Jets and Orography: Idealized Experiments with Tip Jets and Lighthill Blocking

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ABSTRACT

This paper describes qualitative features of the generation of jetlike concentrated circulations, wakes, and blocks by simple mountainlike orography, both from idealized laboratory experiments and shallow-water numerical simulations on a sphere. The experiments are unstratified with barotropic lee Rossby waves, and jets induced by mountain orography. A persistent pattern of lee jet formation and lee cyclogenesis owes its origins to arrested topographic Rossby waves above the mountain and potential vorticity (PV) advection through them. The wake jet occurs on the equatorward, eastern flank of the topography. A strong upstream blocking of the westerly flow occurs in a Lighthill mode of long Rossby wave propagation, which depends on $\beta r^2/U$, the ratio of Rossby wave speed based on the scale of the mountain, to zonal advection speed, $U$ ($\beta$ is the meridional potential vorticity gradient, $f$ is the Coriolis frequency, and $a$ is the diameter of the mountain). Mountains wider (north–south) than the east–west length scale of stationary Rossby waves will tend to block the oncoming westerly flow. These blocks are essentially $\beta$ plumes, which are illustrated by their linear Green function. For large $\beta r^2/U$, upwind blocking is strong; the mountain wake can be unstable, filling the fluid with transient Rossby waves as in the numerical simulations of Polvani et al. For small values, $\beta r^2/U \ll 1$ classic lee Rossby waves with large wavelength compared to the mountain diameter are the dominant process. The mountain height, $h$, relative to the mean fluid depth, $H$, affects these transitions as well. Simple lee Rossby waves occur only for such small heights, $h/H \ll a/f$, that the $f/h$ contours are not greatly distorted by the mountain. Nongeostrophic dynamics are seen in inertial waves generated by geostrophic shear, and ducted by it, and also in a texture of finescale, inadvertent convection. Weakly damped circulations induced in a shallow-water numerical model on a sphere by a lone mountain in an initially simple westerly wind are also described. Here, with $\beta r^2/U \sim 1$, potential vorticity stirring and transient Rossby waves dominate, and drive zonal flow acceleration. Low-latitude critical layers, when present, exert strong control on the high-latitude waves, and with no restorative damping of the mean zonal flow, they migrate poleward toward the source of waves. While these experiments with homogeneous fluid are very simplified, the baroclinic atmosphere and ocean have many tall or equivalent barotropic eddy structures owing to the barotropization process of geostrophic turbulence.

1. Introduction and potential vorticity background

Solar radiation bathes the earth, varying smoothly with latitude, and yet the circulations it produces are filled with small-scale transient eddies, jet streams, and boundary currents. Zonal jet stream generation occurs with thermally forced circulations on a simple, smooth globe. Instability of zonally symmetric, baroclinic circulations is often given as a primary reason for these synoptic-scale features, which are amplified or generated ab initio by many kinds of convective and mechanical turbulence. Such balanced fluid motions have an inherent nonlinear tendency to increase in scale, both laterally and vertically until limited by the $\beta$ effect or dissipation, which can set the width scale of developing jet streams. However, jets also can arise directly from mountainous topography. By concentrating an oncoming westerly flow, a large topographic feature can draw fluid across a great span of latitudes. Topographic structure exists at all lateral scales and is a potent source of circulation structures, from planetary-scale motions to much finer scales. The literature concentrates particularly on hemisphere-filling stationary waves on the one hand (e.g., Held et al. 2002), and lee waves and turbulent wakes on the other. Potential vorticity conservation provides, as in many areas of synoptic dynamics, a logical framework with which to proceed.

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In this paper we present experiments, mostly from our geophysical fluid dynamics (GFD) laboratory, showing qualitatively how topographic features cause jetlike concentrations of flow, occurring naturally in simple, homogeneous fluid on a $\beta$ plane. We believe the results will be relevant to the equivalent-barotropic mode of the atmosphere, though of course full stratification adds much more to the problem.

Potential vorticity (PV) conservation provides the underlying idealized dynamics. A cylinder of water, 1 m in diameter, rotating with a free upper surface, acts as a polar $\beta$ plane for barotropic, shallow-water flows. Constant values of the gravity/rotation potential, $\Phi$, where

$$\Phi = -\frac{1}{2} \Omega^2 r^2 + gz$$

determine the parabolic free surface, lying at a height

$$h_m = H + \frac{1}{2} \frac{\Omega^2}{g} \left( r^2 - \frac{1}{2} R^2 \right).$$

Here, $r$ is radial coordinate, $z$ is vertical coordinate, $h_m$ the fluid depth, $H$ is the mean depth (typically 0.15 m), $R$ is the radius of the cylinder (here 0.95 m), $\Omega$ is the rotation rate (here taking values 2 to 3.5 s$^{-1}$), and $g$ the acceleration due to gravity and planetary rotation. The mean-state PV is $2\Omega/h_m$. Potential vorticity conservation for inviscid shallow-water flow and waves is

$$\frac{Dq}{Dt} = 0; \quad q = \frac{f + \xi}{h}; \quad h = h_m + \eta; \quad \xi = \frac{g}{f} \nabla^2 \eta,$$

where $\xi$ is the vertical relative vorticity, $f = 2\Omega$, $t$ is the time, $h$ the total depth, and $h_m$ the time-averaged depth field, $\eta$ the fluid depth perturbation, $L$ is a characteristic horizontal length scale, and $U$ a characteristic horizontal velocity scale. These are exact equations except in the relation between $\xi$ and $\eta$, which involves errors of order Rossby number, $Ro = U/fL$. For a single layer of homogeneous fluid with a free surface, the linearized wave equation for the surface elevation, $\eta$, combines high-frequency gravity waves with Coriolis modification, Kelvin waves and low-frequency topographic Rossby waves;

$$[\eta_t + f^2 \eta - \nabla \cdot (gh_m \nabla \eta)]_t - fgJ(h_m, \eta) = 0,$$

where $J(\cdot)$ is the horizontal Jacobian operator, in polar coordinates ($r, \theta$),

$$J(A, B) = \frac{1}{r} \left( \frac{\partial A}{\partial r} \frac{\partial B}{\partial \theta} - \frac{\partial B}{\partial r} \frac{\partial A}{\partial \theta} \right).$$

The approximations in this equation are hydrostatic pressure balance and linear (small) amplitude. High-frequency waves, involving the first term in the wave equation, are eliminated for small temporal Rossby number, $fT \gg 1$. The wave equation then becomes

$$\nabla^2 \eta_t + \frac{h_{m,r}}{h_m} \eta_{rr} - \frac{f^2}{gh_m} \eta_r + \frac{f h_{m,r}}{r h_m} \eta_{r} = 0.$$

With a parabolic depth profile and large mean depth, $H \gg \Omega^2 a^2 g^{-1}$, the second term is small [$O(\delta h_m/h_m)$] and the quasigeostrophic wave equation becomes

$$\nabla^2 \eta_t - \frac{f^2}{gH} \eta_r + \frac{f^3}{4gH} \eta_r = 0.$$

Separating out waves in the azimuthal (zonal) direction, $\theta$, and in time with a factor $\exp(in\theta - i\omega t)$ leaves us with Bessel’s equation,

$$\eta_{rr} + r^{-1}\eta_r + \left[ -\frac{n^2}{r^2} - \frac{f^2}{gH} \left( 1 + \frac{fn}{4\omega} \right) \right] \eta = 0$$

with Bessel-function solutions $\eta = J_n(kr) \exp(in\theta - i\omega t)$ and dispersion relation

$$\omega = -\left( \frac{f^3}{4gH} \right) n \sqrt{k^2 + \frac{f^2}{gH}}.$$

Here the effective value of $\beta$ is $\beta = \frac{fh_{m,r}}{h_m} = (\frac{f}{gH}) r + O(\delta h_m/h_m)$, the radial potential vorticity gradient, and the dispersion relation and modes are the same as found for a polar projection of the earth, with a uniform-depth fluid. Traveling waves with spiral wave-crests correspond to the $H_n^1$ and $H_n^2$ complex Bessel functions, which sum together to form standing-wave modes. For the standing modes, eigenvalues for the rigid wall boundary condition at $r = R$ are set by the zeros of $\eta$; at $r = R$: $J_n(kR) = 0$ ($n > 0$). The case of the azimuthal mode 5 wave in Fig. 1 has positive eigenvalues $J_5(kR) = 0$, or $kR = 8.77, 12.34 \ldots$ in this approximation. These solutions have the usual properties of $\beta$-plane Rossby waves at short wavelength and asymptotic shortwave rays that propagate along straight lines, analogous to the great-circle ray paths of Rossby waves on a rotating sphere (Longuet-Higgins 1964). Thus the rays from a point source should approach the pole and then retreat toward lower latitude. They are singular at the origin, but should be understood in terms of a wave generator at a small but finite radius. The short-wavelength limit of the problem amounts to ray-tracing with a gradual variation of $\beta$ with radius. We then recover the familiar $\beta$-plane dispersion relation and gradual amplitude variation expressing energy conservation along a ray.
Mode \((n, m) = (5, 1)\) is shown in Fig. 1, marked by colored dye initially laid down in rings on latitude circles. It is excited by vertically oscillating a small glass cylinder, seen in the lower left quadrant. Linear Ekman friction acts to damp the mode somewhat; its group velocity is principally eastward, that is counterclockwise, or cyclonic. One can see the wave crest amplitude diminishing in this direction. Animations (see www.ocean.washington.edu/research/gfd/) illustrate how rapid rotation endows the fluid with PV elasticity. The Jell-O-like nature appears in real time at \(3.2\) s, and even more vividly in time-lapse videos. The gravest mode, \(R = 8.77\), according to the approximate theory above, has a period of \(10.8\) s compared to the forcing period of \(8.3\) s. The paraboloid in this case is rather big, however, with \(\delta h_m/h_m \sim 1\), so quantitative comparisons require a more complete solution.

The figure suggests the presence of mean zonal flow as well as waves. This is the classic result of stirring a PV gradient, shown so clearly by Whitehead (1975) and elaborated by Rhines (1977). Westward, anticyclonic circulation develops as stirring reduces the PV poleward of any latitude circle not directly intersecting the external forcing agent. The mean Eulerian induced flow for inviscid dynamics is \(-\xi/\beta\) where \(\xi\) is the Lagrangian radial particle displacement of fluid from its rest latitude where its PV is equal to the background, mean-state PV. This activity creates an anticyclonic polar vortex, and a weak easterly flow near the rim of the cylinder. At the latitudes intercepting the wave-maker, a westerly jet forms, roughly speaking, to conserve total angular momentum. Some forcing effects (e.g., the topographic mountains considered below) can exert a mean, as well as oscillatory force on the fluid and thus break the zero-net-angular momentum constraint.

Also important in Fig. 1 are the shards of small-scale mixing evident in the fluid drifting westward from the wave-maker. PV elasticity is scale-dependent, essentially nonexistent at small scales \(L \ll (U/\beta)^{1/2}\) where \(U\) is a scale estimate of the transient velocity. However the elasticity does a remarkably good job at inhibiting mixing of the polar cap (the orange ozone hole). This experiment can be run for hours at relatively large amplitude, with considerable straining of the polar cap, yet without permanent mixing (again, see the videos on our GFD laboratory Web site, above).
## 2. Experimental method

Dye is the traditional observational tool in the GFD laboratory. However dye traces and key dynamical fields (pressure, velocity, PV) are only indirectly related. Below our experiments make use of a new method for imaging the surface pressure field, or geostrophic streamfunction of a fluid, which we call optical altimetry. The coincidence that the mean free surface of a rotating fluid is a paraboloid makes it possible to use the surface as the mirror of a Newtonian telescope. Telescope makers have developed simple, elegant ways to image their mirrors, with the goal of grinding them to perfection. A small light source placed about twice the focal height above the fluid, rather than at infinity, still has a focus that is quite sharp. Anomalous hills and valleys can be measured to within a fraction of the wavelength of light.

For us, the hills and valleys are the field of interest, the height field, or nearly the pressure field close to the upper surface, or the geostrophic streamfunction. For very slow, small Rossby number flows we want to resolve height fields of order 1 μm (10^{-6} m), which is readily done. The standard method is to use a Foucault knife edge to occult the reflected rays near the focus, with a camera placed adjacent to the light source. The method is further described by Rhines et al. (2007) and Afanasyev et al. (2007).

Propagation of Rossby waves from an isolated source is seen with this method in Fig. 2. Rather than an oscillating plunger, we here have a small (15-cm diameter) mountain fixed to the floor of the cylinder, and the rotation rate is oscillated with small amplitude, about a mean of 2.2 s^{-1}, under computer control. At this frequency (ω = 0.2Ω) shorter Rossby waves are excited, and the domain is more like a WKB (geometrical optics) β plane with slowly varying PV gradient. In this circumstance there is a much greater variety of wave-numbers excited. Just as with the Green function for point excitation of an unbounded Cartesian β plane, waves with 1 ≪ k fall into eastward and westward group velocities corresponding to very different wavelengths. The Green function in that Cartesian problem has parabolic wave crests that sweep westward with time, and are here wrapped round latitude circles (see Rhines et al. 2007). Note particularly the long wave crest spiraling inward toward the North Pole. The ray paths for Rossby waves on a sphere are actually great circles, and in the experiment we thus expect the long waves to have a turning point near the North Pole, and to return southward. However, they seem to decay frictionally near the pole, before doing this. As expected, all the wave crests sweep westward with time.

Despite the wide variety of waves generated by this single-frequency forcing, the energy flux is isotropic in the ideal, Cartesian case (and here in the WKB limit). Slower group velocities east of the forcing region correspond to larger oscillatory velocities, and conversely west of the forcing.

## 3. Local topographic waves, wakes, and jets

Another notable feature of the simulation is the spiral wave pattern above the mountain (Fig. 2). This is frequently seen, and is a topographic wave (Rhines 1969), in essence a miniature of the basin-filling Rossby waves. The winding of wave crests into spirals has strong consequences dynamically. With increasing slope of the height field (the free surface), and hence increasing horizontal pressure gradient, the speed of the horizontal currents increases in proportion. Particles drifting with the fluid illustrate this vividly in videos.

Figure 3 shows such a spiral in an analytical theory of Rossby waves/circulation in a shallow-water model, yet with similar properties (Rhines 1989). The analytical model assumes large lateral scales compared with the radius of deformation, hence relating to long, nondispersive Rossby waves. Despite this difference, the winding spiral and enhancement of the flow is very similar. The nondispersive limit has the advantage of being a hyperbolic system with time-evolving characteristics, and hence is highly predictable. The effect is a kind of group velocity based shear dispersion; varia-
tions in the group velocity of the topographic wave in the local azimuthal direction round the mountain cause the free-surface, the pressure and the potential vorticity to wrap into a tight spiral with regions of steep gradient. On the equatorward/eastern side of the mountain the wake concentrates into an intense jet, which is an umbilical cord of PV flowing from the tightly wrapped core. There is in addition a nearly ubiquitous anticyclonic mean circulation induced by PV stirring above such topographic rises, just like the polar vortex in Fig.

1. One of the relatively rare discussions of this effect in the lower atmosphere is given by Hsu (1987).

4. Stationary waves on a westerly flow

The Rossby waves most evident to us are standing waves in the westerly winds of the atmosphere. They exist on a potential vorticity field involving $\beta$ and also the complex flow-induced relative and stretching vorticity, as well as a boundary PV sheet contribution (which will not be discussed in this barotropic context).

Fig. 3. Free surface shown in perspective for an analytical model of Rossby waves in a shallow-water fluid flowing westerly (left to right) above a circular mountain (Rhines 1989). The time series proceeds left to right, then top to bottom. The stripes are potential vorticity contours. An intense pressure gradient in the wake of the mountain is the result of advective dynamics, and topographic-wave concentration. The tightly wrapped potential vorticity field intensifies the velocity field visible here into a trough in the pressure field, which stretches downstream, as we see in both oscillatory and steadily forced flow experiments in this paper. The nondispersive Rossby waves apply literally only to scales $\gg \lambda$, the appropriate radius of deformation. Here the zonal oncoming flow $\sim 35 \text{ m s}^{-1}$ for $\lambda = 1000 \text{ km}$, which is supercritical (faster than the Rossby wave speed); $\delta \theta / h = 0.15$. The circular mountain has diameter of 600 km.
Despite these complications, Rossby’s (1939) original trough formula for the east–west phase speed of waves, \( c \), in the barotropic mode,

\[
c = U - \frac{\beta}{k^2},
\]

is a useful approximation (\( k \) is the east–west wavenumber, and the north–south wavenumber \( l = 0 \)). In this experiment the polar \( \beta \) plane has a stationary mountain and is excited by imposing a solid-body, westerly (eastward) mean zonal flow. This is done with a computer-controlled change of rotation rate; in various experiments the table rotation is ramped steadily downward or changed abruptly, the flow allowed to come back to rest, and then changed again abruptly. Both techniques sweep through the essential parameter space of \( R_\beta = \beta a^2/U \), where \( a \) is the diameter of the mountain. The second essential parameter is its fractional mountain height, \( \delta h/H \). If \( \delta h/H > a\beta/f \) where \( f \) is the Coriolis frequency, the potential vorticity contours (\( f/h = \) constant) of the resting fluid are greatly deformed by the mountain. Closed contours of \( f/h \) then appear. Thus the parameter \( f \delta h/a\beta H \) determines whether deflection of the oncoming westerly flow is significant. Most of earth’s major topographies are tall enough for this parameter to be large, and we take it to be so in all of the simulations presented here.

A fully developed westerly flow, Fig. 4, involves stationary lee Rossby waves to the east of the mountain (which is positioned at “2 o’clock”). The wavelength, about 0.3 m, corresponds to the Rossby trough formula with \( \beta = 4.3 \text{ m}^{-1} \text{s}^{-1} \), which is the value at radius 0.3 m, and a large-scale westerly flow, \( \beta/k^2 \), of \( 10^{-3} \text{ m} \text{s}^{-1} \). It is notable that this \( 1 \text{ mm} \text{s}^{-1} \) flow, which seems so slight, can cause such intense advective dynamics to occur.

The succeeding images, Figs. 4b,c, show Lagrangian particle tracks in this nearly stationary flow. The dots are 1 s apart. The path of the oncoming flow is deflected poleward by a region of blocked fluid, and then wraps round the mountain in the anticyclonic direction. The fluid enters the stationary spiral and exits quickly, in a wake jet that then blends with the Rossby wave-train downstream. It encircles a distinct wake, in which fluid is trapped and slowly moving. This feature itself is interesting; it is a dipole pair, cyclone to the south, anticyclone to the north. In many of our simulations the stationary lee cyclone expands in size and reaches the North Pole, perhaps expressing the PV bridge between high values at the pole and high values above the mountain. The North Atlantic storm track exhibits episodically large cyclonic development in the lee of Greenland, which reaches high into the Arctic Basin (Jung and Rhines 2007).

The laboratory model is rich with nongeostrophic motions. Nonhydrostatic inertial waves are generated in shear zones and trapped by them; a cyclonic shear zone in homogeneous fluid is a potential well that can trap near-inertial waves (because locally the effective value of \( f \) is increased, allowing trapped waves that cannot match with the surrounding fluid; with realistic stratification the internal wave band is altered such that anticyclonic shear zones create such potential wells). To illustrate this effect in isolation, we reverse the zonal flow. Figure 5 shows, with easterly solid rotation of the fluid and no stationary Rossby waves possible, a train of standing inertial waves that are enhanced and ducted by the cyclonic shear zone on the poleward flank of the wake. Once again the linear theory predicts lee inertial waves with circular crests, arcs of which are clear in the figure. These are strongly nonhydrostatic waves, with ray paths that make an angle \( \cos^{-1}(kU/f) \) with respect to the vertical (\( k \) being the azimuthal wavenumber). Here, \( kU/f \) is just the Rossby number appropriate to the

**Fig. 4a.** Nearly stationary waves in a westerly flow (counter-clockwise), encountering a mountain at 2 o’clock, on the polar \( \beta \) plane, viewed with the optical altimetry technique. This is essentially a side-lighted surface height field, proportional to pressure field and geostrophic streamfunction. Besides the stationary waves, the mountain exhibits the height gradient into a spiral jet, essentially an arrested topographic Rossby wave. The flow approaches the mountain on its poleward side, spirals partially around it, and exits in a jet on the equatorward side. Upstream, to the west, a region of blocked fluid has penetrated 1/3 of the way round the latitude circle. The mottled texture within the blocked region is the surface pressure signature of miniature convective cyclones, which develop because of evaporation-induced cooling of the fluid’s upper surface. The amplitude of the height field is of order 1 \( \mu \text{m} \) (10\(^{-6}\) m); \( \Omega = 2.2 \text{ s}^{-1} \).
wave’s lateral scale and the mean zonal velocity. Many other kinds of nongeostrophic unbalanced flow are seen with this laboratory technique; in particular the radiation of internal waves from strongly unstable baroclinic zonal jets (Afanasyev et al. 2007).

5. Transient development of the flow and upwind blocking

The atmosphere is rarely stationary for long, and the development of wakes, traditional standing-wave blocks, and upwind blocks is of great interest. These features were evident in Fig. 4, and their time development is shown another experiment, in Figs. 6a–d. This flow begins with a speed such that \( \frac{R_b}{H} = 3.1 \); that is, the mountain should block the oncoming flow. Thirty-five inertial periods after the eastward flow began, Fig. 6a shows a strong upstream plume of circulation, which establishes the block. This is could be called a Lighthill mode (after Lighthill 1967), essentially the anomalous long Rossby wave that can stem the current and propagate upstream. It is missed out in the linear lee-wave theory of McCartney (1975) by the assumption of no upstream influence.

The theory is simply expressed: including a uniform zonal wind on a \( \beta \) plane, the Rossby wave dispersion relation is

\[
\omega - k U = \frac{-\beta k}{k^2 + f^2},
\]
Standing waves have vanishing frequency, leaving

\[ kU = \frac{\beta k}{k^2 + l^2}. \]

There are two roots: \( k^2 + l^2 = (\beta/U) \) and \( k = 0 \). The first gives a circular locus of possible wavenumbers in \((k, l)\) space (Fig. 7, actually, semicircular when redundancies are removed). All waves have the same wavelength, \( 2\pi/\sqrt{U/\beta} \) and the wave crests are circular. The \( k = 0 \) root, often neglected, is a wave with vanishingly small intrinsic frequency, east–west wave crests, and sizeable westward group velocity relative to the mean flow. The east–west wave crests amount to a pattern of nearly steady zonal currents, which by Fourier superposition of many meridional wavenumbers can express a blocking of the oncoming westerly flow. The total group velocity of this mode is \( U - \beta l^2 \). Hence those Fourier components with sufficiently small north–south wavenumber can penetrate upstream. The locus of possible wave vectors \((k, l)\) for stationary waves in Fig. 7 shows the origin of the semicircular lee-wave crests [which fill out in time in a growing circular region downstream (eastward), tangent to the \( y \) axis]. The downwind speed of development of the lee waves is just twice the mean flow speed, or \( 2U \). The upwind propagating wave, which we term a Lighthill mode, has the effect of blocking the circulation, creating a stagnant region upstream.

![Fig. 6a](image1)

**Fig. 6a.** Surface height field just after onset (at time \( t = 50 \) s or 35 inertial periods) of an eastward (prograde) zonal flow past a solitary mountain (at 3 o’clock in the cylinder), similar to the experiment in Fig. 4. The dominant feature here is the brightly lit, rapidly propagating blocking wave or Lighthill mode that penetrates upstream, west of the mountain.

![Fig. 6b](image2)

**Fig. 6b.** At time \( t = 150 \) s, a fully developed pattern of stationary lee Rossby waves has set up, as well as a field of transient (phase propagating) waves farther to the east. The upstream blocking pattern of Fig. 6a is less distinct, as the large-scale zonal flow has diminished everywhere.

![Fig. 6c](image3)

**Fig. 6c.** At time \( t = 180 \) s, due to blocking and friction, the cyclonic azimuthal mean flow has decreased, and lee Rossby waves diminished in strength while the spiral pressure field over the mountain and jetlike wake persist. These correspond to intense geostrophic flow near the mountain. The pattern is connected to a jetlike concentrated wake, similar to the numerical study shown in Fig. 3.
of the mountain. This has a crucial effect on the entire circulation, for it alters the effective shape of the topography as well as the effective speed of the oncoming flow. Notice that the upstream blocking pattern broadens and moves toward the pole (the center of the apparatus), as we witnessed with long Rossby waves of finite frequency, earlier (Fig. 2). Indeed, this verifies that the upstream block is a Rossby wave. The development of this blocking can also be seen in the time sequence (Fig. 6d). The upstream propagation measured from these images is comparable to the theoretical estimate $U - \beta a^2 = 0.021 \text{ m s}^{-1}$.

The amplitude of the topography must be sufficient to distort the ambient PV contours, $f/h = \text{constant}$, for upwind blocking to occur. This is seen by noting that the $k = 0$ locus of wavenumbers is excited by the zonal ($\chi$) integral of the compact vorticity forcing function for the waves (see Lighthill 1967). To restate this, $\chi$ averaging of a forced wave equation is equivalent, in Fourier space, to examining roots with $k = 0$. Thus it is the $\chi$-averaged vorticity forcing that excites these modes. In flows over topography the forcing effect is the vortex stretching term $-u \cdot \nabla (f/h)$. If the topography is isolated and the streamlines are nearly zonal, this term is dominated by the linearized expression $(fU/H^2)\partial h/\partial x$, whose $\chi$ integral vanishes. Thus the upwind blocking also vanishes with small-height mountains.

Interpretation of the light–dark pattern in the images requires some explanation. In Fig. 6a, the mean rotation is such that the background parabola (including the signal due to the zonal mean flow) is dark; the flow is too fast and the parabolic curvature too great for the reflected light to reach the camera. Bright anomalies appear where the parabolic surface is locally flattened—more horizontal—than the background parabola. This flattening causes two regions of steeper slope at the edges of the “flat” band of latitudes, along which the zonal flow is faster than average: these are flow concentrations to account for lack of zonal flow, and lower dynamical pressure gradient in the white blocked region. Further examples are shown by Rhines et al. (2007).

**Fig. 6d.** By adding background lighting, dye patterns can be viewed with altimetric images. This is the development of initial, solid-body cyclonic flow past the mountain. In this time sequence, proceeding left-to-right then top-to-bottom, the Rossby wave wake downstream of the mountain is seen both in the pressure and dye fields, as is the rapid upstream penetration of the blocking region. Notice the complex dipole in the wake, which eventually becomes dominantly cyclonic. Here dye enters upstream of the mountain from a traversing arm (thin black line) that moves back and forth along a radius. The cyclonic mean flow was suddenly initiated at time $t = 0$, and the images are at 80, 100, 110, 120, 130, and 150 s; $\Omega = 2.2 \text{ s}^{-1}$; initial $U = 7 \times 10^{-3} \text{ m s}^{-1}$ at $r = 0.4 \text{ m}$; blocking parameter $\beta d^2/U = 3.1$. 

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In Fig. 6c, a long train of Rossby waves is visible downstream of the mountain. These are actually propagating (with respect to phase) westward, and are not true lee waves. With $R_p > 1$, instability of the wake jet creates transient waves, as will be described in the next section. Also, the sudden initiation of the zonal flow in this experiment excites both transient (oscillatory) and stationary waves and far downstream the escaped transient waves are dominant. (The flows in Fig. 2 were maintained by computer-controlled, secular spindown of the rotating platform.) Of course this is a useful reminder in interpreting meteorological observations of the hemispheric circulation: with temporal changes in the oncoming eastward flow, both stationary and transient Rossby waves are excited.

6. Transients from wake instability

Some of the features seen here in the stationary waves, wake, and flow above the mountain can be portrayed as exaggerations of the linear Rossby wave solution of McCartney (1975); particularly the lee cyclone. Linear $f$-plane flows, rather like the $\beta$-plane eastward case shown in Fig. 5, are symmetric upstream/downstream, essentially Taylor columns above the mountain. By their symmetry, they exert no wave drag on the topography (though they do have a cross-stream lift force). A stationary anticyclonic high sits over the mountain, with its cyclonic starting vortex having been swept downstream long ago. Slow westerly flow on a $\beta$ plane reacts oppositely to this with $f/h$-following streamlines, which bend equatorward over the mountain and hence form a cycloonic trough, symmetrically above the mountain. As the zonal flow speed is increased, this trough is blown downstream, to sit as a lee cyclone. An anticyclone takes its place above the mountain (this change being analogous to difference in phase (by $\pi$ radians) when a mass-spring oscillator is forced above and below resonance). Now, in fact the displaced pressure field exerts considerable inviscid force on the mountain. The momentum flux, the back reaction on the fluid, is carried by the Rossby wave field to great distance.

However other finite amplitude features in these experiments have little correspondence with linear wave theory. For large values of $R_p$, the Rossby wavelength is rather small, the wake concentrated, and this often turns the basically stationary wave pattern of Fig. 4 into a sea of transient Rossby waves generated by wake instability. It is somewhat difficult to portray this without using videos, yet here we point qualitatively to the result and encourage the reader to look at the work of Polvani et al. (1999) in which this same feature is seen in shallow water integrations on a sphere, or to look at
videos of this phenomenon on our GFD Web site. The impact of this result is that much besides stationary waves can be created in mountain wakes. Polvani et al. follow an elaborate chain of events in which the principal EOFs of the mountain-induced variability resemble transient Rossby waves, the wake develops a jetlike concentration on the equatorward side (as here), and the zonal circulation is reshaped widely (also as here). The hemispheric transient modes give way for still larger mountain height, to small-scale wake instability (as if the mountain begins to resemble a vertical cylinder with Kármán vortex shedding).

The accompanying paper in this volume by Jung and Rhines (2007) explores in detail cyclogenesis in the lee of Greenland. There, rapidly moving transient cyclones, spawned far upwind by deep planetary scale troughs, are captured in the stationary wake, which can expand to fill the entire Greenland/Irminger Sea. A tip jet develops at Cape Farewell, the southern extremity of Greenland. Continuing cyclogenesis sends new transient eddies north to the Nordic Seas and Arctic Ocean [Greenland is 2500 km long, or one-quarter of the distance between pole and equator, and reaches above 3500 m (≈650 hPa) in altitude.]

7. Shallow-water integrations on a sphere

We conclude this discussion of simple orographic effects on the β plane with two integrations of a global shallow-water numerical model. The regime differs from that in the laboratory simulations: \( R_{\beta} \) is of order unity rather than being large, and we add the effect of low-latitude critical lines.

The model has T127 horizontal resolution and a 3-km-tall Gaussian mountain, in a 10-km-deep fluid, is sited at 60°N latitude. Two experiments will be described. In the first a uniform super rotation is imposed at the outset, yet not maintained. However (unlike many such experiments), we choose very weak surface friction, so that the zonal flow is only lightly damped. The life cycle begins with a familiar train of lee Rossby waves propagating on great-circle paths. However both wake instability, described above in section 5, and the Lighthill upwind block sets up quickly. The incident westerly flow declines, and eventually reverses. The sphere is covered with a mixture of transient Rossby waves and eddies. The zonally averaged flow evolves (Fig. 8) from the initial solid rotation to a pair of anticyclonic polar vortices, which are a nearly inevitable consequence of weakly damped turbulence on a sphere (through PV stirring). These vortices overwhelm the upwind block (note the similarity of the zonal flow profile at 60°N and 60°S). The latitudes of the mountain now have easterly mean flow, and hence there is no longer Rossby wave generation. In experiments with the mountain at lower latitude and with larger \( R_{\beta} \) (not shown) the polar vortex no longer interacts with it, and recurring wake instability and a simple Lighthill blocking develop, which indeed is visible (though not commented upon) in the simulations reported by Polvani et al. (1999, their Figs. 11 and 12). The upwind blocking parameter, \( R_{\beta} = \beta a^2/U \), is 1.13 based on the initial west-
erly flow, and the 1000-km diameter of the mountain. With this marginal value, it is the creation of anticyclonic polar vortices by potential vorticity stirring more than Lighthill blocking that reverses the zonal flow at 60°N.

The second experiment (Figs. 9 and 10) explores the presence of meridional shear, $U = U(y)$. We place critical lines at 10°N and 10°S. And, as before, initiate the zonal flow without maintaining it. Here the response is totally different. Rossby wave radiation is absorbed in the Tropics, preventing any significant penetration into the Southern Hemisphere. This absorption feeds easterly momentum into the Tropics, accelerating it. A shelf of easterly momentum moves toward the wave source (Fig. 9), in just the manner of the quasi-biennial oscillation model of Lindzen and Holton (1968). Note that this migration and continued wave absorption contradicts the theorem of Killworth and McIntyre (1985), which argues that nonlinear critical layers eventually become perfect reflectors; their argument is based on the assumption that the critical line stays put. Differences between these simple experiments and those of Brunet and Haynes (1996), whose tropical critical layers do dominantly reflect, may arise because of our weak dissipation and freedom of the zonal flow to evolve (in absence of imposed forces driving the mean zonal flow as in Brunet and Haynes’ experiments).

8. Conclusions

We have described topographic generation of jetlike circulations, forced by an initially uniform angular-velocity zonal flow. Jet formation here is a consequence of arrested topographic waves above the broad mountain topography and will be very sensitive to the form of the topography (as illustrated by Rhines et al. 2007). Upwind blocking of the westerly flow by topography also produced concentrated, intense jets on the edges of the blocked region. While the experiments are unstrati-

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**Fig. 9.** As in Fig. 8, but for initially imposed zonal flow with equatorial easterlies, and critical lines for stationary waves at latitudes 10°N and 10°S. This is a time sequence of zonal-flow profiles, beginning with the nearly sinusoidal flow with two critical latitudes, and ending with the bold curve, in which the critical lines have migrated toward the mountain at 60°N. The momentum flux of Rossby waves drives this zonal flow acceleration. A homogenized band of PV accompanies it. Little energy reaches the Southern Hemisphere, and the Lighthill blocking effect is very weak.

**Fig. 10.** Vorticity (gray shades) and zonal velocity (solid contours, signs indicated) for the experiment in Fig. 9, at day 200. At this time the induced easterly flow south of the mountain corresponds to strong negative relative vorticity. A lee cyclone is visible east of the mountain, exerting continuing wave drag upon it, which appears in the strengthening easterlies.
fied and largely barotropic, they may be useful analogs of equivalent barotropic dynamics of the atmosphere. For large values of the upward blocking parameter, \( R_\beta \gg 1 \), the zonal flow is greatly altered from its initial westerly profile, and jet formation and upward blocking are prominent. The downwind wake tends to be unstable for large or moderate values, \( R_\beta \), filling the domain with transient (nonstationary) Rossby waves and PV stirring. For small values, \( R_\beta \ll 1 \), classic lee Rossby wave generation is the dominant process, with wavelengths large compared to the mountain diameter. Polvani et al. (1999) in their shallow-water study of westerly flow with an isolated mountain also stress that for sufficiently large mountain heights (>2000 m in their model), the wake instability is a dominating process, leading to vacillating jetlike westerly winds. For their experiments, \( R_\beta \) was roughly 3, based on a 20 m s\(^{-1}\) zonal wind and 2000-km meridional extent of the mountain.

For earth topographies (which are not symmetrical mountains) it is the north–south extent that is relevant to the scale \( a \); \( R_\beta \) ranges from \( 11/U \) (\( U \) in m s\(^{-1}\), for \( a = 1000 \) km, at 60°N) to 79/\( U \) (for \( a = 2000 \) km, 30°N). This suggests that major orographic features in subtropical latitudes may be most susceptible to upward blocking, if the appropriately weighted zonal wind is significantly smaller than 11 and 79 m s\(^{-1}\), in the two respective examples. Another way of describing the effect is that when the zonal length scale of stationary waves is smaller than the meridional width of the mountain, upward blocking is possible. A full baroclinic model study is required to say more. All of the experiments described here have mountain heights great enough to distort the PV (\( f/H \)) contours of the resting fluid (i.e., \( \delta h/H > a\beta/\rho \)), as is typical with earth’s prominent topographic features; the Polvani et al. (1999) study focuses on variation of this parameter with fixed \( R_\beta \).

These simple qualitative models cannot of course replace general circulation experiments incorporating the baroclinic physics of the atmosphere. Laboratory experiments can, however, describe interactions of geostrophic and nongeostrophic, even nonhydrostatic fields. Recent developments in our laboratory have made it possible to quantitatively analyze the altimetric images and recover the height field, geostrophic and cyclostrophic velocity field, vorticity and potential vorticity with great spatial resolution (Afanasyev et al. 2007). By combining this technique with another optical measurement of light absorption by dye, we are also able to recover the layer thickness field in a layered, stratified fluid simultaneously with the surface height field. This gives us the internal thermal wind field and layer potential vorticity as well as the barotropic fields.

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