Cliff–Ramp Patterns and Kelvin–Helmholtz Billows in Stably Stratified Shear Flow in the Upper Troposphere: Analysis of Aircraft Measurements

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ABSTRACT

Cliff–ramp patterns (CR) are a common feature of scalar turbulence, characterized by a sharp temperature increase (cliff) followed by a more gradual temperature decrease (ramp). Aircraft measurements obtained from NOAA best aircraft turbulence probes (BAT) were used to characterize and compare CR patterns observed under stably stratified conditions in the upper troposphere, a region for which there are few such studies. Experimental data were analyzed for three locations, one over Wales and two over southern Australia, the latter in correspondence with the Southern Hemisphere winter subtropical jet stream. Comparison of observed CR patterns with published direct numerical simulations (DNS) revealed that they were likely signatures of Kelvin–Helmholtz (KH) billows, with the ramps associated with the well-mixed billows and the cliffs marking the highly stretched braids. Strong correlation between potential temperature and horizontal velocity supported the KH link, though expected correlations with vertical velocity were not observed. The temperature fronts associated with the cliffs were oriented in a direction approximately normal to the mean wind direction. Locally high values of temperature structure constant near these fronts were associated with steep temperature gradients across the fronts; this may be misleading in the context of electromagnetic propagation, suggesting a false positive indication of high levels of small-scale turbulence that would not correspond to scintillation effects. Billow aspect ratios, braid angles, and length scales were estimated from the data and comparisons with published DNS provided a means for assessing the stage of evolution of the KH billows and the initial Richardson number of the layer.

1. Introduction

Cliff–ramp patterns (CR) are common features of scalar turbulence, and have been observed in a variety of turbulent shear flows in both stably and unstably stratified conditions. [The use of the cliff–ramp descriptor is adopted here from a review article by Warhaft (2000).] When flying into the wind or for a fixed sensor, a typical CR structure is characterized by a very rapid increase in temperature, the cliff, followed by a more gradual decrease in temperature, the ramp. The order is reversed for the ramp–cliff (RC) structures, with a gradual increase in temperature followed by a steep decrease. For atmospheric flows with primary gradients in the vertical direction, for example, boundary layer and jet stream, CR patterns will be observed when the product of the vertical gradients of velocity and temperature is positive, while RC patterns will be seen if the product is negative.
The present research deals with CR/RC structures identified in the upper troposphere from aircraft measurements at three different locations and dates. These field experiments were part of a multiyear effort to characterize both refractive turbulence phenomenon and clear-air turbulence events that could impact performance of aerospace systems in the upper troposphere and lower stratosphere. CR/RC structures are of particular interest in both regards because of the large temperature and velocity excursions during the cliffs, which could cause undesirable transients in pitch angle for aircraft with constant Mach number flight control, and because of the increased refractive turbulence caused by higher levels of smaller-scale temperature fluctuations that can accompany the CR/RC structures.

An inherent difficulty associated with experimental studies of atmospheric flow phenomena is the lack of a controlled environment during measurements. Despite this, the aircraft measurements discussed here provided valuable information on the cliff–ramp phenomenon in the upper troposphere, a region of the atmosphere for which there is a lack of such information. In particular, the use of multiple probes on the aircraft facilitated the 3D characterization of the CR structures and comparisons with published direct numerical simulations (DNS) provided more evidence that the CR patterns were signatures of Kelvin–Helmholtz (KH) structures.

Cliff–ramp patterns have been observed in the scalar fields in a variety of turbulent flows. [See Warhaft (2000) and Williams and Hacker (1992) for detailed reviews of cliff–ramp literature.] The characteristic asymmetric features of CRs are considered signatures of large-scale coherent structures (CS), the specific characteristics of which may be unique to the specific flow.

For atmospheric surface layers, the coherent structures are plumelike, featuring an inclined temperature microfront (the cliff) at the upwind edge of the structure. Despite the use of the term plume, buoyancy is thought to play a secondary role to shear (Kaimal and Businger 1970; Gibson et al. 1977; Antonia et al. 1979). In particular, McNaughton (2004) suggests that observed cliff–ramp patterns are consistent with cascading hairpin-shaped vortices that can be enhanced by buoyancy.

Cliff–ramp patterns observed in the mixed layers near the surface of oceans or freshwater lakes have been attributed to billow-like structures that are oriented mainly transverse to the current flow (Thorpe and Hall 1977, 1980; Thorpe et al. 1991; Soloviev 1990), though Ekman flow may lead to a slight crosswind orientation (Thorpe et al. 1991). The cliffs were likened to braids that separate adjacent Kelvin–Helmholtz billows (Thorpe 2005), and although KH instabilities would be expected based on the low Richardson number, other potential causes were suggested for the shear instability (Thorpe 2005), including hairpin vortices similar to boundary layers flows or rotors from breaking waves.

Recently, Whiteway et al. (2004, hereafter W04) presented analysis of CR patterns measured in shear-generated turbulence above the tropopause, concluding that the ramps were due to Kelvin–Helmholtz billows in the process of overturning, providing a mechanism for the transition to turbulence.

Though different explanations for the source of CR patterns have emerged for the various flow types, they share two common features: the convergence-stretching mechanism for cliff/scalar-front formation and the importance of shear. These features are best seen in the detailed mapping of velocity and temperature fields in a plane jet by Antonia et al. (1986), recreated in the rough sketch of Fig. 1. Opposing flow at the boundary between adjacent large-scale structures (with the same sense vorticity) create saddle points where flow converges and diverges. The cliffs represent the temperature fronts that develop along the line of divergence that separates cold and warm fluid convected by the opposing flows of the two structures, and the ramps represent regions of well-mixed fluid within the structures. The sharp nature of the cliffs is maintained by the mechanism of fluid stretching along the line of diverging flow. Sreenivasan (1991) refers to the microfronts as sheets, to reflect their two-dimensional nature, and Thorpe and Hall (1980) showed that these microfronts could be bowed or kinked laterally.

The temperature profile sketch in Fig. 1 shows an
idealized cliff ramp, obtained if the sensor passes through the saddle point. If the sensor is above or below that line, the patterns will still feature a sharp cliff, but the ramps may have additional features due to the regions between the lines of convergence and divergence. The exact shapes of the ramp patterns will likely depend upon the specific coherent structures involved; see Antonia et al. (1986) for profiles obtained in a heated jet.

Measurements of the vertical inclination of the fronts have revealed angles ranging from 26° to 51° (Antonia et al. 1979; Thorpe et al. 1991), though Sreenivasan (1991) indicates that 45° is the generally accepted value. This inclination means that the potential temperature gradients across the microfront are steep in the vertical direction as well as the horizontal. Thus the vertical temperature gradient averaged across an ensemble of these structures represents an average of the large gradients across the cliffs and the weaker gradients across the ramps. In that sense, the cliffs can also be thought of as concentrations of the vertical temperature gradient across the vertical scale of the structures, in a way, reconciling the smaller gradients created by turbulent mixing in the core of the structure with the larger overall mean gradient (Shraiman and Siggia 2000).

The presence of CR structures leads to an inherent intermittency of the scalar field that is somewhat independent of the velocity field (Shraiman and Siggia 2000) and results in a coupling between large-scale structures and smaller-scale behavior; an example of this related to refractive turbulence will be discussed in this paper. This inherent coupling is neglected in the classic isotropic scaling arguments of Kolmogorov (1941), Obukhov (1949), and Corrisn (1951), hereafter referred to collectively as KOC, see Warhaft (2000) for more details. As such, the CR structures have been identified as the source of nonzero temperature derivative skewness (which is of order 1), the nonzero odd moments of the temperature field and the anomalous scaling of higher order structure functions (see Sreenivasan and Antonia 1997; Warhaft 2000, for a review of evidence).

Despite the abundance of studies of CR structures, there is a dearth of information for buoyantly stable flows in the atmosphere above the boundary layer, with W04 representing the primary published work focusing on the cliff–ramp aspect of the turbulence. This paper will attempt to examine the connection between CR patterns in the troposphere and KH billows, using the same dataset as W04 supplemented with two other aircraft datasets that feature CR structures. Section 2 will describe the measurements and the basic flight and atmospheric conditions associated with the three CR/RC observations. Section 3 contains detailed characterizations and comparisons of the CR/RC patterns, including temperature and the three component wind and turbulence fields, and the three-dimensional structure of the temperature microfront associated with the cliff.

In section 4, the CR–KH connection will be explored, primarily through comparisons with previously published numerical simulations of Kelvin–Helmholtz billow evolution in stably stratified shear flow. These include the second-order closure simulation by Sykes and Lewellen (1982), and the DNS by Palmer et al. (1994), Scinocca (1995), Werne and Fritts (1999), Smyth and Moum (2000), and Smyth et al. (2001).

2. Measurements

The aircraft measurements were acquired from the Grob 520T EGRETT, a high-altitude research aircraft operated by Airborne Research Australia. The aircraft is capable of operation at altitudes up to 15 km at airspeeds of approximately 100 m s\(^{-1}\) with an endurance of 8 h. The aircraft is equipped with three National Oceanic and Atmospheric Administration/Field Research Division (NOAA/FRD) best aircraft turbulence (BAT) probes (Crawford and Dobosy 1992)—one located under each wing and one located at the top of the tail. During this study, data were not available from all probes for all of the flights. The probes feature a nine-hole pressure probe for velocity measurements and a microbead thermister for temperature located inside the central dynamic pressure port. All velocity and temperature data were sampled at 50 Hz, providing horizontal spatial resolution of 2 m or less. Wind velocities were calculated using the measured probe velocities and the aircraft velocities and orientation (pitch, yaw, and roll angles) from on-board global positioning system (GPS) receivers and accelerometers (Crawford and Dobosy 1997). For the flight over Wales on 6 June 2000 (designated 000606), a Rosemont five-hole probe with a Rosemont PT50 probe for temperature was installed under the right wing in place of the BAT probe. The frequency response of the BAT’s thermister probe was approximately 3–4 Hz, corresponding to length scales of 25 to 33 m in the EGRETT, while the Rosemont temperature probe had a slightly faster response.

A typical flight featured several level flight segments at altitudes from 7 km up to 14 km, covering wind-relative distances up to 250 km. Generally, the segments were upwind or downwind, but occasionally were crosswind. Data during climb and descent between level segments were also utilized for estimating mean vertical gradients of horizontal velocity and potential temperature.
The three datasets to be discussed in this paper, designated by the dates of the flights, are summarized in Table 1. The selection of these cases does not imply that CR structures are unique to these sets of measurements; they were chosen primarily because of the relatively large temperature variations and large horizontal scales of the CR structures. The Richardson numbers shown in Table 1 were calculated from

$$\text{Ri} = \frac{g \frac{\partial \Theta}{\partial z}}{\Theta} \frac{N^2}{S_Z^2},$$

$$S_Z = \sqrt{\left(\frac{\partial U}{\partial z}\right)^2 + \left(\frac{U \partial \psi}{\partial z}\right)^2},$$

where $\Theta$ is the mean potential temperature and $N$ is the buoyancy frequency, $S_Z$ is the magnitude of the vertical shear, which consists of two components: the vertical gradient of the mean wind speed, $\partial U/\partial z$ and the directional shear due the vertical gradient in wind direction, $U \partial \psi/\partial z$. The latter term represents the component of the shear vector lateral to the wind direction causing the direction of shear and wind to differ. The gradients of $\Theta$, $U$, and $\psi$ were estimated by applying a least square fit to the vertical profiles in the ascent or descent segments immediately before or after the level flight segment. An interesting feature of the three cases shown is that all three Richardson numbers fall in a narrow range between 0.2 and 0.24, just below the critical value of 0.25.

Some other notable features of the three cases are as follows:

- The 000606 case features the same data described in W04. The 11.4-km layer is above the tropopause, and also above the peak in the jet, with the decreasing velocity and stably stratified conditions leading to ramp–cliff structures.
- The 8.3-km level where the CR structures were observed on 020905 was just above an unstable layer that extended down to 7.6 km, which was above a neutral layer extending down to 7 km, as confirmed by three separate descent and ascent segments.
- The 990806 case featured the strongest turbulence levels measured in the multiyear campaign, and has been the subject of previous analyses (Côté et al. 2000, 2003; Wroblewski et al. 2003) but these did not address the cliff–ramp structures directly.

The flight paths and wind directions are shown in Fig. 2a, for the 020905 and 990806 cases, and Fig. 2b for the 000606 case, with the onset of the CR structures marked. As seen in the potential temperature traces, Figs. 2c–e, the appearance of the structures is generally accompanied by a noticeable increase in intensity of the temperature fluctuations. A notable aspect of the 990806 case is the appearance of the CR structures as the aircraft flight path passes from over land to over water. The potential temperature patterns for both the 000606 and 020905 cases indicate significant large-scale variations, on the scale of 100 km or more, throughout the segment, especially the 020905 case with its nearly monotonic increase of 12 K over the length of the flight. These variations, along with changes in level of smaller scale of fluctuations, suggest that conditions are far from homogeneous horizontally through the levels. This casts some doubt on the accuracy of the vertical gradients, and the resulting Richardson numbers and shear directions reported in Table 1, since the gradients found from the climb and descent segments may not be characteristic of the atmosphere in the proximity of the CR.

### 3. Results

#### a. Cliff–ramp structure characterization

Figures 3, 4, and 5 show potential temperature and wind velocity measurements along the flight direction for the 000606, 020905, and 990806, respectively for a portion of the segment near the CR patterns. The horizontal velocity, $U$, is the wind velocity in the mean wind direction and $V$ is the component lateral to the mean wind direction. The wind relative distance is determined by the mean true airspeed for the particular level flight segment, with values measured from the beginning of the level segment. Note that three cliffs have been identified in each of the three cases, and are marked on Figs. 3, 4, and 5 by vertical dashed lines.

Details of the CR/RC structures, obtained from this
Fig. 2. (a) Map showing flight paths (black arrow) and wind direction (white arrow) for 020905 and 990806. White ovals mark location where CR patterns were observed. (b) Same as (a) but for 000606. (c) Potential temperature as a function of distance along flight path for 000606. (d), (e) Same as (c) but for 020905 and 990805, respectively.
data, are shown in Table 2, and include cliff length, temperature change, and horizontal temperature gradients associated with the cliffs and the overall length of the cliff–ramp combination. Note that the lengths are shown both in the flight direction and wind direction.

The 000606 patterns represent ramp–cliff structures, due to the negative velocity gradient combined with stable stratification. These are the smallest of the three cases, both in terms of the potential temperature excursion (2–2.4 K) as well as overall length (1.1 to 1.6 km in the wind direction). The first and third cliffs are nearly identical, as seen from the data in the table, while the second has a larger temperature drop and length, but a longer and less steep cliff. The first and second RCs occur in succession, while the second and third are separated by a mildly asymmetric structure of the same scale as the RCs, but with a very weak cliff (at approximately 78 km). The relation of this structure to the adjoining RC patterns is an issue that requires further investigation.

As noted by W04, there is a negative correlation between potential temperature and horizontal velocity on the scale of the RCs, but little correlation between vertical velocity and potential temperature. The vertical velocity does exhibit a significant increase in the level of fluctuations after the RCs appear, but these levels subside during the cliff passage. The 000606 case also exhibits the smallest level of turbulent fluctuations, facilitating the identification of the cliffs in the temperature signal.

Three CR structures are evident in 020905 data (Fig. 4), the first two occurring in succession followed by a third smaller cliff, separated from the second by a symmetric structure on the same scale as the CRs. The dominant feature of the first two is the large potential temperature excursions associated with the cliffs, on the order of 5 K. As clearly seen in the figure, the first two cliffs are not sharp, extending for 0.6 to 0.7 km (in the flight direction), an order of magnitude longer than those seen in either of the other cases. However, the large angle between the flight path and the wind direction means that the aircraft is flying across the structures; as seen in Table 2, the lengths and gradients in the wind direction are more in line with those of the 000606 structures. The third cliff is more compact, with a length almost an order magnitude smaller than the first two.

The horizontal velocity is positively correlated with

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**Table 2. Details of the cliff–ramp structures:** CW = cliff width, in flight direction/in wind direction; WL = wavelength, in flight direction/in wind direction; HOA = horizontal orientation angle relative to wind direction; VOA = vertical orientation angle relative to horizontal.

<table>
<thead>
<tr>
<th>CR 1</th>
<th>000606</th>
<th>020905</th>
<th>990806</th>
</tr>
</thead>
<tbody>
<tr>
<td>CW (km)</td>
<td>0.058/0.055</td>
<td>0.67/0.38</td>
<td>0.051/0.048</td>
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<tr>
<td>Cliff Δθ (K)</td>
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<td>5.19</td>
<td>3.78</td>
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<td>WL (km)</td>
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<tr>
<td>HOA (°)</td>
<td>−6.6</td>
<td>−32.5</td>
<td>64.6</td>
</tr>
<tr>
<td>VOA (°)</td>
<td>—</td>
<td>—</td>
<td>26.6</td>
</tr>
<tr>
<td>CR 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CW (km)</td>
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<td>0.60/0.29</td>
<td>0.30/0.27</td>
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<tr>
<td>Cliff Δθ (K)</td>
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<td>5.39</td>
</tr>
<tr>
<td>WL (km)</td>
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<td>3.48/1.69</td>
<td>7.16/6.92</td>
</tr>
<tr>
<td>HOA (°)</td>
<td>−5.5</td>
<td>—</td>
<td>4.4</td>
</tr>
<tr>
<td>VOA (°)</td>
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<td>—</td>
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<tr>
<td>CW (km)</td>
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<td>0.078/0.038</td>
<td>0.270/0.27</td>
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<td>4.57</td>
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<tr>
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<td>1.89/0.92</td>
<td>7.00/6.76</td>
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<td>—</td>
<td>−6.7</td>
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<tr>
<td>VOA (°)</td>
<td>—</td>
<td>—</td>
<td>38.6</td>
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temperature, opposite from the 000606 case. However, both are consistent with gradient transport of heat and momentum by vertical motions—the 000606 case with a negative vertical gradient for velocity and the 020905 case with a positive vertical gradient. Like the 000606 case, there is little evidence of the CR structure in the vertical velocity, but there is a substantial increase in vertical velocity fluctuations associated with the appearance of the ramps. The small-scale temperature fluctuations are larger than the 000606 case, with significant fluctuations evident even during the first temperature cliff.

Three CR structures are seen in the 990806 data (Fig. 5), with cliff temperature excursions of 3.8 to 5.4 K and relatively long wavelengths of 6.5 to 8.2 km in the wind direction. These patterns present a unique interpretation problem, particularly the first and second structures, that extend over an interval of about 20 km and appear to be composed of an asymmetrical cliff and ramp followed by a degraded cliff and ramp that has a more symmetrical appearance.

The first cliff is steep, 48 m in length, similar to those seen in the 000606 case, while the second and third cliffs are broader, extending about 0.3 km. A unique feature of this case is the cliff-like behavior of the horizontal velocity coincident with the first cliff in temperature, with a change of 15 m s$^{-1}$ or 25% of the value before the cliff. This is significantly larger than that of any of the other CRs observed. The small-scale turbulence is stronger than the other cases, with fluctuations that obscure some of the features of the cliffs and ramps. This is most evident in the high level of fluctuations in the vertical velocity ($\sigma_w = 1.76$ m s$^{-1}$ compared to 0.51 and 0.81 for 000606 and 020905, respectively.)

b. Geometry of cliffs

The flight heading, wind directions, and shear directions listed in Table 1, are shown graphically in Fig. 6. For 000606 and 990806 flights, the aircraft heading is primarily into the wind, but differed from the wind direction by 61° for 020905 with the wind-direction component of the flight path into the wind. For all three cases, the shear direction differed from the wind direction, about 45° for the 000606 and 020905 flights and 18° for the 990806 flight, indicating directional shear.
As the relative positions measured in the aircraft reference plane (x direction through the centerline of the fuselage) because the aircraft heading differed from the flight direction due to crosswinds. For each cliff, different values were determined for the mean wind speed and direction, aircraft heading, flight direction, and ground speed.

The lag times were estimated using the correlation coefficient between the left ($T_L$) and right ($T_R$) probe temperature signals (Antonia et al. 1979),

$$C_{T_L, T_R}(\tau) = \frac{\langle T_L(t)T_R(t + \tau) \rangle}{\sqrt{\langle T_L^2(t) \rangle \langle T_R^2(t + \tau) \rangle}},$$

where the angle brackets refer to time averaging. The correlation coefficient is calculated for different values of the lag time for data spanning the duration of the cliff. The lag time that produces the highest correlation coefficient is chosen for that cliff. For all cases, an obvious peak in the correlation function versus lag time plot was discernible, with peak correlation coefficients generally between 0.90 and 0.97. No orientation angles were calculated for the 020905 case, due to absence of right probe data. The method was tested by comparison with visual selection of the lag time for the first cliff in the 990806 data, which yielded the same result.

For the 000606 data, the horizontal orientation angle relative to the flight path varied from $-41.1^\circ$ to $-19.8^\circ$, shown graphically in Fig. 6 by the dotted lines. These angles correspond to angles of $-6.6^\circ$ to $12.8^\circ$ relative to the direction normal to the flight direction, suggesting that the fronts line up approximately normal to the mean wind direction. For the 990806 data, there was a wider variation in the front angle, from $-24.4^\circ$ to $19.1^\circ$ relative to the flight direction and $-32.5^\circ$ to $4.4^\circ$ relative to a direction normal to the mean wind.

The vertical orientation of the cliff fronts was estimated using the lag time between the cliff front passage at the tail probe and the left and right wing probes,

$$\Delta z = \Delta z_0 \frac{\sin[\tan^{-1}(\Delta z/\Delta x) - \alpha_{AC}]}{\sin[\tan^{-1}(\Delta z_0/\Delta x)]},$$

where $\Delta z_0$ is the vertical separation distance if the aircraft angle of attack is zero.

Vertical orientation angles were calculated for
990806 case only, the only flight with available data from all three probes. The resulting values for the three cliffs ranged from $26.6^\circ$ to $64.6^\circ$, a range consistent with the values reported by Antonia et al. (1979) for surface layers. In addition the average of the six values calculated was $43.3^\circ$, consistent with the accepted value of $45^\circ$ for both passive scalar fronts and surface layer microfronts. Though the estimated orientation angles seem consistent with previous results, none of the previous studies investigated the stable region of the atmosphere in the upper-troposphere region. This issue will be discussed further in section 4a.

The method for finding the horizontal and vertical angles has a degree of uncertainty because of two factors: the discrete nature of the method ($\tau$ is restricted to multiples of the sampling interval) and the lack of knowledge of the convection velocity of the fronts. The former is the dominant factor in the uncertainty, which was estimated to be approximately $\pm 5^\circ$ for the horizontal angles and for the shallow vertical angles (cliffs 2 and 3) and $\pm 10^\circ$ for the vertical angle for cliff 1. For future flights, higher sampling rates are suggested if detailed ramp characterization is desired.

c. Temperature and velocity correlation

To better study the correlation between temperature and velocity during and around the CR/RC events, bandpass filtering was used to remove the smaller scale turbulent fluctuations and the large scale structures. Although these filters reduce the gradients in the cliffs, they still preserve the basic features at the overall scale of the CR structures. Fourth-order Butterworth filters were used with different pass bands for each of the three cases, which scaled on CR wavelengths. Specifically, the pass bands extended from $0.25L_{\text{min}}$ to $3L_{\text{max}}$, where $L_{\text{min}}$ and $L_{\text{max}}$ were the smallest and largest wavelengths in the flight direction for the three CR structures observed for each case.

The filtered signals were nondimensionalized using the change in the relevant quantity during the cliffs (chosen as the average value for all cliffs for each case). For example, the nondimensional potential temperature was found using

$$\Theta^* = \frac{\theta_{\text{FILT}}}{\Delta \theta_{\text{CLIFF}}}.$$  

Figure 7 shows the filtered potential temperature and horizontal velocity for the three cases, with $-U^*$ shown for the 000606 to better see the correlation between $U^*$ and $\Theta^*$. In all cases, the velocity and temperature are very closely in phase ($180^\circ$ out of phase for 000606), with the rapid change in velocity corresponding closely to the rapid change in temperature for all cliffs. In addition, the temperature and velocity are also correlated during many of the smaller scale motions seen during the ramps, especially for the 990806 case. Correlation coefficients for the two signals were found to be nearly the same for all three cases: $R_{\Theta^*U^*} = -0.77$, 0.77, and 0.83 for the 000606, 029005, and 990806 cases, respectively.

W04 attributed this strong temperature–velocity correlation to vertical gradient transport of fluid—upward motion would bring lower temperature and faster moving fluid (slower moving fluid for a positive vertical gradient of velocity such as those seen for 029005 and 990806). However, W04 also noted an apparent contradiction to this explanation; such transport should lead to a close correlations between vertical velocity and

\[ \text{(6)} \]

FIG. 7. Bandpass-filtered, nondimensional potential temperature (solid lines) and horizontal velocity (dot-dash lines) for (a) 000606 Wales, (b) 029005 Australia, and (c) 990806 Australia. Filter-pass band is $0.25L_{\text{min}}$ to $3L_{\text{max}}$. Vertical lines mark locations of cliffs.
both potential temperature and horizontal velocity, but these were not seen in their measurements. This was attributed to the relatively stable Richardson number, 0.2, such that buoyancy suppressed vertical motion, causing the coherent vertical motions associated with the ramps to be obscured by turbulent motions. Assuming this to be the case, bandpass filtering may provide a means to isolate the CR-scale vertical motions from the turbulent motions, and facilitate the analysis of the correlation between potential temperature and vertical velocity. However, as seen in Fig. 8, even the filtered vertical velocity exhibit large fluctuations with wavelengths that are smaller than that of the CR structures, making it difficult to find consistent patterns associated with the cliffs.

Figure 9 shows the time series of vertical heat flux, based on the filtered nondimensional temperature and velocity, $W^{*}\Theta^{*}$. For the most part, the heat flux is negative, the notable exceptions being the second ramp for 020905 and the first two cliffs for 990806. The average heat fluxes are all negative, consistent with gradient transport across a positive mean potential temperature gradient in the vertical direction. Correlation coefficients for the three cases are $R_{\Theta^{*}W^{*}} = -0.29$, $-0.19$, and $-0.28$ for the 000606, 020905, and 990806 cases, respectively.

d. Structure functions

Second-order structure functions for the velocity and the temperature were found using

$$D_{XX}(r) = \langle [X(t) - X(t + r/t_{\text{TAS}})]^2 \rangle,$$

where $X$ is the temperature or velocity, $r$ is the separation distance in the flight direction so that $r/t_{\text{TAS}}$ is the
lag time between two readings, and the angle brackets refer to time averaging. The structure functions are calculated for the velocities along and lateral to the aircraft flight direction, $D_{UU,A}$ and $D_{VV,A}$, respectively, and for the vertical velocity, $D_{WW}$. For 000606 (Fig. 10a), the horizontal and lateral velocities display a small inertial subrange with the expected $2/3$ slope ($r^{2/3}$ dependence), but the vertical velocity exhibits a steeper slope. For the temperature, a small inertial range is seen from 30 to 50 m, but the dominant feature is a region from 50 to 500 m with a slope close to unity, indicating a structure function that is proportional to $r$. Higher slopes are to be expected when the dominant feature is the cliffs–ramps, since the main contributor to the structure function for $r$ greater than the cliff length will be the temperature change across the cliff. For an ideal case of a signal with a pure temperature shock of $\Delta T$ between two levels, the structure function can be estimated as

$$D_T(r) \approx \Delta T^2 p,$$

where $p$ is the probability that the separation length $r$ will include the ramp, which should be proportional to $r$ divided by the sample length (V. Yakhot 2005, personal communication). This results in a structure function that is proportional to $r$. For signals with real CR/RC patterns, this scaling becomes more obvious for higher order structure functions, as the cliff $\Delta T$ dominates over the contributions from the turbulent fluctuations, and is the source of the anomalous scaling phenomena (Sreenivasan and Antonia 1997). At higher separation distances, the structure function reveals a periodic pattern with minimums at multiples of the cliff CR structure wavelength (average of 1.4 km for 000606).

For 020905 (Fig. 10b), the horizontal and lateral velocities show a more extensive inertial range, but the vertical velocity structure function has a peak around 40 m that exceeds the values for $D_{UU,A}$ and $D_{VV,A}$. The temperature exhibits an inertial subrange from approximately 50 to 300 m, with an increasing slope at lower separation distances due to the thermister thermal lag. Above 300 m, the slope increases, consistent with the higher slopes expected from the CR pattern, as discussed above. The existence of $r^{2/3}$ scaling for temperature inertial range (as opposed to the $r$ behavior seen in the 000606 case) reflects the higher level of temperature fluctuations as well as the longer lengths of the cliffs that shift the CR effect on structure function to larger scales.

The inertial subrange extends over two orders of magnitude for the velocity for 990806 (Fig. 10c). There is no evidence of the cliffs in the temperature structure function; the high level of turbulent fluctuations across a range of scales contributes as much to the structure function as the cliffs, so the characteristic $r$ scaling for the cliffs is not seen. A notable feature of the temperature structure function is the change from $r^{2/3}$ to $r^{2/5}$, the latter being consistent with Bolgiano’s scaling for stably stratified flows (Bolgiano 1959), though the velocity structure function does not exhibit the $r^{6/5}$ dependence predicted by the theory. The $r^{2/5}$ has been observed in other measurements in stably stratified conditions obtained during the campaign (Wroblewski et al. 2003) and is consistent with results of numerical simulations of stably stratified shear flows by Werne and Fritts (2000).
The turbulent dissipation, $\varepsilon$, can be estimated from the velocity structure function in the inertial subrange, $D_{UU,A}(r) = 2\varepsilon^{2/3}r^{2/3}$ yielding the values of $\varepsilon = 2.4 \times 10^{-3}, 6.5 \times 10^{-3}, \text{and } 3.3 \times 10^{-2} \text{ m}^2 \text{ s}^{-3}$ for 000606, 020905, and 990806, respectively. It should be noted that these values are based on the assumption of isotropy, such that the structure constants for the lateral velocities are $4/3$ of the value for the longitudinal velocity. This relationship does not hold for any of the three CR cases, most evident in the structure functions for the vertical velocity seen in Fig. 10 that are mainly lower than those for the longitudinal velocity. Values estimated from the lateral velocity structure functions using the isotropic assumption, $D_{VV,A}(r) = 8/3\varepsilon^{2/3}r^{2/3}$ and $D_{WW}(r) = 8/3\varepsilon^{2/3}r^{2/3}$, are from 35% to 65% of the values found from the longitudinal structure function.

e. Turbulence and short-time structure constants

Variations of turbulence levels within the CR patterns were studied using short-time second-order structure functions for velocities (Fig. 11) and temperature (Fig. 12). These structure functions were calculated for a single value of the separation distance, $r = 45$ m, chosen to be within the inertial subranges for all cases, but well above the scale at which sensor thermal lag would lead to attenuation of the temperature signal. Each data point represents an average over time intervals that correspond to approximately a 1-km segment centered at that location (500 m on either side). This window was chosen to be large enough to obtain rea-
sonable averages, but smaller than the distance between adjacent cliffs. Structure functions are used in lieu of turbulent kinetic energy (TKE) for the velocities, because they provide a better measure of turbulent activity; TKE can include contributions from large-scale structures, for example, a KH billow that may or may not be part of the turbulence field.

Comparison of the levels of the velocity structure functions for the three cases reveals the difference in turbulent activity. Average values of $D_{uu}$ for the 990806 case are about 3 times greater than those for the 020905 case and nearly 7 times higher compared to the 000606 case. Excluding the increased turbulent levels during the last RC structure on 000606, the $D_{uu}$ values for that case are almost an order of magnitude smaller than those of the 990806 case.

For 000606 (Fig. 11a), turbulence levels are highest during the ramps, with the peak values occurring during the third ramp. This trend of higher values during the ramps is also observed for the 020905 structure functions as well (Fig. 11b), except that the first cliff shows a local peak for all three velocities. In contrast, TKE (not shown) reveals peak values near the cliffs, likely due to the large change in horizontal velocity associated with the cliff, thus illustrating that the structure functions are a better measure of turbulent activity. The vertical structure functions exceed those of the other velocities before and after the third cliff for 020905, consistent with the time series shown in Fig. 4. For 990806 (Fig. 11c), the peaks in the velocity structure functions occur at fairly regular intervals (approximately every 5 km), and like the other two case, the peaks are seen within the ramps rather than at the cliffs. For all cases, the lateral velocity structure function often exceeds that of the horizontal velocity, most notably during the third ramp on 000606.

The International Civil Aviation Organization (ICAO) uses short-time estimates of eddy dissipation rate, $EDR = \varepsilon^{1/3}$, as a metric for reporting severity of atmospheric turbulence. The dissipation, $\varepsilon$, is normally estimated from the vertical wind velocity, so EDR values can be found from the maximum values of $D_{ww}$ shown in Fig. 11, using the approach described in section 3d. Applying this method, the 000606 case would be classified as light turbulence ($EDR = 0.21$), the 020905 case would be classified as moderate turbulence ($EDR = 0.31$), while the 990806 case would be categorized as severe turbulence ($EDR = 0.53$).

The behavior of the temperature structure functions (Fig. 12) is significantly different than those of the velocities, with distinct local plateaus near the cliffs for 000606 and 020905. The width of these plateaus is approximately 500 m, half the length of the averaging window. This means that for any window that includes the cliff, the structure function is dominated by the large temperature changes associated with the cliffs themselves; the resulting high values don’t represent smaller-scale turbulent fluctuations. For 990806, local peaks in the structure function occur at locations within the ramps, not just at the cliffs, consistent with higher levels of turbulent fluctuations seen in the velocity structure functions measurements.

The structure function characteristics of the CR patterns are important in refractive turbulence modeling for electromagnetic wave propagation. In particular, the temperature structure constant, $C_T^2 = D_{TT}(r)r^{-2/3}$, is used for determining the level of scintillation of an electromagnetic beam propagating through a turbulent atmosphere, and measurements of temperature structure constants, obtained by balloon or aircraft measurements, are often used to develop and validate models of $C_T^2$ for propagation modeling. High values of $C_T^2$ measured near the cliff temperature fronts (Fig. 12) might represent a false positive indication of high levels of small-scale fluctuations that would contribute to scintillation.

For example, the $D_{TT}$ values (for $r = 45$ m) for the first two peaks for 020905 case correspond to structure constant values of approximately $0.03$ K² m⁻²/³. Extreme values that would be interpreted as intense refractive turbulence. In reality, these high $C_T^2$ levels are features of a much larger-scale structure that would likely result in beam steering errors, rather than scintillation. Thus, differentiating the small-scale refractive turbulence from the large-scale temperature fronts due to CR structures should be a concern for refractive turbulence researchers. The third-order structure function, $D_{TTT}(r) = \langle [T(t) - T(t + r/U_{TAS})]^3 \rangle$ shown in Fig. 12 for $r = 45$ m and a 1-km window, could be used for this purpose, because the large cliff temperature difference is even more dominant than in the second-order structure function. For all three cases, $D_{TTT}$ values are small except near the cliffs (note that $-D_{TTT}$ is shown for the 000606 case). In particular, for the 990806 case, the high $D_{TT}$ values due to the asymmetric cliffs are clearly distinguished from the high values due to the symmetric smaller scale fluctuations in the ramps. Since the large cliff temperature difference is the dominant contribution to $D_{TTT}$, values of $D_{TTT}$ will be inversely proportional to the window size [see the discussion that accompanies Eq. (8)]. However, the choice of window size is not critical here, because $D_{TTT}$ is used solely as a tool to distinguish the cliffs from the background turbulence, so the actual value is less important than the relative difference between the values at the cliff and in the ramps.
4. Kelvin–Helmholtz billows and cliff–ramp patterns

The connection between RC patterns and KH structures was suggested by W04, with the ramps representing the relatively well-mixed billow regions and the cliffs associated with the highly stretched billows separating the billows. This explanation is appealing, because CR/RC patterns are known to be signatures of coherent structures and KH billows are known to be a mechanism for transition to turbulence in the stable region near the tropopause. In addition, the CR/RC structures seen in the aircraft data in the upper troposphere display a more regular and repeatable pattern than those reported in the boundary layer or laboratory flows, which would be consistent with a train of KH billows. This section will examine the CR–KH connection based on the aircraft measurements presented in section 3 and DNS of turbulence in stably stratified shear flows.

The six simulation studies used for comparison are summarized in Table 4. Note that the simulations of Palmer et al. (1994) and Werne and Fritts (1999) assumed a constant potential temperature gradient across the shear layer, as opposed to a hyperbolic tangent profile. Although none of the simulations duplicate the exact initial conditions of the velocity and temperature gradient in temperature along streamwise planes varied between 0 and 2.0 depending on the vertical position within the billow, with an average value of near 1.0.

Table 4. Summary of published numerical simulations of turbulence in stably stratified flows used for comparison with experimental data: SL = Sykes and Lewellen (1982), 2D second-order closure; SC = Scinocca (1995), embedded in domain with $d\theta/dz$ constant; PA = Palmer et al. (1994); WA = Werne and Fritts (1999); SM = Smyth and Moum (2000); SMC = Smyth et al. (2001), Prandtl numbers 1, 2, and 7.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Ri</th>
<th>Re</th>
<th>Profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>SL</td>
<td>0.1</td>
<td>—</td>
<td>$u$: tanh</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>—</td>
<td>$\phi$: tanh</td>
</tr>
<tr>
<td>SC</td>
<td>0.1</td>
<td>2000</td>
<td>$u$: tanh</td>
</tr>
<tr>
<td></td>
<td>0.15</td>
<td>4000</td>
<td>$\theta$: tanh</td>
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<tr>
<td></td>
<td>0.2</td>
<td></td>
<td>$\phi$: tanh</td>
</tr>
<tr>
<td>PA</td>
<td>0.05</td>
<td>500</td>
<td>$u$: tanh</td>
</tr>
<tr>
<td>WA</td>
<td>0.05</td>
<td>2000</td>
<td>$\theta$: linear</td>
</tr>
<tr>
<td>SM</td>
<td>0.08</td>
<td>491</td>
<td>$u$: tanh</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>1244</td>
<td>$\phi$: tanh</td>
</tr>
<tr>
<td>SMC</td>
<td>0.16</td>
<td></td>
<td>$\theta$: linear</td>
</tr>
<tr>
<td></td>
<td>0.096</td>
<td>1244</td>
<td>$u$: tanh</td>
</tr>
</tbody>
</table>

Table 3. Derivative skewness of temperature and velocity.

<table>
<thead>
<tr>
<th></th>
<th>000606</th>
<th>020905</th>
<th>990806</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_T$</td>
<td>$-0.48$</td>
<td>0.58</td>
<td>0.00</td>
</tr>
<tr>
<td>$S_T\cos\phi$</td>
<td>$-0.51$</td>
<td>1.20</td>
<td>0.00</td>
</tr>
<tr>
<td>$S_\phi$</td>
<td>0.24</td>
<td>0.22</td>
<td>0.33</td>
</tr>
<tr>
<td>$S_{T,\text{Filtered}}$</td>
<td>$-1.8$</td>
<td>1.5</td>
<td>1.06</td>
</tr>
</tbody>
</table>

* Filtered refers to bandpass-filtered data (section 3c).

The nonzero values arise because of the steeper gradients associated with the cliffs, compared to the ramps. Sreenivasan (1991) presented data for a wide variety of investigations, with values between 0.55 and 1.25 with a possible trend of lower values for higher Reynolds numbers. Thorpe et al. (1991) showed values from 0.01 to 0.8 for the upper-ocean boundary layer, with a mean value of 0.35. Derivative skewness for temperature and velocity for the three cases are shown in Table 3, including values corrected for the deviation between flight direction and wind direction (following Thorpe et al. 1991). For the 020905 and 000606 cases, the values for temperature are within the range of values reported by Sreenivasan (1991), with the 000606 case yielding a negative value since the cliffs are associated with decreasing temperature. The velocity derivative skewness, as expected, is lower than that of the temperature for these cases because there is less asymmetry in the velocity signal compared to the less steep gradients across the cliff.

Surprisingly, the 990806 data, which featured the strongest cliff in terms of temperature gradient, yielded a derivative skewness value of essentially zero. It is not readily apparent why this is the case; though it may be related to the high levels of smaller-scale turbulence with gradients on the same order as the cliff themselves. This is supported by the high value of skewness, 1.06, for the low-pass-filtered temperature signal, since filtering out smaller scale fluctuations would reveal the asymmetry of the larger-scale cliff–ramps. This result is significant, because low-pass filtering should tend to make structures more symmetric and thus reduce the skewness. Another explanation for the zero skewness for the 990806 case can be found in the Kelvin–Helmholtz DNS results of Smyth and Moum (2000). They showed that the skewness of the streamwise gradient
of the experimental data (and most importantly the level of stability through the Richardson number), such a task would be extremely difficult, since the initial conditions cannot be obtained directly from the aircraft measurements. The main benefit derived from comparisons of DNS with field measurements is the ability to deduce the initial Richardson number of the layer using the flow morphology obtained from the simulations, an idea that the authors attribute to J. Werne (2005, NorthWest Research Associates, personal communication).

a. Layer aspect ratio and cliff temperature front orientation

The apparent lack of correlation between vertical velocity and the CR patterns in temperature is one aspect of the KH explanation that deserves attention. W04 suggest that the high Richardson number, near 0.2, represented a degree of stability that would suppress vertical motions to a level that could be obscured by turbulent fluctuations. The filtered profiles shown in Fig. 8 revealed fluctuations of vertical velocity at slightly smaller scales than the CR structure, making it difficult to draw conclusions about the correlations between vertical velocity and temperature near the cliffs. If strong stability does suppress the vertical motions of the KH billow, one might expect that it would also suppress these fluctuations to a similar extent.

The aspect ratio (the ratio of billow wavelength and billow height) is an indication of the degree to which stability would suppress vertical motions and hence growth of the billows. Figure 13a shows the aspect ratio as a function of initial Richardson number estimated from the contour plots of temperature or density reported for the various DNS studies. The results show a consistent trend, with the higher initial Richardson number cases \( \frac{\text{Ri}}{H} \approx 0.2 \), displaying higher aspect ratios consistent with the idea of flattened billows and suppressed vertical motions.

For comparison, the aspect ratios of the CR/RC structures observed in the aircraft data were calculated. Using an approach suggested by J. Werne (2005, NorthWest Research Associates, personal communication), the height of the layer was estimated from the potential temperature data assuming that the temperature difference across the entire structure is concentrated across the cliff—using the KH model, the billow wraps up the temperature contours, which become concentrated across the braid due to stretching. Thus, the vertical height of the structure can be estimated from

\[
H \approx \frac{\Delta \theta_{\text{CLIFF}}}{\frac{\partial \Theta}{\partial z}},
\]

where the gradient in the denominator is the average background gradient estimated from the climb data. As seen in Table 5, this simple estimate shows compressed/flattened structures for all cases, with average aspect ratios of 4.4, 3.0, and 6.4 for the 000606, 020905, and 990806 cases. By comparing these aspect ratios to the values in Fig. 13a, rough estimates of the initial Richardson numbers for each case can be obtained—0.15 for 000606, 0.10 for 020905, and 0.2 for 990806. Despite the uncertainty of such estimates, they do suggest that the billows for 000606 and 020905 are somewhat fuller than those expected for a layer with an initial Ri number of 0.2, though the 990806 case seems consistent with Ri = 0.2.

This result demonstrates that the values of Ri obtained from the aircraft climb segments are not necessarily the same as those at the beginning of the layer.

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1 Note that the uncertainty associated with estimating the aspect ratio from published plots is as likely a cause of scatter in the data as any difference in the parameters and methods used for simulation.
evolution, and emphasizes the difficulty in matching the initial conditions of the field measurements. DNS have shown that the Richardson number generally evolves during the KH billow development, with values in the 0.25 to 0.4 range during the decaying phase of the layer (SL, WF, SM, and SMC). Thus DNS with initial values of Ri = 0.2 are not necessarily representative of layers, such as those considered here that display Ri = 0.2 at a particular stage of layer development.

Figure 13b shows the braid angle of the billows, also estimated from the temperature or density contour plots of the DNS, showing the expected trend of shallower angles for the flattened billows at higher Richardson number. The estimated vertical angle of 65° for 990806 seems out of line with the trend in Fig. 13b, while the other two values, 27° and 39°, appear consistent with a layer with an initial Richardson number around 0.1. This is in conflict with the above estimate of 0.2 based on the aspect ratio; at that Richardson number, the expected braid angle would be 15° or less. One possible explanation for this discrepancy is that Eq. (10) probably underestimates the billow height, since the vertical gradient of potential temperature across the billow is likely smaller than the horizontally averaged value used and because the temperature drop across the braid may be smaller than that across the entire layer. Potential temperature contours reported by W04 suggest that the billow height estimate may be low by as much as a factor of two for Ri = 0.22, and thus the estimated aspect ratio is high by the same factor. Applying this factor and reducing the estimated aspect ratio for the 990806 case by one-half leads to an estimated Ri of about 0.15, which is closer to the value expected from the braid angles.

The initial Ri values are very coarse estimates, but they do illustrate three important points. First, the calculated values of Ri from climb data are not necessarily representative of the initial stability of the layers. Second, the initial stability of the layers differ, with 020905 being the least stable and 990806 the most stable. Third, and probably most importantly, this approach offers promise for more in-depth investigation of field data using detailed comparisons with DNS results.

As mentioned above, several of the DNS calculations associated with the results in Fig. 11 used hyperbolic tangent profiles for temperature with zero background gradients, which will likely affect the vertical extent of the layer as it develops. The effect of background temperature gradients and the best method for estimating billow heights and initial Ri are issues that require additional study.

Previous studies of cliff–ramp structures generally involved simple flows such as jets, wakes and boundary layers, for which the velocity vector and shear vector were along the same line of action. The atmospheric conditions in the proximity of the CR structures observed near the tropopause exhibit some directional shear, with noncoincident shear and wind directions. As shown in Fig. 6, the alignment of the cliff temperature fronts in the horizontal direction showed some scatter, but was generally close to normal to the wind direction. The lack of exact alignment with the wind direction may be due to the fact that the fronts are three-dimensional (i.e., nonplanar) as suggested by Thorpe et al. (1991), or distorted as a result of transverse-oriented structures, such as those that develop because of KH secondary instabilities (Palmer et al. 1996).

### b. Turbulence and evolution of the layer

During the evolution of KH billows, turbulence levels start out nearly zero, as the billow develops two-dimensionally, and then increase as three-dimensional motions form (WF). If KH structures are the source of the CR patterns, then differences in the turbulence levels among the three cases may reflect different stages of evolution, especially since the Richardson numbers are nearly the same. However, a more rigorous comparison of the turbulence levels, using proper scaling, is needed.

The rms values of the horizontal and vertical velocities were scaled using the characteristic velocity $S$, and the rms of the potential temperature was scaled using the cliff temperature change. As seen in Table 6, the rms values of the horizontal velocity for the three cases collapse well when scaled. However, the rms values are weighted heavily to the large-scale changes associated with the cliff–ramp structures, and do not necessarily represent the smaller-scale turbulence. This is most apparent in the scaled temperature fluctuations;

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**Table 5.** Estimates of CR vertical layer height, $H$, using Eq. (10), where $\Delta \theta_{\text{CLIFF}}$ is the potential temperature change across cliff, $H$ is the estimated height of billow, $L_{\text{CR}}$ is the CR wavelength in wind direction, and AR is the billow aspect ratio, $L_{\text{CR}}/H$.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\partial \theta / \partial z$ (K m$^{-1}$)</th>
<th>$\Delta \theta_{\text{CLIFF}}$ (K)</th>
<th>$H$ (m)</th>
<th>$L_{\text{CR}}$ (m)</th>
<th>AR</th>
</tr>
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<tr>
<td>000606</td>
<td>RC 1</td>
<td>0.007</td>
<td>2.04</td>
<td>291</td>
<td>1077</td>
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<tr>
<td></td>
<td>RC 2</td>
<td>0.007</td>
<td>2.33</td>
<td>333</td>
<td>1617</td>
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<tr>
<td></td>
<td>RC 3</td>
<td>0.007</td>
<td>2.01</td>
<td>287</td>
<td>1333</td>
</tr>
<tr>
<td>020905</td>
<td>RC 1</td>
<td>0.0085</td>
<td>4.3</td>
<td>506</td>
<td>2017</td>
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<tr>
<td>990806</td>
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<td>3.78</td>
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<tr>
<td></td>
<td>CR 2</td>
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<td>1348</td>
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<td></td>
<td>CR 3</td>
<td>0.004</td>
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<td>6761</td>
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Table 6. Turbulence parameter scaling and length scales.

<table>
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<tbody>
<tr>
<td>$\sigma_u$, m s$^{-1}$</td>
<td>1.14</td>
<td>1.87</td>
<td>3.25</td>
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<tr>
<td>$\sigma_u/(S,H)$</td>
<td>0.12</td>
<td>0.12</td>
<td>0.13</td>
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<tr>
<td>$\sigma_w$, m s$^{-1}$</td>
<td>0.514</td>
<td>0.807</td>
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<td>$\sigma_w/(S,H)$</td>
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<td>0.050</td>
<td>0.070</td>
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<tr>
<td>$\sigma_n$, K</td>
<td>0.640</td>
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<td>1.14</td>
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<td>$\sigma_u/\Delta$</td>
<td>0.30</td>
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<td>$\sigma_w\text{, filt}$/(S,H)</td>
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<td>$\sigma_n\text{, filt}$, K</td>
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<td>$L_o/L_E$</td>
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<td>0.31</td>
<td>0.56</td>
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* The filt subscript refers to high-pass-filtered data using cutoff wavelength of $0.25L_{\text{min}}$ where $L_{\text{min}}$ is the smallest wavelength in the flight direction for the three CR/RC featured in each case.

Figs. 4 and 5 clearly show that 990806 features more intense small-scale fluctuations in $\theta$ than 020905, but the overall rms value is higher for the 020905 case. The vertical velocity fluctuations are likely a better indicator of small-scale turbulence. The scaled values of $w$ indicate that the 000606 and 020905 cases have similar turbulence properties, but the 990806 case is more turbulent. This is confirmed by using rms values of high-pass-filtered potential temperature and horizontal velocity (also shown in Table 6), with the filter cutoff frequencies equal to the upper pass band frequency for the bandpass-filtered analysis described in section 3c. The scaled rms values for the filtered $u$ and $\theta$ data display nearly identical trends to those of the unfiltered $w$ results, indicating that the 000606 and 020905 cases may be at similar stages of evolution whereas the 990806 case may be at a later stage, when turbulence levels are higher.

SMC and SM investigated several length scale ratios as indicators of the time evolution of KH billows, using DNS for initial Ri numbers ranging from 0.08 to 0.16. The most promising was the ratio $L_o/L_T$ where $L_o$ is the Ozmidov length scale,

$$L_o \approx \sqrt{\frac{\varepsilon}{N^2}},$$

the smallest scale that is unaffected by buoyancy, and $L_T$ is the Thorpe scale, a measure of the vertical size of overturning motions. SM showed that the ratio $L_o/L_T$ continually increases throughout the evolution, with values less than 0.125 during the initial rollup phase, rapidly increasing up to about 0.5 during the billow collapse and turbulent transition phase, followed by more gradual increases up to 2.0 and higher during the turbulence decay phase. Unfortunately, determination of the Thorpe scale requires analysis of vertical temperature profiles through the KH billow, which are not available from the aircraft data. A similar, but more convenient, length scale is the Ellison scale,

$$L_E \approx \frac{\sigma_u}{\frac{\partial \Theta}{\partial z}},$$

SM show that the ratio of the Ellison and Thorpe scales is close to unity, suggesting that $L_o/L_E$ may act as a surrogate for $L_o/L_T$ for estimating the evolution time of a turbulent layer.

The Ozmidov and Ellison scales and their ratio are included in Table 6 for the three CR/RC cases. Turbulent dissipation, $\varepsilon$, in Eq. (11) was estimated from the velocity structure function, as described in section 3d. The $L_o/L_E$ values for 000606 and 020905 (0.35 and 0.3) are similar, while the value for 990806 is larger (0.56). This is consistent with the simple comparison of the vertical velocity rms values, implying that the 000606 and 020905 layers are in a similar stage of evolution, with the 990806 layer at a later stage. Aside from the relative magnitude of the ratios for the three case, the actual values of $L_o/L_E$ in Table 6 are also consistent with values of $L_o/L_T$ reported by SMC and SM. Comparing to SM, the values near 0.3 for 000606 and 020905 are suggestive of KH billows in the midst of transition to turbulence phase and the value of 0.56 for the 990806 case is indicative of the latter stage of transition.

Since this analysis employs the Ellison scale in lieu of the Thorpe scale, and the scales were found from a horizontal path through the data rather than averaged vertically over the entire layer, its validity may be debatable. However, the reasonable agreement between the magnitude of the length-scale ratio in Table 6 and SM results is encouraging, and provides strong evidence of the KH–CR connection. In particular, the comparisons suggest that strong cliff–ramp structures in the aircraft data seem to be indicators of KH billows in relatively early stages of development, prior to destruction of the cliffs by turbulent diffusion. This raises an interesting question: How long do cliff–ramp patterns persist? If the evolving KH billow is a valid model for turbulence transition in the upper troposphere, then the answer is that they should be observable as long as there are distinguishable braids. One would expect this to span a period between the time the KH billow has rolled up sufficiently to generate steep temperature gradients in the braids, up until the time when turbulent diffusion spreads to the braids and smooths out the gradients. This is difficult to estimate from the pub-
lished DNS results, and remains a subject for further study.

5. Summary

Strong cliff–ramp patterns were identified in aircraft measurements of temperature in the upper troposphere. These showed some similarities to cliff–ramp patterns seen in the atmospheric surface layer and passive scalar laboratory flows, notably the derivative skewness of the temperature. Extracting information regarding the coherent structures responsible for the CR patterns was difficult because only a single horizontal pass through the structures was obtained for each case. Despite this, use of data from multiple probes, estimations of billow heights and aspect ratios, and comparisons with DNS all provided some evidence that the CR structures were generated by KH billows during the turbulent transition phase, though several issues remain to be explored. These include the relatively poor correlation between vertical velocity and temperature near the cliffs and the discrepancy between the estimated vertical angles of the fronts and high aspect ratios of the structures. The authors hope that the analysis in the previous section provides motivation for further experimental campaigns and more detailed comparisons between field data and direct numerical simulations to further our understanding of the development of cliff–ramp patterns in the tropopause region vis-à-vis Kelvin–Helmholtz billow evolution in stably stratified flow.

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