and numerical results needs to be qualified further. Initially, we were unable to reproduce the results of Fig. 10 in Plumb and McEwan (1978) using their precise description and the parameters stated. Although the amplitude and height of the zonal-mean zonal flow reversal were captured well (cf. Fig. 3b), we obtained an oscillation period of \( T = 55 \text{ min} \), in agreement with their general statement but in disagreement with the more precise quantitative comparison. Similarly, we were unable to reproduce the period of their larger-amplitude, lower-frequency “nonlinear” experiment (Fig. 11 in Plumb and McEwan 1978), compare with entries b and f in Table 1. This led us to simulate the laboratory experiment repeated more recently at the University of Kyoto (GFD Dennou Club 2005; Otobe et al. 1998), for which we obtained a smaller discrepancy. The comparison of the two experimental setups identified the key role of the forcing membrane shape. There is a pronounced sensitivity to the forcing wavenumber \( k \) (cf. Plumb 1977), in some cases stronger than \( \sim k^3 \). For example, doubling the domain size increases the period by a factor of \( \sim 10 \) (Wedi and Smolarkiewicz 2005), corroborated by the laboratory experiments of the University of Kyoto (S. Yoden 2004, personal communication). In particular, this indicates that small departures from the prescribed membrane shape can be responsible for large variations in the zonal-mean flow period. A close examination of the oscillating membrane in the movie of the original experiment suggests a less exact shape than in the simulation. Nearly exact comparability with the Plumb and McEwan laboratory results was achieved (Fig. 2), when two randomly chosen membrane chambers were taken as \( e_x = e_{1z} = 0.004 \text{ m} \) in (2).
which represents about 6% uncertainty in the experimental realization of the wavelength of the external forcing. Figure 3 and entries d and e in Table 1 compare the solutions obtained with the altered and the original membrane shape.

The identified discrepancies and the range of laboratory results stimulated a further investigation into a wide variety of possible parametric and numerical sensitivities. Both were found to influence the character and magnitude of the zonal-mean flow oscillation—the vertical extent, the rate of descent of the critical layer, and hence the period. The investigation enabled us to accurately reproduce the range of results from the laboratory experiments, improving our understanding of the mechanism underlying the analog to the QBO. In the next section, we report and quantify the observed sensitivities, while deferring their discussion to sections 4d and 5.

3. Sensitivities

A set of simulations was conducted to examine the sensitivities of the zonal-mean zonal flow oscillation to (a) 2D versus 3D geometry of the problem, (b) initial conditions, (c) boundary conditions, (d) altering the position of the membrane and the size of the computational domain (i.e., the University of Kyoto setup), (e) viscosity and dissipation, (f) stratification, (g) external forcing, and (h) various numerical sensitivities. Selected simulations are highlighted in Table 1 [for the comprehensive summary, see Wedi (2004)].

Fig. 2. Time–height cross section (a) of the observed zonal-mean zonal flow velocity component [adapted from Fig. 10 in Plumb and McEwan (1978), contour lines are in mm s⁻¹] and (b) compared to the results of the 3D numerical simulation at y = L/2 (contour lines are in m s⁻¹). According to Plumb and McEwan (1978), the lowest 2 cm in (a) could not be observed due to restrictions of the viewing window.
a. 2D versus 3D

To optimize the overall computational cost of the study, some of the sensitivities were investigated in the equivalent two-dimensional setup,\(^5\) representative of the midchannel \(y = L/2\) in Fig. 1. While 2D and 3D simulations are qualitatively similar, there are significant quantitative departures in the vertical extent and the mean flow oscillation period (Fig. 4). Two-dimensional simulations agree better with their 3D counterpart, when compared with an “infinite” membrane of uniform shape in the meridional direction (not shown). The comparability to 3D simulations is further enhanced, when only the “energetically averaged” effect of the 3D oscillation is admitted, by reducing the effective forcing amplitude in 2D simulations to \(r_{\text{eff}} = 0.64 \epsilon\) [integrated over the 3D membrane shape (2) and renormalized; cf. appendix C in Plumb and McEwan (1978)]. Nevertheless, in the 2D calculations one observes a successive growth of the vertical extent with each oscillation period, whereas in 3D simulations the oscillation depth and period remain finite.

The phase speed of the waves is fast, compared to the time it takes for the waves to interact. The upper boundary effects become important in the 2D simulations, but not normally in 3D simulations of the laboratory experiment; see the discussion on boundary conditions in section 3c below. Using a very deep tank (3 m), the vertical extent and the period of the mean flow oscillation grew in distinct “steps” (not shown), until the top of the tank was reached. The growing steps found in this simulation are observed at the beginning of almost any 2D simulation (see, e.g., Fig. 4b). We find spatial wave interference predominantly at or above the height of the previous step, which determines the height of the next step and coincides with the height at or below which the critical flow velocity is reached (see section 3g and section 5 for further discussion).

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\(^5\) Our 2D simulations only assume slab symmetry and must not be confused with zonally averaged 2D mechanistic models.
b. Initial conditions

QBO-like oscillations are a canonical example of longtime behavior resulting from short-term fluctuations. Therefore, one may expect the initial conditions to be of minor relevance to the long-term solution. Nevertheless, the choice of initial conditions can be important for practical reasons. As already stated in section 2a, we seeded the simulations with an initial constant environmental zonal wind in the lowest 1–6 cm. This substantially reduced the required overall computing time without compromising the results. If the simulation is not seeded, it is not possible a priori to predict the first onset of a positive or a negative mean flow. An oscillation then develops typically after 1–3 h of simulated time, with an initial asymmetry in the zonal periodicity of the flow enforced randomly by tiny perturbations. It has been found that initial perturbations with a directional bias are as efficient as the initial seed flow. In a 3D simulation with forcing amplitude $e = 0.008 \text{ m}$, without any initial random noise we failed to obtain an oscillation even after 4.5 h of simulation. Noteworthy, in the laboratory experiments of the University of Kyoto, initial difficulties in creating an oscillation were overcome by the addition of “dirty water” (GFD Dennou Club 2005), presumably due to imposing a mechanical disturbance and/or due to restoring the stratification diffused in the vicinity of the membrane (cf. section 3f).

c. Boundary conditions

The role of the upper free-water surface in the Plumb and McEwan experiment was investigated using different physical models for its simulation: an absorbing region, a shallow water model, and a rigid (free slip) lid. The absorbing region was applied by extending the height of the water tank by the absorbing region, thus resulting in $L_z = 0.6 \text{ m}$. The (free slip) shallow water solution at the top of the domain was evaluated at every time step and incorporated [via $H(x, y, t)$] into the coordinate transformation (5) (see Wedi and Smolarkiewicz 2004 for details). The high-resolution solution of the nonlinear shallow water equations with zero forcing wind and oscillating small-amplitude topography essentially remains flat, in agreement with our observation of the upper boundary in the movie of the original Plumb and McEwan experiment. All three models for the upper boundary show negligible differences in the structure or the period of the mean flow oscillation in the 3D simulations, thus justifying the rigid- (free slip) lid approximation adopted in section 2a.

In contrast to 3D calculations, where the amplitude of vertically propagating waves decays with height due to the dispersion, 2D experiments show a significant sensitivity to the type of upper-boundary model applied, including absorbing, free-slip rigid, no-slip rigid, and free-slip shallow water. The differences result from reflected gravity waves that interact with the original waves, effectively controlling the height of maximum zonal-mean flow generation and the period of the oscillation (not shown). In 2D, we found that a no-slip rigid upper lid created the most symmetric regular solution for longtime integrations, whereas the free-slip upper boundary allows reflected waves to propagate back deeper into the domain, thus resulting in enhanced irregularity in the onset of shear flows. Although an absorbing layer (“sponge”) minimized re-
The choice of the boundary condition at the oscillating membrane also affected the period by influencing the effective forcing amplitude. A free-slip condition at the oscillating membrane (case h in Table 1) suppresses a frictional boundary layer and results in a ∼15–25 min shorter period compared to a no-slip condition (case g in Table 1). While less significant in comparison to the influence of changes in the forcing wavelength (cf. section 3g below), it is not negligible. Auxiliary high-resolution studies of the near-membrane boundary layer region showed that with zero stratification the boundary layer grows to approximately 5 cm after a few minutes of simulation. This is not observed when examining the movies of the laboratory experiment of the University of Kyoto, where we cannot find evidence of a significant boundary layer. Simulations of the near-membrane boundary layer with stratification included (and no-slip boundary condition) exhibit a varying boundary layer depth of approximately 0–1 cm.

The width of the boundary layer at the lateral (meridional) no-slip walls is found to be approximately 1 cm and appears to have no effect on the solution in the middle of the simulated tank. Notably, further increased resolution does not change the near-wall behavior.

d. The University of Kyoto setup

The laboratory experiment of the University of Kyoto exhibits two main differences from the original Plumb and McEwan setup: (i) the oscillating membrane was placed at the top of the annulus initiating an oscillation with an apparent upward propagation of the mean flow and (ii) several water tank sizes were available (S. Yoden 2004, personal communication). The setup described in Otobe et al. (1998) is equivalent to the original Plumb and McEwan experiment, whereas the experimental setup described online (GFD Dennou Club 2005) used a domain approximately twice as large ($2L_x, 2L_y$) (case c in Table 1). Their laboratory experiments show a range of observed mean flow oscillation periods of 45–120 min (Otobe et al. 1998), with several

![Figure 4](image-url)
observed periods at 55 min, in close agreement with our results from the original Plumb and McEwan setup. We deduced a period of 76 min from the movie and its description online (GFD Dennou Club 2005). However, since the same movie was found at both references, the precise period for the larger domain case is somewhat uncertain. Notably, after several hours the stratification degraded in the near-membrane layers (S. Yoden 2004, personal communication), potentially influencing the resulting mean flow oscillation period (see section 3f).

Our equivalent numerical simulations used the time-dependent upper boundary entering the coordinate transformation in (5) as

$$H(x, y, t) = H_0 - z_s(x, y, t)$$

The bottom boundary in the simulations was assumed to be rigid no-slip. Two simulations were conducted. The simulation with a no-slip boundary at the membrane shows a period of >100 min. A closer comparison with the filmed time evolution of the experiment is found with a free-slip boundary condition at the membrane (cf. Fig. 5; case h in Table 1), which coincidentally results in a period of 76 min.

The domain size in our simulation influences both the wavelength of the forcing and the curvature of the water tank. We obtain relatively longer periods, a larger vertical extent, and larger zonal-mean velocities compared to the original Plumb and McEwan setup. A potentially better comparability of simulated and laboratory results for larger annuli could indicate that curvature effects (see Read et al. 1997 for a discussion) may be important. However, recent results with a prelimerinary 3D cylindrical curvilinear variant of our model suggest only a small reduction in vertical extent and zonal-mean velocity of the otherwise similar zonal-mean flow oscillation.

e. Viscosity and dissipation

The kinematic viscosity $\nu$ was varied between $1 \times 10^{-5}$ and $1 \times 10^{-7}$ m$^2$ s$^{-1}$, which influences both the vertical extent of the oscillation and the speed of the apparent downward propagation. In particular, lower values of $\nu$ increase the period and limit the vertical extent (cf. Wedi 2004) in agreement with Plumb (1977).

In the asymptotic limit of $\nu = 0$, the simulation shows strongly elongated mean flow layers on top of each other and that an initially enforced oscillation (using a seed flow) is not maintained (not shown). Interestingly, when a Newtonian cooling term of the form $-\beta p'$ with sufficiently large $\beta(=1/180$ s$^{-1})$ is included on the rhs of the thermodynamic Eq. (4), the two layers are broken up and a regular mean flow oscillation emerges. Imposing instead a Rayleigh friction of the form $-\alpha v$ in the momentum equation suppresses even the initial oscillation. Notably in the viscous problem, too large values of kinematic viscosity also suppress the oscillation, in accordance with an overwhelming effect of the viscous term compared to the wave momentum flux divergence in (1).

In contrast, varying the diffusivity of salt $\kappa$ between $1.5 \times 10^{-7}$ and $10^{-9}$ m$^2$ s$^{-1}$ had little impact on the oscillation. However, there is a three orders of magnitude difference between $\nu$ and $\kappa$ (Prandtl number for the saline solution $\approx 685$), and these coefficients appear as a sum in the Wentzel–Kramers–Brillouin (WKB) approximated equations [cf. Grimshaw (1974) and Baines (1995)].

f. Stratification

In the range $1.37$ s$^{-1} \leq N \leq 1.57$ s$^{-1}$ investigated by Plumb and McEwan (1978), our simulations confirm a

---

6 This model option is a work in progress, stimulated by the referees’ comments.

7 A 10 times smaller value of $\beta$ did not affect the $\nu = 0$ solution.
$\approx 1/N$ dependency of the period. However, when half
the Brunt–Väisälä frequency $N = 0.685$ s$^{-1}$ is used
there is no mean flow oscillation. Initially, a sharp
downward-propagating shear layer is produced. But
when it arrives near the forcing membrane, a single
mean flow layer of uniform magnitude remains. De-
spite sufficient amplitude of the forcing membrane, the
reversed flow aloft and the separating shear are weak.
There are no shear-induced wave instabilities observed
and the flow remains stable (see further discussion in
section 3g).

In most simulations presented in this paper a value of
$N = 1.57$ s$^{-1}$ as quoted in the original movie of the
Plumb and McEwan experiment was adopted. We
obtained a stratification $N = 1.88$ s$^{-1}$ from the derived
values of Fig. 10 in Plumb and McEwan (1978), $T_0 = 476$ s, $d_0 = 0.17$ m, and their Eqs. (4.10)–(4.12), which
is inconsistent with the specifications in the original
movie of their experiment and the explanations in the
text [see Wedi and Smolarkiewicz (2005) for simula-
tions using $N = 1.88$ s$^{-1}$]; the Brunt–Väisälä frequency
derived from values of Fig. 11 in Plumb and McEwan
(1978) is $N = 1.71$ s$^{-1}$ with $T_0 = 133$ s, and $d_0 = 0.082$
m.

The qualitative influence of vertically varying strati-
fication was investigated by means of two differently
stratified layers with a continuous but sharp transition
of the form

$$
\rho_x = \begin{cases} 
\rho_0 (1 - 0.25) & \text{if } |z| \leq z_0 \\
\rho_0 (1 - \frac{z}{z_0}) \left[ 1 - 0.65 \frac{z}{z_0} \right] & \text{if } |z| > z_0
\end{cases},
$$

with stability $\mathcal{S} = 0.22$ m$^{-1}$ and the interface at $z_0 = 0.4$
m. The domain was extended to $600 \times (0.43/188)$ m
(such as not to change the vertical resolution) with an
absorbing region starting at 1.1 m, to avoid spurious
reflection from the upper domain boundary. It is found
that gravity waves in part reflect and interfere with the
incoming waves at the height of the stratification
change, initiating a zonal-mean zonal flow below the
interface (not shown). Hence, the interface represents
an effective limiter to the vertical extent and subse-
dently the period of the resulting zonal-mean zonal
flow reversal. Our findings with respect to wave reflec-
tion at the interface of differently stratified fluids are in
qualitative agreement with previous laboratory experi-
ments (Koop 1981) and theory (Baines 1995).

g. External forcing

Because there is a considerable uncertainty with re-
spect to the forcing of the atmospheric QBO, we ex-
plored the effect of various combined wavenumber–
frequency forcing pairs on the oscillation period.
Therefore, we redefine the membrane in terms of
Fourier modes, rather than by elementary shape func-
tions, as

$$
z(x, y, t) = \sum_i \epsilon_i \sin(k_i x) \sin \left( \frac{\pi}{L_y} y \right) \sin(\omega_i t),
$$

where $k_i = 2\pi s/L_y$ denotes the wavenumber and $s_i$ the
mode number. Figure 6 shows the result of the 2D
simulation at $y = L_y/2$, when the system was forced by
three constant-amplitude $\epsilon_i = 0.008$ m wavenumber–
frequency pairs ($s_1 = 4$, $\omega_1 = 0.43$ s$^{-1}$), ($s_2 = 8$, $\omega_2 = 0.37$ s$^{-1}$), and ($s_3 = 16$, $\omega_3 = 0.31$ s$^{-1}$).
This particular forcing excites waves with distinct phase speeds, result-
ing in some irregularity and a change in the mean flow
oscillation toward a longer period and larger vertical
extent. Within the first flow reversal we observe a
zonal-mean velocity of 0.0075 m s$^{-1}$, then a transitional
phase, with a mean flow velocity of 0.011 m s$^{-1}$ and a
mean flow magnitude of 0.023 m s$^{-1}$ after about 100
min. The three consecutive stages indicated by the
change in flow magnitude coincide with the critical flow
velocity $U_\text{crit} = \omega_i/(2\pi s/L_y)$ with respect to the three
forced wavenumber–frequency pairs. Toward the end,
the solution structure suggests a regularity driven by

![Fig. 6. Time–height cross section of the zonal-mean zonal flow for a mixed wavenumber–frequency forcing with \( (s_1 = 4, \omega_1 = 0.43 \text{ s}^{-1}), (s_2 = 8, \omega_2 = 0.37 \text{ s}^{-1}), \) and \( (s_3 = 16, \omega_3 = 0.31 \text{ s}^{-1}) \).]
the longest waves in the simulation (see also discussion in section 4d). Similar results were obtained for other (nonmultiple) wavenumber combinations in the specified forcing. Notably, when multiple combinations are prescribed with the same nominal critical velocity \( \omega_1/k_1 = \omega_2/k_2 = \omega_3/k_3 = 0.013 \text{ m s}^{-1} \) with \( (\omega_1, s_1) = (0.43 \text{ s}^{-1}, 8) \), \( (0.645 \text{ s}^{-1}, 12) \), and \( (0.172 \text{ s}^{-1}, 3.2) \), the resulting solution structure is regular (not shown). In this case all waves contribute to the same apparent downward propagation of the zonal-mean flow and the observed oscillation has a shorter period, compared to the simulation with monochromatic forcing \( (\omega_1, s_1) = (0.43 \text{ s}^{-1}, 8) \), indicative of increased momentum fluxes. This is confirmed by a series of simulations with varying amplitude (thus increased momentum fluxes), corroborating a \( \sim 1/\varepsilon_0 \) dependence of the oscillation period, in accord with Plumb and McEwan (1978). The asymmetry of positive and negative descend rates is a function of the wave momentum fluxes resulting from waves of similar phase. In our experiments this may therefore only arise due to random contributions to a single phase, from reflected waves, or initially, when the seed flow had a slightly larger amplitude than the “steady state” oscillation (e.g., Fig. 2b).

In contrast, when selecting a monochromatic forcing with \( (\omega_1 = 0.43 \text{ s}^{-1}, s_1 = 2, \varepsilon_0 = 0.008 \text{ m}) \), which implies a critical velocity \( U_{\text{crit}} = 0.053 \text{ m s}^{-1} \), we observe a maximum zonal-mean velocity of 0.013 m s\(^{-1}\) and no mean flow reversal develops (during 4.5 h of simulation).

Further, we investigated variations in the external forcing in a series of 2D simulations, by augmenting the monochromatic forcing \( (8) \), \( (s = 8, \omega_0 = 0.43 \text{ s}^{-1}, \varepsilon_0 = 0.64 \times 0.008 \text{ m}) \) with the shape described by Eqs. (2) and (3) at \( y = L_y/2 \), where \( \varepsilon_y = \delta_i \times 0.64 \times 0.008 \text{ m}, \phi_\eta := \phi_\eta + \phi' \), and \( \phi' = \delta_2(\eta) \pi \). The degree of randomness was varied using the parameters \( \delta_1 \) and \( \delta_2(\eta) \). The latter was varied at each time step randomly for one chamber \( \eta \) by the increment \( \delta_2(\eta) = 0.01 \) (and zero for the other chambers), leading after some time to a randomly shaped, forced oscillation of the membrane. The parameter \( \delta_1 \) (fixed throughout a simulation) was used to vary the amplitude \( \varepsilon_y \) of the random perturbations with respect to the monochromatic forcing from 10% to 100%. A limited range of frequencies and wavenumbers is excited, with visible power in wavenumbers 4–10 with \( \delta_1 \leq 0.75 \) and maximum power toward low wavenumbers 1–3 with \( \delta_1 \leq 1 \), and frequencies in the range 0.1–0.5 s\(^{-1}\). Figure 7 shows the corresponding zonal-mean zonal flow oscillation for different choices of the perturbation parameter \( \delta_1 \). A successive weakening of the zonal-mean zonal flow oscillation can be seen and its disappearance at perturbation amplitudes equal (or greater) in magnitude to the amplitude of the monochromatic forcing. However, when the random perturbations in the forcing are stopped at \( t = 24 \text{ min} \), a mean flow oscillation is reestablished (cf. Fig. 7d and Fig. 8), while the magnitude of the observed zonal-mean flow (positive and negative) does not change, or even diminishes. Figure 9 shows the corresponding time evolution of the vertically integrated Reynolds stress, compared to the cases with 10% random forcing, and the 100% forcing together with its 8-min-running average. Large-amplitude variations change to a steady wave momentum flux after the random perturbations have decayed (after \( \sim 31 \text{ min} \)). The spectrum is now dominated by wavenumber 8, for which the prevailing zonal wind is sufficiently critical to sustain the downward propagation. Whereas the low-wavenumber-dominated spectrum for \( t < 24 \text{ min} \) requires greater critical velocities than observed, and higher-wavenumber contributions appear to be of insufficient amplitude. Despite episodically larger wave momentum fluxes (cf. Fig. 9) with 100% random forcing compared to the 10% case, the wave momentum flux is below the threshold for oscillatory solutions with the given forcing spectrum. Hence, increasing the amplitude \( \varepsilon \) of the wave forcing beyond a threshold, or inducing waves, for which the prevailing wind is already locally critical, appears to have the same effect on the onset of oscillatory solutions.

With entirely random forcing \( [\varepsilon_0 = 0 \text{ and } \delta_2(\eta) = 0.001] \), no oscillation had developed after 17 h despite the development of a strong zonal-mean zonal flow opposite to the initially forced lower-level flow, and a very slow apparent downward propagation of the reversed flow (\( \sim 9 \text{ h} \) to arrive at the forcing membrane). It was observed that suitable waves generated by the random forcing maintained the downward-propagating phase, but waves of opposite phase speed were too weak to initiate a reversal.

**h. Numerical sensitivity**

Apart from the sensitivity to upper- and lower-boundary conditions reported before, other aspects of numerical realizability were investigated, using the onset of the zonal-mean flow oscillation and the length of its period as measures of numerical error. Sensitivity studies are detailed below to the resolution, to the advection scheme, to the use of first- or second-order accuracy, to the accuracy of the pressure solver, and to explicit versus implicit formulations of the governing equations.

A series of 2D simulations with varying horizontal resolution were performed. These simulations indicate that too-low horizontal resolution, that is, \( 31 \times 188 \), results in a severely distorted oscillation, with elon-
Fig. 7. Comparison of random external forcing with increasing amplitude of added random perturbations in percent of the underlying regular membrane amplitude $e = 0.008 \text{ m s}^{-1}$. Contour intervals are 3 mm s$^{-1}$; (a) 10% random forcing, (b) 50%, (c) 75%, and (d) 100% (still no mean flow oscillation is observed after 96 min; not shown). Compare also to Fig. 4b for the case of no random perturbations.
gated near-horizontal zonal-mean flow layers. In this case, the enforced dominant mode $s = 8$ is represented with four points per horizontal wavelength. Increasing the resolution to eight points per wavelength results in a qualitatively correct solution but longer period compared to the very high resolution result (80 points per wavelength). With 16 points per wavelength the period is still overestimated by 20%–30%. The solution has converged at 48 points per wavelength and is indistinguishable from the very high resolution reference run. In contrast, variations of the vertical resolution show a shorter period with lower resolution, that is, 639 points per vertical wavelength, and no zonal-mean flow oscillation is observed for resolutions lower than five grid points per vertical wavelength. The period increases by 20%–30% with increased vertical resolution, to converge above 10–15 points per vertical wavelength. With poorer vertical resolution, near-critical and subsequent flow reversals develop earlier (see also discussion in section 5). A converged solution was obtained with $383 \times 38 \times 188$ grid intervals. All simulations in Table 1 were conducted with resolutions in excess of this “convergence threshold” (see Wedi 2004 for details). It is noted that resolution changes impact directly the onset and the speed of the apparent downward propagation.

The comparison of the numerical solution procedure of either flux-form Eulerian or advective-form semi-Lagrangian formulations (albeit based on the same advective transport operator) shows significant sensitivity [see Smolarkiewicz and Margolin (1993) and Smolarkiewicz and Pudykiewicz (1992), respectively, for details of the implementation]. In the semi-Lagrangian framework the numerical dissipation is about three times larger than in the flux-form Eulerian formulation. It is found that numerical dissipation and explicit viscous dissipation act similarly. While the Eulerian simulations exhibit asymptotically the correct behavior, the increased numerical dissipation in the semi-Lagrangian case leads to the development of different bifurcation points in the flow evolution (see Wedi 2006 for a comprehensive discussion). The corresponding time–height cross sections are shown in Fig. 10, where the semi-Lagrangian formalism produces a marked shorter and irregular period. It is noted that the Eulerian framework conserves the total energy of the system to the order of roundoff errors (Wedi 2006).

A similar numerical sensitivity is found when using more dissipative numerical methods such as a first-order upwind scheme, where the increased implicit dissipation also results in shorter periods, accompanied by a more rapid acceleration of the mean flow in relatively lower layers, while reducing the vertical extent of the resulting zonal-mean zonal flow oscillation.

In the explicit case, full $\rho$ is advected, whereas in the implicit case only the perturbation $\rho'$ is advected, resulting in the appearance of the convective derivative on the rhs of the thermodynamic equation in (4). In the latter case the thermodynamic equation has to be solved simultaneously with the momentum equation (see the appendix in Smolarkiewicz et al. 2001 for details). The same time step was used for both integrations. The explicit solution gives only a weak and distorted oscillation period (not shown), similar to the lower-accuracy solution described before (see also Smolarkiewicz et al. 1997). Notably, doubling the ver-

![Fig. 8. Time–height cross section of the zonal-mean zonal flow velocity in the case of 100% random forcing, where the random variation has been stopped at $t = 24$ min (cf. Fig. 7d).](image-url)
tical resolution in the explicit case nearly recovers the implicit solution. From the comparison of the waves generated in both solutions it is concluded that the dispersion of the waves is little effected by the implicit calculations, but the explicit solution procedure is inferior in accuracy when resolving the dissipation processes near the shear layer (see also Dörnbrack et al. 2005).

**Fig. 9.** Vertically integrated Reynolds stress in time for the case with 10% random forcing (heavy dot-dash; cf. with Fig. 7a), 100% random forcing (light gray, solid), running 8-min average (heavy black, solid), and without perturbations in the boundary forcing after \( t = 24 \) min (light dashed); cf. with Figs. 8 and 7d, respectively.

**Fig. 10.** Time–height cross section of (a) the zonal-mean zonal flow velocity in the 2D numerical simulation using a no-slip rigid upper boundary with a flux-form Eulerian advection scheme (case i in Table 1) and (b) the same setup with a semi-Lagrangian advection.
4. Mechanism

a. Preamble

The following three subsections analyze and discuss our numerical simulations of the OBO analog. In section 4b we first review the process of wave–wave and wave–mean flow interaction, by which wave–wave interactions catalyze the onset of strong shear flows (Galmiche et al. 2000) ahead (in time) of dynamic and convective instabilities. In section 4c we provide a detailed analysis of the wave processes in our simulation. We conclude in section 4d with a revised conceptual model of the laboratory analog. It is shown that waves are absorbed through finite depth with greater specificity in time and space than simply approaching a critical level. The smooth profiles of wave drag and mean flow acceleration are a direct consequence, yet the chronological order of events is important. Wave interference and local flow accelerations induced by waves with similar scales provide (themselves) the onset of shear flows at higher levels. Subsequently—but chronologically secondarily—dynamical and convective instabilities, together with kinematic viscosity, act to drive the downward propagation of a shear layer with locally critical flow magnitude. An alternative view of the laboratory experiment emerges, where the mean zonal wind is to be understood as the result of the combined in-phase, irreversible momentum transfer originating from different coherent individual gravity wave sources. The necessary wave filtering for the periodic reversals of the mean zonal wind happens primarily at the forcing level, when the prevailing wind rather abruptly switches sign.

b. Theory

Galmiche et al. (2000) studied wave–wave interactions producing strong horizontal mean currents in stably stratified fluids. By comparing “bidimensional DNS” with calculations that explicitly ignored the retroaction of the mean flow onto the wave field, Galmiche et al. demonstrated that energy can be transferred to the mean flow via nonresonant wave–wave interactions far more efficiently than via elementary wave instabilities. Below, we adopt their semilinear analysis, to elucidate the flow evolution in our simulations.

Consider the simplest case of two interacting internal monochromatic planar waves of the form

\[ w_A = w_{0A} \cos(k_A x + m_A z - \sigma_A t), \]
\[ u_A = -(m_A/k_A)w_{0A} \cos(k_A x + m_A z - \sigma_A t), \]
\[ w_B = w_{0B} \cos(k_B x + m_B z - \sigma_B t), \]
\[ u_B = -(m_B/k_B)w_{0B} \cos(k_B x + m_B z - \sigma_B t), \]

where \( \sigma_A, \sigma_B \) verify the Boussinesq dispersion relation

\[ \sigma_{AB}^2 := \frac{N^2 k_{AB}^2}{(m_{AB}^2 + k_{AB}^2)}, \]

with vertical wavenumber \( m = 2\pi s/L_z \) and mode \( s \), horizontal wavenumber \( k = 2\pi s/L_x \) and mode \( s \), and frequency \( \sigma \) for wave A and B, respectively. The only contribution to the mean flow results from the interference of the two waves if, and only if, \( k_A = k_B \) or \( k_A = -k_B \), with all self-interaction terms identically zero. This can be shown by inserting (9) into the divergence of the averaged Reynolds flux \( \langle u_A' u_B' \rangle \). Using the addition rules of trigonometric functions and noting that 2\( \pi \)-periodic terms do not contribute, one finds for the case \( k_A = k_B = k \)

\[ \frac{\partial}{\partial z} \langle u' w'^x \rangle = \frac{w_{0A}w_{0B}}{2} \left( \frac{m_A^2 - m_B^2}{k^2} \right) \times \sin((m_A - m_B)z - (\sigma_A - \sigma_B)t). \]

In the inviscid 2D case, (1) reduces to

\[ \frac{\partial U}{\partial t} = -\frac{\partial}{\partial z} \langle u' w'^x \rangle. \]

Solving (12) with the initial condition \( U(z, 0) = 0 \) leads to

\[ U(z, t) = \frac{w_{0A}w_{0B}}{2k} \left( \frac{m_A^2 - m_B^2}{\sigma_A - \sigma_B} \right) \times \left[ \cos((m_A - m_B)z - (\sigma_A - \sigma_B)t) \right. \]
\[ \left. - \cos(m_A - m_B)z \right]. \]

In particular, Galmiche et al. (2000) demonstrated that the resulting zonal-mean zonal flow given by (13) can become sufficiently strong to locally develop mean flow magnitudes critical to either wave A or B, with subsequent intense, irreversible energy transfer from the waves to the mean flow. Furthermore, they point out that this mechanism is effective even in the absence of a spatially localized critical level.

c. Analysis of the simulated flow

A detailed spectral analysis of the zonal velocity component is combined with the analysis of local and mean quantities of the emerging flow field to gain insight into the physical processes involved at the various stages of the zonal-mean zonal flow oscillation. We used high temporal and spatial resolution 2D data (case j in Table 1) with 640 \times 295 grid points. The data are stored in a time series of 1-s intervals (every 20th time step) for a period of 1800 s. The forcing membrane
shape is given in (8) with amplitude $\varepsilon = 0.008$ m, frequency $\omega = 0.43$ s$^{-1}$, horizontal mode $s = 8$, $L_x = 1.52$ m, and $L_z = 0.7$ m. We did not employ any seed flow in this simulation. The data are subdivided into 60 intervals of 30 s each, to follow the temporal flow evolution in spectral space.

Initially, there is no mean flow and a symmetric standing wave pattern emerges (Fig. 11), which is equivalent to two opposite traveling gravity waves with horizontal modes $s = +8$ and $s = -8$, and vertical mode $s_z = 12$, in agreement with the linear dispersion relation (10) with frequency $\omega_0 = \sigma = 0.43$. The spontaneous symmetry breakdown and the initial conditions were discussed in section 3b. As a result, a mean flow $U$ develops in some initial direction. Figure 12 shows the developing zonal-mean zonal flow reversal in the period 14–30 min. Figure 13 illustrates the flow field in the $x$–$z$ plane at representative times, and the corresponding horizontal and vertical wavenumber and frequency distribution for selected 30-s time intervals during that period.

At time $t = 16$ min we predominantly observe the horizontal mode $s = -8$ and a discrete set of vertical wavenumbers (Fig. 13d), which arises from the oscillatory membrane motion. The horizontal phase speed is positive, and the waves propagate to the right (Fig. 13a). We observed a single dominant frequency at all times of $\approx 0.42$ s$^{-1}$ (Figs. 13d–f), which is nearly equivalent to the forcing frequency of the oscillating lower boundary. Indicated at $t = 16$ min (Fig. 13a) and at $t = 17$ min (Fig. 14a), one observes wave interference patterns of reinforcement and cancellation between the peaks and troughs of the vertically propagating waves—oriented from the lower right to the upper left. For clarity, we also show another phase of the mean flow evolution (Figs. 15 and 16) at $t = 25$ min, when the waves cover more of the domain in the vertical; note, the mean flow is reversed. Figures 15 and 16 display the zonal velocity field and the corresponding local Richardson number $\text{Ri} = N^2/(audz)^2$, respectively. Interference patterns are oriented from the lower left to the upper right, while $\text{Ri} > 1$ everywhere. In regions of reinforcement we find nonnegligible variance in the zonal-mean velocity field (cf. Fig. 17), indicating the onset of shear.

Returning to the flow evolution before the mean flow reversal at $t = 19$–20 min, Figs. 14b and 14c show that the line of reinforcement and enhanced variance nearest to the oscillating membrane has strengthened compared to the earlier time in Fig. 14a, while the discrete spectrum of vertical wavenumbers broadens toward higher modes (Figs. 14d and 14e). At the same time, the wave activity above the increasingly horizontally shaped line diminishes, and subsequently disappears.
The local Richardson number $Ri$ is diagnosed for times $t = 17$ min and $t = 20$ min (see Wedi 2004 for other times). Figure 18 exhibits local regions of the flow with $Ri \sim O(1)$ and $Ri < 0$ within the forming positive mean wind $+U$ shear layer (Fig. 20a). The Reynolds stress divergence is negative (Fig. 20b), consistent with Eq. (12). In the further flow development at times $t = 21$–22 min the shear layer propagates downward (Fig. 20a and Fig. 13b). On arrival at the oscillating membrane at approximately $t = 22$ min, we observe solely positive or zero mean wind throughout the entire domain (Fig. 12). At time $t = 24$ min we observe only horizontal mode $s = +8$, and a similar vertical wavenumber spectrum as at $t = 16$ min (cf. Figs. 13d and 13f), the horizontal phase speed is negative, and the waves propagate to the left (cf. Figs. 13a and 13c). More details of the flow evolution may also be found in Wedi (2004).

### d. Conceptual model

Based on the flow analysis in the preceding subsection and the results in section 3, we revise the interpretation of the laboratory analog of the QBO given in

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(Figs. 14g–i). The local Richardson number $Ri$ is diagnosed for time $t = 17$ min and $t = 20$ min (see Wedi 2004 for other times). Figure 18 exhibits local regions of the flow with $Ri \sim O(1)$ and $Ri < 0$ within the forming positive mean wind $+U$ shear layer (Fig. 20a). The Reynolds stress divergence is negative (Fig. 20b), consistent with Eq. (12). In the further flow development at times $t = 21$–22 min the shear layer propagates downward (Fig. 20a and Fig. 13b). On arrival at the oscillating membrane at approximately $t = 22$ min, we observe solely positive or zero mean wind throughout the entire domain (Fig. 12). At time $t = 24$ min we observe only horizontal mode $s = +8$, and a similar vertical wavenumber spectrum as at $t = 16$ min (cf. Figs. 13d and 13f), the horizontal phase speed is negative, and the waves propagate to the left (cf. Figs. 13a and 13c). More details of the flow evolution may also be found in Wedi (2004).

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*Fig. 13. Comparison of zonal wind, horizontal and vertical wavenumbers, and dominant frequencies diagnosed at selected time intervals by spectrally analyzing the zonal velocity (case j in Table 1) for waves traveling to the (a) right and (b) left. Bar length indicates the relative power of each vertical wavenumber.*

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8 Figures 14g–i were obtained by comparing the power spectrum of individual horizontal wave modes in nine different heights for different time intervals. By relating the individual power spectra to the maximum power observed within a given time interval, we are able to visualize the relative strength of the wave signal in a particular height band (i.e., only modes with a power of at least 10% of the maximum are shown).
Fig. 14. Comparison of zonal wind, horizontal and vertical wavenumbers, and dominant horizontal wavenumbers diagnosed at selected vertical heights by spectrally analyzing the zonal velocity (case j in Table 1). (d)–(f) Bar lengths indicate the relative power of the individual wave components. (g)–(i) Bar lengths symbolize the relative signal strength of the analyzed wave signals in each particular height in percent of the dominant power in a particular time interval. Only vertical levels larger than 10% are plotted.
Plumb (1977) and Plumb and McEwan (1978). The mechanism of the mean flow reversal is sketched in Fig. 19.

In Fig. 19a the negative mean flow $-U$ near the oscillating membrane enforces gravity waves with a single dominant horizontal wave mode $s \approx -8$. The phase speed is positive and the waves propagate to the right. The continuously changing forcing shape (i.e., the ups and downs of the oscillating membrane) provides regular shifts in the excitation, and thus in the phase of the vertical propagation, which leads to linear vertical wave interference with subsequent breakup of the coherent wave trains (cf. Figs. 13a and 13d and Figs. 14a and 14d). Notably, no significant power has been found in higher wavenumbers with half the forcing frequency, thus precluding an interpretation in terms of parametric subharmonic instability (Staquet and Sommeria 2002).

Subsequently, nonlinear wave–wave interactions transfer wave-transported momentum to the mean flow according to Eq. (12). When the wave interference pattern appears first, $Ri > 1$ documents no apparent wave instability (Fritts and Alexander 2003), yet localized zonal-mean flow acceleration is observed. The resulting mean flow retroacts with the waves and catalyzes the onset of shear-induced instabilities (Galmiche et al. 2000). Later, as shown in Fig. 18a, episodes of wave instabilities occur in the flow, although no critical level is observed. This is evident in our 3D simulations, where the zonal mean may never reach critical magnitude. Nevertheless, instability occurs intermittently and the waves experience local “critical behavior,” thus defining a “critical layer” environment (Andrews et al. 1987; McIntyre 2003). That is, wave amplitudes reduce over some depth, determined by the locally induced shear.

Figure 19b indicates the next phase of the flow evolution, where the shear layer with positive mean wind $+U$ develops at the lowest level of the wave interference pattern. The zonal-mean flow is oriented in the same direction as the propagating waves. The zonal-mean flow grows, approaching asymptotically $U_{\text{crit}} = \omega_0/k = 0.43/(2\pi \beta/L_x) \approx 13 \text{ mm s}^{-1}$, which represents a critical level ($\omega_0 - uk \to 0$) to the horizontally propagating waves. Aloft, vertical propagation of the wave group $G_z = \partial \omega_0/m = -m(\omega_0 - uk)/(m^2 + k^2) \to 0$, whereas the discrete spectrum of vertical wavenumbers broadens toward higher modes (cf. Figs. 14e and 14f) consistent with critical-level theory (Bretherton 1966; Booker and Bretherton 1967). Within the critical layer, effectively all momentum carried by the $s \approx -8$ wave is transferred to the mean flow. With the subsequently enhanced convergence of the wave-induced stress through wave breaking, the shear layer grows downward trailing behind a positive zonal-mean wind above the descending layer (Fig. 20). Notably, in the 2D simulations the downward growth is not observed until the magnitude of the zonal-mean flow reaches $U_{\text{crit}}$. Viscosity (explicit, cf. section 3e; numerical, cf. section 3h) acts in support of the downward progression of the mean zonal flow. The effect of such viscosity is primarily to act as a “mixing agent” by locally enhancing the wave momentum flux convergence in regions of strong nonlinearity and wave breaking.

When the shear layer with $+U$ arrives at the forcing level (Fig. 19c), the oscillation of the membrane induces vacillations in its vicinity, as can be seen by the positive

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**Fig. 15.** Zonal velocity at $t = 25$ min.

**Fig. 16.** Local Richardson number $Ri = -(g/\rho)(\partial \rho/\partial z)/(\partial u/\partial z)^2$ at $t = 25$ min. There are no occurrences, where $Ri \ll 1$, but distinctive layers of wave interference can be observed. Richardson numbers greater than 9 (where one may assume the validity of $Ri \gg 1$) and lower than $-1$ are plotted in the maximum or minimum scale, respectively.
and negative contributions of the Reynolds stress at $t = 22$ in Fig. 20. The shear layer is not attenuated (Plumb 1977) but the prevailing wind rather abruptly switches sign. Consistently, the observed transition is more rapid with larger membrane amplitude or in 2D geometry. The flow above the oscillating membrane is now from the opposite direction to allow waves with horizontal wave mode $s = +8$, propagating to the left. A short period of uniformly positive or zero mean wind throughout the domain is followed by the onset of the next phase (Fig. 19d). Wave interference, critical-layer formation, and apparent downward propagation of the zonal-mean zonal flow proceed as before but with opposite sign compared to Fig. 19a. The mechanism may be interpreted such that both horizontal wave modes $s = +8$ and $s = -8$ are present, but where one wave mode is filtered out immediately by the prevailing mean wind through absorption just above the forcing level. Therefore, the oscillation in the laboratory analog propels itself through the subsequent arrival of a reversed mean wind at the forcing level and not through “viscous” destruction of the mean shear layer as proposed by Plumb (1977). In brief, the zonal-mean wind in the laboratory experiment can be understood as the result of the combined in-phase, irreversible momen-

tum transfer originating from different coherent individual gravity wave sources.

5. Discussion

In section 3 we reported on numerical and parametric sensitivities of simulated QBO-like oscillations. Here, we discuss our observations in the context of earlier studies, and implications for the realizability in numerical weather prediction and climate models.

a. Forcing

With insufficient forcing amplitude, no mean flow oscillation emerges; see section 3g. The occurrences of stable (no mean flow reversal) solutions were discussed in Yoden and Holton (1988), using bifurcation theory with either symmetric or asymmetric forcing. In the former case, the wave momentum flux forcing must exceed a particular threshold to attain an oscillatory solution. In the latter case, the wave forcing contributing to both, positive and negative mean flow accelerations, has to exceed the threshold. Both conclusions are confirmed by our experiments. The physical reasoning for the threshold value appears to be, whether there is
sufficient flow acceleration generated to become locally critical for any of the waves observed in the fluid. In agreement with Yoden and Holton (1988), the amplitude of the zonal-mean wind oscillation becomes “saturated” in our simulations. Furthermore, the effectiveness of the forcing may be diminished by 3D wave dispersion (Smith 1980). Unlike the 2D calculations, the zonal-mean flow oscillation in the 3D simulations and in the laboratory experiments is most visible only in the lower part of the domain (section 3a). This observation is consistent with linear theory (cf. Smith 1980), where the amplitude of the propagating three-dimensional waves decays with height $\propto z^{-1/2}$ (cf. Fig. 2); yet in the atmosphere, 3D wave dispersion will be overwhelmed by the density decay with height. The effectiveness of the forcing may also be adversely influenced by the presence of boundary layers (section 3c), or weak stratification (section 3f). Notably, in the case of weak stratification (section 3f) only in the first oscillation phase is there sufficient negative acceleration. When the shear layer arrives at the membrane, and the near-membrane flow switches sign, the solution becomes stable.

Conclusions drawn in this paper with respect to the local critical-layer environment (section 4) may add to discussions on severe downslope windstorms (Smith 1985; Laprise and Peltier 1989a,b). We note a close affinity of our flow past the undulating “mountains” of the oscillating membrane, when comparing the Froude number parameter range. For example, investigations into the instability of Long’s model are based on the parameter $(Nh/U) = 0.85$ (Baines 1995, p. 259), where $N$ symbolizes the Brunt–Väisälä frequency, $h$ is the obstacle height, and $U$ the flow speed. In Laprise and Peltier (1989a), $Nh/U = 0.95$ and the aspect ratio of the mountain $h/a = 0.095$. In the laboratory experiment $Nh/U = 0.96$, and the aspect ratio of a single membrane chamber is $h/a = 0.08$.

b. The onset

Both 2D and 3D simulations show wave interference patterns that precede the onset of shear flow. There

![Waves propagating right](image)

Fig. 19. The principal mechanism of the oscillation in the laboratory experiment of Plumb and McEwan (1978) is shown in four representative phases of the flow. (a) There is a zonal-mean zonal flow $-U$ allowing gravity waves with a single dominant horizontal wavenumber $s = -8$; (b) wave–wave mean flow interaction generates a critical layer that propagates downward, generating a zonal-mean zonal flow $+U$; (c) the reversed mean flow arriving at the oscillating membrane filters the waves of opposite direction to allow only waves with horizontal wavenumber $s = +8$; and (d) wave–wave mean flow interaction generates a downward-propagating zonal-mean zonal flow $-U$. 
are, however, other mechanisms that may influence the onset of shear in numerical simulations. With inferior vertical resolution, the onset mechanism appears different, as interference patterns are not observed within the specified sampling frequency. Instead, dynamic or convective wave instabilities appear to develop instantly. Moreover, in 2D simulations, the no-slip upper boundary condition (section 3c), or vertically varying stratification (section 3f), may catalyze the onset of the next phase of the mean flow oscillation at the boundary or interface, respectively, when the vertical extent of the oscillation equals the height of the boundary/interface. Concomitantly, the wind of the current oscillation phase appears to suppress wave interference patterns throughout the entire depth of the mean flow layer. Since interference patterns are observed only above the established mean flow, it produces the “growing steps” in the vertical extent of the oscillation (section 3a).

**c. Wave dissipation**

Based on our results and beyond previous interpretations, the laboratory experiment is driven by locally induced shear and subsequent wave instabilities with viscosity effective at the Kolmogorov scale. In our DNS of the laboratory experiment the only physical dissipative mechanism is due to molecular viscosity, whereas in the real atmosphere there are molecular effects as well as radiative damping (Fels 1982). In section 3e we tested the sensitivity of inviscid (\(\nu = 0\)) solutions to Newtonian cooling (an archetype for radiative damping) and Rayleigh friction (often used in mechanistic models to parameterize wave breaking). While Newtonian cooling promoted the onset of a zonal-mean flow oscillation, equally sized Rayleigh friction suppressed its development. The difference may be understood from linear theory. Inserting a monochromatic wave \(e^{ikx+imz-\omega t}\) into the linearized x-z-slice Boussinesq equations, one may derive the dispersion relation

\[
\omega^2 - i\omega \left( \frac{\alpha m^2}{m^2 + k^2} + \beta \right) - \sigma^2 - \alpha \beta \frac{m^2}{m^2 + k^2} = 0,
\]

where \(\alpha\) and \(\beta\) denote the Rayleigh friction and Newtonian cooling, respectively, and \(\sigma = N^2 k^2/(m^2 + k^2)\). For either \(\alpha = 0\) or \(\beta = 0\), (14) implies a decay \(e^{-\sigma t}\) and a phase distortion \(e^{-\xi t}\), where \(\xi = \beta\) for \(\alpha = 0\) and \(\xi = \alpha m^2/(m^2 + k^2)\) for \(\beta = 0\). Except for the hydrostatic regime, the latter case evinces a substan-
tially smaller decay rate, which is insufficient to sustain a mean flow oscillation in the $v = 0$ experiments. In turn, Newtonian cooling mimics molecular viscosity, thus corroborating the utility of the parametric WKB model of the laboratory experiment (Plumb and McEwan 1978).

Several parameterization schemes have been designed to account for the lack of resolved nonorographic gravity wave drag, in which research has concentrated on different representations of the dissipation processes [e.g., see McLandress and Scinocca (2005), for an analysis of various parameterization schemes, and references therein]. Incidentally, McLandress and Scinocca find that the hydrostatic GCM response is largely insensitive to the dissipation mechanism, and the parameterized mechanism of critical-layer absorption is found to dominate the GCM response below 50 km.

d. The mean flow reversal

The evolution of the QBO in the equatorial stratosphere, as analyzed by the ERA-40 project (Pascoe et al. 2005; Uppala et al. 2005), provides a good representation of the tropical winds up to 2–3 hPa (Baldwin and Gray 2005). In their analysis Pascoe et al. (2005) have found the coexistence of a weak third QBO phase at about 1 hPa. In our experiments with symmetric forcing, the filtering at the forcing level starts the next phase of the QBO only once the previous phase has arrived near the membrane. Incidentally, Gabis and Troshichev (2005) find that in only 3% of the months (of a 50-yr dataset) does the westerly wind phase exist throughout the entire height range. This coincides approximately with 21 observed reversals from westerly to easterly during this period and would be more analogous to the behavior in the standard laboratory experiment. However, the available data are currently not sufficiently reliable at 1 hPa due to the intrinsic dependence of the ERA-40 analysis on the model background, which is influenced by the top of the model domain near the stratosphere with increasingly poor resolution, the addition of artificial Rayleigh friction and an increased horizontal diffusion applied toward ECMWF’s model top. Nevertheless, our sensitivity experiments do not exclude the possibility of the onset of a third QBO phase afoot. In the experiments with random forcing in section 3g, we were able to produce a third weak mean zonal wind phase afoot above the descending main shear layer. Also in experiments with vertically varying stratification (not shown), the onset of various phases afoot could be achieved (in which reflected waves contributed to the wave momentum flux). This result indicates that waves of similar phase direction to the prevailing mean wind may provide mean flow acceleration of the same phase afoot, if unaffected by critical-layer absorption below.

e. Limitations

Effects of rotation, the lateral confinement of QBO-like oscillations, mean density changes, and the effect of a mean tropospheric and stratospheric upwelling (Plumb 1977; Saravanan 1990; Dunkerton 1997a) have not been investigated in this paper. The two-wave model (Holton and Lindzen 1972; Plumb 1977) had been extended to investigate the QBO driven by a continuous atmospheric wave spectrum (Saravanan 1990). Saravanan concludes that the period is “not too sensitive” to the waves’ phase speed, damping coefficient, and zonal wavenumber, when forced by a red-noise wave spectrum (with power concentrated at low phase speeds). In contrast, all of our simulations are dominated by critical-layer shear and are characterized by a distinct dependency on the waves’ phase speed and zonal wavenumber. The random variations introduced in section 5g excite only a limited range of wavenumbers and frequencies, although different shapes of the external forcing may be explored in the future.

6. Conclusions

The simplicity of the laboratory experiment together with recent advances in numerical methods facilitate a revealing quantitative analysis of the zonal-mean zonal flow oscillation and its mechanism (sections 4c and 4d). The chronological importance of individual wave processes, as observed in our DNS studies, further the analogy of the laboratory setup to the atmosphere (Dunkerton 1981a,b, 1997b; McIntyre 2003), where mesoscale gravity waves and inertia–gravity waves (Dunkerton 1997b,a), which are typically not well resolved in global circulation models, may provide part of the QBO forcing.

Our results provide a closer link between the laboratory analogs of Plumb and McEwan (1978) and Otobe et al. (1998), other numerical simulations, and the atmospheric QBO. As GCMs are starting to reproduce the QBO, our work gives reassurance in view of earlier works while providing a reference for future high-resolution numerical simulations, since current computational limitations (section 3h) inhibit a “direct” numerical simulation of the atmospheric QBO with existing climate models.

In conclusion, our studies show a strong sensitivity of the zonal-mean flow oscillation to numerical detail, which can easily obscure or alter the observed mecha-
nism in the simulations (cf. sections 3h, 4d, and 5). The DNS results are less dependent on resolution issues, which is particularly important, as various authors caution on the interpretation of GCM results (Lane and Knievel 2005; A. Tompkins 2005, personal communication) in connection with wave propagation and excitation. This is primarily due to the competition between parameterized deep convection and resolved disturbances associated with large-scale precipitation (Scinocca and McFarlane 2004), which also influences significantly medium-range weather prediction (Tompkins et al. 2004). Equivalent sensitivities have been found in investigations on the excitation of equatorial Kelvin wave–like disturbances and their often-suggested connection to the wave-driven tropospheric Madden–Julian oscillation (MJO) (Madden and Julian 1971 1972). Hence, research is under way to explore the possibility of a DNS for the virtual laboratory analog to the MJO.

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