NOTES AND CORRESPONDENCE

Virtualization

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ABSTRACT

The virtual temperature of a moist air parcel is defined as the temperature of a dry air parcel having the same mass, volume, and pressure. It is shown here that a virtual air parcel can be formed diabatically by warming the parcel to its virtual temperature while replacing its water vapor with the equivalent mass of dry air under isobaric, isochoric conditions. Conversely a saturated virtual air parcel can be formed diabatically by cooling the parcel to its saturated virtual temperature while replacing some of its dry air with the equivalent mass of water vapor under isobaric, isochoric conditions. These processes of virtualization can be represented on a vapor pressure–temperature diagram. This diagram facilitates the comparison of the relative density of two moist air parcels at the same pressure. The effects of liquid and/or solid water can also be included.

1. Introduction

The *Glossary of Meteorology* (Glickman 2000) defines the virtual temperature of a moist air parcel as the temperature of a dry air parcel having the same pressure and density. Because the virtual temperature is useful in the calculation of the thickness of a pressure layer and in the determination of the buoyancy of an air parcel, it is necessary that the dry air parcel have the same volume of the moist air parcel that it represents. The *Federal Meteorological Handbook No. 3* (OFCM 1997) makes this issue explicit in its definition of virtual potential as the "temperature that a volume of dry air must have in order to have the same density as an equal volume of moist air at the same pressure." This definition is adopted here. Although the virtual temperature is often considered a fictitious temperature, it is the purpose of this note to demonstrate that it is possible to create a virtual air parcel by diabatically warming the moist air parcel to its virtual temperature while replacing its water vapor with an equivalent mass of dry air under isobaric, isochoric conditions. The creation of a virtual air parcel may be considered a process, defined here as virtualization, and as such may be displayed as lines on a vapor pressure–temperature diagram. Thus, the virtual temperature may be depicted graphically in a manner similar to the dewpoint, wet-bulb, and equivalent temperatures.

Section 2 presents a derivation of the virtual temperature. Section 3 interprets the definition as the process of virtualization. Following Betts (1982) saturated as well as dry virtual temperatures are defined. Section 4 discusses the effect of liquid and solid water in the parcel. Section 5 offers some conclusions and a virtual graphic calculator.

2. Virtual temperature

Consider a parcel of moist air with total pressure p, volume V, and temperature T. The total number of gas molecules in the parcel, N, is the sum of the number of dry air, N_d, and water vapor, N_v, molecules

$$N = N_d + N_v. \tag{1}$$

The equation of state for the moist parcel is

$$pV = NkT, \tag{2}$$

where k is the Boltzmann constant. The equation of state for the virtual air parcel of temperature T_v and

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N_{virt} molecules of dry air is, by the isochoric and isobaric constraints,

$$pV = N_{\text{virt}} k T_v. \tag{3}$$

The virtualization process requires that the mass of water vapor be replaced with dry air, so

$$m_d N_{\text{virt}} = m_d N_d + m_v N_v, \tag{4}$$

where m_d and m_v are the molecular masses of dry air and water vapor. Dividing by m_d, one finds

$$N_{\text{virt}} = N_d + \varepsilon N_v, \tag{5}$$

where $\varepsilon = m_v/m_d = 0.622$. The constraint that the process be isochoric ensures that the density of the moist parcel and the virtual parcel are the same. The isobaric and isochoric constraints imply, from (2) and (3),

$$N_{\text{virt}} T_v = NT. \tag{6}$$

Then from (1) and (5),

$$T_v = \left(\frac{N_d + N_v}{N_d + \varepsilon N_v}\right) T, \tag{7}$$

or, last,

$$T_v = \left(\frac{1 + r_v/\varepsilon}{1 + r_v}\right) T, \tag{8}$$

where $r_v = \varepsilon N_v / N_d$ is the mixing ratio of the moist air parcel. The result (8) is the standard mathematical definition of the virtual temperature (Glickman 2000) and may be accurately approximated by

$$T_v = (1 + 0.608 r_v) T. \tag{9}$$

Approximations of similar magnitude are used below.

3. Virtualization

To interpret the creation of the virtual parcel as a process, the result (9) may be written in terms of the vapor pressure e and pressure p using $r_v \cong \varepsilon e/p$. Then one finds

$$e = \frac{p}{0.608\varepsilon}\left(\frac{T_v - T}{T}\right) \simeq \frac{p}{0.608\varepsilon}\left(\frac{T_v - T}{T_v}\right). \tag{10}$$

The virtualization process takes the parcel from temperature T and vapor pressure e to the temperature T_v and zero vapor pressure. During the process, intermediate values of the vapor pressure and temperature, denoted by primes, are described by

$$e' - e = \frac{p}{0.608\varepsilon}\left(\frac{T - T'}{T_v}\right). \tag{11}$$

In analogy with the duality between the wet-bulb and equivalent temperatures, one can define a saturated virtual temperature, T_{sv}, as the temperature of a saturated parcel having the same mass, volume, and pressure of a moist air parcel. A saturated virtual parcel can be formed diabatically by cooling the parcel to its saturated virtual temperature while replacing some of its dry air with the equivalent mass of water vapor under isobaric, isochoric conditions. From (11), the saturated virtual temperature is defined as

$$e_{\text{sat}}(T_{sv}) - e = \frac{p}{0.608\varepsilon}\left(\frac{T - T_{sv}}{T_v}\right), \tag{12}$$

where the subscript "sat" denotes saturation. Betts and Bartlo (1991) call the saturation virtual temperature the density temperature, but the *Glossary of Meteorology* (Glickman 2000) defines the later appellation as being equivalent to the virtual temperature (8).

Figure 1 presents a vapor pressure versus temperature diagram. It represents a synthesis of Figs. VII-5 and -6 of Iribarne and Godson (1981) with an extension to the process of virtualization. The sloping concave curve is the saturation vapor pressure with respect to liquid water. The dashed vertical lines are isotherms; the dashed horizontal lines are isobars of vapor pressure. The three straight solid lines describe distinct thermodynamic processes. The solid horizontal line represents the process of isobaric cooling of an air parcel of temperature T with vapor pressure e to its dewpoint temperature, T_d. The tilted solid line represents the isobaric moistening of an air parcel to its wet-bulb temperature, T_w, or, conversely, the isobaric drying of

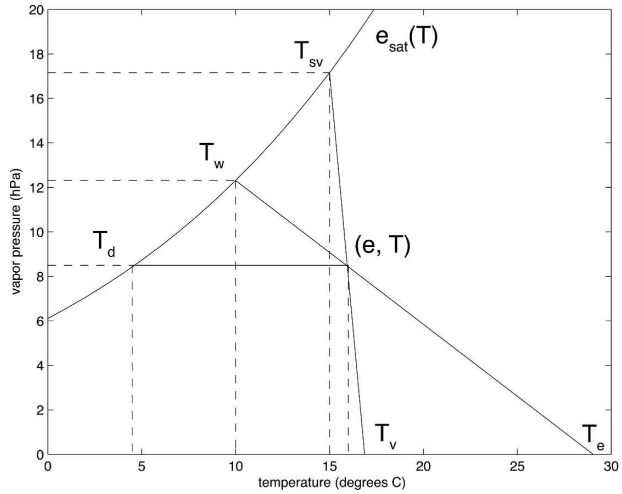

FIG. 1. A vapor pressure vs temperature diagram for $p = 1000$ hPa indicating the relationship between the virtual and saturated virtual temperatures, T_v and T_{sv}, between the wet-bulb and equivalent temperatures T_w and T_e, and between the temperature and dewpoint temperature T and T_d for an unsaturated air parcel at the temperature T and vapor pressure e.

the parcel to its (isobaric) equivalent temperature, T_e. The nearly vertical solid line represents the virtualization process that either dries the parcel to its virtual temperature or moistens it to its saturation virtual temperature. The isopleth of virtual temperature, deemed an isovirtual, is accurately described by the approximate Eq. (10). The process of virtualization takes a moist parcel originally with vapor pressure e and temperature T along an isovirtual to zero vapor pressure and the temperature T_v. It is readily seen that the inequality

$$T_d < T_w < T_{sv} < T < T_v < T_e \tag{13}$$

holds for unsaturated air parcels with temperature T. It is noted that a virtual temperature may also be assigned to parcels that are supersaturated.

4. Effect of liquid or solid water

It is straightforward to extend the analysis of section 2 to include liquid and/or solid water (henceforth, liquid water). In that case, the mass conservation relation (5) is replaced by

$$N_{virt} = N_d + \varepsilon(N_v + N_l), \tag{14}$$

where N_l is the number of liquid water molecules in the moist parcel. The relation (6) still holds, but (8) is replaced with the familiar result

$$T_{wv} = \left(\frac{1 + r_v/\varepsilon}{1 + r_v + r_l}\right)T, \tag{15}$$

where r_l is the liquid water mixing ratio. Here (15) is the wet virtual temperature in contrast to the (dry) virtual temperature (8). [Emanuel (1994) calls (15) the density temperature.] Again the result (15) is the standard mathematical definition of the virtual temperature (Glickman 2000) and may be accurately approximated by

$$T_{wv} = (1 + 0.608r_v - r_l)T. \tag{16}$$

The effect of the liquid water is to increase the density of the parcel and decrease its virtual temperature. It is important to note that neither (15) or (16) requires that the vapor mixing ratio be saturated.

The depiction of this process on a thermodynamic diagram is facilitated by defining a pseudo vapor pressure, e_p, for the liquid water

$$e_p \equiv pr_l/(0.608\varepsilon). \tag{17}$$

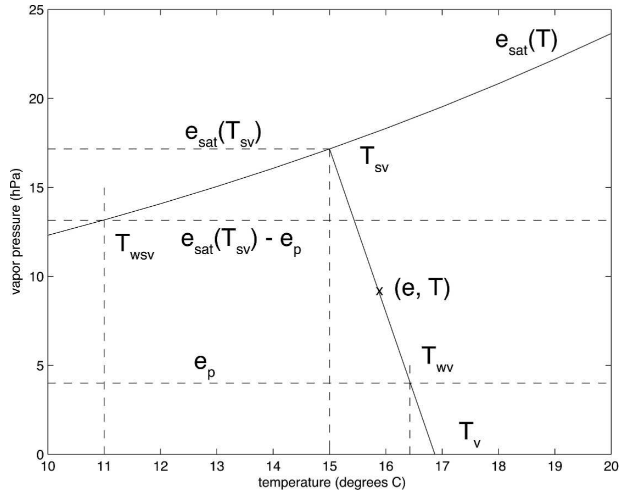

FIG. 2. A vapor pressure vs temperature diagram for $p = 1000$ hPa indicating the relationship among the virtual, saturated virtual, wet virtual, and wet saturated temperatures, T_v, T_{sv}, T_{wv}, and T_{wsv}, for an unsaturated air parcel at the temperature T, vapor pressure e, and pseudo vapor pressure e_p, and for a saturated air parcel at the temperature T_{sv}, vapor pressure $e_{sat}(T_{sv})$, and pseudo vapor pressure e_p.

Then, in place of (10) one has

$$e - e_p = \frac{p}{0.608\varepsilon}\left(\frac{T_{wv} - T}{T}\right) \simeq \frac{p}{0.608\varepsilon}\left(\frac{T_{wv} - T}{T_v}\right). \tag{18}$$

Here, the virtualization process takes the parcel from temperature T and effective vapor pressure $e - e_p$ to the temperature T_{wv} with zero effective vapor pressure. Then, the wet virtual temperature of a parcel can be found graphically (see Fig. 2) by the intersection of the isovirtual with the isobar e_p. This diagram is for the case that $e_p < e_{sat}(T_{sv})$. If $e > e_p$, then $T_v > T_{wv} > T$ and the effect of the water vapor dominates the wet virtual temperature. Conversely, if $e < e_p$, then $T_{wv} < T$ and the effect of the liquid water dominates. If $e_p > e_{sat}(T_{sv})$ (not shown), then $T_{wv} < T_{sv} < T$ and again the effect of the liquid water dominates. In this latter case, the wet virtual parcel is supersaturated.

Lastly, a wet saturated virtual temperature, T_{wsv}, may be defined analogous to (12). It is the temperature of a saturated parcel with no liquid water having the same mass, volume, and pressure of a saturated air parcel with liquid water. A wet saturated virtual parcel can be formed diabatically by cooling the parcel to its wet saturated virtual temperature while replacing some of its water with the equivalent mass of dry air under isobaric, isochoric conditions. Assume that the parcel of interest is saturated at the temperature T with pseudo vapor pressure e_p and seek to describe a virtual parcel that is saturated at the temperature T_{wsv} with N_d^* molecules of dry air and N_v^* molecules of water vapor but with no liquid water. Then, the mass conservation constraint implies

$$N_d^* + \varepsilon N_v^* = N_d + \varepsilon(N_v + N_l), \tag{19}$$

but the isobaric and isochoric constraints imply

$$(N_d + N_v)T = (N_d^* + N_v^*)T_{wsv}. \tag{20}$$

Proceeding in a similar manner to the derivations of (8) and (15), one finds

$$\left(\frac{1 + r_v^*/\varepsilon}{1 + r_v^*}\right)T_{wsv} = \left(\frac{1 + r_v/\varepsilon}{1 + r_v + r_l}\right)T, \tag{21}$$

or, approximately,

$$(1 + 0.608r_v^*)T_{wsv} = (1 + 0.608r_v - r_l)T, \tag{22}$$

where $r_v^* = \varepsilon N_v^*/N_d^*$. Rewriting the mixing ratios in terms of vapor pressures yields, approximately,

$$e_{sat}(T) - e_p - e_{sat}(T_{wsv}) = \frac{p}{0.608\varepsilon}\left(\frac{T_{wsv} - T}{T_v}\right). \tag{23}$$

This expression is the extension of (12) to saturated parcels with liquid water. Then, the wet saturated virtual temperature of a parcel can be found graphically (see Fig. 2) by the intersection of the saturation vapor pressure curve with the isobar of the effective vapor pressure $e_{sat}(T) - e_p$. Here, the virtualization process takes the parcel along the saturation vapor curve from temperature $T = T_{sv}$ to T_{wsv}. It is readily seen that the inequality

$$T_{wsv} < T_{sv} < T_{wv} < T_v, \tag{24}$$

holds for saturated air parcels with temperature T_{sv} and liquid water such that $e_p < e_{sat}(T_{sv})$. If the effective vapor pressure is negative [i.e., $e_p > e_{sat}(T_{sv})$], then a wet saturated virtual temperature cannot be assigned to the parcel.

5. Conclusions

This note advances the concept of virtualization as an open process that takes an air parcel diabatically to alternative thermodynamic states of differing moisture contents but always with the same pressure, volume, and mass. The process can be represented graphically on a vapor pressure–temperature diagram by drawing isopleths of virtual temperature, named isovirtuals. [See Betts and Bartlo (1991) for the depiction of saturated and wet saturated virtual temperatures on a tephigram.]

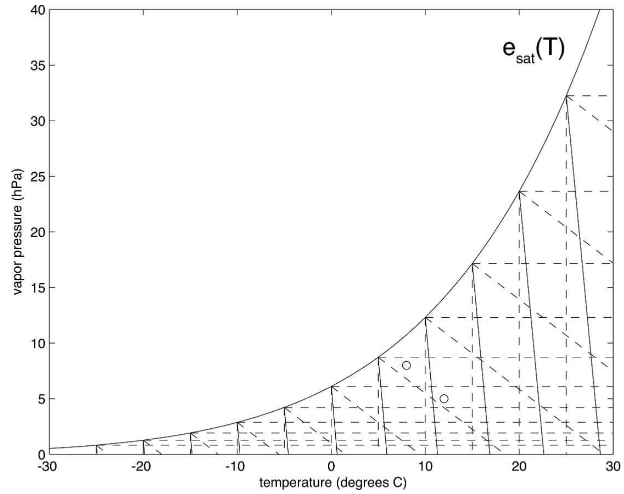

FIG. 3. A vapor pressure vs temperature diagram for $p = 1000$ hPa suitable as a graphical calculator of virtual, wet-bulb, equivalent, and dewpoint temperatures. The two circles denote two parcels with different temperatures and vapor pressures. The warmer, drier parcel is less dense than the cooler, moister parcel because its virtual temperature is larger.

Figure 3 presents an example of a virtual calculator. This vapor pressure–temperature diagram plots the saturation vapor pressure with respect to liquid water as a solid concave curve and contains dashed isobars and isotherms and solid isovirtuals. The tilted dashed lines represent the adiabatic isobaric process associated with the wet-bulb and equivalent temperatures. Based on the preceding discussion, it is straightforward to calculate the various virtual temperatures as well as the dewpoint, wet-bulb, and equivalent temperatures from Fig. 3 for a particular parcel of interest. It is important to note that, although the chart is valid for a pressure $p_{00} = 1000$ hPa, the virtual temperatures at other pressures p may be calculated using the relation

$$T_v(p) = T + \frac{p_{00}}{p}[T_v(p_{00}) - T], \tag{25}$$

after first determining the virtual temperature depression, $T_v - T$, at p_{00} from the chart. Although accurate determination of the virtual temperature should be done numerically using the exact form of (10), the virtual calculator has pedagogical value as an aid to visualize the process of virtualization.

It is noted that the isovirtuals also correspond to isopycnals with density increasing to the left. Thus the relative density and buoyancy of two moist air parcels at the same pressure can be quickly ascertained visually by plotting their locations on the calculator. Parcels to the right of an isovirtual are less dense than those to the left.

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