Comparison of Bulk and Bin Warm-Rain Microphysics Models Using a Kinematic Framework

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(Manuscript received 5 May 2006, in final form 15 November 2006)

ABSTRACT

This paper discusses the development and testing of a bulk warm-rain microphysics model that is capable of addressing the impact of atmospheric aerosols on ice-free clouds. Similarly to previous two-moment bulk schemes, this model predicts the mixing ratios and number concentrations of cloud droplets and drizzle/raindrops. The key elements of the model are the relatively sophisticated cloud droplet activation scheme and a comprehensive treatment of the collision–coalescence mechanism. For the latter, three previously published schemes are selected and tested, with a detailed (bin) microphysics model providing the benchmark. The unique aspect of these tests is that they are performed using a two-dimensional prescribed-flow (kinematic) framework, where both advective transport and gravitational sedimentation are included. Two quasi-idealized test cases are used, the first mimicking a single large eddy in a stratocumulus-topped boundary layer and the second representing a single shallow convective cloud. These types of clouds are thought to be the key in the indirect aerosol effect on climate. Two different aerosol loadings are considered for each case, corresponding to either pristine or polluted environments. In general, all three collision–coalescence schemes seem to capture key features of the bin model simulations (e.g., cloud depth, droplet number concentration, cloud water path, effective radius, precipitation rate, etc.) for the polluted and pristine environments, but there are detailed differences. Two of the collision–coalescence schemes require specification of the width of the cloud droplet spectrum, and model results show significant sensitivity to the specification of the width parameter. Sensitivity tests indicate that a one-moment version of the bulk model for drizzle/rain, which predicts rain/drizzle mixing ratio but not number concentration, produces significant errors relative to the bin model.

1. Introduction

Clouds and their impact on the transfer of solar (shortwave) and thermal (longwave) radiation are the most challenging aspect of climate and climate change (e.g., Stephens 2005 and references therein). One of the most uncertain aspects is the indirect effect of atmospheric aerosols (e.g., Rostayn and Liu 2005; Lohmann and Feichter 2005). The indirect impact of aerosols on climate concerns the influence through cloud processes. These aerosol indirect effects have been hypothesized to impact the radiative properties of clouds in several ways, including the impact on droplet size and hence the optical depth (the first effect: Twomey 1974, 1977), and impact on liquid water path, cloud lifetime, and extent (second effect; e.g., Albrecht 1989). Estimates of the combined global first and second indirect effects exhibit a wide range of values in recent general circulation model (GCM) studies, from about $-1.0$ to $-4.4\text{ W m}^{-2}$ (Rostayn and Liu 2005, and references therein).

A major source of difficulty in assessing the indirect aerosol effect using GCMs is that they must rely on subgrid parameterizations to represent clouds and cloud processes, convection in particular. These parameterizations are able to capture some key characteristics of clouds but struggle to represent interactions between cloud processes and other components of the climate system (e.g., radiative transfer, surface processes). As discussed by Grabowski (2006a), these multiscale interactions and their impact on the dynamics and climate can be studied with better confidence using models that

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* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

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DOI: 10.1175/JAS3980

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are able to resolve convective-scale and mesoscale processes [e.g., cloud system–resolving models (CRMs)]. Although such models have to parameterize small- and microscale processes, the removal of issues directly concerning the convective and cloud-scale dynamics is a major step forward. Convection-resolving resolutions are already feasible for continental-scale numerical weather prediction [e.g., the Weather Research and Forecasting (WRF) Model; see information online at http://www.wrf-model.org]. For climate, convection-resolving modeling of atmospheric general circulation is in its infancy, with the first global CRM simulation (only 10 days long) recently completed in Japan (Tohmita et al. 2005). CRMs are also used as “superparameterization” of small- and mesoscale processes in GCMs, an approach proposed by Grabowski and Smolarkiewicz (1999; see also Grabowski 2001) and already proven useful in either realistic (e.g., Khairoutdinov and Randall 2001; Khairoutdinov et al. 2005) or idealized (Grabowski 2003, 2006b) simulations of atmospheric general circulation.

Most CRMs rely on bulk cloud microphysics schemes. As far as the indirect effects are concerned, these schemes must parameterize the droplet effective radius and coalescence rate (including drizzle/rain formation) using some functional form of the particle size distributions. It follows that the use of bulk microphysics schemes introduces additional uncertainty in assessing the first and second indirect aerosol effects. Bin-resolving microphysics models, on the other hand, explicitly calculate the particle size distribution and therefore provide more rigorous solutions than bulk models. However, the computational cost associated with bin microphysics is significantly higher than bulk schemes, and there are still unresolved issues related to the application of such models to CRMs with relatively low spatial resolutions [e.g., droplet nucleation, see Saleeb and Cotton (2004); or the impact of entrainment and mixing on cloud droplet spectra, see discussion in Grabowski (2006a)]. It follows that bulk microphysics schemes are currently the only viable approach for many applications, in particular for the cloud-resolving and superparameterized GCMs.

A recent improvement in bulk microphysics schemes has been the prediction of two moments of the hydrometeor size spectra rather than just one (i.e., the mixing ratio). Several such schemes have already been developed and used in a variety of applications (e.g., Koenig and Murray 1976; Ferrier 1994; Meyers et al. 1997; Khairoutdinov and Kogan 2000, hereafter KK2000; Seifert and Beheng 2001, hereafter SB2001; Milbrandt and Yau 2005; Morrison et al. 2005). These two-moment schemes predict the number concentration and mixing ratio of the hydrometeor species, which increases the degrees of freedom and improves representation of the microphysical processes. The prediction of cloud particle number concentration and explicit treatment of droplet activation from a distribution of aerosol in a two-moment scheme can potentially provide more physically robust estimates of the indirect aerosol effect than allowed using simpler schemes (see discussion in Grabowski 2006a).

In this study, a two-moment warm-rain bulk microphysics scheme is developed. Various formulations for coalescence processes described in the literature are implemented and tested. These results are compared to simulations using a detailed bin-resolving microphysics scheme. Evaluation of bulk parameterizations against bin models has been a traditional approach (e.g., Beheng 1994, hereafter B1994; Berry and Reinhardt 1973; SB2001). Wood (2005b) tested various autoconversion and accretion parameterizations using rates derived from numerical solution of the collision–coalescence equation combined with observed particle size distributions. Here we use a kinematic (i.e., prescribed flow) model to test the bulk scheme in a two-dimensional (2D) flow field allowing for advective and gravitational transport of hydrometeors. The kinematic framework allows for testing of the schemes in a realistic flow field without added complexities associated with dynamical–microphysical feedbacks. This framework allows us to examine how different formulations of the coalescence processes impact results when all relevant microphysical and transport processes are included; most of the previous studies have tested different formulations in a parcel or 1D framework and neglected other microphysical processes. Two sets of simulations are performed, corresponding to polluted and pristine aerosol conditions. In addition, two different configurations of the kinematic model are employed, mimicking microphysical processes in a drizzling stratocumulus cloud and in a heavily precipitating shallow cumulus. Shallow cumulus and stratocumulus are important components of the climate system, and as such are expected to play a critical role in the climate impact of the first and second indirect aerosol effects. The goal is to characterize uncertainties and validate the bulk approach for modeling shallow cumulus and stratocumulus in the context of different aerosol regimes.

The paper is organized as follows. The next section briefly describes the bulk and bin microphysics models. Section 3 gives an overview of the kinematic model and its configuration for the stratocumulus and cumulus regimes. Section 4 presents results. Concluding discussion is given in section 5.
2. Model description

a. Bulk microphysics scheme

A bulk two-moment, warm-rain microphysics scheme has been developed based on the approach of Morrison et al. (2005). This scheme predicts the number concentrations \( N_c \) and mixing ratios \( q_c \) of cloud droplets (subscript \( c \)) and drizzle/rain (subscript \( r \)). Cloud droplets and drizzle/raindrops are assumed to follow gamma size distributions,

\[
N(D) = N_0 D^{\mu} e^{-\lambda D},
\]

where \( D \) is diameter, \( N_0 \) is the “intercept” parameter, \( \lambda \) is the slope parameter, and \( \mu = 1/\eta^2 - 1 \) is the spectral shape parameter (\( \eta \) is the relative radius dispersion: the ratio between the standard deviation and the mean radius). Parameters \( N_0 \) and \( \lambda \) are derived from the specified \( \mu \) and predicted number concentration and mixing ratio of the species (see Morrison et al. 2005). Drizzle/raindrops are assumed to follow a Marshall–Palmer (exponential) size distribution, implying \( \mu = 0 \).

Wood (2005b) shows that the exponential distribution for drizzle/rain agrees well with observations for stratocumulus.

The parameter \( \mu \) for cloud droplets is specified from \( \eta \) following Martin et al. (1994). Their observations of maritime versus continental warm stratocumulus have been approximated by the following \( \eta - N_c \) relationship:

\[
\eta = 0.000\,571\,4N_c + 0.2714,
\]

where \( N_c \) is the cloud droplet number concentration (cm\(^{-3}\)). The upper limit for \( \eta \) is specified to be 0.577, corresponding to \( N_c = 535 \) cm\(^{-3}\) using (2). A quantitative understanding of the relationship between \( N_c \) and \( \eta \) remains uncertain. A decrease of \( \eta \) with \( N_c \) for given flight paths is evident from observations of stratocumulus described by Pawlowska et al. (2006). Note that this \( \eta - N_c \) dependence is opposite in sign to that of (2). The decrease of \( \eta \) with increasing \( N_c \) for a given set of aerosol characteristics due to vertical velocity fluctuations is consistent with the analytical expression for \( \eta \) derived by Liu et al. (2006). Note that parameters of \( \eta \) used in large-scale models (e.g., Rotstain and Liu 2003) are typically based on mean microphysical parameters and therefore do not include this local variability of \( \eta \). Some limited testing is done here to highlight sensitivities to \( \eta \) using the \( \eta - N_c \) relationship from Grabowski [1998; Eq. (9)], which gives the opposite sign of the change in \( \eta \) with \( N_c \) compared to (2):

\[
\eta = 0.146 - 5.964 \times 10^{-2} \ln \left( \frac{N_c}{2000} \right).
\]

The droplet effective radius \( r_e \), which is the relevant parameter for determining the cloud optical properties, is obtained by dividing the third by the second moment of the droplet size distribution given by (1), giving

\[
r_e = \frac{\Gamma(\mu + 4)}{2\lambda\Gamma(\mu + 3)},
\]

where \( \Gamma \) is the Euler gamma function.

The evolution of \( N_c \) and \( q_c \) for each species is given by

\[
\frac{\partial N}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a(u - V_N)k]N = \mathcal{J}_N = \left( \frac{\partial N}{\partial t} \right)_{\text{act}} + \left( \frac{\partial N}{\partial t} \right)_{\text{cond}} + \left( \frac{\partial N}{\partial t} \right)_{\text{acc}} + \left( \frac{\partial N}{\partial t} \right)_{\text{auto}} + \mathcal{D}(N),
\]

\[
\frac{\partial q}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a(u - V_q)k]q = \mathcal{J}_q = \left( \frac{\partial q}{\partial t} \right)_{\text{act}} + \left( \frac{\partial q}{\partial t} \right)_{\text{cond}} + \left( \frac{\partial q}{\partial t} \right)_{\text{acc}} + \left( \frac{\partial q}{\partial t} \right)_{\text{auto}} + \mathcal{D}(q),
\]

where \( u \) is the wind velocity vector, \( \rho_a \) is the air density profile, \( V_N \) and \( V_q \) are the number- and mass-weighted mean particle fall speeds, respectively, \( k \) is a unit vector in the vertical direction, and

\[
\mathcal{D} = \frac{1}{\rho_a \partial \mathcal{X}} \rho_a K \frac{\partial}{\partial \mathcal{X}}
\]

is the horizontal diffusion operator mimicking turbulent mixing in natural clouds with the mixing coefficient \( K \) given as \( K = 0.2E^{1/2} \Delta x \) [where \( E \) is the turbulent kinetic energy, assumed 1 m\(^2\) s\(^{-2}\) for the stratocumulus case and 4 m\(^2\) s\(^{-2}\) for the cumulus case, and \( \Delta x \) is the horizontal grid length; cf. Eq. (2.26) in Klemp and Wilhelmson (1978)]. Results are fairly insensitive to the specified value of \( E \). The symbolic terms on the right-hand side of (5) and (6) represent the source/sink terms for \( N_c \) and \( q_c \). These include activation of aerosol (subscript act; cloud water only), condensation/evaporation (subscript cond), accretion of cloud droplets by rain (subscript acc), autoconversion of cloud droplets to rain (subscript auto), and self-collection of cloud water and rain (subscript self; \( N \) only). Autoconversion re-
sults from the artificial separation between cloud droplets and drizzle/rain in bulk schemes, and does not correspond to any real physical process. Self-collection represents coalescence of particles such that the resulting particle remains within the same hydrometeor category (i.e., leading to a change in \( N \) but not \( q \)).

Collectively, autoconversion, accretion, and self-collection will be referred to as the coalescence processes. Three different parameterizations for the coalescence processes are implemented into the present scheme and tested here. These parameterizations include B1994, SB2001, and KK2000. These schemes have been chosen here because they include tendencies for both \( N \) and \( q \) resulting from the coalescence processes. All three schemes are based on numerical solution to the collision–coalescence equation using a detailed (bin) microphysics approach. B1994 is based on curve fitting of bin-resolving parcel simulations, while KK2000 was developed from curve fitting of large-eddy model simulations of a stratocumulus-topped marine boundary layer. SB2001 developed explicit rate equations based on the collision–coalescence equation using the Long (1974) collection kernel and universal functions based on similarity arguments. The universal functions were estimated from numerical solution using bin microphysics. Since the SB2001 scheme was developed using droplet distribution functions in mass rather than size space, a corresponding gamma mass distribution is derived for a given gamma size distribution from (1). This is done by assuming equal \( q, N \), and \( \eta \) between the gamma representations of the mass and size distributions, similar to the approach of B1994. Under this assumption, there is a relationship between the width parameter in size space \( \nu \) and the equivalent width parameter in mass space \( \nu \):

\[
\mu = \left[ \frac{\Gamma \left( \nu + \frac{5}{3} \right) \Gamma (\nu + 1)}{\Gamma \left( \nu + \frac{4}{3} \right)^2} - 1 \right]^{-1} - 1. \tag{7}
\]

Note that the coalescence terms using B1994 and SB2001 depend explicitly on \( q_c, N_c, q_r, N_r \), and \( \mu \). Rates using KK2000 depend explicitly on \( q_c, N_c, N_r \), and \( q_r \) only. Since self-collection of rain is not included in the KK2000 parameterization, the B1994 formulation is used to complete this parameterization. Wood (2005b) describes in detail differences in the autoconversion and accretion rates for several different schemes. The focus here is to determine how these differences translate into differences in the simulated cloud micro- and macroscopic properties when all microphysical processes are simultaneously considered together with advective and gravitational transport.

Condensation/evaporation and droplet activation require explicit treatment of the supersaturation field. In most bulk schemes supersaturation is neglected, instantaneous adjustment to water-saturated conditions is used to calculate condensation/evaporation of cloud water, and droplet nucleation is parameterized based on the vertical velocity near the cloud base using bulk characteristics of aerosol particles. Herein, supersaturation is predicted, which typically requires small model time steps (around 1 s) to capture rapid changes in supersaturation resulting from droplet growth. The change in \( q \) resulting from droplet or drizzle/rain condensation/evaporation is

\[
\frac{\partial q}{\partial t}_{\text{cond}} = \frac{q_v - q_s}{\tau} \left( 1 + \frac{dq_r L_v}{dT c_p} \right)^{-1}, \tag{8}
\]

where \( q_v \) is the water vapor mixing ratio, \( q_s \) is the water vapor mixing ratio at saturation, \( L_v \) is the latent heat of vaporization, \( c_p \) is the specific heat of air at constant pressure, and \( \tau \) is the phase relaxation time associated with each hydrometeor species, which depends on the mean particle size and number as well as environmental conditions (see Morrison et al. 2005).

The reduction of \( N_c \) and \( N_r \) during evaporation follows from KK2000. For resolved, adiabatic vertical motion, \( N_c \) decreases by evaporation only when \( q_c \) falls below a small threshold value and when the supersaturation is negative. Different formulations for the evaporation of \( N_c \), more appropriate under nonadiabatic conditions (e.g., reducing \( N_c \) to maintain constant mean volume radius), produce fairly small changes in the domain-average cloud characteristics (e.g., cloud water path, cloud optical depth, etc.) even when applied to the resolved vertical motion for the cases here. During evaporation \( N_r \) is reduced so that a constant mean drizzle/raindrop size is maintained. The appropriateness of this assumption for cloud types other than stratocumulus, especially for evaporation of rain in low relative humidity environments (i.e., when the cloud base is elevated), will be explored in future work. This may involve modification of the approach in such a way as to change the mean drizzle/raindrop size according to the environmental conditions.

The droplet activation parameterization is developed by applying Kohler theory to a lognormal aerosol size distribution of aerosol \( f_a \):

\[
f_a = \frac{dN_a}{dr_d} = \frac{N_t}{\sqrt{2\pi} \ln \sigma_{d,t}} \exp \left[ - \frac{(\ln r_d - \ln r_{d,0})^2}{2 \ln^2 \sigma_d} \right], \tag{9}
\]

where \( r_d \) is the dry aerosol radius, \( N_t \) is the total aerosol number, \( \sigma_d \) is the standard deviation, and \( r_{d,0} \) is the geometric mean radius of the dry particles.
The number of activated cloud condensation nuclei (CCN) \( N'_c \), as a function of supersaturation \( s = q_s/q_s^* - 1 \) is (Khvorostyanov and Curry 2006)

\[
N'_c = \frac{N_r}{2} \left[ 1 - \text{erf}(u) \right] ; \quad u = \frac{\ln(s/s^*)}{\sqrt{2} \ln s_0},
\]

where erf is the Gaussian error function, and

\[
s_0 = r_{ab}^{-1(1+\beta)} \left( \frac{4A^3}{27b} \right)^{1/2},
\]

\[
\alpha_s = a_s^{(1+\beta)}.
\]

Here \( A \) is the Kelvin parameter and \( b \) and \( \beta \) depend on the chemical composition and physical properties of the soluble part of the dry aerosol (see Khvorostyanov and Curry 2006).

Since aerosol is not predicted in the present scheme, the number of aerosol previously activated is assumed to be equal to the number of pre-existing droplets, \( N_c \). The actual activation rate is therefore given by

\[
\left( \frac{\partial N_c}{\partial t} \right)_{act} = \max \left( \frac{N'_c - N_c}{\Delta t}, 0 \right),
\]

where \( \Delta t \) is the length of the model time step. Note that in regions with significant collision–coalescence, the resulting depletion of \( N_c \) could result in overprediction of droplet activation with this approach (probably an issue relevant mainly to in-cloud rather than cloud base activation). Future work will involve testing this simplified approach versus the traditional approach of adding another prognostic variable to track CCN depletion. The change in \( q_s \) resulting from activation is calculated by assuming that newly formed droplets have an initial radius of 1 \( \mu \)m, following KK2000. The results were insensitive for values of initial radius between 1 and 3 \( \mu \)m. Gas kinetic effects on droplet growth (i.e., accommodation coefficient) are neglected in both bin and bulk models for simplicity. The accommodation coefficient may impact the phase relaxation time scale and hence droplet activation rate. However, the main point here is to have a consistent treatment between the bin and bulk models.

Terminal fall speeds (in the bin as well as in the bulk models) are calculated using the data of Gunn and Kinzer (1949) and Beard (1976) modified by Simmel et al. (2002). Since raindrop fall speed parameters vary widely as a function of drop diameter over the size distribution, lookup tables for \( V_N \) and \( V_g \) are used for the bulk scheme.

b. Bin microphysics scheme

A warm-rain bin-resolving microphysics scheme has been developed to provide a benchmark for testing the bulk scheme. The bin-resolving scheme solves the equation for the spectral density function \( \phi(r, x) = dN(r)/dr \), where \( dN \) is the concentration of droplets/drops at spatial location \( x \) and in the radius interval \( r, r + dr \). The equation is

\[
\frac{\partial \phi}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a(u - V(k))\phi] + \frac{\partial}{\partial r} \left( \frac{dr}{dt} \right)\phi
\]

\[
= \left( \frac{\partial \phi}{\partial t} \right)_{act} + \left( \frac{\partial \phi}{\partial t} \right)_{coal} + D(\phi),
\]

where the third term on the left-hand side represents growth of droplets by condensation of water vapor \( (dr/dt) \) is the rate of change of droplet radius \( r \), and the two terms of the right-hand side represent sources due to droplet activation, collision–coalescence, and horizontal diffusion, respectively.

In the discrete system consisting of \( N \) bins (or classes) of droplet/drop sizes, the spectral density function for each bin \( i \) is defined as \( \phi^{(i)} = N^{(i)}(r^{(i)}) \), where \( N^{(i)} \) is the concentration of droplets/drops in the bin \( i \) and \( r^{(i)} \) is the width of this bin. This transforms the continuous equation (14) into a system of \( N \) coupled equations:

\[
\frac{\partial \phi^{(i)}}{\partial t} + \frac{1}{\rho_a} \nabla \cdot [\rho_a(u - V^{(i)})\phi^{(i)}] \]

\[
= \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{act} + \left( \frac{\partial \phi^{(i)}}{\partial t} \right)_{coal} + D(\phi) \text{ for } i = 1, \ldots, N
\]

where the first term on the right-hand side represents the condensational growth term in (14) (i.e., the advective transport in radius space). The activation term is relevant only for the bin corresponding to the initial droplet radius (specified here to be 1 \( \mu \)m, consistent with the bulk model). The coalescence term consists of two separate terms—the first one representing a source of droplets in bin \( i \) owing to collisions of two droplets from different bins that result in formation of a droplet in bin \( i \) and a sink term that represents collisions of droplets from bin \( i \) with all other droplets (e.g., Pruppacher and Klett 1997). Breakup of raindrops is neglected for simplicity since we investigate only stratocumulus and shallow cumulus. We note that previous modeling studies have suggested that breakup has an impact on the production of giant drops (>4 mm) under some conditions (Reisin et al. 1998).

For the stratocumulus case, 54 bins between 1 and 835 \( \mu \)m are employed. For the cumulus case (owing to
the occurrence of larger drops), 69 bins are used, extending to 5512 μm. The grid in radius space is linear–exponential, with the mean radius \( r_i \) (μm) for each bin \( i \) given by

\[
r_i = 0.25(i - 1) + 10^{0.055(i - 1)}
\]

(16)

with linear spacing important for small sizes (less than about 15 μm) to minimize spectral dispersion during condensational growth and exponential spacing dominant for larger radii to provide a stretched grid incorporating drizzle/rain sizes. Some limited testing was performed with increased bin resolution (50% more bins over the same size range) to document that the grid spacing given by (16) provides a reasonable approximation to the microphysical processes in both stratocumulus and cumulus regimes. In terms of the bin model output, drops with radius less than 40 μm are assumed to be cloud droplets, those greater than this size are assumed to be drizzle/rain. This size separation is generally consistent with the various coalescence parameterizations implemented into the bulk model.

The system (15) is integrated on split time steps, with advective and gravitational transport together with coalescence calculated on each primary time step, and condensation/evaporation treated as advection in radius space over variable sub–time steps (to ensure numerical stability in terms of the Courant–Friedrichs–Levy criterion) using the 1D advection scheme of Smolarkiewicz (1984). The tabulated values of Hall (1980, and references therein) are used to specify collection efficiencies over the range of droplet sizes. Numerical solution of the collision–coalescence term is provided by the linear flux method (Bott 1998). The treatment of droplet activation is the same as in the bulk scheme.

### 3. Kinematic model configuration

The bin and bulk microphysics schemes were implemented in a 2D kinematic modeling framework, presented in Szumowski et al. (1998) and subsequently applied in Grabowski (1998, 1999). The kinematic framework employs a specified flow field, which allows for testing of the microphysics in a framework that includes advective transport and droplet sedimentation. In addition to the equations describing conservation of the condensed water and drizzle/rain discussed above, the kinematic model solves equations for the potential temperature and water vapor mixing ratio. These equations include advective transport, sinks/sources due to condensation/evaporation and latent heating, and additional sources/sinks needed to obtain the quasi-equilibrium conditions (e.g., large-scale advection).

Transport in the physical space is calculated using the 2D version of the multidimensional positive-definite advection transport algorithm (MPDATA) scheme (Smolarkiewicz 1984; Smolarkiewicz and Margolin 1998). Entrainment of clear air into the clouds is neglected. The primary model time step (substepped as needed for condensation/evaporation as described above) is 1 and 0.5 s for stratocumulus and cumulus cases, respectively. Sensitivity tests using a time step of 0.25 s produce little difference in the results.

Two sets of simulations corresponding with polluted and pristine aerosol environments are run for each configuration described below. Here, we assume a monomodal lognormal aerosol distribution following (9). The mean radius is 0.05 μm and the geometric standard deviation is 2 (unitless). The parameter \( \beta \) is 0.5, meaning that the soluble fraction is a function of particle volume. The parameter \( b \) is given by Khvorostyanov and Curry (1999), assuming that the soluble portion of the aerosol consists of ammonium sulfate. For the pristine case, the total aerosol number concentration is \( N_r = 100 \text{ cm}^{-3} \) (hereinafter referred to as PRISTINE); for the polluted case, the total aerosol concentration is \( N_r = 1000 \text{ cm}^{-3} \) (referred to as POLLUTED). Note that changes in the aerosol resulting from scavenging and other cloud–aerosol processes may play an important role in the indirect aerosol effects (e.g., Feingold et al. 1999). However, here we focus on differences between the bin and bulk model results, and the background aerosol is assumed to be constant in time and space in both the bin and bulk models. Neglecting cloud–aerosol feedbacks considerably simplifies the approach and allows us to focus on the cloud microphysical formulations and coalescence processes in particular.

#### a. Stratocumulus regime

For the stratocumulus case, the grid spacing is 20 m in the vertical and horizontal directions, with periodic lateral boundary conditions. The domain size is 2 km wide and 1 km deep. Initial conditions are based on an idealized stratocumulus-topped boundary layer, with constant equivalent potential temperature of 288 K. The flow field is time invariant and consists of a single eddy spanning the entire depth of the computational domain, with maximum values of the vertical velocity \( -1.7 \text{ m s}^{-1} \) (Fig. 1). A large-scale moisture flux \([\text{latent heat flux (LHF)}]\) into the domain equivalent to either 3 or 30 W m\(^{-2}\) is applied to the water vapor mixing ratio field, with a sensible heat flux (SHF) of \(-3\) or \(-30\) W m\(^{-2}\) similarly applied to the temperature field to maintain a quasi balance of heat in the domain.
ture fluxes are applied evenly across the domain so that the tendencies of $q_v$ and $T$ owing to these fluxes are

$$\frac{\partial q_v}{\partial t_{\text{fix}}} = \frac{\text{LHF}}{L_v \rho_a H},$$

(17)

$$\frac{\partial T}{\partial t_{\text{fix}}} = \frac{\text{SHF}}{c_p \rho_a H},$$

(18)

where $H = 1000$ m is the height of the domain.

Simulations are run to the point of near equilibrium, generally requiring a time integration of about 6–12 hours. Note that in equilibrium values of LHF of 3 and 30 W m$^{-2}$ correspond to a precipitation rate of about 0.1 and 1 mm day$^{-1}$, respectively. This configuration is intended to approximate the response of the cloud microphysics to aerosol in a slowly developing cloud system that maintains near equilibrium with its large-scale environment. The initial total water mixing ratios and LHF are varied in the simulations to hasten the integration time required to attain equilibrium. These modifications have negligible impact on the equilibrium cloud microphysics attained in the various runs. The initial cloud water is given by a simple saturation adjustment; no drizzle/rainwater is initially present.

The specific forcing applied in the stratocumulus case results in a strong link between microphysical processes and macroscopic characteristics of a simulated cloud. In essence, the quasi-equilibrium cloud has to be sufficiently deep to provide a precipitation rate that balances the prescribed LHF forcing. For instance, the PRISTINE cloud is anticipated to be shallower than the POLLUTED cloud (i.e., featuring a higher cloud base, as the cloud top is prescribed by the depth of the domain). It follows that the differences in microphysical processes (i.e., autoconversion of cloud water into rain) feed back into the cloud depth and other macroscopic properties (liquid water path, optical depth, etc.). The differences between these properties will be the key in the analysis of model results.

b. Shallow cumulus regime

For the cumulus case, the flow field varies in time, representing the development of a quasi-idealized shallow convective plume. This case was developed and described in detail in Szumowski et al. (1998) and was simulated using a detailed microphysics model by Reisin et al. (1998). Initial thermodynamic profiles (Fig. 2) are based on aircraft data from 10 August 1990 during the Hawaiian Rainband Project (HaRP) (Szymowski et al. 1998). These profiles represent typical upstream summertime conditions off the windward shore of Big Island, Hawaii, and include 1) a 200–300-m-deep well-mixed surface layer, 2) lifting condensation level between 600 and 700 m, and 3) a weak inversion or isothermal layer at about 2 km with a dry layer above that limits the vertical development of convection (Szymowski et al. 1998). In situ aircraft observations from the University of Wyoming King Air were made during this case, providing cloud water content data and drop size spectra using a forward-scattering spectrometer probe (FSSP) and a 2D precipitation (2D-P) optical array probe (see Szymowski et al. 1998 for details). The horizontal and vertical grid spacing is 50 m over a domain 9 km wide and 3 km deep. The flow pattern consists of low-level convergence, upper-level divergence, and a narrow updraft in the center of the domain. A detailed description of the time-varying flow field is given in the appendix. The maximum updraft speed is held constant.
at 1 m s\(^{-1}\) for the first 15 min, intensifies to a peak value of 8 m s\(^{-1}\) at 25 min, later decays to a value of 2 m s\(^{-1}\) at 40 min, and is held constant for the remainder of the 60-min simulation. Figure 3 shows the velocity field at 25 min, the time of the maximum updraught. There is no initial cloud water for this regime.

4. Results

a. Stratocumulus case

All simulations are initialized with zero rainwater, but after coalescence begins the precipitation rate eventually balances the influx of water specified through LHF and the simulations attain near equilibrium. The bin and bulk models produce similar results, particularly using KK2000 and SB2001. At equilibrium, \(q_c\) increases with height while \(N_e\) is fairly constant throughout the depth of the cloud (Figs. 4 and 5). This structure is consistent with observations of marine stratocumulus (e.g., Nicholls 1984; Duynkerke et al. 1995; Pawlowska and Brenguier 2000; Wood 2005a). The values of \(q_c, N_e\), and cloud-base height vary considerably between the PRISTINE and POLLUTED simulations even though the overall structure is consistent. The droplet size spectra produced by the bin model vary as expected between the POLLUTED and PRISTINE runs (Fig. 6) and are consistent with observed spectra in boundary layer stratiform clouds (e.g., Wood 2005a).

The domain-averaged equilibrium cloud depth, cloud optical depth, mean (“effective”) effective radius \(r_e\), cloud water path \(CWP = \int_0^H \rho_w q_c \, dz\), and droplet concentration \(N_e\) for the various simulations are compared in Table 1. The effective \(r_e\) is defined following Grabowski (2006a):

\[
r_e = \frac{3}{2} \frac{CWP}{\rho_w \tau_e},
\]

where \(\tau_e\) is the cloud optical depth defined as

\[
\tau_e = \frac{3}{2} \frac{1}{\rho_w} \int_0^H \frac{\rho_w q_c}{r_e} \, dz,
\]

with the integration covering the vertical extent of the computational domain. It has to be stressed that \(CWP, r_e,\) and \(\tau_e\) are calculated independently for each model column and subsequently averaged over all columns. Thus, the domain-averaged \(r_e\) differs from the value obtained simply by using domain-averaged values of \(CWP\) and \(\tau_e\) in (24). Hereafter, the variables represent equilibrium, domain-averaged values unless otherwise stated.

Both the bulk and bin simulations exhibit significant differences between the PRISTINE and POLLUTED runs in terms of the cloud depth, \(CWP, r_e,\) and \(N_e\) (see Table 1). As expected, \(N_e\) is much larger in the POLLUTED regime compared to PRISTINE due to the larger aerosol number concentration and higher droplet activation rate. For a given cloud microphysics configuration (i.e., using either bin microphysics or bulk microphysics with B1994, SB2001, or KK2000), the \(r_e\) is greater for the PRISTINE run compared to POLLUTED. The bulk simulations produce a decrease in \(r_e\) between the PRISTINE and POLLUTED runs slightly smaller than in the bin model (Table 2). The relative increase in \(CWP\) between the bulk PRISTINE and POLLUTED simulations is slightly larger than in the bin simulations. These increases in \(CWP\) correspond primarily to an increase in the cloud depth (i.e., a lowering of the cloud base height), allowing for more accretional growth of falling rain so that the surface rain rate balances the influx of moisture through LHF. Increasing LHF from 3 to 30 W m\(^{-2}\) results in larger values of cloud depth, \(CWP, r_e,\) and \(\tau_e\), but produces similar results in terms of the relative changes in the relevant variables (see Table 2). Overall, the KK2000 scheme produces results closest to the bin model in terms of overall performance and its sensitivity to aerosol.

Differences in the simulations are further highlighted by examining vertical profiles of the rain microphysics for the PRISTINE runs (an analysis of the POLLUTED runs gives a similar picture). The horizontally averaged rainwater mixing ratios near the surface are similar between the simulations (Fig. 7a), which is expected given that all of the simulations are run to equilibrium, which requires the same surface precipitation rate (values of \(q_c\) near the surface differ slightly be-
between the bin and bulk simulations owing to differences in the mean rain fall speed). The rain mixing ratios differ more significantly higher in the domain. KK2000 produces a \( q_r \) profile similar to the bin simulation. SB2001 and B1994 produce smaller values of \( q_r \) within most of the cloud layer. The mean raindrop size (Fig. 7b) varies by about a factor of 2–4 between the bin and bulk simulations. However, some of this difference may be due to the use of complete exponential functions (from 0 to \( \infty \)) in the bulk model to represent the raindrop size distribution (which allows analytic expression for the moments of the size distribution), while in the bin model only drops larger than the separation radius of 40 \( \mu m \) are considered as rain. Interestingly, all of the bulk simulations produce an increase in the mean raindrop size below cloud base in contrast to the nearly flat profile produced by the bin model. This occurs because number- and mass-weighted fall speeds are applied in the bulk model to \( N_r \) and \( q_r \), respectively, resulting in size sorting through sedimentation, as shown by a sensitivity test with \( V_{N_r} = V_{q_r} \). The difference in the shape of the vertical profiles of mean raindrop size suggests that the bulk scheme may overestimate size sorting during sedimentation, a conclusion also reached by Wacker and Seifert (2001).

Horizontally averaged equilibrium profiles of the various microphysical process terms in (5) and (6) for rain/drizzle and PRISTINE are shown in Fig. 8. Similar to the findings of Wood (2005b), accretion dominates autoconversion in the production of \( q_r \) except near cloud top. Since autoconversion is mostly confined to a narrow zone near cloud top, high vertical resolution is needed to adequately simulate drizzle production. The different schemes produce substantially different pro-

![Fig. 4. Plot (x-z) of the (top) equilibrium cloud water mixing ratio and (bottom) droplet number concentration for the bin and bulk (using KK2000) PRISTINE stratocumulus simulations with LHF = 3 W m\(^{-2}\). A similar cloud structure is produced by the bulk model using the SB2001 and B1994 parameterizations.](image-url)
cess rates for most of the terms. The KK2000 scheme has the largest autoconversion production term for $N_r$. The smaller, more numerous drizzle drops in KK2000 lead to a more rapid evaporation rate of $q_r$, and hence $N_r$ below cloud base, so that near the surface all of the bulk schemes produce similar values of $q_r$, $N_r$, and mean raindrop size. One might speculate that if microphysical–dynamical feedbacks were included, the larger evaporation rate below cloud base using KK2000 would impact stability and hence boundary layer dynamics, which could in turn feed back to the cloud macrophysical and microphysical characteristics. The depletion of $N_r$ due to self-collection of rain is negligible and can be neglected for this case.

The impact of predicting only one moment for rainwater (i.e., mixing ratio but not number concentration) is also tested. One-moment schemes typically specify a constant intercept parameter $N_0$ and then diagnose the mean raindrop size from $q_r$ and $N_0$ (e.g., Dudhia 1989; Reisner et al. 1998; Szumowski et al. 1998; Grabowski 1999). In two-moment schemes, $N_0$ and mean raindrop size are free parameters that vary with the predicted $N_r$ and $q_r$. Using the two-moment scheme here, $N_0$ varies widely (over four orders of magnitude) across the domain for a given simulation (see Fig. 9). The domain-averaged $N_0$ is almost an order of magnitude smaller for POLLUTED compared to PRISTINE. To test the impact of $N_0$, we have modified the bulk scheme to predict only one moment (mixing ratio) for rain. The equilibrium cloud microphysics are highly sensitive to the specification of $N_0$ using the one-moment scheme. Increasing $N_0$ means that the mean raindrop size is decreased for a given $q_r$, resulting in slower sedimentation and more evaporation. Thus, a deeper cloud layer, more cloud water, and reduced autoconversion and accretion rates are needed to produce the same surface precipitation rate to balance LHF, as required by equilibrium. Using a value of $N_0$ of $10^7$ m$^{-4}$, following Szumowski et al. (1998), the one-moment scheme is able to produce results similar to the two-moment

**Fig. 5.** As in Fig. 4 but for the POLLUTED stratocumulus simulations.
scheme for PRISTINE (see Table 1). However, the cloud depth, CWP, and \( \tau_e \) are substantially larger than that produced by the two-moment scheme for POLLUTED. Increasing \( N_0 \) to \( 10^8 \) m\(^{-3}\) improves the simulation for POLLUTED, but substantially degrades CWP and \( \tau_e \) for PRISTINE. These results imply that one-moment schemes with constant \( N_0 \) are inadequate when applied over a range of microphysical conditions.

Finally, sensitivity of model results to the specification of the width of cloud droplet spectrum is illustrated in simulations marked as SB2001 in Table 1. In these simulations, the \( \eta - N_c \) relationship (2) was replaced by (3), that is, the one used in Grabowski (1998). For the PRISTINE case, the results are consistent with other simulations. For the POLLUTED case, however, SB2001 \( \eta \) has the deepest and the optically thickest cloud. This is consistent with the fact that (2) and (3) give similar widths for \( N_c \) around 60 cm\(^{-3}\) [0.31 for (2) and 0.36 for (3)], but different values for \( N_c \) around 300 cm\(^{-3}\) [0.44 for (2) and 0.26 for (3)].

Width of the cloud droplet spectrum can be calculated directly from the bin simulations. The domain-averaged CWP produced by the bin model (for regions with \( q_c > 0.01 \) g kg\(^{-1}\)) decreases slightly between PRISTINE and POLLUTED (from 0.27 to 0.25). This is in contrast to (2), but the decrease is not as strong as suggested by (3). Spatial variability of \( \eta \) in both PRISTINE and POLLUTED cases is significant, with \( \eta \) decreasing

![Fig. 6. Example of equilibrium drop size spectra (at \( z = 0.9 \) km, \( x = 1.25 \) km) from the stratocumulus bin run with LHF = 3 W m\(^{-2}\) for the PRISTINE (solid) and POLLUTED (dotted) regimes.](image)

---

**Table 1.** Equilibrium domain-averaged cloud depth, cloud optical depth \( \tau_e \), cloud water path (CWP), droplet number concentration \( N_c \) and “effective” \( \tau_e \) for the stratocumulus regime. For \( N_c \), only in-cloud regions with cloud water mixing ratio larger than 0.1 g kg\(^{-1}\) are included in the averaging. Cloud depth is calculated by defining cloud boundaries using a droplet number concentration of 1 cm\(^{-3}\); \( N_0 \) indicates the one-moment scheme (using KK2000) with the rain intercept parameter \( N_0 \) specified at the given value. SB2001 \( \eta \) indicates the sensitivity test with the formulation for relative dispersion \( \eta \) given by Grabowski (1998).

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<th>Cloud depth (m)</th>
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<th>CWP (g m(^{-2}))</th>
<th>( N_c ) (cm(^{-3}))</th>
<th>( \tau_e ) (( \mu )m)</th>
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strongly with height in both ascending and descending branches of the circulation, and substantially smaller values present at a given height in the descending branch (not shown). The decrease of $\eta$ with height is mostly due to the increase of the mean droplet radius, with changes of the standard deviation of the cloud droplet spectrum (e.g., due to spectral narrowing associated with condensational growth) of secondary importance. The similarity of $\eta$ between the PRISTINE and POLLUTED cases is a result of compensating effects of the standard deviation of cloud droplet spectrum and the mean droplet size. In the PRISTINE case, standard deviation at a given height is larger than in POLLUTED case (possibly due to the impact of collision–coalescence on the spectrum of cloud droplets), but so is the mean droplet size. Consequently, the ratio of the two ($\eta$) decreases only slightly between the two regimes. The similarity of $\eta$ between PRISTINE and POLLUTED, and its strong decrease with height, contrasts the observations of Martin et al. (1994, see their Fig. 4). However, a key point is that here we have neglected cloud-top entrainment, which is expected to broaden the spectra. It should also be kept in mind that Martin et al. calculated spectral width for droplets less than 25 $\mu$m in radius, while our bin model results include droplets smaller than the separation radius of 40 $\mu$m.

b. Cumulus case

The cloud macrophysical and microphysical properties in the cumulus regime are strongly driven by the time-varying vertical velocity field. Time series of the domain-averaged surface precipitation rate, CWP, $N_c$, and $\tau_e$ are shown in Figs. 10 and 11. Table 3 presents the time- and domain-averaged surface precipitation rate, CWP, $N_c$, $\tau_e$, and $\eta$ between the time of maximum updraft velocity and the end of the simulation ($t = 25$–60 min). In all of the simulations, cloud water begins to form in the domain at $t = 6$ min, followed by a rapid increase in cloud water amount as the vertical velocity increases and the eventual development of a narrow rain shaft. The domain-averaged CWP reaches a maximum at around the time of the maximum updraft velocity. As expected, $N_c$ is much higher for POLLUTED compared to PRISTINE runs due to the higher aerosol concentration. Figure 12 shows an example of size spectra produced by the bin model for the time and location near the peak updraft, with the spectra differing between PRISTINE and POLLUTED as expected. The PRISTINE size spectrum is fairly consistent with ob-

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Table 2. As in Table 1 except for the difference in equilibrium domain-averaged effective $\tau_e$ and CWP between POLLUTED and PRISTINE for the stratocumulus regime. Difference in $\tau_e$ ($\mu$m) is calculated as PRISTINE minus POLLUTED. Relative difference in CWP (%) is calculated as (POLLUTED – PRISTINE)/PRISTINE.
Fig. 8. Equilibrium, horizontally averaged vertical profiles of rain/drizzle microphysical process rates for the KK2000, SB2001, and B1994 stratocumulus PRISTINE runs with LHF = 3 W m$^{-2}$. 
served size spectra for this case (see Szumowski et al. 1998). In all of the simulations, the time of maximum surface precipitation rate occurs 8–17 min after the time of the peak updraft velocity. The POLLUTED bin run shows a delayed onset of maximum precipitation rate by about 7 min compared to the PRISTINE run. The initial onset of precipitation occurs earlier in the bulk simulations, particularly for POLLUTED. The POLLUTED simulations all show a decrease in the overall surface precipitation rate compared to PRISTINE. The time-averaged magnitude of this suppression ranges between 0.66 and 0.93 mm h\(^{-1}\) (Table 4).

In all of the simulations the time-averaged difference in \(\tau_r\) is about 6–7 \(\mu\)m (see Table 4). However, all of the bulk simulations have values of \(\tau_r\) larger than the bin model, especially KK2000. The time-averaged relative increase in CWP between PRISTINE and POLLUTED is somewhat larger in the bin compared to bulk simulations. Differences in \(\tau_r\) between the runs generally track differences in the CWP, and there is almost no difference in fractional cloud cover between the runs.

Figures 13 and 14 show time–height plots of the maximum cloud water and rainwater mixing ratios at a given level, following the approach of Szumowski et al. (1998). These plots are created by combining columns at which the maximum for a given field occurs at a given time. Note that they trace the trend of field maxima but do not give information about the position in the horizontal plane. The largest values of cloud water mixing ratio \(q_c\) are generally consistent among the bulk and bin PRISTINE simulations, ranging between 2.51 and 3.33 g kg\(^{-1}\) and occurring around the time of the maximum updraft velocity. A similar picture emerges for the POLLUTED simulations, although the largest values of \(q_c\) are slightly higher, ranging between 3.87 and 4.08 g kg\(^{-1}\). Aircraft observations for this case indicate a maximum \(q_c\) of \(\sim 1.5–2\) g kg\(^{-1}\), although these measurements may underestimate \(q_c\) in the middle and upper portion of the cloud by as much as a factor of 2 (Szumowski et al. 1998). Somewhat larger values of \(q_c\) in the bulk compared to the bin simulations occur mostly after \(t \sim 40\) min. The maximum values of rainwater mixing ratio \(q_r\) are also similar between the bin and bulk simulations, although the largest values tend to be somewhat greater using the bulk model, particularly for KK2000 (maximum of 5.95 versus 3.60 g kg\(^{-1}\) in the bin simulation). Nonetheless, these values are significantly less than reported by Szumowski et al. (1998), who indicated values of \(q_r\) exceeding 20 g kg\(^{-1}\) using a one-moment bulk scheme. This occurred in their simulation owing to spurious accumulation of rainwater in the updraft (in bulk models rain mass all falls at the mass-weighted terminal fall speed). Our contrasting results likely reflect a combination of horizontal diffusion (which was not used by Szumowski et al.), different mean rain sizes, and different values for the rain parameters (e.g., different terminal fall speeds versus drop size relationships).

Vertical profiles of rain microphysical process rates in (5) and (6) for the bulk model simulations and the column with the maximum updraft speed are shown in Fig. 15. SB2001 and B1994 produce similar process rates, while accretion and especially autoconversion rates are much smaller using KK2000 (it should be kept in mind that KK2000 was originally developed for stratocumulus). Since the evaporation rate of \(q_r\) below cloud base is small relative to \(q_r\), the evaporation of \(N_r\) is negligible. Condensation onto rain within the cloud region is also small relative to autoconversion and accretion and could be neglected for this case. Similar to
the stratocumulus case, the decrease of $N_r$ resulting from self-collection of rain is much smaller in magnitude compared to the increase in $N_r$ from autoconversion. Similar to the stratocumulus case, the intercept parameter for rain, $N_0$, plays a critical role in the evolution of the cloud microphysics. Figure 16 shows a time-height plot of $N_0$ at the location of the maximum $q_r$ in the horizontal plane using the two-moment scheme; $N_0$ varies several orders of magnitude in space and time, is largest within the cloud layer, and decreases toward the surface, reflecting the impact of size sorting (different mean fall speeds are applied to $N_r$ and $q_r$) and evaporation. As in the stratocumulus case, the CWP and $\tau_e$ are highly sensitive to $N_0$ using the one-moment scheme (see Table 3). Using a value for $N_0$ of $10^7$ m$^{-4}$, following Szumowski et al. (1998), this scheme produces significantly greater CWP and $\tau_e$ and significantly less surface precipitation, relative to the two-moment scheme and the bin model. Increasing $N_0$ to $10^9$ m$^{-4}$ decreases the CWP and $\tau_e$ and thus improves simulation of the cloud microphysics, but the surface precipitation remains weak. These results occur because increasing $N_0$ in the one-moment scheme decreases the mean raindrop size and hence the fall speed as described for the stratocumulus case. In the cumulus case, the reduced rain fall speed leads to larger values of $q_r$ within the cloud layer and hence greater cloud water accretion and reduced CWP. At the same time, the smaller raindrop size limits the precipitation rate at the surface. The two-moment scheme, which allows $N_0$ to vary with the predicted $N_r$ and $q_r$, produces relatively small raindrops within the cloud layer, which limits CWP, but large raindrops near the surface, which enhances the surface precipitation rate. These results further illustrate the inadequacy of one-moment schemes using a specified value for $N_0$. Note that the response of the CWP and $\tau_e$ to $N_0$ in the one-moment scheme here is opposite in sign to that of the stratocumulus case. This difference occurs because the stratocumulus case was examined in the context of cloud characteristics that were in equilibrium with the applied forcing.

Fig. 10. Time series of domain-averaged surface precipitation rate PREC, CWP, droplet number concentration $N_c$, and effective $\tau_e$ for the PRISTINE cumulus runs, for SB2001, B1994, KK2000, and the bin model simulations.
As in the stratocumulus case, SB2001 simulations in Table 3 illustrate the impact of the specification of the relative dispersion of the cloud droplet spectrum \( \eta \). The impact resembles the stratocumulus case as well, with relatively small effect in the PRISTINE case [owing to the similar \( \eta \); 0.29 for (2) and 0.38 for (3)] and a large effect in the POLLUTED case [consistent with \( \eta \approx 0.41 \) for (2) and 0.27 for (3)].

The width of cloud droplet spectrum \( \eta \) varies widely in bin simulations. Similar to the stratocumulus case, the time- and space-averaged \( \eta \) decreases slightly (by about 0.05) between the PRISTINE and POLLUTED cases. It also appears that \( \eta \) varies inversely with the vertical velocity, with values less than 0.2 when the updraft is strongest and values greater than 0.4 when the updraft is weak (~2 m s\(^{-1}\)). This is consistent with the theoretical expression for \( \eta \) developed by Liu et al. (2006). However, the impact of entrainment on \( \eta \) is neglected in both our study and that of Liu et al. Moreover, Liu et al. do not consider droplet collision–coalescence, which may play an important role in the standard deviation of the spectrum in the PRISTINE case as already alluded to in the stratocumulus case.

5. Summary and conclusions

In this paper, a two-moment warm-rain bulk microphysics parameterization for application in high-resolution cloud models was developed and tested against a detailed bin microphysics scheme. Three different formulations (KK2000; B1994; SB2001) described in the literature for the coalescence processes (autoconversion, accretion, and self-collection) were implemented in the bulk model. Testing was done using a kinematic framework with a specified flow field. The kinematic framework is an effective way to compare microphysical models without added complexities due to feedbacks between the microphysics and dynamics. At the same time, it allows the microphysics schemes to operate in a realistic framework including gravitational and advective transport. The schemes were applied in two quasi-idealized regimes: (i) boundary layer stra-
tocumulus and (ii) shallow cumulus. In the stratocumulus case with a time-invariant flow field, simulations were run to near equilibrium with the specified forcing. A time-varying flow field was employed for the cumulus regime. Emphasis was placed on testing the models under both polluted and pristine aerosol conditions.

The analysis focused in particular on differences in the mean ("effective") radius \( r_e \) [see Eq. (24)] and cloud water path between the POLLUTED and PRISTINE cases. There was almost no change in fractional cloud cover across the domain among the various simulations (or change in cloud lifetime in the time-varying cumulus case). Differences in \( r_e \) and CWP between PRISTINE and POLLUTED were similar between the bin and bulk simulations (especially using KK2000 and SB2001). However, the values of \( r_e \) tended to be somewhat larger for all of the bulk simulations. KK2000 performed best for the stratocumulus case, which is not surprising given that it was developed in the context of curve fits to bin model simulations of boundary layer stratocumulus. The SB2001 scheme also performed quite well for this case. For the cumulus case, there was generally less difference among the bulk simulations, except that KK2000 tended to produce much larger values of \( r_e \).

Uncertainty in the bulk model results may have resulted from our specification of the relative dispersion of the droplet size distribution \( \eta \) as a function of \( N_c \). Sensitivity tests using a formulation with different sign of the change of \( \eta \) with \( N_c \) following Grabowski (1998) indicated significant sensitivity of \( r_e \) and CWP using the B1994 and SB2001 schemes.

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**Table 3.** Time- and domain-averaged surface precipitation rate \( \text{PREC} \), cloud optical depth \( \tau_c \), CWP, droplet number concentration \( N_c \), and effective \( r_e \) for the cumulus regime. For \( N_c \), only in-cloud regions with cloud water mixing ratio larger than 0.1 g kg \(^{-1} \) are included in the averaging. Time averaging is between the time of the maximum updraft velocity and the end of the simulation (\( t = 25-60 \) min); \( N_0 \) indicates the one-moment scheme (using SB2001) with the rain-intercept parameter \( N_0 \) specified at the given value. SB2001\( \eta \) indicates the sensitivity test with the formulation for relative dispersion \( \eta \) given by Grabowski (1998).

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Aerosol</th>
<th>( \text{PREC} ) (mm h (^{-1} ))</th>
<th>( r_e )</th>
<th>CWP (g m (^{-2} ))</th>
<th>( N_c ) (cm (^{-3} ))</th>
<th>( \tau_c ) (( \mu )m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bin</td>
<td>POLLUTED</td>
<td>2.17</td>
<td>103.4</td>
<td>724.5</td>
<td>256.7</td>
<td>11.1</td>
</tr>
<tr>
<td>KK2000</td>
<td>POLLUTED</td>
<td>2.50</td>
<td>97.6</td>
<td>885.5</td>
<td>239.6</td>
<td>15.7</td>
</tr>
<tr>
<td>SB2001</td>
<td>POLLUTED</td>
<td>2.70</td>
<td>96.1</td>
<td>771.5</td>
<td>241.6</td>
<td>12.7</td>
</tr>
<tr>
<td>B1994</td>
<td>POLLUTED</td>
<td>2.56</td>
<td>109.7</td>
<td>911.5</td>
<td>246.7</td>
<td>13.1</td>
</tr>
<tr>
<td>( N_0 = 10^7 )</td>
<td>POLLUTED</td>
<td>0.80</td>
<td>165.2</td>
<td>1439.8</td>
<td>337.7</td>
<td>13.6</td>
</tr>
<tr>
<td>( N_0 = 10^9 )</td>
<td>POLLUTED</td>
<td>0.96</td>
<td>103.2</td>
<td>830.5</td>
<td>265.7</td>
<td>12.5</td>
</tr>
<tr>
<td>SB2001( \eta )</td>
<td>POLLUTED</td>
<td>1.84</td>
<td>176.9</td>
<td>1349.3</td>
<td>334.8</td>
<td>12.2</td>
</tr>
<tr>
<td>Bin</td>
<td>PRISTINE</td>
<td>3.07</td>
<td>29.2</td>
<td>323.7</td>
<td>41.4</td>
<td>17.5</td>
</tr>
<tr>
<td>KK2000</td>
<td>PRISTINE</td>
<td>3.32</td>
<td>34.7</td>
<td>436.9</td>
<td>35.3</td>
<td>22.1</td>
</tr>
<tr>
<td>SB2001</td>
<td>PRISTINE</td>
<td>3.36</td>
<td>37.6</td>
<td>428.3</td>
<td>37.6</td>
<td>18.7</td>
</tr>
<tr>
<td>B1994</td>
<td>PRISTINE</td>
<td>3.49</td>
<td>38.2</td>
<td>443.9</td>
<td>35.9</td>
<td>19.0</td>
</tr>
<tr>
<td>( N_0 = 10^7 )</td>
<td>PRISTINE</td>
<td>2.53</td>
<td>60.6</td>
<td>762.1</td>
<td>42.5</td>
<td>20.0</td>
</tr>
<tr>
<td>( N_0 = 10^9 )</td>
<td>PRISTINE</td>
<td>1.92</td>
<td>43.0</td>
<td>493.4</td>
<td>41.0</td>
<td>18.9</td>
</tr>
<tr>
<td>SB2001( \eta )</td>
<td>PRISTINE</td>
<td>3.44</td>
<td>33.9</td>
<td>391.3</td>
<td>37.9</td>
<td>19.4</td>
</tr>
</tbody>
</table>

---

**Table 4.** As in Table 3 but for the difference in time- and domain-averaged surface precipitation rate \( \text{PREC} \), CWP, and effective \( r_e \). The difference in \( \text{PREC} \) (mm h \(^{-1} \)) and \( r_e \) (\( \mu \)m) is calculated as PRISTINE minus POLLUTED. Relative difference in CWP (%) is calculated as \((\text{POLLUTED} - \text{PRISTINE}) / \text{PRISTINE}\).

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Diff PREC</th>
<th>Diff ( r_e )</th>
<th>Diff CWP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bin</td>
<td>-0.90</td>
<td>6.4</td>
<td>123.8</td>
</tr>
<tr>
<td>KK2000</td>
<td>-0.82</td>
<td>6.4</td>
<td>102.7</td>
</tr>
<tr>
<td>SB2001</td>
<td>-0.66</td>
<td>6.0</td>
<td>80.1</td>
</tr>
<tr>
<td>B1994</td>
<td>-0.93</td>
<td>5.9</td>
<td>105.3</td>
</tr>
<tr>
<td>( N_0 = 10^7 )</td>
<td>-1.73</td>
<td>6.4</td>
<td>88.9</td>
</tr>
<tr>
<td>( N_0 = 10^9 )</td>
<td>-0.96</td>
<td>6.4</td>
<td>68.3</td>
</tr>
<tr>
<td>SB2001( \eta )</td>
<td>-1.60</td>
<td>7.2</td>
<td>244.8</td>
</tr>
</tbody>
</table>

---

**Fig. 12.** Example of the size spectra near the time and location of the maximum updraft velocity (\( t = 25 \) min, \( x = 4.5 \) km, \( z = 2 \) km) from the bin model cumulus simulations for the PRISTINE (solid) and POLLUTED (dotted) regimes.
parameterizations. There was much less sensitivity using KK2000 since this scheme has no explicit dependence on $\eta$. Note that both formulations for $\eta$ [i.e., Eqs. (2) and (3)] produced similar results for PRISTINE, but using the Grabowski (1998) formulation substantially degraded results relative to the bin model for POLLUTED. This is because the two formulations predict similar $\eta$ for low droplet concentrations (smaller than 100 cm$^{-3}$), but differ significantly for higher droplet concentrations. Work is underway to develop more robust formulations of the relative dispersion using both idealized modeling with bin microphysics (of the type presented in this paper) and aircraft observations (Pawlowska et al. 2006).

There was a strong link between the microphysical and macrophysical properties of the quasi-equilibrium cloud in the stratocumulus case. The cloud layer had to be sufficiently deep to produce enough precipitation at the surface to balance the specified forcing. Thus, simulations with weak rain production for a given amount of cloud water (e.g., the runs with polluted aerosol) had lower quasi-equilibrium cloud base and higher cloud water paths. In the cumulus regime, the microphysics had less impact on the macrophysical cloud characteristics, but nonetheless had a large impact on the precipitation rate and cloud microphysics. The most important difference between bulk and detailed microphysics simulations was a premature onset of significant surface precipitation (by several minutes) in all bulk schemes. Increasing the aerosol loading resulted in a suppression of the total precipitation rate at the surface, as well as significant changes in $r_e$ and CWP as described above.

Sensitivity tests explored the role of rain microphys-
ics using a one-moment scheme that predicted rain mixing ratio but not number concentration, as is the case for most bulk microphysics schemes. Results were highly sensitive to the specification of rain size distribution intercept parameter $N_0$ using this scheme. A key point is that no single value of $N_0$ in the one-moment scheme was found that could produce results consistent with the two-moment scheme and the bin model for both cumulus and stratocumulus regimes. The large variability in $N_0$ using the two-moment scheme, combined with the large sensitivity of the one-moment scheme to $N_0$, suggests the need to predict both $q_r$ and $N_r$, and hence allow $N_0$ and mean raindrop size to vary as free parameters in a physically consistent way with $q_r$ and $N_r$. This may be especially important for microphysics schemes that are intended for use across a wide range of cloud types and conditions, as in regional or global climate simulations using CRMs. The drawback, of course, is increased computational cost associated with the added prognostic variable. It may also be possible to diagnose $N_0$ for the one-moment scheme as a function of rainwater content, height, or some combination of variables that realistically captures the evolution of $N_0$, based on the results of the two-moment simulations.

This study was intended to gauge the ability of the bulk model to reproduce the bin model results when applied to both polluted and pristine aerosol conditions and was not meant to quantify actual indirect effects. Thus, several simplifications were made, including neglect of cloud–aerosol feedbacks and cloud-top entrainment, which may be particularly important for the second indirect effect (Ackerman et al. 2004). These results could also have some relevance to GCMs.
Fig. 15. Vertical profiles of rain microphysical process rates at $t = 30$ min for the KK2000, SB2001, and B1994 cumulus PRISTINE runs and the column with the maximum updraft speed.
parameterizations that include a subgrid distribution of
cloud water and sophisticated treatment of the micro-
physics, although in the absence of coupling between
the subgrid cloud/precipitation water and subgrid ver-
tical velocity, important processes such as enhanced
collection of droplets due to lofting of rain in updrafts
will be neglected. For application of the new micro-
physics scheme into CRMs, with a time step of
10 s and vertical grid spacing on the order of 100 m or more,
the scheme will need to be modified. In particular, the
approach taken here based on explicit prediction of
droplet activation requires a time step less than about 1
s and vertical grid spacing less than about 50 m. The
role of mixing between a cloud and its environment and
the subsequent reduction of droplet number concentra-
tion versus size also plays a key role in the microphysics
of clouds and their radiative impact (Grabowski 2006a).
Mixing may also provide fresh nuclei that initiate drop-

Acknowledgments. This work was supported by the
NOAA Grant NA05OAR4310107 (W. Grabowski, PI)
and NASA GWEC Grant NAG 5-11756 (G. McFarqu-
har, PI). Comments on the manuscript by W. Hall and
R. Rasmussen are acknowledged.

APPENDIX

Formulation for the 2D Flow for Tests Described
in Szumowski et al. (1998)

The idealized flow velocities \((u, w)\) in a 2D test mim-
icking a shallow convective cloud is prescribed using a
streamfunction \(\psi\), where

\[
\rho_u u = -\frac{\partial \psi}{\partial z}, \quad \rho_w w = \frac{\partial \psi}{\partial x},
\]

with \(\psi\) given by a general formula,

\[
\psi = -A_1 \cos\left(\alpha \pi \frac{x - x_o}{h_x}\right) \sin\left(\beta \pi \frac{z - z_o}{h_z}\right)
\]

\[
+ \frac{A_2}{2} \left(\frac{z}{H_2}\right)^2.
\]

In (A2), the first term describes the time-evolving up-
draft in the center of the domain and the vertically
varying horizontal flow outside the central part, whereas the second term describes a horizontal flow
with weak vertical shear across the domain. In (A2),
\(0 \leq x \leq L_x\) and \(0 \leq z \leq L_z\), where \(L_x = 9.0\) km and \(L_z = 2.7\) km are extents of the flow region in \(x\) and \(z\)
directions, respectively; \(h_x = 1.8\) km, \(x_o = 3.6\) km; and
\(z_o = 0, \ h_z = 3.4\) km if \(z \leq 1.7\) km,
\(z_o = 0.7\) km, \(h_z = 2.0\) km if \(1.7\) km \(\leq z \leq 2.7\) km.

Moreover, \(\alpha = 1\) for \(|x - x_o| \leq 0.9\) km (\(x_o = 4.5\) km is
the center of the domain in the \(x\) direction) and \(\alpha = 0\)
otherwise; \(\beta = 1\) for \(x \leq 5.4\) km and \(\beta = -1\) for \(x > 5.4\)
km. The scale height for the shear layer is \(H_2 = L_z\). The
time-dependent amplitudes \(A_1\) and \(A_2\) are prescribed in
the following way:

\[
A_1 = a_{10} + a_1 f_1(t),
\]
\[
A_2 = a_{20} + a_2 f_2(t),
\]

where the amplitudes and functions describing time
evolutions are (all SI units)

Fig. 16. Rain size distribution intercept parameter \(N_0\) for the
cumulus PRISTINE and POLLUTED simulations using the two-
moment bulk model with SB2001.
for $t \leq 300$ s:

$$a_{10} = 0; \quad a_4 = 5.73 \times 10^2; \quad a_2 = 0;$$

$$f_1(t) = 1; \quad f_2(t) = 1;$$

for $300$ s $\leq t \leq 900$ s:

$$a_{10} = 0; \quad a_4 = 5.73 \times 10^2; \quad a_2 = 6.00 \times 10^2;$$

$$f_1(t) = 1; \quad f_2(t) = 1 + \cos(\pi(t - 300.0)/600.0 - 1);$$

for $900$ s $\leq t \leq 1500$ s:

$$a_{10} = 5.73 \times 10^2; \quad a_4 = 2.02 \times 10^3;$$

$$a_2 = 6.00 \times 10^2;$$

$$f_1(t) = 1 + \cos(\pi(t - 900.0)/600.0 + 1);$$

$$f_2(t) = 1 + \cos(\pi(t - 300.0)/600.0 - 1);$$

for $t > 1500$ s, $t_1 = \min(2400, t)$:

$$a_{10} = 1.15 \times 10^3; \quad a_4 = 1.72 \times 10^3;$$

$$a_2 = 5.00 \times 10^2;$$

$$f_1(t_1) = 1 + \cos(\pi(t_1 - 1500.0)/900.0);$$

$$f_2(t_1) = 1 + \cos(\pi(t_1 - 1500.0)/900.0 - 1);$$

Note that (A2) provides the flow only up to $z = 2.7$ km. Vanishing flow is prescribed above.

REFERENCES


