Large-Eddy Simulation of the Onset of the Sea Breeze

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ABSTRACT
This paper describes results from a large-eddy simulation (LES) model used in an idealized setting to simulate the onset of the sea breeze. As the LES is capable of simulating boundary layer–scale, three-dimensional turbulence along with the mesoscale sea-breeze circulation, a parameterization of the planetary boundary layer was unnecessary. The basic experimental design considers a rotating, uniformly stratified, resting atmosphere that is suddenly heated at the surface over the “land” half of the domain. To focus on the simplest nontrivial problem, the diurnal cycle, effects of moisture, interactions with large-scale winds, and coastline curvature were all neglected in this study. The assumption of a straight coastline allows the use of a rectangular computational domain that extends to 50 km on either side of the coast, but only 5 km along the coast, with 100-m grid intervals so that the small-scale turbulent convective eddies together with the mesoscale sea breeze may be accurately computed. Through dimensional analysis of the simulation results, the length and velocity scales characterizing the simulated sea breeze as functions of the externally specified parameters are identified.

1. Introduction

The land–sea breeze circulation is caused by the differential heating of the air mass over land with respect to that over sea (e.g., Dutton 1976, p. 375). During the daytime, because of their different heat capacities, solar radiation heats the land surface faster than it does the adjacent ocean surface, creating a horizontal temperature gradient in the air above, and consequently a horizontal pressure gradient. As a result, cool air from the ocean blows toward the land close to the surface, while warm air rising over land flows toward the ocean above the cool-air layer. The sea-breeze circulation is known to significantly influence the climate of coastal regions and to contribute to pollutant and energy transport, thus playing an important role in the air quality of coastal cities (Rotunno et al. 1996; Miller et al. 2003). For this reason it has been widely investigated in theoretical, numerical, and experimental studies.

In the literature there exists a large number of numerical studies of the sea breeze, both in real and idealized conditions (Simpson 1994; Miller et al. 2003). For the most part the idealized simulations have been two-dimensional (invariant in the along-coast direction) as they were intended to shed light on the idealized sea-breeze circulation described above. Of course, in reality the sea-breeze circulation is much more complex, as it can be modified by many processes such as the synoptic forcing, the shape of the coast line, complex coastal orography, the Coriolis force, turbulence, cloud cover, and others (for a recent review of observational work see Miller et al. 2003). In the present work, we take a small step toward greater realism by using a model capable of simulating three-dimensional turbulence to study the sea-breeze circulation. More specifically, we have used a model that can simulate the mesoscale sea-breeze circulation together with the small-scale...
turbulent convective boundary layer flow over the land.

The interaction between the sea breeze and turbulence is obviously very complicated to examine through an analytical approach because the nonlinear terms, which characterize the turbulent motion in the governing equations, cannot be neglected. Therefore, to achieve our purpose we need a model that is able to reproduce the required horizontal inhomogeneities in surface heat flux and drag coefficient as well as the turbulent eddies in the planetary boundary layer (PBL). For this reason we use the nonhydrostatic Weather Research and Forecasting (WRF) Model (Skamarock et al. 2005) in a large-eddy simulation (LES) configuration. By “LES configuration” we refer to three-dimensional simulations with high-enough resolution in the lowest few kilometers of the atmosphere so that there is no need for a PBL parameterization, since the turbulent motion of eddies, responsible of the growth of the PBL, is explicitly solved.

In the present study, we use the numerical experiments as a virtual laboratory to determine the dependence of the solutions on the external parameters (such as the surface heat flux and the Coriolis parameter). Our aim is to use this virtual laboratory to capture the essential physics of the sea breeze and thus determine some of its basic features, such as its horizontal and vertical scales as a function of the external parameters. Our basic experimental design considers a rotating, uniformly stratified, resting atmosphere that is suddenly heated at the surface over the “land” half of the domain, which is also characterized by a larger drag coefficient. To focus on the simplest nontrivial problem, the diurnal cycle, effects of moisture and interactions with large-scale winds were all neglected in this study. By means of dimensional analysis we were able to identify the length and velocity scales characterizing the horizontal and vertical motion as functions of the fundamental external parameters governing the dynamics of the sea breeze.

This paper is organized as follows. In section 2 we give an outline of the WRF model and how we use it as a LES model. In section 3 a detailed description of the sea-breeze simulations is provided. Section 4 is dedicated to the analysis of results through dimensional analysis and the determination of length and velocity scales. A summary and the conclusions are given in section 5.

2. The WRF model as an LES model

The numerical simulations presented in this paper were performed using the WRF model. The WRF model is constructed so that it could be used for both forecast and research purposes, and so can be run in either real or idealized situations (with real or ideal initial data). A detailed description of the basic equations and numerical scheme of the WRF model can be found in Skamarock et al. (2005). To summarize briefly, the WRF model solves the fully compressible, nonhydrostatic equations of motion in a terrain-following hydrostatic-pressure coordinate, using a Runge–Kutta second- or third-order time-integration scheme and a second- to sixth-order spatial-discretization scheme for the advection terms. The WRF model uses a time-splitting scheme, in which fast acoustic and gravity wave modes are computed with a small time step. In the present application, we have chosen a third-order Runge–Kutta scheme for the time integration, and the fifth-order advection scheme for the advection terms.

The WRF model can be used for a broad range of applications. In fact, without the constraint of the hydrostatic assumption, the model does not have any theoretical restriction on spatial resolution and can be used, for example, to investigate dry convection or moist convective clouds that can be explicitly computed. To simulate the turbulent convective heat transfer that drives the sea breeze, we ran the WRF model as an LES model so as to avoid any parameterization of the PBL. Without the PBL parameterizations, the spatial resolution has to be high enough to assure that turbulent eddies are explicitly computed down to scales small enough to accurately transport heat and momentum, and thus to create the PBL. In our simulations the spatial resolution used is ~100 m, which is in the inertial subrange for atmospheric convective boundary layers (e.g., Nieuwstadt et al. 1993) as required by the LES technique (Mason 1994). To represent the subgrid-scale motions, we use a well-known closure scheme in which a prognostic equation for the subgrid-scale turbulent kinetic energy (TKE) is solved.

To carry out the simulations to be described here, we needed to change the formulation of the eddy heat- and momentum-flux terms in the TKE equation so that they would be consistent with the formulation of surface heat and momentum flux in the prognostic equations for temperature and momentum, respectively. For the sake of illustration, we write here the TKE equation in the case of dry, plane, and horizontally homogeneous flow (i.e., all horizontal derivatives are neglected):

\[
\frac{\partial \theta}{\partial t} = -w\theta \frac{\partial U}{\partial z} - u'w' \frac{\partial V}{\partial z} + \frac{g}{\theta} \frac{\partial w'}{\partial \theta} - \frac{\partial (w'(e + p'/p))}{\partial z} - \epsilon, \tag{1}
\]
where the TKE is \( e = \frac{(u'^2 + v'^2 + w'^2)}{2} \); \( (u', v', w') \) is the departure of the velocity from its grid-volume average \((U, V, W)\); \( \theta' \) and \( p' \) are the departures of potential temperature and pressure, respectively, from their grid-volume averages; and \( e \) is the dissipation rate. The overbar signifies the grid-volume average.

To explain how we computed the heat flux at the surface in (1), we focus on the buoyancy-production term, \( B_c = (g/\theta)\overline{w}'\overline{\theta}' \). The turbulent temperature flux is parameterized by the flux-gradient closure

\[
B_c = -\frac{g}{\theta} K_H \frac{\partial \theta}{\partial z},
\]

where \( K_H \) is the eddy heat conductivity, and we have dropped the overbar for convenience in writing.

For the finite-difference form of (2), the WRF model uses a staggered Arakawa-C grid in which \( \theta \) and \( e \) are defined at the same vertical grid level and offset from the \( w \) level by one-half the vertical grid distance. With this system of staggering, \( w_1 \) denotes the vertical velocity at the surface (zero in the present case), while \( (\theta, e, K, u)_1 \) denotes the values of these quantities at \( z = \Delta z/2 \).

For the calculation of the buoyancy-production term (2), the vertical derivative of temperature is computed by means of a finite difference such that the buoyancy-flux term at the vertical level \( k \) is defined at the same \( k \) level as that of the temperature; that is,

\[
B_{ek} = -K_{HR} \frac{\theta_{k+1} - \theta_{k-1}}{2\Delta z}.
\]

At the lowest grid level \((k = 1)\), the buoyancy contribution to the TKE equation in the WRF code was originally computed as

\[
B_{e1} = -K_{HR} \frac{\theta_2 - \theta_5}{3/2\Delta z},
\]

where \( \theta_5 \) is the temperature extrapolated to the surface. Computed in this way the buoyancy flux \((4)\) at \( z = \Delta z/2 \) is not consistent with the given buoyancy flux at \( z = 0 \).

To make the TKE equation consistent with the temperature equation, we changed the computation of \( B_{e1} \) so that the buoyancy flux at the lowest level is the average of the prescribed surface buoyancy flux \( Q_S \), and the flux computed at the second level; that is,

\[
B_{e1} = \frac{-K_{HR} \frac{\theta_2 - \theta_1}{\Delta z} + Q_S}{2}.
\]

We now focus attention on the shear-production term \( S_c = -\overline{u'w'/(\partial U/\partial z)} \) [the other shear term \( -\overline{v'w'/(\partial V/\partial z)} \) is obviously treated in an analogous way]. The turbulent momentum flux is also parameterized by a flux-gradient closure so that the shear term becomes

\[
S_c = \frac{-(K_M \frac{\partial U}{\partial z})}{2}.
\]

To test the WRF model as an LES model we performed a simulation that reproduces a well-established result: the growth of the convective boundary layer (CBL) heated from the ground in horizontally homogeneous conditions. Because of the horizontal homogeneity (HH) we refer to this simulation as the HH-LES simulation and compare it with results from the four different LES codes presented in Nieuwstadt et al. (1993). The simulation was performed on a \( 5 \times 5 \times 5 \) km\(^3\) domain, covered by \( 50 \times 50 \times 50 \) grid points corresponding to a grid spacing of \( dx = dy = 100 \) m, and \( dz \) is approximately 100 m (with slight grid stretching in the vertical). The time step is 5 s. The initial potential temperature increases linearly with height and corresponds to the constant Brunt–Väisälä frequency \( N = 0.01 \) s\(^{-1}\), the surface heat flux is given by \( Q_S = 0.06 \) m K s\(^{-1}\), and the drag coefficient is \( C_d = 0.007 \). A random perturbation, with amplitude 0.1 K, is also added to the temperature field in order to speed the transition to turbulent convection. In accordance with the test problem presented in Nieuwstadt et al. (1993) there is no mean wind in the initial condition. The results from our simulation, shown in Fig. 1, are consistent with those of the model results shown in Nieuwstadt et al. (1993) and other past studies. Figure 1a shows a well-mixed layer growing with time, while Fig. 1b shows that the total heat flux decreases linearly with height and that most of

\[1\] From this point forward we refer to potential temperature as simply “temperature.”
it is carried by the resolved turbulent motions. The plan view of the vertical velocity near the lower surface in Fig. 1c shows the well-known cellular pattern (cf. Fig. 14 of Schmidt and Schumann 1989). In an independent study using the WRF model, Serafini (2004) carried out a more detailed comparison with the results reported in Nieuwstadt et al. (1993) and found the quantitative behavior of the WRF model generally consistent with that of the other models.

3. Sea-breeze simulation

a. Description of the basic problem

Now that we have verified that the chosen numerical resolution (~100 m)—and more generally the model setup—is able to reasonably reproduce CBL growth, we will describe changes to the model setup that allow the simulation of an idealized three-dimensional sea-breeze circulation.

As the typical sea-breeze cross-coast extent is of the order of 50–100 km, we increased one of the horizontal dimensions (x) of the computational domain to 100 km. Since we are considering a straight coastline, the only relevant length scale in the along-coast direction is that of the convective eddies (Fig. 1); hence we keep the other dimension (y) of the computational domain as it was in the test case described in the previous section.

Thus the resulting domain, sketched in Fig. 2, is a volume of dimension 5 km × 100 km in the horizontal and 4 km in the vertical direction. This domain is covered by a uniform grid of ~100 m in the vertical and horizontal directions, just as in the test case described in the previous section. However, one of the horizontal dimensions is increased to 100 km to allow the simulation of an idealized three-dimensional sea-breeze circulation.

As a sea-breeze flow resembles a gravity current, other relevant length scales may be that of the Kelvin–Helmholtz billows along the top of the sea-breeze front and that of the lobe and cleft structure along the sea-breeze front (e.g., Simpson 1994, 193–195). It is shown in the latter reference that the scale of these features is only a fraction of the gravity current depth and as such lie well within the 5-km domain chosen for the y direction.

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Fig. 1. (a) Potential temperature profiles from $t = 0$ to $t = 6$ h. (b) Total (solid line) and subgrid (dashed line) heat flux at $t = 6$ h. (c) Horizontal section of vertical velocity field $\mathbf{w}$ at the lowest interior grid level.

Fig. 2. Schematic diagram of the domain of the sea-breeze simulation. This domain is covered by a uniform grid of approximately (since stretching is used) 100 m in the vertical and 100 m in the horizontal direction.
above. As we assume an infinitely long straight coastline in the $y$ direction, we can retain periodic boundary condition in the $y$ direction but impose open conditions in the $x$ direction (Skamarock et al. 2005, p. 39).

The land and sea properties of the surface are distinguished by different values of the surface heat flux and drag coefficients; hence they are defined as functions of the distance from the coast. We choose a hyperbolic tangent function in order to make the surface heat flux and the drag coefficient practically constant over the land and sea parts of the domain but with a numerically resolved gradient at the coastline ($x = 0$); that is,

$$\text{Heat Flux}(x, y) = \frac{1}{2} Q_L^* \tanh(x/\lambda) + 1$$

$$\text{Drag Coefficient}(x, y) = \frac{1}{2} C_d \tanh(x/\lambda) + 1.$$  

In this formulation, the surface heat flux and drag are zero over the sea part of the domain, and assume the values $Q_L^*$ and $C_d$, respectively, over the land part. The length scale $\lambda = 100 \text{ m}$, and is not varied for the experiments reported here; with this choice, the effective transition between the asymptotic values $0.1Q_L^*$ and $0.9Q_L^*$ (for the surface heat flux, e.g.) is approximately 6 grid intervals.

The initial conditions for our experiments are a dry (no humidity) and calm (zero velocity) atmosphere and an initially stable stratification (characterized by the constant static stability $N^2$). Complex orography and time variation of the surface heat flux were also neglected, while the Coriolis effect [specified through the parameter $f (\text{s}^{-1})$] was taken into account. The model was integrated with a 5-s time step. We integrated the model for 6 h in order to focus on the onset of the sea-breeze flow and its interaction with the growing CBL over the land.

b. Basic structure of the “control” run solution

From the preceding description we see that the external parameters of the problem are therefore $Q_L^*$, $C_d$, $N$, $f$; we first show the results from the “control” case, which we define here as the case with $C_d = 0.007$, $N = \sqrt{2} \times 10^{-2} \text{ s}^{-1}$, $f = 10^{-4} \text{ s}^{-1}$, and $Q_L^* = 0.078 \text{ m K s}^{-1}$ (corresponding to case_e in Table 1). This “control” run is one of several carried out (see Table 1) that will be analyzed in the following.

Figure 3 shows a vertical section ($x-z$) of the con-
tours of $u$ component of the horizontal velocity field (i.e., the cross-coast component) superimposed on the potential temperature field at $t = 3, 6$ h (Figs. 3a,b) at an arbitrarily chosen $y$ location from the control run. In Figs. 3a,b we observe the sea-breeze circulation increasing in time in both spatial extent and intensity. Since in the present simulation both the turbulent motion and sea-breeze circulation are explicitly solved, the fluctuations in the velocity field representing the turbulent eddies characterizing the CBL are clearly in evidence over the land part of the domain not yet reached by the sea-breeze flow.

Thus, while the sea breeze is penetrating inland, the CBL is growing farther inland, as we would expect from the HH-LES simulation; it is worth observing that vertical increase of the sea breeze in time is closely related to the growth of the CBL height. This correspondence is in agreement with Simpson (1994, Fig. 11.2), who observed from the laboratory experiment of Mitsumoto et al. (1983) that the height of the returning flow approximately coincides with the CBL depth. Although idealized, the present simulated field of cross-coast velocity is similar both in its vertical structure and magnitude to that measured by Doppler radar (Simpson 1994, Fig. 10.9).

In Fig. 4 we present a contour plot of the vertical velocity field $w$ over a horizontal section ($x$-$y$) at $z = \Delta z$ (the first vertical grid level). Figures 4a and 4b correspond to $t = 3$ h and $t = 6$ h, respectively, as in the previous figures, but here we selected a region of the inland domain ($10$ km < $x$ < $40$ km) in order to focus on the turbulent motion. Figure 4a indicates that after 3 h of integration, the vertical velocity field over land exhibits the typical turbulent behavior of the CBL characterized by hexagonal convection cells shown in Fig. 1c. Comparing the convection cells at $t = 3$ h (Fig. 4a) and $t = 6$ h (Fig. 4b), we observe that the average cell size is also increasing in time as long as the CBL is growing.

Since the breeze originates as a stably stratified mass of air investing an unstable boundary layer constantly heated from below, there will be a transitional zone between the stable marine air and the convectively unstable air far inland in which the sea-breeze flow tends to stabilize the CBL and, at the same time, is itself destabilized by positive surface heat flux from the ground. In Fig. 4b we can observe that, for $x < 20$ km, the contours of the vertical velocity field have already lost their cellular pattern typical of the CBL, while for $x > 20$ km, although this zone is already invested by the sea breeze, we can still recognize the cellular pattern of the CBL, but with the cells appearing as bands. In their water tank simulation of sea breeze, Mitsumoto et al. (1983) also recognized these “longitudinal vortices” as a consequence of a colder fluid moving horizontally over a heated surface. Based on the literature concerning banded convection (e.g., Sykes and Henn 1989; Moeng and Sullivan 1994), we conjecture that the tendency to organize the convection into bands is through the action of the vertical wind shear associated with the sea breeze. Finally, we note in Fig. 4b the effect of the Coriolis force in curving these bands toward the right (facing the land), as would be expected in the Northern Hemisphere (i.e., for $f > 0$).

4. Analysis of the parameter dependence of the averaged fields

a. Along-coast averaged structure of temperature and velocity field

To focus on the mesoscale features of the sea-breeze circulation, we discuss here the results obtained by averaging the temperature and velocity fields in the along-coast ($y$) direction. Most of the turbulent motion will thus be filtered out.

In Figs. 5a,b we show the vertical section of the $y$-averaged potential temperature and $u$ component of velocity field at $t = 3$ and 6 h, respectively. As discussed above, by the end of the simulation, the sea breeze has invested most of the land part of the domain and has expanded in the vertical direction. The $u$ component of velocity increases in time and, because the Coriolis effect is included, the sea-breeze flow changes direction, resulting in an increasing alongcoast component of velocity ($v$). As the turbulent motion is mostly filtered out by the $y$ averaging, here we notice more clearly (with respect to Fig. 3) the correspondence between the increase of the sea-breeze depth and the growth of the CBL.

In Fig. 6 we show the $y$-averaged $v$ component at $t = 6$ h, which illustrates that a circulation is developing in
the vertical plane parallel to the coastline. Thus the overall result is that the vertical plane containing the main circulation rotates counterclockwise (if $f > 0$) because of the Coriolis force (see Fig. 10.12 of Dutton 1976). As a consequence, it is reasonable to expect this veering of the wind will subtract energy from the main circulation in the vertical plane orthogonal to the coastline.

In our control experiment we observe a sea-breeze circulation in the vertical plane expanding in the horizontal and vertical directions, increasing in velocity intensity and veering in time. Next we ask, which are the external parameters mainly controlling these behaviors and how does the solution depend on these parameters? To answer these questions, we performed a series of simulations varying the set of input parameters with respect to those of the “control” run with the aim of using the results to perform a dimensional analysis of the problem. The parameters of the performed simulations are summarized in Tables 1 and 2. Basically, we performed one set of simulations (Table 1) varying the surface heat flux and the initial stability, and a second set of simulations (Table 2) similar to the first one but with the Coriolis force set to zero.

b. Dimensional analysis

Dimensional analysis is based on Buckingham’s Pi theorem (e.g., Birkhoff 1960, chapter IV), which states that an experimental outcome involving $n$ input dimensional parameters and $m$ independent dimensions can be expressed in terms of $n - m$ dimensionless groups.

We have assumed that the only relevant external parameters affecting the solution are the surface heat flux $(w'\theta')_0$, the parameter $g/\theta_0$, the initial static stability parameter $N$, the Coriolis parameter $f$, and the time $t$ since heating started. With these choices we have implicitly assumed that, for instance, the finite horizontal scale of the transition between land and sea needed for the numerical simulations is not a crucial parameter (in principle the step change in surface conditions at the coast has no intrinsic scale); through numerical experimentation we have also observed that $C_d$ does not exert a very strong influence on the solution features of interest here, and thus we have also chosen not to vary the drag coefficient $C_d$ over land in the present experiments.

We begin our analysis by seeking the vertical scale of

![Fig. 5](image-url) Same as in Fig. 3, but for the $y$-averaged potential temperature field and cross-coast velocity field for case_e.

![Fig. 6](image-url) A vertical cross section of the $y$-averaged $u$ component (along-coast) of the velocity field (contour interval $= 1 \text{ m s}^{-1}$) for case_e at $t = 6 \text{ h}$. Contours in solid lines represent positive values of the along-coast component of velocity, while dashed lines refer to negative values.
the sea breeze, $Z$. Given the $n = 6$ dimensional variables $Z, (w')_0, g/\theta_0, N, f, t$, and the $m = 3$ fundamental dimensions $T$ (time), $\Theta$ (temperature), and $L$ (length) we can build the 3 dimensionless groups:

$$G_1 = Z \sqrt{N^3/Q}, \quad G_2 = Nt, \quad G_3 = \frac{f}{N},$$

where $Q = (g/\theta_0)(w')_0$. Hence dimensional analysis indicates that

$$Z \sqrt{\frac{N^3}{Q}} = G\left(\frac{Nt}{f/N}\right).$$

To find the function $G(Nt, fN^{-1})$ that best describes the data, we choose $Z$ as the height at which the cross-coast component of the $y$-averaged horizontal velocity goes to zero (see, e.g., Fig. 5) at the coastline and calculated it from our first set of simulations (Table 1). Note that in this first set of simulations, by varying the initial stability $N^3$ we are implicitly varying the dimensionless quantity $f/N = G_3$. In Fig. 7, we plot $Z \sqrt{(N^3/Q)}$ versus $Nt$ on a log–log diagram, for all the simulations presented in Table 1. The data indicate no systematic variation with $fN^{-1}$, hence we deduce that $G(Nt, fN^{-1}) = F(Nt)$ in (11). In Fig. 7 we also show the curve obtained by a least squares fit of the data to a simple power law $f(x) = ax^p$ (where $x = Nt$). The best fit yields $b \approx 0.5$; that is,

$$Z \sqrt{\frac{N^3}{Q}} = F(Nt) \propto \sqrt{Nt},$$

from which we can finally deduce that the vertical scale of the sea-breeze circulation is

$$Z \propto \frac{\sqrt{Qt}}{N}.$$  

The dependence (13) on the given parameters $N$, $Q$, and $t$ is the same as that expected for the CBL height. Assuming an idealized boundary layer heated at the ground, the height of the growing CBL can be estimated from the argument that the time change of vertically integrated buoyancy [defined as $b = (g/\theta_0)\theta$] equals the input at the ground (i.e., the surface heat flux); that is,

$$\int_0^d [(b_S + N^2d) - (b_S + N^2z)] dz = Qt,$$

where $b_s = (g/\theta_0)\theta_S$ is the buoyancy at the surface, and $b_s = b_s + N^2z$ is simply a rescaled version of the potential temperature profile, $\theta = \theta_S + (\partial \theta/\partial z).z$. This argument, widely used in the literature dealing with the classic problem of the growth of the mixed layer (Ball 1960; Stull 1988; Sullivan et al. 1998; Lilly 2002), gives that the mixed-layer depth $d = \sqrt{2Qt/N}$. Thus, this classical result, together with the dimensional analysis leading to the expression (13) for $Z$, confirms our previous observation that the sea-breeze vertical extent and the growth of the convective boundary layer are closely related, and more precisely they have the same dependence on the external parameters $Q$, $N$, and $t$.

We now use the dimensional analysis to find the cross-coast horizontal velocity scale $U$. In this case the dimensional variables are $U$, $(w')_0$, $g/\theta_0$, $N$, $f$, and $t$.
The dimensionless groups deriving from the combination of these variables are

\[ G_1 = \frac{U}{\sqrt{Q/N}}, \quad G_2 = Nt, \quad G_3 = ft. \]  

(15)

As for the vertical scale, we now look for the universal function that provides the relationship between the velocity scale and the external parameters, namely,

\[ \frac{U}{\sqrt{Q/N}} = H(Nt, \frac{f}{N}). \]  

(16)

First we analyze the simple case in which the Coriolis force is neglected (\( f = 0 \)). In this case, (16) reduces to

\[ \frac{U}{\sqrt{Q/N}} = H_0(Nt). \]  

(17)

To evaluate the function \( H_0 \), we performed a second set of simulations in which the Coriolis parameter \( f \) is set to zero. These simulations, described in Table 2, are analogous to those of the first set, except that the Coriolis parameter \( f \) is set to zero. (In our naming convention, “case_a_f0,” e.g., has the same values of \( Q \) and \( N \) as in case_a but \( f = 0 \).)

We take the maximum value in the vertical of the cross-coast component of the \( y \)-averaged horizontal velocity at the coast (\( x = 0 \)) as a representative velocity scale. In Fig. 8a we first show the dimensional velocity \( U \) as function of \( t \) on a log–log diagram; this figure shows how the rough data (i.e., without any scaling) exhibit a dependence on \( Q \) and \( N \) that varies from case to case. Figure 8b shows the dimensionless velocity \( U/\sqrt{Q/N} \) as a function of the dimensionless time \( Nt \) on a log–log diagram. The data now collapse onto a nearly straight line corresponding to a power-law function of \( Nt \) with exponent 0.6,

\[ \frac{U}{\sqrt{Q/N}} = H_0(Nt) \propto (Nt)^{0.6}. \]  

(18)

In cases relevant to midlatitudes, the Coriolis parameter \( f \) is not zero, thus we would like to determine the function \( H \), which provides the functional form of the dependence on both \( Nt \) and \( f/N \). The function \( h \) has to generalize the previous case and thus has to reduce to \( H_0 \) in the case where \( f = 0 \). Based on the control run and past experience we expect that the effect of the Coriolis force, after a few hours, is to transfer energy from the \( u \) component to the \( v \) component of the horizontal wind (Figs. 5 and 6), and thus to have a slower growth in time of the \( u \) component of velocity with respect to the case with \( f = 0 \). To make further progress we recognize that there are many possible forms of the function \( H \) satisfying this behavior, and so in order to estimate an appropriate scale for \( u \) we use the momentum equation in the \( x \) direction at \( z = 0 \):

\[ \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} - f\bar{u} = - \frac{\partial \bar{\phi}}{\partial x}, \]  

(19)

where we have neglected the friction term \( \partial \tau^x/\partial z \), which is reasonable in the sea part of the domain and where the overtilde represents the along-coast average.

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3 Case_c was excluded from our analysis as it seems to behave as an “outlier” with respect to the other simulations; this behavior is most likely because case_c had the smallest surface heat flux \( Q \) and the highest initial stability \( N \), which factors inhibited a very well-developed turbulence for the grid resolution used here.
Denoting scales by capital letter, a scale analysis of (19) gives
\[ \frac{U^2}{L} - fV \sim \frac{\Delta \Phi}{L}, \tag{20} \]
where \( L \) is the horizontal length scale and we have assumed that the relevant time scale \( T \) is \( \frac{U}{L} \).

For the general case \( f \neq 0 \), we need estimates for \( V \) and \( L \). To estimate \( V \), we use the momentum equation in the \( y \) direction at \( z = 0 \):
\[ \frac{\partial \tilde{u}}{\partial t} + \tilde{u} \frac{\partial \tilde{u}}{\partial x} + f\tilde{u} = 0. \tag{21} \]
Scale analysis of (21) (again using \( U/L \) as the time scale) gives immediately \( V \sim fL \); substituting the latter into (20) then yields
\[ U^2 + f^2 L^2 \sim -\Delta \Phi. \tag{22} \]
The simulations suggest further that \( L \sim U t \), or \( L = \alpha U t \) (\( \alpha \) is a constant of proportionality); substituting this into (22) gives finally
\[ U \sim \sqrt{-\frac{\Delta \Phi}{1 + (\alpha ft)^2}}. \tag{23} \]

Recalling that we are looking for the universal function \( H(Nt, f/N) \), and using the relationship (23), we expect the following relation between the dimensionless groups:
\[ \frac{U}{\sqrt{Q/N}} \sim \frac{(Nt)^{0.6}}{\sqrt{1 + \left( \frac{f}{N} \right)^2}}, \tag{24} \]
where the parameter \( \alpha \) is to be determined. Note that we have imposed that expression (24) in the limiting case \( f = 0 \) coincides with the relationship (18) [which implies that \( \sqrt{-\Delta \Phi} = (Nt)^{0.6} \)]

We use the data from our set of experiments and a best-fit analysis in order to verify this relationship and to determine the parameter \( \alpha \). In Figs. 9a,b we show \( u/U \) as a function of \( Nt \) for all cases; the experiments are divided in two groups characterized by the same value of the parameter \( f/N \): cases a, b, and \( f/N = 10^{-2} \) in Fig. 9a and cases d and e \( (f/N = 10^{-2}/\sqrt{2}) \) in Fig. 9b. The solid lines superimposed results from the best fit performed using the function
\[ S(x) = \beta \frac{x^{0.6}}{\sqrt{1 + \left( \frac{f}{N} \right)^2}}, \tag{25} \]
where \( x = Nt \). The parameters \( \alpha \) and \( \beta \) derived from best fit in both cases are compatible with the values \( \alpha = 0.37 \) and \( \beta = 0.32 \).

Finally we investigate the potential temperature field. The external parameters in this case are \( \Theta \), \( (w'\theta)'_0 \), \( g/\theta_0 \), \( N \), and \( t \) (not considering the Coriolis effect), from which the following dimensionless groups may be derived:
\[ G_1 = \frac{g}{\theta_0 N (w'\theta)'_0}, \quad G_2 = Nt. \tag{26} \]
Hence it must follow that
\[ \frac{g}{\theta_0} = \frac{\Theta}{\sqrt{NQ}} = R(Nt). \tag{27} \]
Analysis of the experiments (not shown) indicates that \( R(x) = x^{0.5} \), and that there is no variation with Coriolis effect; hence the temperature scale is
We note that this temperature scale is precisely that which comes from the simple CBL model embodied in (14): the temperature of the mixed layer is $\theta = \theta_s + (\theta_0/g)N^2d$; with $d = \sqrt{QtN}$, (28) follows.

At this point we have found velocity and potential temperature scales and also vertical and horizontal scales, and we check our scaling by plotting the nondimensional vertical and horizontal profile of velocity and temperature of all the cases. In Fig. 10a we show the nondimensional velocity $u/U$, and in Fig. 10b we show the nondimensional potential temperature $\theta/\Theta$ versus the nondimensional height $z/Z$ for all the cases described in Table 1. The collapse of the curves corresponding to the different cases is evidence that this scaling captures the dependence on the external parameters we have varied. In Fig. 10c we show the nondimensional velocity $u/U$, and in Fig. 10d we show the nondimensional potential temperature $\theta/\Theta$ versus the nondimensional horizontal distance from the coast $x/L$.

These figures indicate that there might be a slight dependence on some other parameters, and we speculate that, given that there are no other external parameters, this could be an effect of some implicit numerical-modeling parameters.

c. Discussion

Pearson (1973) carried out two-dimensional (no alongshore variation) simulations of the sea breeze driven by a prescribed heating profile (i.e., depth scale given in addition to the surface heat flux) over the land. He found that the velocity scale of the sea breeze was $\sim \sqrt{D}$ where $D$ is the time-integrated surface heat flux (in this study $D = Qt$). Using a CBL model similar to that embodied in (14), Richiardone and Pearson (1983) developed an analysis of the availability of potential energy (from the CBL) to be converted to the kinetic energy of the mesoscale sea breeze. They applied that analysis to two-dimensional numerical experiments varying the entrainment factor in the CBL parameterization, the heat input over the land, the Coriolis pa-
parameter and the initial Brunt–Väisälä frequency. Richiardone and Pearson (1983) found that the efficiency of
conversion of the CBL potential energy into sea-breeze kinetic energy is $A_f(t)/A_0(t)$ where $A_f(t)$ contains the
dependence on the Coriolis parameter $[A_f(t) = \text{const.}; A_f(0) \text{ decreases with } t]$.

As in Pearson (1973) and Richiardone and Pearson (1983), the present study considers the surface heat flux
and Coriolis parameters as external input parameters. In this study, however, we deduce length, velocity, and
temperature scales from the LES data. In comparison with the velocity scale $\sqrt{\mathcal{Q}_t}$ found by Pearson (1973),
our analysis (18) gives $\sqrt{\mathcal{Q}_t(N_t)^{0.1}}$, which indicates a weak dependence on $N$.$^4$ Our analysis of the vertical
scale of the sea breeze (13) is consistent with the CBL model used in Richiardone and Pearson [1983, their Eq.
(1)]. Finally we note the qualitative agreement of $A_f(t)$ (Fig. 8 of Richiardone and Pearson 1983) with both (25)
and the results shown in Fig. 9.

A number of different scaling laws were developed using data from both observations and mesoscale nu-
(1987) by eliminating the eddy diffusivities in favor of the sensible heat and momentum fluxes as external pa-
rameters. Steyn (1998) further developed velocity and depth scales of the sea breeze based on data from a
single location. In a recent paper Steyn (2003) revisits his previous set of scalings by analyzing data from three
different locations in order to examine the effect of latitude and by using the time-integrated (instead of instan-
taneous) heat flux as proposed by Tijm et al. (1999). In the present study we have examined the sim-
plest time dependence for the surface heating ($Q = \text{const. for } t > 0$) and so the diurnal frequency, used as
an external parameter by Steyn (1998, 2003) and Niino (1987), does not enter in our analysis. In agreement
with Steyn (2003), Tijm et al. (1999), Richiardone and Pearson (1983), and Pearson (1973) we find that the
time-integrated heat flux ($\mathcal{Q}_t$), initial static stability $N$, and the Coriolis parameter $f$ are the important external
parameters.$^5$

5. Summary and conclusions

In this paper we report on numerical simulations of the sea breeze using the large-eddy simulation (LES)
technique, which explicitly computes the turbulent convective eddies of the convective boundary layer over
land as well as the mesoscale sea-breeze circulation.

$^4$ If the temperature over the sea is considered to be undis-
turbed from its initial state, then the difference in hydrostatic
pressure at the ground between land and sea would be given by
(15), that is, $\Delta \Phi = Q_t$; the scale-analyzed momentum Eq. (21) with $f = 0$ indicates therefore that $U = \sqrt{\mathcal{Q}_t}$, as found by Pearson
(1973). We speculate that the weak additional dependence on $N$ is
related to the presence of internal waves that may change the
temperature profile with time over the sea.

$^5$ In Steyn (1998, 2003) the land–sea temperature contrast is also
specified as an external parameter. Within the context of the
present model with prescribed heat flux, the temperature contrast
between land and sea emerges from the calculation and is not an
independent external parameter.
The focus of our simulations is what is arguably a canonical problem of sea-breeze research: starting from a rest state characterized by a constant stability $N$ on an $f$ plane a constant heat flux $Q$ over land is switched on at $t = 0$. Figure 11 illustrates the major features of the simulated sea-breeze circulation, which are the mesoscale sea-breeze circulation in the vertical plane, its southward turning as seen in the horizontal plane, and its transition to convective boundary layer turbulence over land.

We conducted a number of experiments varying $Q$, $N$, and $f$ in this problem and have analyzed the results to arrive at the length, velocity, and temperature scales of the developing sea breeze, which scales we summarize here. The vertical length scale is

$$Z = \frac{\sqrt{Q t}}{N},$$

which corresponds to the classic result for the depth of the well-mixed layer; the horizontal velocity scale is

$$U = 0.32 \sqrt{\frac{Q t (N t)^{0.1}}{1 + \left(0.37 \frac{f}{N} N t\right)^2}},$$

which takes into account the turning of the wind away from the cross-coast plane by the Coriolis effect; the horizontal length scale is simply $L = Ut$; the scale of the departure of the temperature from its initial value is

$$\Theta = \frac{\theta_0}{g} \sqrt{2Q t N},$$

which is also consistent with classic CBL theory.

Although limited in scope, these results are purely deductive from the analysis of the LES experiments and provide definitive answers concerning what we think should be considered a canonical problem in sea-breeze studies. The analysis performed here suggests how to proceed systematically with design and analysis of LES experiments on the sea breeze to study more complex and realistic situations concerning, for example, nonzero initial mixed-layer depth and wind, diurnal heating, and the effects of humidity.

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