Orographic Flow Response to Variations in Upstream Humidity

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ABSTRACT

The effects of upstream relative humidity (RH) on low-level wind and precipitation patterns for low-speed, statically stable flows over a mountain are investigated using idealized two- and three-dimensional numerical-simulation experiments in which RH is increased from 0% to 100%. For RH less than some critical threshold, the flow upstream becomes less decelerated as RH is increased; for RH greater than this threshold, the flow upstream becomes more decelerated as RH is increased. This increasing deceleration with RH is due to locally enhanced static stability resulting from enhanced condensation near the freezing level. Analyses from the simulations indicate that the lifted condensation level and the height of the freezing level are significant control parameters for the upstream-flow deceleration in the steady-state solutions. Dimensional analysis using these control parameters (as well as others) brings forth new nondimensional parameters that are shown to enter into analytic formulas for the orographic upstream-flow deceleration in a moist atmosphere.

1. Introduction

From a theoretical perspective, orographic blocking of dry air is well understood. Moist flow has received less attention, but latent-heat release is known to alter wind patterns over a mountain with saturated flow tending to suffer less upstream-flow deceleration than dry flow. Attempts to quantify moist effects using moist nondimensional control parameters that are simple extensions of known dry ones have so far been unsuccessful. In this paper, the flow dependence on moisture content is reconsidered using idealized numerical experiments and the results are viewed through the lens of an extended range of nondimensional control parameters. As will be demonstrated, microphysical effects associated with enhanced condensation near the freezing level can have significant impacts on the steady-state flow patterns affecting the relation between steady-state conditions and moist nondimensional control parameters based solely on latent heating.

Consider the following scenario: dry, inviscid, two-dimensional (2D), hydrostatic flow over an isolated ridge. The dimensional control parameters for this situation are the mountain height $h$, mountain half-width $a$, buoyancy frequency $N$, basic-state wind speed $U$, and Coriolis parameter $f$. These five dimensional control parameters can be combined into three independent, nondimensional control parameters (Birkhoff 1960, chapter 4); the Froude number $F = U/Nh$, the Rossby number $R_o = U/Na$, and the hydrostatic number $U/Na$. Based on numerous studies under the above conditions, it is known that the strength of low-level upstream-flow deceleration and deflection is a function of $F$ and $R_o$ (e.g., Smith 1982; Pierrehumbert and Wyman 1985; Trüb and Davies 1995; Peng et al. 1995; Lin and Wang 1996; Olafsson and M. Bougeault 1996). [For hydrostatic flows, steady-state solutions are independent of $U/Na$ (Smith 1979).] There are a multitude of ways flow in the real atmosphere can depart from the simple scenario described above; each departure [e.g., $N = N(z)$] would

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need to be accounted for in a dimensional analysis, resulting in a greater number of nondimensional control parameters and possibly affecting the relation between flow deceleration and $F$ and $R_o$.

One effect not included above, but that is known to affect flow properties, is latent-heat release. Increasing the moisture, and, as a result, the potential for latent-heat release, has been found to cause flows to have less upstream-flow deceleration (Katzfey 1995; Buzzi et al. 1998; Miglietta and Buzzi 2001; Rotunno and Ferretti 2001; Jiang 2003; Colle 2004; Colle and Zeng 2004). The buoyancy frequency has historically been used as a way to account for latent-heat effects. For moist but unsaturated flow, it is given by

$$N^2 = \frac{g}{\theta_v} \frac{\partial \theta_v}{\partial z}$$

(e.g., Emanuel 1994, p. 166), where $g$ is the acceleration due to gravity and $\theta_v$ is the virtual potential temperature. Saturated flow has the buoyancy frequency

$$N_m^2 = \frac{1}{1 + q_w} \left[ \Gamma_m (c_p + c_i q_w) \frac{1}{\theta_v} \frac{\partial \theta_v}{\partial z} + \left( \Gamma_m c_i \ln \frac{\theta_v}{T} - g \right) \frac{\partial q_w}{\partial z} \right],$$

where $\Gamma_m$ is the moist adiabatic lapse rate, $c_p$ is the specific heat at constant pressure for dry air, $c_i$ is the specific heat at constant pressure for liquid water, $T$ is the temperature, $g$ is gravity, and $q_w$ is the total liquid and frozen water mixing ratio plus the saturation–water vapor mixing ratio. [Eq. (2) is derived from the expressions for $N_m^2$ provided in Emanuel (1994) and Kirshbaum and Durran (2004), but with the vertical gradients of $\theta_v$ and $q_w$ separated.] The quantity $N_m$ is generally smaller than $N$.

There is no evidence to suggest $N$ or $N_m$ will give meaningful information about steady-state conditions for flows that are unsaturated upstream but become saturated near the mountain through orographic uplift. For saturated nonprecipitating flows, it has been suggested that the relations between steady-state flow patterns and $F$ and $R_o$ derived for dry flow still hold when $N_m$ is used in place of $N$, all else being equal (Fraser et al. 1973; Barcilon et al. 1979; Durran and Klemp 1982). However, recent studies indicate this may not be strictly true for flow with precipitation (Jiang 2003; Colle 2004). Possibly this is because latent-heat release is only one of several processes associated with the condensation of water vapor that can affect orographic flow. In general, one can expect condensate to load the air or, if there is precipitation, to unload it and deplete moisture from it (Miglietta and Rotunno 2005, 2006). This complex chain of cause and effect, resulting from the interaction of the orographic flow and microphysical processes, poses a significant challenge to the identification of those parameters in the initial data that control the steady-state solution.

To gauge the utility of nondimensional control parameters based on either $N$ or $N_m$, one must first determine the effects of water vapor on flow deceleration and deflection. As relative humidity (RH) is increased from 0% to 100%, does a flow become progressively less decelerated as suggested by generally smaller values of $N_m$? Is a simple substitution of $N_m$ for $N$ sufficient in a dimensional analysis of saturated flow or do other dimensional control parameters exist? Can nondimensional control parameters be used to determine the steady-state upstream-flow deceleration and/or deflection when there is precipitation? These are the questions addressed herein.

In this study, several sets of idealized two- and three-dimensional (3D) experiments were conducted to assess how flow patterns differed as RH was increased from 0% to 100%. The results of the numerical experiments were considered in light of what we find to be the relevant nondimensional control parameters. To limit the number of dimensional control parameters to a reasonable-sized collection, this study is limited to stable, frictionless flow with no radiation. Additionally, only flows with $F < 1$ are considered. Such flows are traditionally associated with strong upstream-flow deceleration (Smith 1979) in the nonrotating ($R_\ast \gg 1$) case. This paper is organized as follows: The design of the numerical experiments is discussed in section 2. The effects of RH on flow and precipitation patterns are explored in section 3. A dimensional analysis of the experiments is performed and analytic expressions for flow deceleration are presented in section 4. Concluding remarks are provided in section 5.

2. Design of numerical simulations

The Advanced Regional Prediction System (ARPS) version 5.2.5 (Xue et al. 2001) is used to perform several sets of idealized simulations. The two-dimensional
simulations have 150 grid points in the (along-stream) \( x \) direction, four grid points in the (across-stream) \( y \) direction, and a horizontal grid spacing of 10 km. There are 28 grid points in the (vertical) \( z \) direction, with a stretched vertical grid spacing that has a minimum distance of 100 m near the ground and a total domain depth of 25 km. Other relevant model settings are a Coriolis parameter (on an \( f \) plane) of \( 10^{-4} \) \( \text{s}^{-1} \), a time step of 6 s, fourth-order advection in the horizontal and vertical directions, radiation boundary conditions on the along-stream edges of the domain, periodic boundary conditions on the across-stream edges of the domain, and rigid lower and upper boundaries. Rayleigh damping is imposed above 14 km. The microphysics are handled by the Weather Research and Forecast (WRF) single-moment, six-class scheme (WSM6; Hong et al. 2004). There is no friction or radiation. All simulations are initially horizontally homogeneous. The 3D simulations have 70 grid points in the \( y \) direction, radiation conditions on all four lateral boundaries, and either a 300- or 100-km elongated ridge, but are otherwise identical to the 2D simulations.

Table 1 shows the different types of simulations performed. The values of \( F \) and \( N \) in Table 1 and elsewhere in the text are 0–2 km averages because the initial profiles have \( N = N(z) \) (following the recommendation of Reinecke and Durran 2008). A bell-shaped mountain with a height of 2 km located 800 km from the inflow boundary is used with \( a \) given in Table 1. Each series is comprised of 20 simulations with RH varying from 0%−100% in 5% increments. The RH is constant throughout the depth of the domain. Figure 1 shows the different sounding choices for this study. The temperature profiles are deliberately chosen so that initial profiles of \( N^2 \) and \( N^2_m \) are positive. The simulations are integrated to a nondimensional time, \( Ut/a \), of 10; unless otherwise noted, all results are shown at this time. The 2D (3D) simulations are referred to as SX-YY (3DSX-YYL or 3DSX-YYS) where X is the series number, YY is the relative humidity, and L or S indicates whether the ridge length is 300 km (L) or 100 km (S).

### 3. The effect of RH on flow and precipitation patterns

#### a. Two-dimensional flow

We start with consideration of series 1 and 2 (S1 and S2, respectively; Table 1). The initial sounding for these simulations (Fig. 1) has a temperature profile that is nearly moist neutral (constant \( \theta_c; N_m^2 \approx 1.0 \times 10^{-5} \text{s}^{-2} \)), similar to that used in Rotunno and Ferretti (2001) and Miglietta and Rotunno (2005, 2006). S1 and S2 have \((F, R_\alpha) = (0.5, 2.0)\) and \((0.25, 1.0)\), respectively. Figures 2a–d show vertical cross sections of \( u' \) and \( \nu' \) for S1–90, S1–50, S2–90, and S2–50, respectively. In the RH = 90% cases (Figs. 2a,c), \( u' \) and \( \nu' \) are near 0 m s\(^{-1}\) on the upstream side of the mountain. In the RH = 50% cases (Figs. 2b,d), there is upstream-flow deceleration \((u' < 0 \text{ m s}^{-1})\) and leftward flow deflection \((\nu' > 0 \text{ m s}^{-1})\). To see if more stably stratified flows have a similar dependence on RH, another series, S3 (Table 1), is performed. Figures 2e,f show \( u' \) and \( \nu' \) for S3–90 and S3–50, respectively. Although both cases exhibit a region of deceleration on the upstream side of the mountain, S3–50 has the stronger deceleration \((u' \approx -2.8 \text{ versus})

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**Table 1. Description of properties of the experimental series.**
The properties include the buoyancy frequency \( N \), basic-state wind speed \( U \), mountain half-width \( a \), surface temperature \( T_{sfc} \), Froude number \( F \), and Rossby number \( R_\alpha \).

<table>
<thead>
<tr>
<th>Series</th>
<th>( N ) (s(^{-1}))</th>
<th>( U ) (m s(^{-1}))</th>
<th>( a ) (km)</th>
<th>( T_{sfc} ) (K)</th>
<th>( F )</th>
<th>( R_\alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.0100</td>
<td>10.00</td>
<td>50</td>
<td>276</td>
<td>0.50</td>
<td>2</td>
</tr>
<tr>
<td>S2</td>
<td>0.0100</td>
<td>5.00</td>
<td>50</td>
<td>276</td>
<td>0.25</td>
<td>1</td>
</tr>
<tr>
<td>S3</td>
<td>0.0135</td>
<td>6.75</td>
<td>50</td>
<td>276</td>
<td>0.25</td>
<td>1.35</td>
</tr>
<tr>
<td>S4</td>
<td>0.0130</td>
<td>13.00</td>
<td>50</td>
<td>290</td>
<td>0.50</td>
<td>2.6</td>
</tr>
<tr>
<td>S5</td>
<td>0.0100</td>
<td>10.00</td>
<td>100</td>
<td>276</td>
<td>0.50</td>
<td>1</td>
</tr>
<tr>
<td>S6</td>
<td>0.0120</td>
<td>12.00</td>
<td>50</td>
<td>270</td>
<td>0.50</td>
<td>2.4</td>
</tr>
<tr>
<td>S7</td>
<td>0.0125</td>
<td>12.50</td>
<td>50</td>
<td>276</td>
<td>0.50</td>
<td>2.5</td>
</tr>
<tr>
<td>S8</td>
<td>0.0115</td>
<td>11.50</td>
<td>50</td>
<td>276</td>
<td>0.50</td>
<td>2.3</td>
</tr>
<tr>
<td>S9</td>
<td>0.0135</td>
<td>13.50</td>
<td>50</td>
<td>276</td>
<td>0.50</td>
<td>2.7</td>
</tr>
<tr>
<td>S10</td>
<td>0.0110</td>
<td>16.50</td>
<td>50</td>
<td>276</td>
<td>0.75</td>
<td>3.3</td>
</tr>
<tr>
<td>S11</td>
<td>0.0120</td>
<td>18.00</td>
<td>50</td>
<td>276</td>
<td>0.75</td>
<td>3.6</td>
</tr>
</tbody>
</table>
and also has a considerably stronger deflection ($u' = 21.1$ versus $12.7$ m s$^{-1}$).

Figure 3 shows the steady-state rain rate for the six cases profiled above. The S1–90 and S2–90 cases have distributions expectable for weakly decelerated flows with isolated maxima of about 1.6 and 0.8 mm h$^{-1}$, respectively, positioned about 25 km upstream of the mountain peak ($x = 0$ km). The S3–90 case is similar to
that of strongly decelerated saturated flows (Miglietta and Buzzi 2001, 2004; Jiang 2003; Colle 2004) in that moderate accumulations extend far upstream (200 km) of the mountain peak and the distribution is more even. In the cases with RH = 50%, parcels did not reach the LCL (1081 m) until they were over the mountain, and so they have smaller isolated maxima ranging from 0.2 to 0.8 mm h\(^{-1}\) at approximately 25 km upstream of the mountain peak.

A more complete picture of the way flow is modified by changes in RH is provided in Fig. 4, which shows the minimum \(u'/U\) (\(u'_{\text{min}}/U\); Fig. 4a) and maximum surface \(v'/U\) (\(v'_{\text{max}}/U\); Fig. 4b) within four nondimensional units, \(x/a\) (which corresponds to a distance of 200 km for S1), upstream of the mountain peak as a function of RH for all simulations in S1–S3, and three additional series, S4, S5, and S6 (Table 1). In S4, the surface temperature is 290 K and \(N\) is 0.013 s\(^{-1}\) (Fig. 1). The mountain half-width \(a\) in S5 is 100 km. In S6, the surface temperature is 270 K and \(N\) is 0.012 s\(^{-1}\) (Fig. 1). For all flows with RH less than about 40%, \(u'_{\text{min}}/U\) and \(v'_{\text{max}}/U\) are insensitive to changes in RH. This is referred to as the dry regime. Based on nonrotating flow dynamics, one expects the cases with higher \(F\) (S1, S4, and S6) to have weaker flow deceleration in this regime; however, as will be demonstrated in section 4, a decrease in \(U\) has little effect \(u'_{\text{min}}/U\). Hence, all of the cases except S5 have a similar steady-state flow deceleration of about \(-0.45\). When RH is greater than about 40%, but less than about 75%, \(u'_{\text{min}}/U\) strongly increases and \(v'_{\text{max}}/U\) decreases as RH is increased. We refer to this as moist subregime 1. Cases with RH above about 75% in S1–S3 and S5 are in moist subregime 2, which is characterized by decreasing \(u'_{\text{min}}/U\) with increasing RH.

The analyses in this section answer one of the questions posed in the introduction. As RH is increased from 0%–100%, flow does not monotonically become less decelerated. Cases that are very moist may actually be more decelerated as RH approaches 100%.

b. An explanation for the existence of moist subregime 2

What causes the flow to become more decelerated in the high-RH cases (S1, S2, S3, and S5)? The key to this unexpected behavior is in the height of the freezing level \(z_{\text{fr}}\). Vertical cross sections of \(z_{\text{fr}}\), the equivalent potential temperature \(\theta_e\), and the cloud and rainwater mixing ratio (which includes both frozen and liquid hydrometeors) \(q_i\) for S3–95 are shown in Figs. 5a and 5b, respectively. In both cases, there is a \(q_i\) maximum just below \(z_{\text{fr}}\), but this maximum is located at low levels (i.e., below mountain top) and extends about 350 km upstream of the mountain peak in S3–95 (Fig. 5a). In S4–95 (Fig. 5b), the \(q_i\) maximum is predominantly...
over the mountain slope. In either case, the $q_c$ maxima are composed predominantly of cloud water $q_c$. This is demonstrated in Fig. 6, which shows a vertical profile of the primary $q_c$ constituents for S3–95 and at $x/a = -4$. A budget of the terms that act to enhance or deplete $q_c$ was performed. These terms are included in Fig. 6 and show that vapor condensation is the primary cause for the $q_c$ maximum. Enhanced condensation and production precipitate just below $z_f$ has been observed in the real atmosphere and different theories exist to explain it. These include localized cooling due to melting and enhanced accretion on melting hydrometeors (Woods et al. 2008).

Enhanced upstream low-level condensation and precipitation, as in S3–95, has a chain reaction effect. First, it acts to increase $q_w$ locally. This is demonstrated in Fig. 7a, which shows a vertical profile of $q_w$ at $t = 0$, 10 and at $x/a = -4$. Note that a low-level $q_w$ maximum forms in S3–95. Consequently, both terms A and B from (2) increase, leading to a large low-level maximum in $N_m$ (Fig. 7b). This, in turn, causes greater flow deceleration. For comparison, a vertical profile of $q_w$ at $t = 10$ for S3–75 is included in Fig. 7a. The low-level $q_w$ profile decreases slightly with time and does not form a maximum. Ultimately, this causes S3–75 to have smaller $N_m$ (Fig. 7b) and therefore less flow deceleration. Every series with a moist subregime 2 has some alteration to the upstream profiles of $q_w$ in the cases with the most moisture. In contrast, the S4 low-level $q_w$ and $N_m$ profiles change very little with time (Figs. 7c,d).

The effects of upstream condensation and precipitation on flow deceleration were tested by repeating S3 except with the hydrometeor fall speeds set to zero and setting $q_v$ at all grid points back to the original values after each time step. This allows for conversion of vapor to condensate and permits the resulting latent heat to be released but does not allow $q_w$ to change with time. This series is called MS3. The values of $u_{\text{min}}/U$ for S3 and MS3 are shown in Fig. 8. Because hydrometeor growth due to collision–coalescence was limited by setting the fall speeds to zero, MS3 has less latent-heat release than S3, leading to a dry-to-moist transition at RH = 50%. More important, though, is the fact that MS3 does not have two moist subregimes. For all cases with RH greater than 50%, $u_{\text{min}}/U$ increased with increasing RH.

There is some sensitivity to the choice of microphysical parameterization in moist subregime 2. To demonstrate this, the S3 series was repeated with the Lin et al. (1983) microphysical parameterization scheme in place of WSM6 (the LS3 series). The values of $u_{\text{min}}/U$ for LS3

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1 Experiments with $z_f$ ranging from below ground level (S6) to 3000 m above ground level (S4) indicate that a low-level upstream $q_c$ maximum only occurs if 100 m < $z_f$ < $h$ (not shown). The minimum height of 100 m for $z_f$ was determined using the simulations with the WSM6 microphysical parameterization scheme. Sensitivity tests using other schemes indicate that the minimum height is different when the scheme is changed.
are included in Fig. 8. Note that the values of $u_{\text{min}}/U$ are slightly different from those in S3 and the transition from subregime 1 to subregime 2 occurred at a slightly higher RH in LS3. This difference is due to the differing treatment of ice crystals. In WSM6, ice crystals have a prescribed fall speed while small liquid droplets do not fall. This sometimes leads to an accumulation of condensate just below the freezing level, as in S3–95 (e.g., Fig. 6). The Lin et al. (1983) scheme, on the other hand, assumes that the fall speed of ice crystals is $0 \text{ m s}^{-1}$. This allows more time for snow to develop, which can later fall and affect condensation below $z_{\text{fp}}$.

c. Three-dimensional effects

Do the above findings hold as new layers of complexity are added to the problem? Studies of dry flow indicate there are important distinctions between flows with an infinitely long ridge and flows with a finite ridge length. When there exists an infinitely long ridge, air parcels essentially have an infinite amount of time to

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**Fig. 6.** Vertical profiles of various water species and generation–depletion terms for cloud water in S3–95 at $x/a = -4$.

**Fig. 7.** Vertical profiles of (a), (c) water vapor mixing ratio and (b), (d) $N^2_a$, together with terms A and B from (2) for select S3 and S4 cases.
accelerate along the face of the mountain and eventually cross to its lee. For cases with a finite ridge length, deflected air parcels have only a finite length of time to accelerate and cross over the barrier before reaching the end of the ridge (Pierrehumbert and Wyman 1985; Peng et al. 1995; Petersen et al. 2003 and citations therein) and the precipitation maximum is shunted to the left of the mean flow (Jiang 2006). To better understand how moist, rotating flow behaves in the presence of a 3D obstacle, the S1 and S3 simulations were repeated in 3D with either a 300- (3D-L) or 100- (3D-S) km-long ridge.

Figure 9 shows the rain rate and surface wind barbs for the RH = 90% cases from 3DS1 and 3DS3. In all panels, there is some degree of flow deflection on the upwind side of the mountain, resulting in the rain rate maximum being positioned toward the left end (facing downwind) of the ridge. The horizontal extent of the rain shield in 3DS1 (Figs. 9a,b) extends about 150 km upstream of the mountain peak. This is also true of the S1–90 experiment (Fig. 3). The precipitation distribution in 3DS3 shows some sensitivity to barrier length. In 3DS3–90S (Fig. 9d), the precipitation shield extends about 100 km upstream of the ridge peak toward the left end of the ridge, and the rain rates are weak. In 3DS3–90L (Fig. 9c), the precipitation shield extends about 150 km upstream of the mountain at the left end of the ridge, and the rain rates are only slightly higher than in 3DS3–90S. In S3–90 (Fig. 3), the rain shield extends about 200 km upstream of the mountain peak. The 3DS3 cases also have some semblance of barrier-jet formation, with wind speeds increasing toward the left and a near-stagnation point toward the right end of the ridge.

Streamlines from the RH = 90% 3D simulations (Figs. 10a,c) show that the degree of deflection is more or less independent of the ridge length. Consequently, the 3DS3 streamline, which crosses the mountain in 3DS3–90L (Fig. 10a), flows around the mountain in 3DS3–90S (Fig. 10c). Streamlines for the RH = 50% cases are provided in Figs. 10b,d. Comparison of these streamlines to their RH = 90% analogs shows that as RH is decreased, the degree of deflection increases. Ultimately, this affects the location of precipitation maxima. Note that the precipitation maximum is shifted further to the left of the mean flow as RH is decreased in 3DS1. (There was no precipitation in the 3DS3–50 simulations.)

The value of $u'_{\text{min}}/U$ as a function of RH for the S3 and 3DS3 simulations is shown in Fig. 11. The dry-to-moist regime transition was dependent on the ridge length with the transition occurring at a higher RH as the ridge length was decreased. This transition occurred at $(U/N\text{LCL}) = 1$ (see section 4) in all 3D cases except 3DS3-S, which did not have a moist subregime 2. In 3DS3-S, a significant proportion of the low-level flow went around, rather than over, the ridge (Fig. 10c), leading to less upstream precipitation in this simulation compared to 3DS3-L and S3 (Figs. 3 and 9). Ultimately, the upstream vertical gradient in 3DS3-S was not significantly altered (not shown).

In summary, the flow dependencies found in the 2D simulations were generally well reproduced in 3D, but for very stable flows the cross-stream ridge length can be an important dimensional control parameter.

4. The moist versus the dry regime and nondimensional control parameters

Is $N_m$ is a reasonable substitute for $N$ in saturated flow? To answer this question, an analytic relation of the form $u'_{\text{min}} = s(F, R_o)$ is sought by performing an independent set of experiments with varying $F$ and $R_o$. These simulations have RH = 0%, $U = 10$ m s$^{-1}$, $a$ ranging from 25 to 200 km, and $h$ ranging from 988 to 2963 m. These simulations are otherwise identical to S3. Figure 12a shows $u'_{\text{min}}$ as a function of $F$ and $R_o$ (solid contours). These curves are similar to those in Fig. 14 of Pierrehumbert and Wyman (1985). When $u'_{\text{min}}/U$ is multiplied by $F$ (dashed lines in Fig. 12a), the resulting curves are approximately collapsed, indicating that a universal solution of the form $Fu'_{\text{min}}/U = s(R_o)$ exists. These collapsed curves have been fit with a second-order polynomial given by

$$F u'_{\text{min}}/U = -0.0235 R_o^2 - 0.0075 R_o - 0.0674. \quad (3)$$

The analytic relation (3) and $Fu'_{\text{min}}/U$ for the RH = 0% cases in S1–S6 as well as from five other series,
S7–S11 (Table 1), are plotted in Fig. 12b. These points lie approximately on the curve given by (3). The values of $F u_{\text{min}} / U$ for the S1–S11 simulations with RH = 95% are plotted in Fig. 12c. These cases are effectively saturated and therefore $N_m$ is used in the formulation for $F$. If $N_m$ is indeed a reasonable substitute for $N$, the steady-state values of $F u_{\text{min}} / U$ for these simulations should sit on or near the analytic curve given by (3). As Fig. 12c demonstrates, some of the simulated values of $F u_{\text{min}} / U$ are close to the analytic curve, but S3–S4, and S9–95 are far removed from it. Hence, substituting $N_m$ for $N$ in (3) does not allow it to describe these simulations. The values of $u'_{\text{min}} / U$ in moist subregime 1 are generally higher than in the dry regime, and as a consequence the analytic relation (3) (using $N$) underestimates $u'_{\text{min}} / U$ in this regime. This is demonstrated in Fig. 12d, which shows $F u_{\text{min}} / U$ (black dots) for the S1–S11 cases with RH = 70%. [The analytic relation (3) was also calculated using $N$ for dry layers and $N_m$ for saturated layers, but the resulting $F u_{\text{min}} / U$ fails to fit the curve (not shown)]. The differences between the steady-state $F u_{\text{min}} / U$ and those predicted using (3) indicate that flows in the moist regime must have additional nondimensional control parameters that need to be accounted for in (3).

To find any additional nondimensional control parameters in the moist regime requires reconsideration of the dimensional control parameters. We start with moist subregime 1. The list of dimensional parameters in this regime includes $N_m$ or $N$, $U$, $h$, $a$, $f$, and an

Fig. 9. The steady-state rain rate (shaded as in legend) and surface wind barbs.
additional parameter that provides information about moisture content. Here, we use the LCL because it gives an indication of the potential for an orographic flow to reach saturation and its inclusion does not introduce any new dimensions to the parameter space. The addition of this parameter requires that another nondimensional control parameter be devised. One candidate for this is \((U/NLCL)\). This number is a variant on the Froude number in that it measures whether there exists sufficient kinetic energy for parcels to reach the LCL.\(^2\) Figure 13a shows the maximum \(q_i\) at any height within four nondimensional units upstream of the mountain peak as a function of \((U/NLCL)\). Cases

\(^2\) A similar formulation was used in Wang et al. (2000), only with LCL replaced with the level of free convection (LFC) to determine whether flow had the necessary potential for parcels to reach the LFC and for convective cloud development.
with \((U/N_{\text{LCL}})\) less than about 0.6 do not have clouds. The \(u'_{\text{min}}/U\) as a function of \((U/N_{\text{LCL}})\) is plotted in Fig. 13b. All of the cases with \((U/N_{\text{LCL}})\) of about 0.6 or less are in the dry regime and have constant \(u'_{\text{min}}/U\). For \((U/N_{\text{LCL}})\) greater than about 0.6, flows are in the moist regime and \(u'_{\text{min}}/U\) varies strongly with increasing \((U/N_{\text{LCL}})\) up to about \((U/N_{\text{LCL}}) = 5\), where the \(u'_{\text{min}}/U\) curves tend to asymptote to a constant value. Hence, \((U/N_{\text{LCL}})\) may be used to determine in advance whether flow will be in the dry or moist regime and, as a result, whether there will be condensation and latent-heat release.

Some sensitivity to \(z_{\text{fr}}\) for flows in moist subregime 2 was found in section 3b. If \(z_{\text{fr}}\) is included in the dimensional space for flows in moist subregime 2, then an additional nondimensional control parameter for flows in this regime must exist. Consideration of \(u'_{\text{min}}/U\) as a function of \((L_{\text{CL}}/z_{\text{fr}})\) (Fig. 13c) shows that an \((L_{\text{CL}}/z_{\text{fr}})\) of about 0.9 represents a critical number and that S1, S2, S3, and S5, which can have \((L_{\text{CL}}/z_{\text{fr}}) < 0.9\) and 100 m < \(z_{\text{fr}} < h\), have a moist subregime 2.

A correction for latent heat effects and upstream water vapor depletion of the form

\[
F_{\text{min}}/U = \left( F_{\text{min}}/U \right)_\text{dry} + 0.04 \times \min\left( 5, \frac{U}{N_{\text{LCL}}} \right) \times s \left( \frac{L_{\text{CL}}}{z_{\text{fr}}} \right),
\]

where

\[
s \left( \frac{L_{\text{CL}}}{z_{\text{fr}}} \right) = \begin{cases} 
5.5 + 4.5 \tanh \left( 7.5 \frac{L_{\text{CL}}}{z_{\text{fr}}} - 4 \right) & \text{if } 100 \text{ m} < z_{\text{fr}} < h \text{ and } \frac{L_{\text{CL}}}{z_{\text{fr}}} < 0.9, \\
1 & \text{otherwise}
\end{cases}
\]

Fig. 12. (a) Values of \(u'_{\text{min}}/U\) as a function of \(R_{\text{e}}\) (solid contours of constant \(F\)) and \(F_{\text{min}}/U\) (dashed contours of constant \(F\)), and (b)–(d) averaged \(u'_{\text{min}}/U\) (black contour), the analytic solution to Eq. (3) (gray contour) and \(F_{\text{min}}/U\) for the S1–S11 cases with RH = 0%, 95%, and 70%, respectively.
Subtracting 0.04 \((U/NLCL)\) from the \(Fu_{\text{min}}/U\) in the RH = 70% cases yields the points given by the diamonds in Fig. 14a. These points are close to the analytic curve (3). Equation (5) provides reasonable estimates for \(Fu'_{\text{min}}/U\) for all cases in moist subregime 1. Figure 14b shows \(Fu'_{\text{min}}/U\) for the RH = 95% cases in S1–S11 after the correction (4) is applied. These points are on or near the analytic curve (3). The above correction, (4), provides reasonable estimates of \(Fu'_{\text{min}}/U\) for all cases in moist subregime 2.

Using a similar methodology to the above, analytic relations for other dependent variables (i.e., \(v'\) and \(w\)) could be devised. Such is beyond the scope of this paper. Moreover, the analytic relations (3)–(5) are unlikely to provide reasonable predictions for more complex flows than the 2D cases used here to derive these relations because the full spectrum of dimensional control parameters for realistic flows was not considered and because there is some sensitivity for those terms in (4) and (5) involving the LCL to the choice of microphysical parameterization. However, the above exer-
cise does serve to answer the final question posed in the introduction: Can nondimensional control parameters be used to determine steady-state flow patterns when precipitation is present? For these simple experiments, the answer is yes.

One final question is whether the above methodology is useful for determining a priori the maximum rain rate \( R_{\text{max}} \). In the simplest conception, \( R_{\text{max}} \) is given by the following:

\[
R_{\text{max}} = \int_{h}^{z_{\text{fr}}} w(x, z) \frac{\partial \rho_{w}}{\partial z} \, dz,
\]

where \( w \) is the vertical velocity and \( \rho_{w} \) is the saturation vapor density (Smith 1979). The above formulation assumes that all condensate immediately falls to the ground as precipitation and that the incident airstream is saturated, which is frequently true in cases of orographic rain (see Miglietta and Rotunno 2006 and references therein). An important issue arises when attempting to adapt this simple model to our results, namely that the simplification assumes that an airstream will remain saturated as it ascends the mountain. When there exists upstream precipitation and moisture depletion, this is not strictly true (Miglietta and Rotunno 2006) and the vertical gradient of moisture (i.e., \( \partial \rho_{w}/\partial z \)) will be altered (as for example is shown in Fig. 7a). Ultimately, changes to \( \partial \rho_{w}/\partial z \) may be proportional to the new nondimensional control parameters (LCL/\( z_{\text{fr}} \)) and (\( U/N \text{LCL} \)), but devising a pure connection between \( \partial \rho_{w}/\partial z \) and the nondimensional control parameters is made difficult by the fact that hydrometeors sometimes drift downwind and sometimes re-evaporate back into the atmospheric column before they reach the ground, microphysical parameterization sensitivities notwithstanding.

5. Conclusions

In this paper, the effects of moisture on flow and precipitation patterns for low wind speed, statically stable, nearly moist-neutral, and moderate Rossby number flows were studied and attempts were made to determine the most relevant dimensional and nondimensional control parameters for moist flow. As relative humidity is increased from 0% to 100%, a flow does not become monotonically less decelerated. For some of the very moist flows studied here, enhanced condensation near the freezing level leads to changes in the upstream vertical profile of \( q_{w} \) and hence causes increased upstream-flow deceleration as RH approaches 100%. The degree of upstream-flow deceleration as RH approaches 100% is somewhat sensitive to the choice of microphysical parameterization (sections 3a and 3b). Some of the simulations were repeated in three dimensions to insure the results have generality to more realistic flows. Most effects noted for two dimensions were reproduced in three dimensions. The three-dimensional simulations also revealed that the location of the precipitation maximum was sensitive to the upstream RH profile with decreases in RH leading to a leftward (facing downstream) displacement of the maximum (section 3c).

A simple substitution of \( N_{m} \) for \( N \) did not provide reasonable predictions of steady-state flow deceleration for all cases. These include cases where the freezing level was below mountain top and where there was strong, upstream, low-level water-vapor depletion. Analyses reveal that the dimensional control parameters for these moist, 2D flows include the LCL and \( z_{\text{fr}} \), parameters, which are unimportant for dry flows. These dimensional parameters were incorporated into analytic relations that predict flow deceleration based on an extended set of nondimensional control parameters. These relations yield reasonable estimates of flow deceleration for 2D cases with latent-heat release and precipitation (section 4).

This study has been limited to low-speed, statically stable, and nearly moist-neutral flows; additionally, friction and radiation effects have been neglected. The restriction to statically stable and nearly moist-neutral flows does limit the application of these results somewhat. However, previous research has shown that moist-neutral conditions have often been observed in conjunction with orographic precipitation (Miglietta and Rotunno 2005 and references therein). The sensitivity of flow to the height of the freezing level is consistent with Colle’s (2004) simulations, which did include friction. Finally, our simulations reach a steady-state condition within 7–9 h of integration time, so it is unlikely that radiation would have had a large impact on these results.

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