Maximum Entropy Production in Climate Theory

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ABSTRACT

R. D. Lorenz et al. claim that recent data on Mars and Titan show that planetary atmospheres are in unconstrained states of maximum entropy production (MEP). Their model as it applies to Venus, Earth, Mars, and Titan is reexamined, and it is shown that their claim is not justified. This does not necessarily imply that MEP is incorrect, and inapplicable to atmospheres, but it does mean that the difficult and unexplored problem of dynamical constraints on the MEP solution must be understood if it is to be of value for climate research.

1. The MEP hypothesis

Paltridge (1975, 1978) first proposed that Earth's climate structure might be explained from a hypothesis of maximum entropy production (MEP). If correct, this proposal would be of crucial importance to future climate research because it provides the hitherto missing global constraint of the second law of thermodynamics. Subsequent investigations have generally supported Paltridge's work, but not to the degree that MEP is accepted as a useful principle in modern climate research.

A recent review by Ozawa et al. (2003) summarizes the literature since Paltridge's first contribution, and concludes with an optimistic assessment of the value of the MEP hypothesis for climate systems. In my estimation, in addition to Paltridge's work and directly related researches, the most important contributions to Ozawa et al.'s conclusion are the following: first, a theoretical paper by Dewar (2003) demonstrating the correctness of MEP for an unconstrained system (i.e., constrained only by energy and mass conservation); second, 50 yr of theoretical investigations of laboratory flows, based on MEP, which Ozawa et al. interpret as supportive of the hypothesis; and third, a claim by Lorenz et al. (2001) that the horizontal thermal structures of Mars and Titan support the MEP hypothesis.

Dewar's is a statistical theory; his work has not been challenged to my knowledge, although such theories have been frequently and unsuccessfully discussed in the past (Jaynes 1980). The proof is *unconstrained*, apart from energy and mass conservation. Thus for any atmospheric problem in which the fluid equations, planetary rotation, gravitational constraints on vertical structure, thermal inertia, etc. are known to be important, unconstrained MEP is inapplicable. Since fundamental dynamical theory and numerical models indicate that all of these are crucial parameters for climate, it is not surprising that little advance has been made in this field. This point was made by Rodgers (1976) in his comments on Partridge's first paper. It is, therefore, disturbing that Dewar needs to claim successes with climate theory as supportive of his work.

I have a different view from Ozawa et al. (2003) as to the value of the evidence from investigations of laboratory flows, and I shall comment further in section 4. Finally, the investigation of Lorenz et al. (2001) is an order-of-magnitude study using a two-box, unconstrained model with gray emission and absorption. They demonstrate that MEP gives a better account of the horizontal temperature structure for Mars and Titan than one particular scaling approximation. If we are to claim verification for a fundamental thermodynamic principle, however, quantitative agreement with observed data must be demonstrated rather than improvements over debatable approximations.

The primary objective of this paper is limited. In the next two sections I shall examine whether the model of Lorenz et al. (2001) does, in fact, agree with the best data from Venus, Earth, Mars, and Titan. I find that it

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does not. In section 4 I discuss the imposition of constraints on an MEP calculation.

2. A gray, box model for MEP

Following Lorenz et al. (2001) I examine two latitude zones: zone 1 is tropical (30°N–30°S), and zone 2 is extratropical (30°–90°N and 30°–90°S). Each is characterized by a single value of the temperature, T, the incoming solar radiation, S, and the outgoing thermal radiation, E (assumed equal to σT^4 where σ is Stefan's constant). Energy balance requires

$$S_1 + S_2 = E_1 + E_2. \tag{1}$$

The flux of energy between zones is

$$F = S_1 - E_1 = E_2 - S_2. \tag{2}$$

The rate of entropy production created by this flux is

$$\dot{s} = F(1/T_2 - 1/T_1). \tag{3}$$

From (2) and the definition of $E(=\sigma T^4)$

$$\dot{s} = \sigma T_2^3 - \frac{S_2}{T_2} + \sigma T_1^3 - \frac{S_1}{T_1}. \tag{4}$$

For maximum entropy production

$$\frac{d\dot{s}}{dT_1} = \frac{dT_2}{dT_1}\left(3\sigma T_2^2 + \frac{S_2}{T_2^2}\right) + 3\sigma T_1^2 + \frac{S_1}{T_1^2} = 0. \tag{5}$$

From (1) it can be shown that

$$\frac{dT_2}{dT_1} = -\left(\frac{T_1}{T_2}\right)^3. \tag{6}$$

Consequently,

$$\left(\frac{E_1}{E_2}\right)^{5/4} = \left(\frac{S_1}{S_2}\right)\left(\frac{1 + 3E_1/S_1}{1 + 3E_2/S_2}\right). \tag{7}$$

From (1),

$$\frac{S_1}{E_1} = \frac{1 + E_1/E_2}{1 + S_1/S_2}, \tag{8}$$

and similarly for the ratio S_2/E_2. Substituting (8) into (7),

$$\left(\frac{E_1}{E_2}\right)^{1/4} = \left(\frac{S_1}{S_2}\right)\left(\frac{4 + E_2/E_1 + 3S_2/S_1}{4 + E_1/E_2 + 3S_1/S_2}\right). \tag{9}$$

Equation (9) is an algebraic equation for E_1/E_2 in terms of S_1/S_2. The results shown in Table 1 have been obtained from a numerical solution.

This solution is identical to that of Rodgers (1976), except that Rodgers' solution is continuous and requires use of the calculus of variations. Rodgers also conjectured that the solution would not be greatly changed if E were to replace $T(\propto E^{1/4})$ in (3). With the two-box model this yields exactly

$$\frac{E_1}{E_2} = \left(\frac{S_1}{S_2}\right)^{1/2}, \tag{10}$$

the same result as obtained by Rodgers. In the parameter range of solar ratio that occurs in Table 1, there is no significant difference between results from (10) and (9).

Lorenz et al. (2001) linearize both F and E with respect to temperature. They obtain an *approximate* relationship between the linearity coefficients. In the linear limit $E_1 \rightarrow E_2$ and $S_1 \rightarrow S_2$, it can be shown that the solution of Lorenz et al. is the same as (10).

TABLE 1. Solar and thermal emissions for Venus, Earth, Mars, and Titan. The numbers in parentheses are probable errors for measured quantities. Where no errors are indicated, no data were available.

		Fluxes (W m^{-2})		
		E_{equ}	E_{mep}	E_{obs}
Venus	Mean	157.3 (2.2)	157.3	157.3 (3.8)
	Zone 1	191.9 (2.7)	174.8	161.4 (3.8)
	Zone 2	122.8 (1.7)	139.8	152.9 (3.8)
	Zone 1 / Zone 2	1.56 (0.03)	1.25 (0.01)	1.06 (0.04)
Earth	Mean	237.0 (5.9)	237.0	237.0 (5.9)
	Zone 1	300.4 (7.5)	269.7	258.5 (6.5)
	Zone 2	173.6 (4.3)	204.3	215.5 (5.4)
	Zone 1 / Zone 2	1.73 (0.06)	1.32 (0.03)	1.20 (0.04)
Mars	Mean	116.7	116.7	116.7
	Zone 1	140.2	128.7	139.0
	Zone 2	93.2	104.7	94.1
	Zone 1 / Zone 2	1.51	1.23	1.48
Titan	Mean	2.81	2.81	2.81 (0.012)
	Zone 1	3.43	3.12	2.96 (0.012)
	Zone 2	2.20	2.50	2.67 (0.012)
	Zone 1 / Zone 2	1.56	1.25	1.11 (0.06)

3. Data for Venus, Earth, Mars, and Titan

Table 1 shows the incoming solar flux and outgoing thermal flux for Earth, Mars, Venus, and Titan for three different situations: radiative equilibrium ($E_{equ} = S$); MEP (E_{mep}); and observed (E_{obs}). The following procedure was used. The best mean values of S and E were chosen from a study of all of the available data on both incoming and outgoing radiances. Since the two means must be equal [Eq. (1)] the most reliable was chosen and used for both. I place most confidence in the ratios between zones. The emission ratios for MEP were calculated from the solar ratio. From the ratios and the mean fluxes, the MEP fluxes for the two zones separately were calculated. Thermal radiance ratios must lie between 1 and the solar value; a higher value would imply heat transport by motions into tropical regions, which would not be acceptable thermodynamically. The solar ratio for a planet with constant albedo is 1.56, as it is observed to be for Venus, and as it is assumed to be for Titan. Some details of the calculations follow.

a. Venus

The solar constant at Venus is 2624 W m^{-2}, and the planetary albedo is 0.76. The source of both solar and thermal data is Taylor et al. (1983) and Schofield and Taylor (1982). The data are from the Pioneer Venus mission and are only presented for the Northern Hemisphere but Schofield and Taylor express the view on the basis of individual filter measurements that the two hemispheres are similar. Schofield and Taylor assign a probable error of 1.4% to solar measurements and 3.8 W m^{-2} to the thermal radiances. The radiance ratio for MEP is assigned half the error of the solar ratio on the basis of Eq. (10). The difference between the MEP and observed radiance ratios is 0.19 ± 0.04.

b. Earth

The solar constant at Earth is 1373 W m^{-2}, and the albedos are as follows: zone 1, 0.26; zone 2, 0.35. The source of observed data is Vonder Haar and Suomi (1971). These authors state that the accuracy of both solar and thermal radiances is 2%–3%. I have adopted 2.5% in Table 1. The radiance ratio for MEP is assigned half the error of the ratio for solar radiance on the basis of Eq. (10). The difference between measured and MEP values for the thermal radiance ratio is 0.12 ± 0.05.

c. Mars

The solar constant at Mars is 588.98 W m^{-2}. Data on albedos and radiances are based on unpublished data from the Thermal Emission Spectrometer (TES) instrument on *Mars Orbiter*, for which I am indebted to M. D. Smith (2005, personal communication). Averages were taken over one complete Martian year that had no major dust storms. No data on errors were available. Measurements are available at two times of day only, 0200 and 1400 h, and these were averaged. North and South Hemispheres and all longitudes were also averaged. A small correction was applied for the incomplete TES spectral coverage. The solar data should be as reliable as for Venus and the error in the MEP radiance ratio should also be about the same. The thermal radiances are, however, subject to huge diurnal changes and averages of 0200 h and 1400 h probably differ significantly from the true mean (aliasing), mainly due to the amplitude of the semidiurnal temperature variation. As a result of aliasing, the global mean thermal radiances exceed the global mean solar radiances in the TES data even though both measurements are of high quality. In Table 1 I have scaled the thermal radiances (keeping the zone 1 to zone 2 ratio constant) to make the average equal to the average for the solar radiances. The error involved in this uncertainty can be discussed but cannot be evaluated objectively. On the basis of the measured semidiurnal pressure tide of about 1% of the total pressure (Zurek et al. 1992, their Fig. 2) the amplitude of the semidiurnal thermal tide is negligible. However, the solid surface semidiurnal amplitudes are larger. On the basis of the early theoretical treatment of Gierasch and Goody (1967) the aliasing could be so large that, if applied to one zone only, it could perhaps remove the discrepancy between the measured (1.48) and MEP (1.23) radiance ratios. This is unlikely but cannot be ruled out. The observed radiance ratio of 1.48 receives some support from numerical climate model calculations of the equator-to-pole dynamical heat fluxes. From Zurek et al. (1992, their Fig. 22) an estimate of the globally averaged poleward flux at 30° for clear skies is, when distributed over one climate zone, 4 W m^{-2}, which would result in a radiance ratio of 1.4.

d. Titan

The solar constant at Titan is 15.2 W m^{-2} and the albedo is 0.26 (Samuelson 1983). The ratio of solar radiances is based on a sphere with constant albedo. Measured radiances at a limited range of frequencies exist for several latitudes on Titan but the data are insufficient to yield the total thermal radiance. Instead Samuelson et al. (1997) have derived surface air temperatures by means of best fits to models with up to 10 free parameters. I have used the temperatures for a four-parameter model given in their Fig. 21. Error bars in this figure and information from their Table 1 suggest a temperature error of approximately 1 K, and the errors in Table 1 of this paper are estimated on this basis. Surface temperatures should be higher than emission temperatures. The global emission based on surface temperatures is 3.94 W m^{-2} while the global solar absorption is 2.81 W m^{-2}. I have scaled radiances derived from the near-surface temperatures to give the correct global solar radiance. The temperature decrease is

7 to 8 K, which is consistent with the opacity data given Samuelson et al. If the solar radiance errors for Titan are similar to those for Venus, as is plausible, the error in the MEP radiance ratio is trivial compared to the error in the observed thermal radiance ratio. The difference between MEP and observed radiance ratios is therefore estimated to be 0.14 ± 0.06.

Taken together, these data for Earth, Venus, Mars, and Titan do not give quantitative support to the hypothesis of unconstrained MEP states. The data for Earth and Venus are not consistent with unconstrained MEP. Mars data may also be inconsistent, but the problem of aliasing diurnal changes needs more attention. The Titan data appear to be inconsistent but the indirect method of inferring thermal radiances leaves room for larger errors than I have estimated.

This does not dispose of the matter. Even this limited model can be improved by using temperatures continuously changing with latitude, an atmosphere with vertical structure, and more sophisticated nongray radiative transfer. In addition, the question of imposing appropriate dynamical constraints remains open. But the question under discussion is whether the data analyzed with a gray, two-box, unconstrained model support the MEP hypothesis for planetary atmospheres. They evidently do not.

4. Constraints on MEP

If MEP is a correct thermodynamic principle the crux of a useful application is to identify and incorporate in a calculation the constraints appropriate to the problem. The more constraints that are applied, the smaller becomes the parameter space containing the solution and, for practical purposes, the solution may be achieved without invoking MEP at all. The choice of which, and how many, constraints to apply is a subtle one.

Paltridge (1975) and Pujol and Fort (2002) have both combined MEP with empirical climate constraints. Paltridge (1975) treated the entire climate system with MEP subject to enough empirical constraints to reduce the problem to a single variable (the horizontal heat flow). The full significance of these constraints is hard to understand, although the solution is impressive. Pujol and Fort (2002), in an interesting variation on conventional radiative–convective modeling, discuss a tropospheric model with all of the usual constraints, but with the surface temperature discontinuity maintained as a free parameter, to be determined from MEP. In both of these cases the constraints are not imposed analytically and may not respond correctly to parametric variations so that the fundamental significance of these treatments is difficult to evaluate.

The correct way to proceed is to be found in 50 yr of efforts to solve laboratory flow problems using MEP. Rayleigh convection and shear flows can be measured in the laboratory with precision and parameters can be varied in a controlled manner. In addition, the theoretical problems are well defined and far simpler than the climate problem. In the first application of MEP to the Rayleigh convection problem, Malkus (1954) introduced constraints (rigid boundaries, positive definite time-mean temperature gradients, a smallest scale of motion derivable from stability theory) to obtain a partial solution. Since then there have been many attempts to obtain solutions for laboratory heat flows and shear flows incorporating constraints imposed by the Navier–Stokes and continuity equations. The mathematical treatments are usually very complex, sometimes elegant, and interesting qualitative behaviors have been obtained, but quantitative agreement of the kind that would be required for climate prediction has not yet been demonstrated (see Howard 1972 and Busse 1978 for some early work, and Doering and Hyman 1997 and Kerswell 1998, 2002 for more recent contributions). Both Howard and Busse express serious reservations about the value of MEP for simple flows.

These are the reasons for my doubts concerning the lessons from laboratory flows that I expressed in section 1. In recent years fluid dynamicists (e.g., Kerswell 1998) have explored extrema other than MEP, and Malkus (2003) has expressed a nuanced view based on a principle of statistical stability. He claims quantitative, objective results based on this principle, and shows that statistical stability leads to a thermodynamic statement that is similar to, but not the same as, MEP.

Another aspect of constraints on climate systems is the mix of steady and fluctuating motions that is known to be present on Earth and Mars, and is probably present in all planetary atmospheres. Both component motions contribute to the meridional heat flux, but only the fluctuating component can be subject to MEP. To apply MEP to the whole heat flux will certainly lead to error. On Earth, the two classes of motion have different structures and it is difficult to speak of a simple ratio between the two processes, but Lorenz (1967, see his Fig. 46) suggests approximate equality. More recent data from Peixoto and Oort (1992) suggest that chaotic motions (see their Fig. 13.11) transport about twice as much heat as steady-state circulations and standing eddies. The atmospheric dynamics of Mars is now fairly well observed and understood; see Zurek et al. (1992). Recent work with general circulation models incorporates, in addition to familiar terrestrial transports, a pole-to-pole condensation flow of carbon dioxide. One series of calculations gives the maximum meridional energy fluxes expressed as a fraction of the solar flux: planetary-scale, steady-state motions, 3.3%; condensation flow, 2.4%; standing eddies, 0.9%; transient eddies, 1.4%. Only transient eddies (less than one-fifth of the total) might be subject to MEP. Venus rotates very slowly, which allows a direct circulation to extend to the poles (see Gierasch et al. 1997 for a review of Venus dynamics). Not only can this steady-state circulation account for the observed small temperature contrast, but steady-state theories tend to predict an atmosphere that is even more isothermal than is observed. As matters stand there is no demand for transport by chaotic motions to explain the Venus climate.

I conclude that there is little evidence in support of unconstrained MEP for calculating climate states of Earth and the other planets. MEP may be helpful as a replacement for scaling arguments in order-of-magnitude planetary discussions. It may be useful for treating individual climate processes, such as small-scale turbulence, but experience with laboratory flows suggests that this may be very difficult. It is possible that MEP could be used in a diagnostic mode, for example, to choose between alternate, independent climate projections. But Malkus' use of a statistical stability criterion might be as useful, and closer to the traditional reasoning of the atmospheric community.

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