Rossby Waves and Jets in Two-Dimensional Decaying Turbulence on a Rotating Sphere

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ABSTRACT

Jet formation in decaying two-dimensional turbulence on a rotating sphere is reviewed from the viewpoint of Rossby waves. A series of calculations are performed to confirm the behavior of zonal mean flow generation on the parameter space of the rotation rate $\Omega$ and Froude number $Fr$. When the flow is nondivergent and $\Omega$ is large, intense easterly circumpolar jets tend to emerge in addition to the appearance of a banded structure of zonal mean flows with alternating flow directions. When the system allows surface elevation, circumpolar jets disappear and an equatorial easterly jet emerges with increasing $Fr$. The appearance of the intense easterly jets can be understood by the angular-momentum transport associated with the generation, propagation, and absorption of Rossby waves. When the flow is nondivergent, long Rossby waves tend to be absorbed near the poles. In contrast, when $Fr$ is large, Rossby waves can hardly propagate poleward and tend to be absorbed near the equator.

1. Introduction

The problem of two-dimensional turbulence on a rotating sphere has attracted researchers since the pioneering work by Williams (1978). The series of spherical plots of streamfunction (Fig. 16 of Williams 1978) are quite impressive as they conjure images of the circulation of Jovian atmosphere. A fascinating feature is that, when the rotation rate and the radius of the sphere for Jupiter are adopted, a zonally banded structure of zonal mean flows with alternating flow directions appears spontaneously in a randomly forced two-dimensional nondivergent fluid. It was a vivid validation of the prediction by Rhines (1975) that the $\beta$ effect tends to produce a banded structure in two-dimensional nondivergent turbulence.

Even with the huge computational resource available at the Geophysical Fluid Dynamics Laboratory (GFDL) when compared to the standard at the time, the actual computational domain utilized by Williams (1978) was not the entire sphere but was limited to only a ½ longitudinal sector covering a latitudinal region from $0^\circ$ to $80^\circ$N with a resolution of $128 \times 128$. The spherical plots were made by assuming longitudinal periodicity and equatorial symmetry. Although the results were attractive, it was impossible for ordinary scientists to follow in those days due either to the lack of access to sufficient computer resources or for the necessity to pursue more practical issues such as development of global models for numerical weather prediction or climate simulation. We had to wait about 15 yr, until a certain level of completion was achieved in the development of global atmospheric models along with a great improvement of computer resource availability, to be able to tackle the problem of two-dimensional turbulence on a sphere. The bright side to this blank period was that images of Rossby wave propagation on
a sphere had been accumulated, as exemplified in such works as Grose and Hoskins (1979) and Hoskins and Karoly (1981), with some remarks by Hayashi and Matsuno (1984), Hayashi (1987), and so on. The results of these past studies are useful in understanding the behavior of zonal mean flow formation on a rotating sphere as described in this paper.

Successive studies on two-dimensional turbulence on a rotating sphere were carried out in the 1990s. Yoden and Yamada (1993) investigated decaying, instead of forced, two-dimensional nondivergent turbulence with full spherical geometry by utilizing a spectral transformation model with a resolution of T85, in which variables are evaluated by the sum of spherical harmonics up to total wavenumber 85, and nonlinear terms are calculated on $256 \times 128$ longitude–latitude grids. A full spherical version of forced turbulence was investigated by Nozawa and Yoden (1997a,b) for the same setup as Williams (1978), and by Huang and Robinson (1998) for a different type of forcing to realize a constant energy input rate. They reconfirmed the spontaneous formation of a banded structure of zonal mean flows and recognized its persistent characteristics. A peculiar feature found by Yoden and Yamada (1993) is that, in addition to the appearance of the banded structure of zonal mean flows, intense easterly (retrograde) circumpolar jets tend to emerge especially at large rotation rate. The appearance of an easterly circumpolar jet has been confirmed as a robust feature in decaying turbulence, while in forced turbulence, its appearance seems to depend on the forcing scale. When the forcing scale is sufficiently small, an intense easterly circumpolar jet fails to appear in forced turbulence (Nozawa and Yoden 1997a).

When surface elevation is permitted, the nature of easterly momentum accumulation changes dramatically. Cho and Polvani (1996) investigated decaying two-dimensional shallow-water turbulence on a rotating sphere. One interesting point in their results is that, in contrast to the nondivergent cases, an intense easterly jet does not appear around the pole, but appears at the equator when the radius of deformation is small. Furthermore, the situation becomes even more complicated when the rotation rate is decreased; it seems that the direction of equatorial jet is dependent on the initially given field (Kitamura and Ishioka 2007).

In the following sections, we are going to review the results of decaying turbulence on a rotating sphere with some additional results of parameter study carried out in order to interlink the situations investigated by Cho and Polvani (1996), Yoden and Yamada (1993), and others. The formation of easterly jet can be roughly understood as a result of Rossby wave accumulation. The asymptotic behavior of jet formation for the nondivergent case with increasing rotation rate is discussed in a separate paper Takehiro et al. (2007a), and the behavior of wave activity is described in Takehiro et al. (2007b). The detailed analysis of equatorialjet direction for the shallow-water case at intermediate rotation rates is also discussed in a separate paper (Kitamura and Ishioka 2007). Here, we will focus on the general trend in the behavior of jet formation of decaying turbulence in the parameter space of the rotation rate and the surface gravity wave speed (or, normalized rotation rate and Froude number).

2. Model

The system we are going to consider is a two-dimensional flow on a rotating sphere. When the system is appropriately scaled, fluid motion is governed basically by two parameters, which in the present case are the nondimensional rotation rate $\Omega = a\Omega_0/U$ and Froude number $Fr = U/\sqrt{gH}$, where $a$ is the radius of the sphere, $\Omega_0$ is the dimensional rotation rate of the sphere, $g$ is gravity acceleration at the surface of sphere, $H$ is the depth of the fluid layer, and $U$ is a velocity scale; $\Omega_0$ may be referred to as the Rossby number. The nondimensionalized governing equations are then given as

$$\frac{du}{dt} - u\nu\tan\phi - 2\Omega tu\sin\phi = -\frac{1}{Fr^2\cos\phi}\frac{\partial \eta}{\partial \lambda} + F_{\lambda},$$

(1)

$$\frac{dv}{dt} - u^2\nu\tan\phi + 2\Omega tu\sin\phi = -\frac{1}{Fr^2}\frac{\partial \eta}{\partial \phi} + F_{\phi},$$

(2)

$$\frac{d\eta}{dt} + (1 + \eta)\left(\frac{1}{\cos\phi}\frac{\partial u}{\partial \lambda} + \frac{1}{\cos\phi}\frac{\partial v}{\partial \phi}\right) = -(1)^{\nu+1}\kappa_{2\nu}\nabla^2\eta,$$

(3)

where $(u,v)$ is eastward and northward component of velocity normalized by $U$, $\eta$ is surface elevation normalized by $H$, $t$ is time normalized by $a/U$, $(\lambda, \phi)$ is longitude and latitude, $d/dt = \partial/\partial t + u/\cos\phi\partial/\partial \lambda + \omega/\partial \phi$, $\nabla^2$ is the spherical Laplacian, $F_{\lambda}$ and $F_{\phi}$ are the hyperviscosity terms, and $\kappa_{2\nu}$ is the hyperdiffusivity coefficient. We have adopted hyperviscosity instead of normal viscosity to ensure that dissipation occurs at the small-scale edge of the spectral space to realize a dissipation-free turbulent flow over a majority of the spectral space. For numerical calculations, (1) and (2) are transformed into the equations of vorticity and divergence. The system is composed of equations of vorticity, divergence, and surface elevation, as can be seen...
from (3). The viscosity terms \( F_\chi \) and \( F_\phi \) are defined such that the dissipation of vorticity \( \zeta \) and divergence \( D \) are given as \((-1)^{n+1}v_{2p}(\nabla^2 + 2)\zeta \) and \((-1)^{n+1}v_{2p}\nabla^2 \)

\( D \), respectively, where \( v_{2p} \) is the hyperviscosity coefficient. Typical values of hyperviscosity and diffusivity are \( p = 8 \) and \( v_{2p} = \kappa_{2p} = 10^{-34} \) in our recent calculations with resolution of T170 (512 \( \times \) 256) (Takehiro et al. 2007a). Nondivergent system is given as the limit of \( Fr \rightarrow 0 \) and is described by the following vorticity equation:

\[
\frac{\partial \zeta}{\partial t} + J(\psi, \zeta) + 2\Omega \frac{\partial \psi}{\partial \lambda} = (-1)^{n+1}v_{2p}(\nabla^2 + 2)\zeta,
\]

(4)

where \( \psi \) is streamfunction \( (u = -\partial \psi/\partial \phi, v = \partial \psi/\cos \phi \), \( \partial / \partial \lambda) \), \( J(\psi, \zeta) = (\partial \psi / \partial \lambda) (\partial \zeta / \partial \phi) - (\partial \psi / \partial \phi)(\partial \zeta / \partial \lambda) ]/ \cos \phi \).

The radius of deformation \( L_D \) and the equatorial radius of deformation \( L_{De} \) are the important quantities in considering flow behaviors. Here \( L_D \) is a measure of the distance fluid can spread from a mass source on a rotating system, and \( L_{De} \) is a measure of the latitudinal distance fluid can spread from a mass source placed at the equator. They are expressed in our nondimensional system as \( L_D = L_{Dp}/\sin \phi \) where \( L_{Dp} = \sqrt{2}\pi Fr\Omega \) and \( L_{De} = 1/\sqrt{2Fr\Omega} = \sqrt{L_{Dp}}. \) Also, \( L_{De} \) is the radius of deformation at the pole, and \( L_{Dp} \) is defined as the radius of deformation given by substituting \( \sin \phi = L_{De} \).

The magnitudes of these numbers may be inferred from those of real planets. The value of \( \Omega \) is about 40 when assuming the values of Earth’s atmosphere as \( \Omega_s = 7 \times 10^{-5} \) s\(^{-1}\), \( a = 6 \times 10^6 \) m, and \( U = 10 \) m s\(^{-1}\); about 300 when assuming the values of Jovian atmosphere as \( \Omega_s = 2 \times 10^{-2} \) s\(^{-1}\), \( a = 7 \times 10^7 \) m, and \( U = 50 \) m s\(^{-1}\); and about 4000 when assuming the values of Earth’s ocean as \( U = 0.1 \) m s\(^{-1}\). The value of \( Fr \) is about 0.005 when assuming the values of the external mode of Earth’s ocean as \( H = 4000 \) m and \( g = 10 \) m s\(^{-2}\) with \( U = 0.1 \) m s\(^{-1}\), and about 0.025 when assuming the values of an internal mode of Earth’s ocean as \( H = 800 \) m and the reduced gravity \( g' = 0.02 \) m s\(^{-2}\) with \( U = 0.1 \) m s\(^{-1}\).

Most of the initial velocity fields of the calculations presented in the following sections are randomly given with the energy spectral distribution of Cho and Polvani (1996),

\[
E(n) = \frac{A n^{\gamma/2}}{(n + n_0)^\gamma},
\]

(5)

where \( n \) is total wavenumber of spherical harmonics. Note that \( n_0 \) and \( \gamma \) give the peak wavenumber and the width of the energy spectrum, respectively. The coefficient \( A \) is given such that the global mean kinetic energy be unity, that is, \( E = \sum_{n=2}^{N} E(n) = 1 \), which means that the global mean root-mean-square velocity is \( \sqrt{2} \).

Numerical calculations were performed by a spectral transformation method. The details are described in Ishioka et al. (2000). The fourth-order Runge–Kutta method was utilized for time integration. The numerical model used in section 4 is the SPMODEL, which is available from the SPMODEL Web page (http://www.gfd-dennou.org/library/spmodel/). The concept of SPMODEL is described in Takehiro et al. (2006).

3. Barotropic decaying turbulence

a. Circumpolar easterly jet

What Yoden and Yamada (1993) found for the nondivergent decaying turbulence on a rotating sphere was the spontaneous appearance of easterly circumpolar jets. By conducting an ensemble experiment starting from 48 initial fields (randomly generated 12 different distributions \( \times \) 4 north–south and plus–minus symmetries), they showed that the appearance of circumpolar easterly jets is a statistically robust feature (Fig. 1). They carried out a parameter study for the values of \( \Omega \) from 0 to 400, and found that the circumpolar jet appears more clearly as the rotation rate increases. However, the initial energy spectrum they adopted was \( E(n) = An^{5/2}e^{-n/2} \), which displays a peak at the total wavenumber \( n = 10 \). This means that, as is argued in Hayashi et al. (2000) and described below, the fields are not very turbulent especially when \( \Omega \) is large.

An important quantity that characterizes a flow with a background potential vorticity gradient is the transition wavenumber, or the Rhines wavenumber (Rhines 1975; Vallis and Maltrud 1993; Rhines 1994). It is defined as the wavenumber at which the magnitude of the nonlinear advection term is comparable to that of the linear background potential vorticity advection term in the potential vorticity equation. When the characteristic wavenumber of a flow is larger than the transition wavenumber, the nonlinear term dominates and the flow is regarded as wavy; the disturbances in such cases are referred to as Rossby waves.\(^1\) The linear term of the potential vorticity equation is often referred to as the \( \beta \) term, and various effects caused by the existence of the \( \beta \) term are ge-

\(^1\) In this paper, the term “Rossby wave” is used to denote a disturbance when the linear term of the potential vorticity equation can be assumed to be dominant in its evolution.
numerically referred to as the $\beta$ effect. For the system given by (4), provided that the rotation rate of the system is sufficiently large and the vorticity gradient caused by the flow is neglected, the transition wavenumber may be locally defined as

$$k_\beta = \frac{\beta}{\sqrt{\langle u \rangle}}, \quad \beta = 2\Omega \cos \phi, \tag{6}$$

where $\langle u \rangle$ is root-mean-square velocity associated with the eddies of the spatial scale $k_\beta^{-1}$ and is often substituted simply by the root-mean-square velocity of the field. A factor of $\sqrt{\zeta}$ may be multiplied depending on how one counts the wavenumber $k_\beta$.

Because of the component $\cos \phi$ in (6), or the latitudinal variation of $\beta$ effect on a sphere, the transition wavenumber varies with latitude; it vanishes at both poles and reaches maximum at the equator. This implies that, for a disturbance with a given characteristic scale, $K^{-1}$, there appears a wavy (Rossby wave dominant) region around the equator between the latitudes of $\phi_\beta = \pm \cos^{-1}(K^2/\zeta/2\Omega)$. For the case of Yoden and Yamada (1993), $\langle u \rangle = \sqrt{\zeta}$, since the mean kinetic energy given initially is unity. As the value of $\zeta$ ranges from 0 to 400, the corresponding transition wavenumber ranges from 0 to 24 at the equator. Assuming also that $K = 10$ from their initial spectral profile, the initial fields are Rossby wave dominant in the lower latitudes for $\zeta > 50 \sim 100$. For $\zeta = 400$, the initial fields are wavy up to quite high latitudes, $\phi_\beta = \pm 80^\circ$.

The formation of the easterly circumpolar jet may be regarded as a result of turbulent mixing of potential vorticity in the polar region where the flow regime is turbulent. Since north (south) pole is the place where potential vorticity takes the maximum (minimum) on a rotating sphere, a latitudinal mixing of potential vorticity causes negative (positive) anomaly of potential vorticity there. This indicates that easterly anomaly appears at the pole. However, in order to realize a potential vorticity mixing in such a way, it is necessary for a net easterly angular momentum to be supplied from the outside of the polar region to maintain the conservation of angular momentum. Actually, the easterly angular momentum can be delivered by Rossby waves traveling from the lower latitudes. A brief illustration can be given by considering the weak nonlinear version of (4):

$$\frac{\partial^2 \zeta}{\partial t} + \frac{\pi}{\cos \phi} \frac{\partial \zeta}{\partial \lambda} + \frac{\beta}{\cos \phi} \frac{\partial \psi'}{\partial \lambda} = F', \tag{7}$$

$$\frac{\partial}{\partial t} (\bar{\pi} \cos \phi) + \frac{1}{\cos \phi} \frac{\partial}{\partial \phi} \left( \cos^2 \phi \bar{u}' \bar{v}' \right) = F_\zeta \cos \phi, \tag{8}$$

where $\beta = \beta - (\partial \phi)(\langle 1/\cos \phi \rangle \partial \phi) \cos \phi \bar{u}]$, and $F$ and $F_\zeta$ are the viscosity terms; the overbar and prime symbols denote the zonal mean and the deviation from it, respectively. The conservation of wave activity, $\mathcal{A}$, can be derived immediately from (7):

$$\frac{\partial}{\partial t} \mathcal{A} + \nabla \cdot \mathcal{J} = \frac{\cos \phi}{\beta} \zeta F', \quad \mathcal{A} = \frac{\zeta^2}{2} \cos \phi \frac{\beta}{\beta},$$

$$\mathcal{J} = \left[ \bar{u} \bar{A} + \frac{1}{2} \cos \phi (\bar{v}'^2 - \bar{u}'^2) - \cos \phi \bar{u}' \bar{v}' \right]. \tag{9}$$

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where $\nabla \cdot$ is spherical divergence operator. Equations (8) and (9) indicate that $\mathbf{A}$, the second-order mean conserved quantity associated with a Rossby wave propagating on a sphere, corresponds to easterly angular momentum.

When the rotation rate is large and the vorticity gradient of the mean flow can be neglected, Wentzel–Kramers–Brillouin–Jeffries (WKBJ) theory (ray theory) shows that a Rossby wave packet propagates along a great circle if observed from a relatively rotating frame having the speed of eastward angular phase velocity of the wave (Hoskins and Karoly 1981; Hayashi and Matsuno 1984; Hayashi 1987). The dispersion relation of a nondivergent Rossby wave is given as $\omega = -\beta k/(k^2 + l^2)$, where $\omega$ and $(k, l)$ are frequency and wavenumbers in the longitudinal and latitudinal directions, respectively. The latitudinal and longitudinal components of group velocity of the wave packet are given as $c_{\phi,k} = (c, 0) + 2\omega^2 / k\beta(k, l)$, where $c = \omega / k$ is longitudinal phase speed. Since $\omega$ and $k$ are conserved along a ray, the dispersion relation indicates that the total wavenumber $\sqrt{k^2 + l^2}$ is also conserved, and that the longitudinal phase speed $c$ is proportional to $\cos \phi$. When observed from a relatively rotating frame with angular velocity of $c / \cos \phi$, the wave packet travels along a path $[1 / \cos \phi] (d\phi / d\lambda) = c_{,\phi} / (c_{,\phi} - c)$ that is equal to $l / k$ and can be integrated easily to yield a great circle, $\tan \alpha \sin(\lambda - \lambda_{eq}) = \tan \phi$, where $\alpha$ and $\lambda_{eq}$ are the pitch angle and longitude of the wave packet at the equator, respectively. Note that the magnitude of the relative group velocity is constant along the great circle ray, since $\sqrt{(c_{,\phi} - c)^2 + c_{,\phi}^2} = 2\omega^2 / \sqrt{k^2 + l^2} / \beta k$. A Rossby wave packet simply travels straight along a great circle at a constant group velocity.

Let us assume a wave packet whose pitch angle $\alpha$ is sufficiently large so that it can propagate into the higher latitudes where $\beta$ is small. The propagation of Rossby wave packet induces second-order easterly flow, whose magnitude is enhanced in the higher latitude because the angular momentum associated with the wave packet is conserved during its propagation. On the other hand, the local westward phase speed of Rossby wave decreases as the wave packet approaches the higher latitudes. We can expect that the large easterly fluctuation of zonal mean flow and small westward phase speed of Rossby wave in the higher latitudes are suitable for the occurrence of critical latitude absorption there, which contributes to the easterly angular-momentum accumulation in those latitudes.

Figure 2 is an example of comparison between weak-nonlinear and full-nonlinear evolutions of zonal mean angular momentum starting from one of the initial conditions utilized by Yoden and Yamada (1993). Once the easterly acceleration is initiated in the polar region, it continues steadily. The nonlinear and the weak-nonlinear evolutions of circumpolar jets are quite similar to each other. The spatial distribution of the amplitude of initial disturbance is statistically uniform on the sphere, which means that the initial distribution of $\mathbf{A}$ is also statistically uniform as is observed in Fig. 2 (lower left), since $\mathbf{A} \sim \xi^2 / \Omega$ when the rotation rate is large. Singularity appears only in the higher-latitude regions where the field is in a turbulent regime. Critical latitude absorption occurs around the latitude of the regime transition from wave to turbulent. As time progresses, the amount of $\mathbf{A}$ decreases smoothly except for the singular polar regions. Correspondingly, the easterly angular-momentum transfer from the mid- and low latitudes to the polar regions occur (not shown here, but see also Fig. 5c and its description in the next subsection). Westerly angular momentum increases slightly and uniformly in the mid- and lower latitudes.

The emergence of easterly circumpolar jets becomes a robust feature with increasing $\Omega$. They develop faster with increased magnitudes and decreased widths, and the location of their axes shift toward the higher latitudes (Takehiro et al. 2007a,b).

**b. Banded zonal mean flow formation**

Figure 2 also indicates that the result of $\Omega = 400$ of Yoden and Yamada (1993) does not represent a formation of a banded structure from turbulence through two-dimensional inverse cascading of energy and the Rhines effect. The banded zonal mean flows exist there nearly in the initial field. The evolution of a field whose initial energy spectral peak is located in a high wavenumber region compared to $k_{\mu}(\phi = 0) = 24$, the transition wavenumber at the equator for $\Omega = 400$, were visualized by Ishioka et al. (1999) using higher-resolution models, T341 (1024 × 512) and T682 (2048 × 1024). The results were further described by successive works: Yoden et al. (1999), Hayashi et al. (2000), and Yoden et al. (2002). The features of Yoden and Yamada (1993), the banded structure of zonal mean flows and circumpolar jets, were successfully reproduced through inverse cascading of energy in the spectral space.

Figure 3 is an example of the vorticity fields at $t = 5$ for various values of $\Omega$ from the same initial field of the energy spectrum (5) with $(n_0, \gamma) = (50, 1000)$. Zonal mean angular momentum is plotted in the right-hand side of each panel. Time evolutions of the fields can be seen in the form of movies in Ishioka et al. (1999). In the nonrotating case (left), a number of coherent vortices appear and then their population decreases gradually through mergers. Zonal mean angular momentum
may seem to be fairly large but it is not steady; the profile changes as the coherent vortices move. In the high-rotation case $\Omega = 400$ (right), the vorticity field shows zonally elongated structures. The profile of zonal mean angular momentum shows the emergence of easterly circumpolar jets, while in the mid- and low latitudes, zonal mean flow tends to be westerly biased. In addition to these jets and weak westerly bias, the banded structure of zonal mean flows that has evolved from a turbulent initial field appears in the mid- and low latitudes.

The spontaneous development of the banded structure in the mid- and low latitudes occurs fairly early on in the time evolution, by about $t = 1 \sim 2$ as is shown in Hayashi et al. (2000) for $\Omega = 400$. Figure 4e (Fr = 0 case, the details of Fig. 4 are described later in section 4) is another example of the development of zonal mean flow from an initial condition of the energy spectrum (5) with $(n_0, \gamma) = (50, 100)$, which has the broader spectral width compared to the case of Fig. 3. A high characteristic latitudinal wavenumber of the zonal mean angular-momentum profile, which corresponds to

Fig. 2. Comparison between the (upper left) nonlinear and (upper right) weak nonlinear evolutions of angular momentum starting from the same initial field with energy peak wavenumber $n = 10$ for $\Omega = 400$. Also shown are (lower left) $\lambda$ and (lower right) $\beta$ of the nonlinear calculation. Adapted from Hayashi et al. (2000).
the initial energy spectral peak given at \( n_0 = 50 \), evolves rapidly by around \( t \sim 1 \), and the banded structure appears. After this initial stage of rapid development, the banded structure in the mid and low latitudes reaches a near-steady state; the width of each band remains almost unchanged afterward, although a slight change in its magnitude and shape continues on top of the slowly growing small westerly bias. The formation of an easterly circumpolar jet, which appears around the southern pole in the case of Fig. 4e, proceeds gradually and continuously. The poleward accumulation of easterly momentum from the mid- and low latitudes is exemplified in the cumulative eddy angular momentum flux in Fig. 5e (the details of Fig. 5 are described later in section 4). The accumulation of momentum flux also proceeds gradually and continuously throughout the integration period. Note that the eddy momentum flux is quite asymmetric around the equator in correspondence with the growth of the southern polar jet for this case.

The energy spectrum is characterized by the appearance of anisotropy. Two-dimensional energy spectra corresponding to the cases of Fig. 3 are shown in Fig. 6. With increasing \( \Omega \), two distinct features become more and more evident; energy density becomes prominent in the region of small azimuthal wavenumber \( m \), and the disappearance of energy is observed in an airfoil-shaped region at the lower wavenumber edge on the side of \( m = n \), where \( n \) is the total wavenumber. The appearance of these anisotropic features is associated with the development of zonal elongation of vorticity patterns and the banded structure of zonal mean flows. The airfoil-shaped region is considered to correspond to the dumbbell region described by Vallis and Maltrud (1993) for the \( \beta \)-plane turbulence (Nozawa and Yoden 1997b; Huang and Robinson 1998). The dumbbell region is the area where the \( \beta \) term dominates in the potential vorticity equation. Inside this area, phase desynchronization resulting from the \( \beta \) term makes nonlinear interactions less efficient. It is the area where the generation of Rossby waves by the inverse cascading of energy from the larger wavenumber region are hardly observed. Vallis and Maltrud (1993) argued that the size of the dumbbell region can be roughly estimated by the transition wavenumber \( (6) \) and that the dumbbell shape may be explained by considering the anisotropy in the magnitude of the \( \beta \) term in \((k, l)\) plane. Plotted in Fig. 6 is the line defined by Nozawa and Yoden (1997b) and Huang and Robinson (1998), where the magnitudes of nonlinear advection term and the linear \( \beta \) term represented by the spherical harmonics expansion are balanced as \( \langle u \rangle n = 2 \Omega m/[m(n + 1)]; \langle u \rangle = \sqrt{2} \) is used in Fig. 6. The shape of this line and the airfoil region of small energy in Fig. 6 have some resemblance. However, the area covered by the line is much larger. This means that there are disturbances whose evolutions are controlled mainly by the linear \( \beta \) term; they can be referred to as Rossby waves.

Circumpolar easterly jets do appear even from the initial condition whose energy spectral peak is located at the wavenumber higher than the transition wavenumber. In such initial conditions, the amount of disturbances that can propagate as linear Rossby waves is quite small at the beginning. It seems that long Rossby waves, whose longitudinal wavenumbers are small and group velocity directions are poleward, are excited as a result of energy inverse cascading. According to the ensemble mean wave property analyses by Takehiro et al. (2007b), it has been confirmed that there are two stages in the evolution of wave activity; the early stage
of rapid dissipation most likely caused by local cascading of enstrophy and potential vorticity mixing associated with the formation of the banded structure, and the later slow stage of generation, propagation, and polar absorption of long Rossby waves associated with the development of circumpolar jets. The scenario of circumpolar jet formation presented in the previous subsection seems to be applicable to the development after the generation of long Rossby waves of this later stage.

The number of banded easterly and westerly zonal mean flows agrees roughly with the transition (Rhines) wavenumber (Hayashi et al. 2000). It seems to be determined by the mixing ability of local potential vorticity of the initially given field. Indicated in Fig. 6 by an arrow on the $\phi$ axis is $n_{\phi}$, which was introduced by Nozawa and Yoden (1997a) as a measure of the global mean of the transition wavenumber. It is defined as the global average of $k_{\phi}^2$, and substituting $\langle u \rangle = \sqrt{2}$, we

**Fig. 4.** Time evolutions of zonal mean angular momentum $\pi \cos \phi$ with $\Omega = 400$ for various values of Fr. Lines are the profiles at selected times $t = 0.1, 0.2, 0.3, 0.6, 1.0, 3.0, 6.0, \text{and} 10.0$. The magnitude corresponding to the value of 0.5 is indicated by the arrow at the bottom left of each panel. Shaded areas in the background indicate that $\pi \cos \phi$ is negative at the respective latitude and time. Cross marks on the latitudinal axes indicate the latitudes $\pm \phi$ listed in Table 1.
have \( n_\beta = \sqrt{\pi \Omega / (4\sqrt{2})} \) where the numerical factor \( 1/\sqrt{2} \) is retained following their definition. The values of \( n_\beta \) is 5 for \( \Omega = 50 \) and 15 for \( \Omega = 400 \).

It might be speculated that the banded structure in the mid- and low latitudes of decaying turbulence disappears with longer integration periods. According to our calculations, the banded zonal flows in the mid- and low latitudes are relatively steady. Takehiro et al. (2007a,b) also confirmed that the banded structure, once it is established, does not seem to change even after \( t = 10 \). Since there is no source of small-scale eddies, the development of the banded structure ceases once it consumes the energy of small-scale eddies. After this early development, there remains little reason for further mixing of zonal mean flows. The obtained zonal mean flow profiles in the mid- and low latitudes are barotropically stable, and they decay quite slowly due to the effect of friction. One remaining possibility is

![Diagram of zonal mean flux evolution](image)

**FIG. 5.** Time evolutions of cumulative eddy angular-momentum flux \( \int_{t_0}^{t} \bar{u} \bar{v} \cos \phi \, dt \) with \( \Omega = 400 \) for various values of Fr. Lines are the profiles at selected times \( t = 0.2, 0.3, 0.6, 1.0, 3.0, 6.0, \) and 10.0. The magnitude corresponding to the value of 1.0 is indicated by the arrow at the bottom left of each panel. Shaded areas in the background indicate that \( \int_{t_0}^{t} \bar{u} \bar{v} \cos \phi \, dt \) is negative at the respective latitude and time. Cross marks on the latitudinal axes indicate the latitudes \( \pm \phi \) listed in Table 1.
the type of instability discussed by Manfroi and Young (1999). They considered a nonlinear evolution of parallel flow on a β plane and showed that a merger of banded zonal flows may occur very slowly. We have not checked the stability of the banded structure from their viewpoint. The integration period of Takehiro et al. (2007a,b) may still be too short.

c. Forced versus decaying

The major difference in the appearances of jet formation between forced and decaying turbulence is the appearance of intense circumpolar easterly jets. According to the results of Nozawa and Yoden (1997a), Huang and Robinson (1998), Huang et al. (2001), and other recent studies, intense circumpolar easterly jets are rarely observed in forced turbulent flows, except for the few cases presented in Nozawa and Yoden (1997a) where the forcing scale or the rotation rate of the system are rather large.

A notable difference between the natures of forced and decaying turbulence is the behavior of the total energy of the system. For the case of forced turbulence, the total energy increases through two-dimensional upward cascading to the value determined by the details of the dissipation, which is usually introduced at large scales. Consequently, the transition wavenumber decreases with increase in total energy until the system reaches a statistically steady state. The number of the banded zonal mean flows roughly coincides with the transition wavenumber and follows its evolution. This is in sharp contrast with the case of decaying turbulence, where the transition wavenumber is nearly constant or even increases, since the total energy dissipates quite slowly or the turbulent part of the energy decreases. For the exceptional cases of Nozawa and Yoden (1997a), the total energy remains small and the transition wavenumber is rather large. When the forcing wavenumber is smaller than the transition wavenumber, the disturbances generated can propagate as Rossby waves and be absorbed around the poles. If the integration time was longer and the amount of total energy increased, the transition wavenumber would become smaller than the forcing wavenumber and the results would be similar to the other cases.

We have not yet solidly figured out the reason why an intense easterly circumpolar jet does not emerge when the forcing scale is small. As mentioned briefly above, the transition wavenumber in a statistically steady state of forced turbulence roughly corresponds to the largest scale of energy inverse cascading determined by the dissipation of the system. However, longer Rossby waves still seem to be excited to a certain extent. Energy accumulates near the $m = 0$ axis in the spectral space and has the spectral shape of $n^{-5}$ (Huang et al. 2001). The intensity of those Rossby waves might be too small for the creation of a prominent circumpolar jet. Or, there might be some obstacles that prevent the propagation of Rossby waves that otherwise would have reached the polar regions. The dissipation acting on the large scales introduced for the cases of forced turbulence might be an obstacle. A large change in the potential vorticity profile might be another obstacle. Since small-scale eddies are continuously supplied by the forcing, the evolution of zonal mean flows continues steadily up to the point where a staircase profile of potential vorticity is realized. The resultant banded zonal mean flows are intense for the cases of forced turbulence and might affect the propagation property of Rossby waves. This contrasts with the banded structure of decaying turbulence, where the evolution of
Table 1. Summary of experimental parameters. These are the parameters for the experiments described mainly in section 4 and shown in Figs. 4, 5, 7–13. Note that the initial spectral peak wavenumber \( n_0 \) is 50 and the resolution of the numerical model is T170; \( k_p(\phi = 0) \) is the transition (Rhines) wavenumber at the equator defined by (6) with \( (\phi) = \sqrt{2} \), \( L_{DP} = 1/(2Fr(\Omega)) \) is the radius of deformation at the pole, \( L_{De} = \sqrt{L_{DP}} \) is the equatorial radius of deformation, and \( \phi \) is the latitude where the radius of deformation is equal to the Rhines scale.

<table>
<thead>
<tr>
<th>( \Omega )</th>
<th>( Fr )</th>
<th>( k_p(\phi = 0) )</th>
<th>( L_{DP}^{-1} )</th>
<th>( L_{De}^{-1} )</th>
<th>( L_{DP} )</th>
<th>( L_{De} )</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/( \sqrt{10} )</td>
<td>0</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>0</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>25</td>
<td>1/( \sqrt{10} )</td>
<td>5.9</td>
<td>16</td>
<td>4.0</td>
<td>0.063</td>
<td>0.25</td>
<td>0.37 (21°)</td>
</tr>
<tr>
<td>50</td>
<td>1/( \sqrt{10} )</td>
<td>8.4</td>
<td>32</td>
<td>5.6</td>
<td>0.032</td>
<td>0.18</td>
<td>0.26 (15°)</td>
</tr>
<tr>
<td>100</td>
<td>1/( \sqrt{10} )</td>
<td>12</td>
<td>63</td>
<td>8.0</td>
<td>0.016</td>
<td>0.13</td>
<td>0.19 (11°)</td>
</tr>
<tr>
<td>400</td>
<td>1/( \sqrt{10} )</td>
<td>24</td>
<td>250</td>
<td>16</td>
<td>0.004</td>
<td>0.063</td>
<td>0.09 (5°)</td>
</tr>
<tr>
<td>400</td>
<td>1/100</td>
<td>24</td>
<td>80</td>
<td>8.9</td>
<td>0.013</td>
<td>0.11</td>
<td>0.30 (17°)</td>
</tr>
<tr>
<td>400</td>
<td>1/10000</td>
<td>24</td>
<td>25</td>
<td>5.0</td>
<td>0.04</td>
<td>0.2</td>
<td>0.86 (49°)</td>
</tr>
<tr>
<td>400</td>
<td>1/100000</td>
<td>24</td>
<td>8</td>
<td>2.8</td>
<td>0.13</td>
<td>0.35</td>
<td>1.47 (84°)</td>
</tr>
<tr>
<td>400</td>
<td>0</td>
<td>24</td>
<td>0</td>
<td>0</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>—</td>
</tr>
<tr>
<td>4000</td>
<td>1/( \sqrt{10} )</td>
<td>75</td>
<td>2500</td>
<td>75</td>
<td>0.0004</td>
<td>0.02</td>
<td>0.03 (1.7°)</td>
</tr>
</tbody>
</table>

Zonal mean flows ceases when the energy of small-scale eddies is consumed (Takehiro et al. 2007b) and the potential vorticity profile in the mid- and low latitudes is affected less severely (see also Fig. 7 for the potential vorticity gradient profile).

4. Shallow-water decaying turbulence

a. Jets and disturbances with finite Fr

As for the shallow-water decaying turbulence on a rotating sphere, what Cho and Polvani (1996) found was the spontaneous appearance of the easterly equatorial jet. This tendency was also confirmed by Iacono et al. (1999a,b). The appearance of the equatorial jet contrasts with the results of nondivergent decaying turbulence where easterly momentum is accumulated in the polar regions. By utilizing a spherical shallow-water model with resolution of T170, Cho and Polvani (1996) illustrated that an equatorial easterly jet having a magnitude of about 150 m s\(^{-1}\) is produced for \( \Omega_n = 1.8 \times 10^{-4} \text{ s}^{-1} \), \( a = 7.2 \times 10^7 \text{ m} \), and \( L_{DP} = 2 \times 10^6 \text{ m} \) (~0.03\( \alpha \)), which were values selected assuming the case of Jupiter.

To understand the difference between the shallow-water and the nondivergent systems, we have attempted to trace the behavior of the easterly jet in the nondimensional space connected continuously with the nondivergent system discussed in the previous section. The basic set of equations is now given by (1), (2), and (3). Numerical integrations are conducted for the parameters \( Fr = 1/\sqrt{10}, 1/\sqrt{100}, 1/\sqrt{1000}, 1/\sqrt{10000}, \) and \( 0, \) with the values of \( \Omega \) from 0 to 4000. For all runs, the same initial condition is adopted, which consists of a given vorticity field, and both the divergence and surface elevation fields are set to be zero. The kinetic energy spectrum is given by (5) with the values of \( (n_0, \gamma) \) as (50, 100). The mean energy density \( \mathcal{E} \) is unity. The initial fields of vorticity and streamfunction are presented in Fig. 8f and Fig. 13f, respectively. The values of radius of deformation at the pole \( L_{DP} \) and equatorial radius of deformation \( L_{De} \) are summarized in Table 1. The shortcoming of our calculation might be that the resolution of our model, T170 (512 \( \times \) 256), is not enough to resolve \( L_{DP} \) for some of the cases. However, since \( L_{De} \) is properly or almost properly represented and the initially given field has the larger resolvable scales, we can expect to obtain some information about the features near the equator even for those cases.

Figure 7 shows the zonal mean profiles of angular-momentum \( \tilde{\pi} \) \( \cos \phi \), surface elevation \( Fr^{-2} \tilde{\eta} \), potential vorticity \( \tilde{\zeta} = (\tilde{\zeta} + 2\Omega \sin \phi)/(1 + \tilde{\eta}) \), and potential vorticity gradient \( \partial \tilde{\zeta}/\partial \phi \) at \( t = 10 \) for various values of \( Fr \) with \( \Omega = 400 \) and also for the initial state. The profiles of \( \tilde{\eta} \) \( \cos \phi \) change from the equatorial easterly jet like that of Cho and Polvani (1996) for \( Fr = 1/\sqrt{10} \) (Fig. 7a) to the easterly circumpolar jet for the nondivergent case \( Fr = 0 \) (Fig. 7e). An intense jet appears around the southern pole for this nondivergent case as described in section 3b. An interesting characteristic shown in Fig. 7 is that the latitudinal region where intense zonal mean flows are observed is confined to a band around the equator, the width of which increases with decreasing \( Fr \) or the increase of the radius of deformation.

From the profiles of potential vorticity gradient (Fig. 7), it appears that potential vorticity is not thoroughly mixed in each zonal flow band. As for the intense equatorial easterly jet region, a signature of mixing may be identified as the appearance of the minimum of potential vorticity gradient near the equator in the case of \( Fr = 1/\sqrt{10} \). Potential vorticity seems to be well mixed inside of the circumpolar jet of the nondivergent case \( Fr = 0 \). However, intense mixing is not a necessity for
the appearance of circumpolar easterly jet of the non-divergent case either (e.g., see Takehiro et al. 2007b).

Figures 4 and 5 show time evolutions of zonal mean angular momentum and cumulative eddy angular-momentum flux \( \int_0^t \overline{\nu' \nu'} \cos \phi \, dt \), respectively. Integrated flux is plotted to smooth out temporal variations and to pick up features of momentum transfer relevant to the long-term development of equatorial or circumpolar easterly jet of the non-divergent case either (e.g., see Takehiro et al. 2007b).

Fig. 7. Zonal mean fields at \( t = 10 \) with \( \Omega = 400 \) for various values of Fr and initial fields. Each panel shows the zonal mean profiles of velocity \( \pi \), angular momentum \( \pi \cos \phi \), surface elevation \( \eta \), potential vorticity \( q = (\zeta + 2 \Omega \sin \phi)/(1 + \eta) \), and potential vorticity gradient \( \partial q/\partial \phi \), from the left to the right. Cross marks on the latitudinal axes indicate the latitudes \( \pm \hat{\phi} \) listed in Table 1.
polar jets. It should be noted that our data sampling interval is $\Delta t = 0.1$, and that we omitted the flux calculated from the initial data $t = 0$, since its inclusion resulted in relatively large noises, especially for the cases with large values of $\text{Fr}$ and $\Omega$. As seen in Figs. 4a and 4b, the equatorial easterly jets develop rather slowly. The time scale of their development is similar to that of the circumpolar easterly jet in the nondivergent case (Fig. 4e). Correspondingly, the evolutionary trend of cumulative eddy angular-momentum flux becomes clear at the later period of integration. The direction of the cumulative eddy easterly angular-momentum flux at the later period changes with decreasing Fr from equatorward (Figs. 5a,b) to poleward (Fig. 5e) in the nondivergent case.

Figure 8 shows vorticity fields at $t = 5$ with $\Omega = 400$ for various values of Fr and the initial field. When the value of Fr is large, coherent vortices appear in the mid- and/or high latitudes. Most of them are anticyclonic, as is emphasized by Iacono et al. (1999a,b). According to the time series (not shown here), the vortices appear and are established in the early stage of integration by about $t = 1$. In the higher latitudes, they last coherently over extended periods of time, while in the lower latitudes, they are stretched and elongated in the zonal direction, and disappear. The latitudinal boundary where such vortices appear approaches equatorward with increasing Fr. It seems that the latitudes of the appearance of coherent vortices coincide with the latitudes at which the zonal mean flows become weak. The amplitudes of vortices also decrease with increasing latitude. Especially in the high latitudes for the case of $\text{Fr} = 1/\sqrt{10}$, it is quite difficult to identify the signatures of disturbances in the streamfunction field (shown in Fig. 13d). Even in the vorticity field (Fig. 8a), disturbances have very small amplitudes.

When Fr is large, a banded structure of zonal mean flows seems to appear even in the mid- and high latitudes as indicated by the persistence of persistent shaded areas in Fig. 4a. The corresponding latitudinal variation in the plot of zonal mean surface elevation is also observed in Fig. 7a, although its amplitude in the off-equatorial latitudes is quite small. These might be considered as a signature of the inverse cascading of energy and the Rhines effect. However, zonal mean flows in the higher latitudes, where amplitudes of disturbances are quite small, are actually caused by the initial geostrophic adjustment process. For $\text{Fr} = 1/\sqrt{10}$, there is little evolution in the profile of zonal mean potential vorticity except at low latitudes. This can be more clearly observed by comparing the initial and $t = 10$ profiles of potential vorticity gradient $\nabla \Phi$ (Figs. 7a,f).

The zonal mean flows in the high latitudes, including the regions where $\nabla \Phi$ changes signs, hardly evolve after the initial geostrophic adjustment process. They seem to be linearly stable because the latitudinal scale of $\nabla \Phi < 0$ is much larger than the radius of deformation (Waugh and Dritschel 1991). As for the zonal mean flows in the lower latitudes where coherent vortices appear in Fig. 8a, it may also be difficult to observe the signature of development of zonal mean flows in Fig. 4a. However, if we examine carefully the development of zonal flows on both sides of the equatorial jet, their amplitudes, although they are quite small, seem to increase, and the latitudes at which the small zonal flows exist seem to extend poleward with time. The period of time integration in the present study may be too short to confirm the evolution of zonal mean flows.

Figure 9 shows the zonal mean profiles at $t = 10$ for various values of $\Omega$ with $\text{Fr} = 1/\sqrt{10}$, the large Fr cases of our numerical calculations. The corresponding time developments of zonal mean angular momentum from $t = 0.1$ to $t = 10$ are shown in Fig. 10, and those of cumulative eddy angular-momentum flux are shown in Fig. 11. With increasing $\Omega$, the latitudinal regime of the intense zonal mean flows becomes confined near the equator, which is the same trend observed in Fig. 7 with increasing Fr. The emergence of the equatorial easterly jet is a fairly robust feature. Its existence is confirmed for $\Omega \geq 50$, and becomes instantly recognizable at $\Omega \approx 100$. Correspondingly, the cumulative eddy easterly angular-momentum fluxes for $\Omega \geq 50$ become equatorward. The width of the equatorial easterly jet narrows with increasing $\Omega$. It seems that the width of the equatorial jet is roughly equal to the equatorial radius of deformation.

Figure 10 confirms what has already been seen in Fig. 4; the formation of the equatorial easterly jet is slow and continuous. As with the development of the circumpolar easterly jet (Takehiro et al. 2007b), the development time scale of the equatorial easterly jet also decreases with increasing $\Omega$. In Fig. 10, this is observed as the earlier appearance of the equatorial easterly with increasing $\Omega$. When $\Omega = 4000$, an equatorial easterly jet forms almost immediately and is established at around $t = 1$. Compared to the nondivergent case (Fig. 5e), the transport of angular momentum due to eddies is not so global for the large Fr cases, but its extent is still significantly wider than the area of the equatorial jet and the counterwesterly flows on both sides of the equatorial jet. These suggest that the easterly momentum associated with Rossby waves is transported into the equatorial region from the off-equatorial latitudes. It may be noted that when Fr is large, the contribution of $\eta$ to angular-momentum flux is not negligible. Accord-
According to our calculations (not shown here), however, its contribution to the cumulative momentum flux is not so large, especially at the later period of integration of the cases for $\Omega \geq 100$ except around the equator within the radius of deformation. The overall features of angular-momentum flux do not change even when the contribution of $\eta$ is included.

A noteworthy feature in Fig. 10 is that, at the beginning of the integration, there sometimes appears a westerly jet at the equator. As time progresses, the east-

*Fig. 8.* Vorticity at $t = 5$ for various values of $Fr$ with $\Omega = 400$, and the initial vorticity field. Contour intervals are 20 in (a)–(e) and 100 in (f). The zero contour is suppressed. Gray to bluish colors indicate negative values.
erly momentum accumulates equatorward and an easterly jet is eventually established. However, it seems that an equatorial westerly jet remains for $\Omega = 25$ (Fig. 10b). Our calculation also shows that an equatorial westerly jet appears in the case of $(\Omega, Fr) = (25, 1/\sqrt{10})$ (not shown here). There are actually some ranges of $Fr$ and $\Omega$ in which an equatorial westerly jet is established depending on the initial condition. A detailed parameter study with ensemble runs on the occurrence of the westerly equatorial jet was performed by Kitamura and Ishioka (2007).

Figure 12 shows vorticity fields at $t = 5$ for various values of $\Omega$ with $Fr = 1/\sqrt{10}$. When $\Omega$ is large (Figs.
12e,f), equatorward-westward phase tilts are evident in the low latitudes, suggestive of the equatorward propagation of Rossby waves. The phase tilts correspond to the appearance of equatorward easterly angular-momentum fluxes from the off-equatorial areas shown in Figs. 11e and 11f. When $\Omega$ is small (Figs. 12b–d), the appearance of coherent vortices are more remarkable.

Coherent vortices, when they do appear, tend to be anticyclonic, as have already been seen in Fig. 8, except in the case of $\Omega = 0$. In addition to those vortices, equatorial waves are prominent. They are more clearly observed in the fields of streamfunction as shown in Figs. 13c and 13d.

As for the coherent vortices, the preferred appear-

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**Fig. 10.** Same as in Fig. 4 but for various values of $\Omega$ with $Fr = 1/\sqrt{10}$.
ance of anticyclonic vortices occurs only for large values of Fr. When the value of Fr decreases, or the radius of deformation increases, the cyclonic vortices become dominant (not shown) as is described by Theiss (2004). According to our calculation starting from the same initial field, when Fr = 1\(\sqrt{10}\), anticyclonic vortices are preferred for Fr = 1\(\sqrt{100}\), 1\(\sqrt{1000}\), and 1\(\sqrt{10000}\). When Fr = 100, cyclonic vortices are preferred for Fr = 1\(\sqrt{1000}\), and 1\(\sqrt{10000}\).

b. Effect of the radius of deformation

The latitudes at which the coherent vortices appear are also the latitudes in which zonal mean flows are

![Fig. 11. Same as in Fig. 5 but for various values of \(\Omega\) with Fr = 1\(\sqrt{10}\).](image)
weak. It seems that they roughly correspond to the latitudes where the radius of deformation is smaller than the Rhines scale (\(k^{-1}\)). The latitude \(\Phi\) at which the radius of deformation is equal to the Rhines scale is given by \(k_{\mu}(\Phi) = \sin \Phi / L_{DP}\). The values of \(\Phi\) for our experimental parameters of Fr and \(\Omega\) obtained by assuming \(\langle \mu \rangle = \sqrt{2} \) in (6) for \(k_{\mu}\) are summarized in Table 1. The values of \(\Phi\) seem to indicate roughly the boundary of significant zonal mean flow appearance in Figs. 5 and 9. It seems that when the inverse cascading energy encounters the deformation radius prior to the Rhines scale (transition wavenumber), the banded structure of

![Vorticity plots for various \(\Omega\) values](image)

**Fig. 12.** Same as in Fig. 8 but for various values of \(\Omega\) with Fr = \(1/\sqrt{10}\).
zonal mean flows fails to develop but a number of coherent vortices emerge instead.

It is worth noting that a naive estimate of the transition wavenumber for the case of $Fr/\beta H_0^2 11005$ is same as that of nondivergent case, if the quasigeostrophic approximation, $\nabla \times \nabla \times \psi = 0$, is applicable. Since we usually neglect the latitudinal variation of $L_D$, we have $J(\psi, q) = J(\psi, \zeta)$. It can be

$$\frac{\partial q}{\partial t} + J(\psi, q) + \beta \frac{1}{\cos \phi} \frac{\partial \psi}{\partial \lambda} = F, \quad q = \zeta - L_D^{-2} \psi, \quad (10)$$

Fig. 13. Streamfunctions at $t = 5$ for various values of $Fr$ and $\Omega$, and the initial streamfunction field. Contour intervals are (a), (b), (e) 0.1; (c), (f) 0.05; and (d) 0.025. The zero contour is suppressed. Gray to bluish colors indicate negative values.
concluded that the transition wavenumber at which the magnitudes of the nonlinear term and the linear term are comparable is also given as $k_p$ in (6). However, numerical results indicate that this does not hold true for small $L_d$. The transition from the nonlinear regime to the linear regime does not seem to occur.

An intuitive reason why a banded structure of zonal mean flows fails to develop when $L_d < k_p^{-1}$, while the magnitude of the linear term overwhelms that of the nonlinear term so long as energy inverse cascading continues, is that Rossby waves become nondispersive (Okuno and Masuda 2003). When the linear term is dominant, all disturbances simply move westward at the same velocity $\beta L_d^2$ when $L_d \ll k^{-1}$. The linear term may be large, but it does not interfere with the nonlinear term; there is no reason for a vortex to disperse as linear waves at the limit of small $L_d$. Okuno and Masuda (2003) showed that by using the Galilean transformation with the westward velocity of $\beta L_d^2$, the uniform drift caused by the linear term can be eliminated and the nonlinear term dominates in the transformed vorticity equation when $L_d$ is small.

The effect of the linear term is often evaluated by the use of the Rossby wave frequency. It might seem reasonable to assume that the linear effect is dominant when the time scale of a Rossby wave is shorter than the time scale of turbulent mixing. Since the dispersion relation for a Rossby wave is now given as $\omega = -\beta k/(k^2 + \lambda^2 + L_d^{-2})$, by equating this frequency with the turbulent frequency $\langle u \rangle k$ and neglecting the anisotropy between $k$ and $l$, the transition wavenumber may be modified to $\tilde{k}_p = \sqrt{\beta/(\langle u \rangle - L_d^2)}$. One might conclude from this expression that the transition to wavy regime will never occur when $L_d$ is smaller than $k_p^{-1}$ (e.g., Smith 2004; Theiss 2006). However, this seems to contradict with the above naive estimation of each term in (10); the linear term overwhelms the nonlinear term when the scale of a disturbance increases.

We suspect that the reason for the absence of the transition is not that the frequency of Rossby waves vanishes as the scale of a disturbance increases when $L_d$ is finite, but that the Rossby waves become nondispersive.

It is worth mentioning that the energy inverse cascading of two-dimensional turbulence progresses quite slowly beyond the radius of deformation (e.g., Yanase and Yamada 1984; Watanabe et al. 1997). Vortices hardly interact with each other beyond the distance farther than the radius of deformation. Once energy inverse cascading reaches the scale of deformation radius, coherent vortices of that size form, and merging between them occurs only occasionally. Longer time integrations may be required in order to observe the evolution of coherent vortices and zonal mean flows on a sphere when $L_d$ is small.

As for the small amplitudes of disturbances in the higher latitudes when $Fr$ and $\Omega$ are large, it should be noted that the radius of deformation $L_D$ is smaller than the typical length scale of the initial field. Since the initial field is given by a relative vorticity field, the magnitude of streamfunction actually realized after the initial geostrophic adjustment process is $K^2(K^2 + L_D^2)$ times that of the nondivergent case. This ratio is obviously quite small when $L_D$ is small compared to the characteristic scale of the initial disturbances $K^{-1}$. Based on the evolution of the fields (not shown), it is confirmed that the initial geostrophic adjustment is completed by around $t = 1$ in the high latitudes for $\Omega = 400$. Relative vorticity anomaly diminishes by that time, while potential vorticity anomaly remains in those latitudes.

c. Equatorial easterly jet

The emergence of equatorial easterly jet found by Cho and Polvani (1996) is an extremely robust character of decaying shallow-water turbulence. According to our calculations, zonal mean flow at the equator tends to be easterly when $Fr \approx 1/\sqrt{100}$ and $\Omega \approx 50$. When $Fr$ and $\Omega$ are sufficiently larger than these values, an equatorial easterly jet appears without exception. As mentioned in section 4a, the development of equatorial easterly jet proceeds rather slowly (Fig. 10), and easterly angular momentum is accumulated from a wider latitudinal area outside of the equatorial radius of deformation (Fig. 11) where equatorward-westward phase tilts are observed (Figs. 8a,b). These may be an indication that the propagation of Rossby waves and their absorption near the equator contribute to the development of equatorial easterly jet.

When $Fr$ is not zero, a Rossby wave packet does not propagate along a great circle. With decreasing radius of deformation, the maximum latitude it reaches decreases. Consequently, easterly angular momentum is no longer transferred toward the poles. Ray theory again requires that $\omega$ and $k \cos \phi$ are conserved along a ray; hence, assuming that quasigeostrophic approximation is applicable, $k^2 + \lambda^2 + L_D^{-2} \sin^2 \phi$ is now conserved. This indicates that when $L_D$ is small compared to the wavelength of the packet, its latitudinal propagation is restricted to near the equator where $\sin \phi$ is small. In such cases of latitudes with small $L_D$, a Rossby wave packet reaches its turning latitude where $l = 0$ immediately upon its poleward propagation, and $l$ increases rapidly as it propagates equatorward. In-
crease in \( l \) may be favorable for the occurrence of dissipation.

As described in the previous subsections, when the radius of deformation is small (\( F_r \) and/or \( \Omega \) are large), the equatorial area within the equatorial radius of deformation remains a region of high eddy activity. In these latitudes, after the initial geostrophic adjustment process, most of turbulent energy inverse cascaded from small scales is expected to be trapped as equatorial waves. On the other hand, the phase speed of a long Rossby wave but with large \( l \) coming from the off-equatorial latitudes may be small and comparable to the velocity fluctuation scale \( \langle u \rangle \), that is, \( \sqrt{2} \), since the phase speed is bounded by \( F_r^{-1/3} \), the long Rossby wave limit at the equator, and is inversely proportional to the latitudinal wavenumber. These indicate that the possibility for the occurrence of wave–mean flow interaction is high at the equatorial area within the equatorial radius of deformation (Theiss 2004). Once an easterly zonal mean flow is established at the equator, further absorption of Rossby waves occurs and the resultant easterly momentum accumulation continues. According to this scenario, the maximum width of the equatorial jet is roughly on the order of the equatorial radius of deformation, since the eddy mixing remains high and wave mean-flow interaction can occur only in that area when the radius of deformation is small. Note that the jet width may further grow with the accumulation of easterly momentum due to critical level absorption; however, it will progress slowly, as shown in our calculations. More detailed analyses with weak-nonlinear theory is presented in Kitamura and Ishioka (2007).

It may be noted that in many cases where the radius of deformation is not large, the activity of equatorial waves are prominently observed (e.g., Figs. 13c,d). It seems that, when the initial conditions are favorable for producing antisymmetric (say, negative vorticity to the north and positive vorticity to the south) equatorial disturbances, westerly acceleration occurs at the early stage of integration, while when they are favorable for symmetric disturbances, easterly acceleration occurs. Antisymmetric equatorial disturbances do not mix potential vorticity across the equator, which causes a westerly anomaly at the equator compared to the center of mixing latitudes. In contrast, symmetric equatorial disturbances tend to mix potential vorticity across the equator, which causes an easterly anomaly at the equator (Dunkerton and Scott 2008). The initial condition of the numerical runs presented in the previous subsections seems to be favorable for producing a westerly anomaly at the early stage of the time integration (Fig. 10). However, Rossby waves propagating from the off-equatorial latitudes continuously deposit easterly momentum in the equatorial region. Once easterly zonal mean flow is generated at the equator, the critical latitude absorption of waves seems to promote further development of easterly zonal mean flow at the equator. Equatorial easterly jets eventually appear for the cases \( \Omega \approx 50 \) in Fig. 10.

5. Concluding remarks

Decaying turbulence on a rotating sphere tends to produce a banded structure of zonal mean flows and a peculiar intense easterly jet, especially when the rotation rate of the system \( \Omega \) is large. This peculiar jet appears as a circumpolar jet when the radius of deformation is large, and as an equatorial jet when the radius of deformation is small. The latitudinal region where the intense easterly jet appears, polar or equatorial, may be determined by examining where the turbulent mixing is active and Rossby waves can be absorbed. When the radius of deformation is large, it is the polar region where \( \beta \) approaches zero, while when the radius of deformation is small, it is the equatorial area within the equatorial radius of deformation where kinetic energy can remain large.

It must be noted that there seems to be no distinguished intense easterly jet observed in the real atmospheres or the oceans. The strong south polar vortex of Saturn is westerly (Vasavada et al. 2006; Sánchez-Lavega et al. 2006), and so are the equatorial zonal mean flows of Saturn and Jupiter. Those of Uranus and Neptune are easterly; however, the zonal mean flows of Uranus and Neptune do not appear to be confined to the equatorial radius of deformation, as can be seen from Fig. 2 of Sukoriansky et al. (2002). The oceans of earth are found to have a banded structure (Galperin et al. 2004; Nakano and Hasumi 2005; Maximenko et al. 2005), and if they are caused by turbulent mixing, there may be a possibility of turbulence-driven easterly (westward) flow also at the equator. However, it will not be an easy task to isolate the contribution of turbulent mixing from those of direct thermal or wind forcing in the origin, for instance, of the equatorial intermediate current like that of Ollitrault et al. (2006). At the present, the benefit of investigating decaying turbulence on a rotating sphere is limited to understanding the behaviors of turbulent flows in general.

Although the setup of decaying turbulence is expected to provide a comprehensible model of turbulence on a rotating sphere, its behavior is still not very well understood. We have not clearly demonstrated the
reason for the limited tendency of an intense easterly circumpolar jet to appear in the forced nondivergent turbulence case. Generation, propagation, and/or dissipation of long Rossby waves must be different for decaying and forced turbulence. However, even in the case of decaying turbulence, we presented only some evidence for wave propagation, and have not given a sufficient description on the formation of a banded structure and generation of long Rossby waves. We have yet to understand, for instance, the possible magnitudes of long Rossby waves that propagate away and of the remaining eddies that contribute locally to the formation of a banded structure. These are issues associated with the spectral evolution of energy power near the \( k = 0 \) (or \( m = 0 \)) axis. Studies made from the viewpoint of wave mean-flow interaction like those of Shepherd (1987) and Ishioka et al. (2007) describe only the maintenance of zonal mean flows after they have appeared. Recently, Balk (2005) presented a mathematical description of the accumulation of energy near the \( k = 0 \) axis by using a new invariant quantity. However, it still seems difficult to envisage what is going on in the physical space from manipulation of the invariant.

As for the cases where the radius of deformation is small, there seems to be some resemblance between the forced case and the decaying case. The forced case also tends to produce easterly zonal mean flows at the equator when the radius of deformation is small (Scott and Polvani 2007). Coherent vortices appear in the mid- and high latitudes, and the magnitudes of zonal mean flows decrease with increasing latitude. It seems to be a fairly strong feature that the equator acts as an easterly-momentum absorber when the radius of deformation is small. Although the appearance of coherent anticyclonic vortices in the higher latitudes and the decreasing trend in the intensities of zonal mean flows with latitude may correspond to the features of the atmospheres of Jupiter and Saturn, as emphasized by Scott and Polvani (2007), no such correspondence can be found for the appearance of the easterly zonal mean flow at the equator having the width of the equatorial radius of deformation. As for the westerly flows around the equatorial areas of Jupiter and Saturn, angular-momentum redistribution due to thermal convection in a rotating spherical shell seems to be the mechanism receiving more interest in recent years (Busse 1983a,b; Christensen 2001; and review by Vasavada and Showman 2005). Heimpel et al. (2005) shows that, in addition to the westward flow around the equator, a banded structure of zonal flows caused by the \( \beta \) effect appears in the mid- and high latitudes inside of the tangent cylinder. Based on the Taylor–Proudman theory, the convective cells should be aligned in the direction of the rotation axis and can be approximately described as a two-dimensional flow on an equatorial plane perpendicular to the rotation axis. Now, a scenario of momentum transfer similar to those described in this paper is applicable also to the spherical case (Takehiro and Hayashi 1999; Yano et al. 2005). We have produced a two-dimensional flow on the equatorial plane with a potential vorticity gradient, but with the opposite sign. The convective cells propagate as Rossby waves on the equatorial plane from the inner area of their generation to the outer region. The momentum associated with these Rossby waves is westerly because of the sign of potential vorticity gradient. Consequently, westerly momentum is accumulated at the equatorial outer boundary. According to the weak nonlinear calculation of Takehiro and Hayashi (1999), the equatorial westerly appears unless parameters such as the Prandtl number and shell depth are chosen so that the thermal effect becomes dominant and the easterly acceleration of zonal mean meridional circulation overwhelms the westerly acceleration.

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