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Stratified Turbulence: A Possible Interpretation of Some Geophysical Turbulence Measurements

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ABSTRACT

Several existing sets of smaller-scale ocean and atmospheric data appear to display Kolmogorov–Obukov–Corrsin inertial ranges in horizontal spectra for length scales up to at least a few hundred meters. It is argued here that these data are inconsistent with the assumptions for these inertial range theories. Instead, it is hypothesized that the dynamics of stratified turbulence explain these data. If valid, these dynamics may also explain the behavior of strongly stratified flows in similar dynamic ranges of other geophysical flows.

1. Introduction

The term "stratified turbulence" was introduced by Lilly (1983) to describe the dynamics of flows dominated by stable density stratification. Such flows can consist of internal waves, but also of quasi-horizontal, meandering motions; they possess potential vorticity and are strongly nonlinear. Recent numerical simulations (Riley and deBruynKops 2003; Lindborg 2005) and scaling arguments (Lindborg 2005, 2006) addressing the dynamics of stratified turbulence establish that a strong downscale transfer of energy exists in the horizontal and, along with this, that the development of a horizontal spectral inertial range can occur. This downscale transfer of energy can then lead to smaller-scale instabilities and turbulence, providing new "pathways" to turbulence in geophysical flows.

In this paper we hypothesize that stratified turbulence dynamics explain several sets of seemingly anomalous ocean and atmospheric data. In the next section we give a brief description of stratified turbulence, summarizing some results from numerical simulations as well as from scaling analysis. In the third section we present several independent sets of ocean and atmospheric data whose horizontal spectra appear to be consistent with an inertial range interpretation; we argue that these results are inconsistent with a classical Kolmogorov–Obukhov–Corrsin interpretation but are consistent with the hypothesis of stratified turbulence. In the final section we conclude by summarizing our results and discuss more general possible implications.

2. Stratified turbulence

Following Lindborg (2006), we consider a flow system satisfying the Navier–Stokes equations subject to the Boussinesq approximation. Energy is assumed to be input at horizontal length scales of order $\mathcal{L}_L$, with corresponding velocity scale σ_u, an rms horizontal velocity, and buoyancy frequency N. The principal requirement for the existence of stratified turbulence is that the Froude number F_h defined by these parameters is small; that is,

$$F_h = \frac{\sigma_u}{N\mathcal{L}_L} \ll 1. \tag{1}$$

In the ocean cases to be considered, $\mathcal{L}_L$ is on the order of at least several hundred meters, σ_u is on the order of 0.1 m s^{-1}, and the buoyancy period is on the order of

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tens of minutes. These values of the parameters generally satisfy the condition for stratified turbulence. Furthermore, the direct numerical simulations by Riley and deBruynKops (2003) indicate that the inequality may not need to be necessarily very strong, but only "less than order 1."

There are several important ramifications of the assumption of low Froude number. First of all, from mathematical theory (Billant and Chomaz 2000a,b), from physical arguments (Lilly 1983), and from numerical simulations (Riley and deBruynKops 2003; Lindborg 2005), it is known that, for large enough Reynolds numbers, strongly stratified flows can develop strong vertical shearing of the horizontal velocity, leading to vertical differential length scales ℓ_v of the order

$$\ell_v \sim \sigma_u/N \tag{2}$$

on time scales of order $\mathcal{L}_L/\sigma_u$. There are at least two types of instabilities known to lead to this. One is the decorrelation instability of Lilly (1983), and the other the zigzag instability of Billant and Chomaz (2000a,b). Because of either mechanism, shear will develop in the layers and become so strong that the local Froude number based upon this vertical scale, $F_v = \sigma_u/N\ell_v$, will become of order 1, and hence the local Richardson number Ri will also become order 1, as pointed out by Billant and Chomaz (2001). Estimates suggest that this scale is on the order of tens of meters in the ocean.

The development of these local, strong shear layers leads to two main effects. First, the instabilities will tend to break up the larger structures of order $\mathcal{L}_L$ in the horizontal, a phenomenon observed in the direct numerical simulations of Riley and deBruynKops (2003). The process is repeated in many steps, which means that energy will pass from large horizontal scales of motion to small scales of motion in a forward energy cascade. Although the dynamics of this energy cascade are controlled by the strong stratification, the process has some important features in common with the three-dimensional Kolmogorov cascade. The kinetic energy dissipation rate ϵ_K is still expected to scale with σ_u and $\mathcal{L}_L$, so that the dissipation rate can be estimated as (Taylor 1935)

$$\epsilon_K \sim \frac{\sigma_u^3}{\mathcal{L}_L}. \tag{3}$$

Combining (1), (2), and (3), the ratio between ℓ_v and $\mathcal{L}_L$ can be estimated as

$$\frac{\ell_v}{\mathcal{L}_L} \sim F_h \sim \left(\frac{\ell_O}{\mathcal{L}_L}\right)^{2/3} \ll 1, \tag{4}$$

where $\ell_O = (\epsilon_K/N^3)^{1/2}$ is the Ozmidov scale, which can be interpreted as the largest horizontal scale that has sufficient kinetic energy to overturn. (See the appendix for additional discussion of the Ozmidov scale.) This relation illustrates the very elongated, or low aspect ratio, character of the structures of motion. In this respect stratified turbulence is very different from isotropic, Kolmogorov inertial range turbulence.

The second effect of the local regions of strong shear will be the tendency to develop local patches of three-dimensional turbulence, causing further cascading of energy to smaller scales. At some specific vertical and horizontal length scales the influence of stratification will become weak enough that the strongly stratified turbulent cascade can no longer prevail, and there will be a transition to more classical three-dimensional turbulence. To see this, in Fig. 1 we have reproduced a typical vertical wavenumber spectrum $\Phi_s(k)$ of vertical shear in the ocean from Gargett et al. (1981). [Note that the vertical shear spectrum $\Phi_s(k) = k^2E_K(k)$, where E_K is the kinetic energy spectrum discussed below.] The spectrum takes a local minimum around the wavenumber $k_b = 1/\ell_O$. Consistent with the discussion of the Ozmidov scale in the appendix, the minimum marks a transition between those scales of motion that are strongly influenced by stratification and those that are not. To the right of the minimum the influence of stratification becomes asymptotically weaker with increasing wavenumber and the spectrum approaches the spectrum of isotropic Kolmogorov turbulence. According to the established interpretation, the spectrum to the left of the minimum, which follows a k_v^{-1} behavior for more than one decade, is a spectrum of weakly nonlinear internal gravity waves, although strong nonlinear effects may not be negligible in the neighborhood of the minimum (Müller et al. 1986; Lindborg and Riley 2007). The observations reported in Fig. 1 are made at different depths at two different locations, one outside the island of Bermuda and one at the northern edge of the Gulf Stream. In all records $k_b \approx 1$ cycle per meter (cpm), which means that the spectrum range to the left of the minimum corresponds to spatial scales between 1 and 100 m. In these measurements the Ozmidov length scale varied between 0.7 and 1.3 m (see Table 3 from Gargett et al. 1981).

The minimum in the vertical shear spectrum in Fig. 1 at $k_b = 1/\ell_O$ suggests that the transition to Kolmogorov inertial range turbulence occurs at a vertical length scale near the Ozmidov scale. Based upon the results from numerical simulations (Riley and deBruynKops 2003; Lindborg 2005) it is expected that the transition from stratified turbulence to more classical three-dimensional turbulence occurs at a horizontal scale $\mathcal{L}_S$

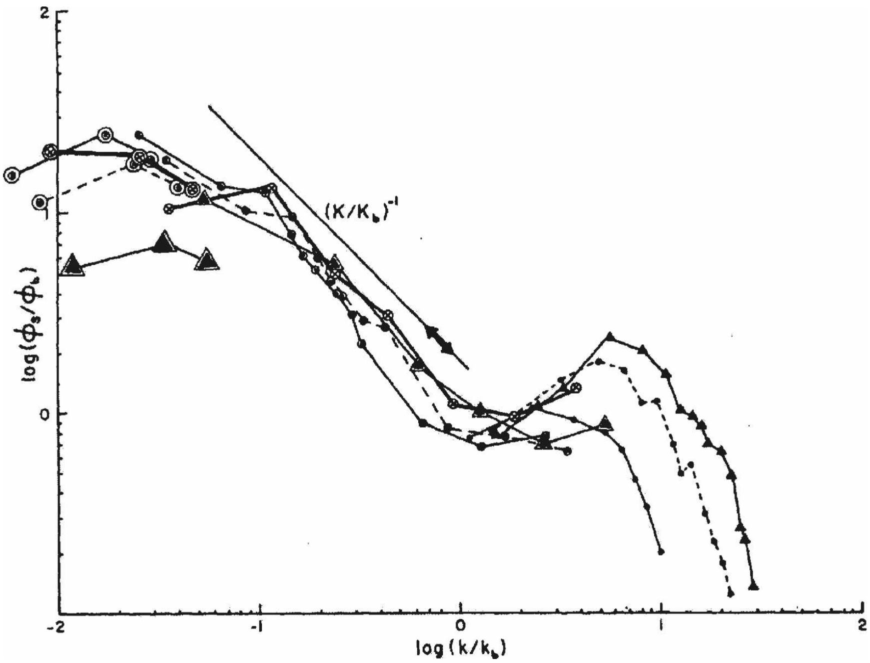

FIG. 1. Vertical wavenumber spectra of vertical shear in the ocean. The spectra are normalized by $\phi_b = (\epsilon_K N)^{1/2}$ and the wavenumber is normalized by $k_b = N^{3/2}/\epsilon_K^{1/2}$. Reproduced from Gargett et al. (1981).

that is not much larger than the Ozmidov scale. Therefore, from (4),

$$\frac{\mathcal{L}_S}{\mathcal{L}_L} \ll 1,$$

implying at low Froude number a large separation of scales from the energy-containing scale to the classical three-dimensional turbulence scale.

Assuming a low Froude number of the larger-scale motions, implying a large separation of scales, assuming a Reynolds number high enough that viscous effects are unimportant over these scales, and making plausible scaling arguments [e.g., Eq. (3) above], Lindborg (2005, 2006) reasoned that a spectral inertial range could exist in the horizontal spectra within the horizontal scales from $\mathcal{L}_L$ to $\mathcal{L}_S$. This inertial range is analogous to the Kolmogorov inertial range in three-dimensional, nonstratified turbulence, with the one-dimensional horizontal spectra of the horizontal kinetic energy and potential energy given by

$$E_K(k_h) = C_K \epsilon_K^{2/3} k_h^{-5/3} \quad \text{and} \tag{5}$$

$$E_P(k_h) = C_P \epsilon_K^{-1/3} \epsilon_P k_h^{-5/3}. \tag{6}$$

Here ϵ_K is the average dissipation rate of the horizontal kinetic energy, ϵ_P is the average dissipation rate for potential energy, and C_K and C_P are universal constants. Note that the original arguments of Obukhov (1949) and Corrsin (1951) for the inertial-convective subrange are invalid for this case, since the flows in this range are clearly highly anisotropic, and since other physics, namely stable density stratification, are strongly influencing the dynamics.

Lindborg performed three-dimensional numerical simulations of forced, strongly stratified flow subject to the above conditions, and found inertial ranges as predicted (see Fig. 2). The constants were found to be $C_K \approx C_P \approx 0.5$. Riley and deBruynKops (2003) performed direct numerical simulations of decaying, strongly stratified flows and also found inertial range behavior, with coefficients consistent with those found by Lindborg.

Horizontal inertial range spectral behavior has often been observed in the middle atmosphere. In Fig. 3 the horizontal wavenumber spectra from the upper troposphere and lower stratosphere are reproduced from Nastrom and Gage (1985). It is a remarkable feature of these spectra that they exhibit $k_h^{-5/3}$ spectra in the wavenumber range corresponding to wavelengths between 1 and 500 km. Lindborg (2006) argued that the flow in this range satisfied the conditions for stratified turbulence, and that these dynamics were then what leads to the development of the $k_h^{-5/3}$ horizontal spectra. We here argue that flow on even smaller scales can meet

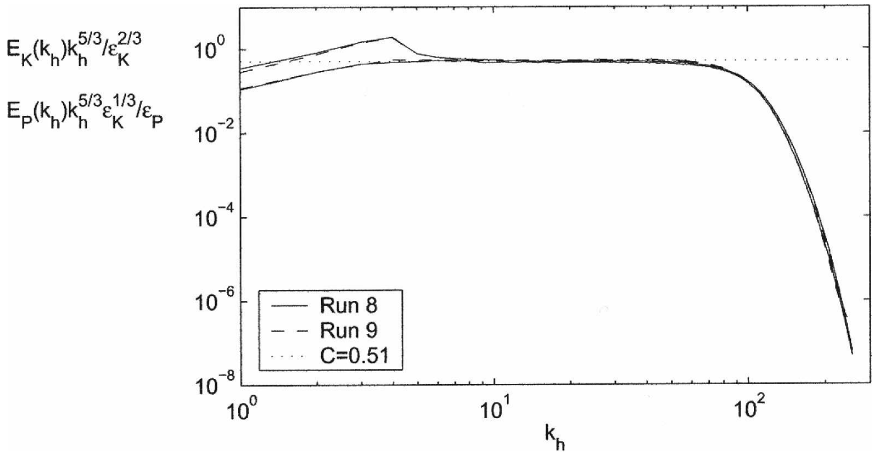

FIG. 2. Scaled kinetic and potential energy horizontal spectrum (from Lindborg 2005) showing the predicted inertial range behavior.

these requirements, and therefore exhibit similar behavior. We shall now investigate the observational evidence for this statement.

3. Ocean and atmospheric data

There is some evidence that stratified turbulence exists in stably stratified regions of the ocean on horizontal scales from those that are not much larger than Ozmidov scale (usually around 0.1–1 m) up to at least a few hundred meters. It is possible that the energy on the few hundred–meter scale is a result of a forward cascade due either to nonlinear internal wave interactions or stratified turbulence at larger scales, or it may be the remnants of turbulent patches, as observed in laboratory experiments (e.g., Spedding 1997; Praud et al. 2005). In the latter cases at late times the Froude number becomes very low, which is a requirement for stratified turbulence. The Reynolds number in these laboratory flows is also very low, however, probably inhibiting the development of smaller-scale turbulence that might occur in the ocean.

Klymak and Moum (2007a,b) have collected temperature data near the Hawaiian ridge from horizontal tows of a microstructure platform; measurements were made at a variety of depths ranging from 700 to 3000 m. They obtained displacement spectra by interpreting temperature in terms of vertical displacement; they also obtained direct measurement of energy dissipation rates over a very large dynamic range, with dissipation rates varying over four orders of magnitude. In particular, in addition to the spectra of larger-scale motions, they presented horizontal spectra in the range from several hundred meters down to below one meter. Figure 4 gives typical spectra of the horizontal gradient of vertical displacement (slope) obtained from their experiments. (Note that k_x in this and the next figure corresponds to k_h in the present paper.) The spectra have been binned by the value of the turbulent diffusivity defined by $K_\rho = 0.2\epsilon/N^2$, and have been normalized by the Wentzel–Kramers–Brillouin (WKB) scaling N_0/N, where N is the local value of the buoyancy frequency and $N_0 = 5.3 \times 10^{-3}$ s^{-1}. Note that N varies by only about a factor of 2 for the spectra plotted here. It is seen in the figure that, for wavenumbers below a few tenths of a cycle per meter (horizontal lengths above a few meters), the spectra approximately follow a $k_h^{1/3}$ curve, especially in the more energetic cases. (The $k_h^{1/3}$ spectra for slope correspond to $k_h^{-5/3}$ spectra for temperature.) Klymak and Moum (2007b) remark that the "horizontal spectra exhibit a turbulent shape ($\phi\zeta_x \sim k_x^{1/3}$) to surprisingly low wave numbers," for the most intense turbulence "to scales exceeding 500 m."

It is tempting to consider the horizontal spectra somewhat below 1 cpm as exhibiting an Obukhov/Corrsin inertial-convective subrange. Some physical reasoning and the data of Gargett et al. (1981) are of value in this consideration. Most of the data collected by Klymak and Moum were at a depth of about 700 m, where the mean buoyancy frequency was $N^2 = 2 \times 10^{-5}$ s^{-2}. The dissipation rate ϵ_K varied between 4.3×10^{-10}, for the lowest curve in Fig. 4, to 1.2×10^{-7} m^2 s^{-3}, for the upper curve. Using these values, the Ozmidov scale is found to range from approximately 7 cm for the lower curve to approximately 1.2 m for the upper curve. Therefore, at wavenumbers below the Ozmidov wavenumber of order 1 cpm, the effects of density stratification should be strong, and in fact overturning should be difficult. [As discussed in the appendix, the local Froude number at a horizontal scale ℓ can be estimated as $(\ell_0/\ell)^{2/3}$, so that the Froude number

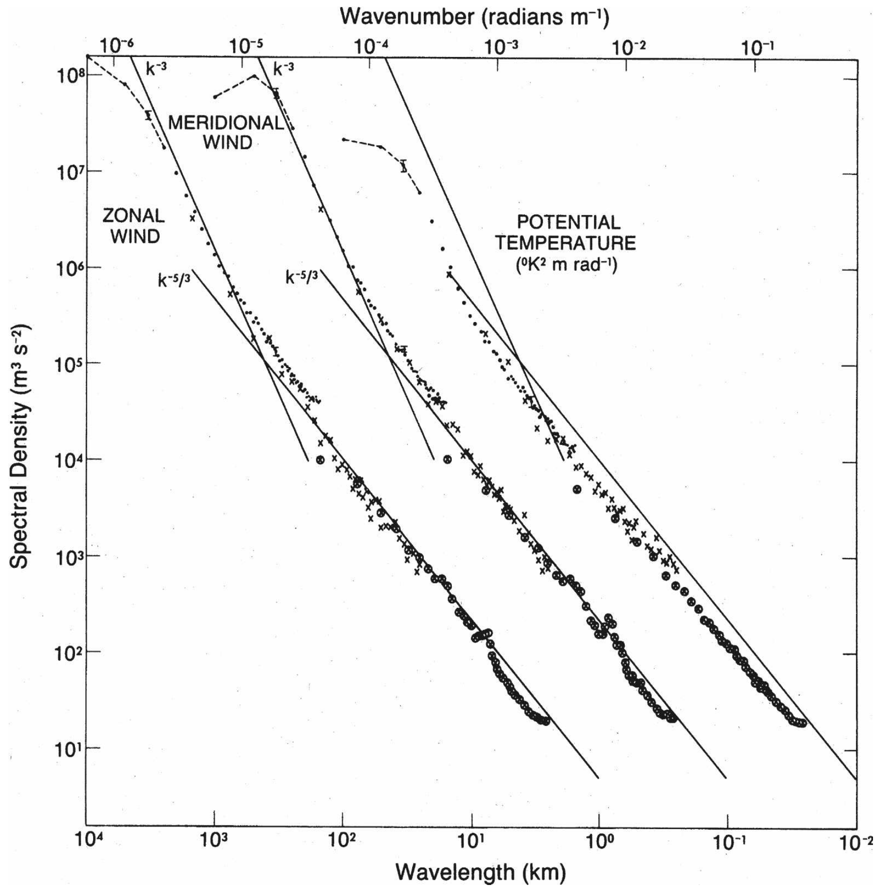

FIG. 3. From left to right, variance power spectra of zonal wind, meridional wind, and potential temperature near the tropopause from Global Atmospheric Sampling Program aircraft data. The spectra for meridional wind and temperature are shifted 1 and 2 decades to the right, respectively. Reproduced from Nastrom and Gage (1985).

would be small for horizontal scales larger than the Ozmidov scale.] This would violate one of the principal assumptions of the theory of Obukhov and Corrsin, namely that other physical mechanisms not considered in their theory, in particular stable density stratification, are not operative.

In addition, these values of the Ozimdov scale are in the same range as the values obtained by Gargett et al.

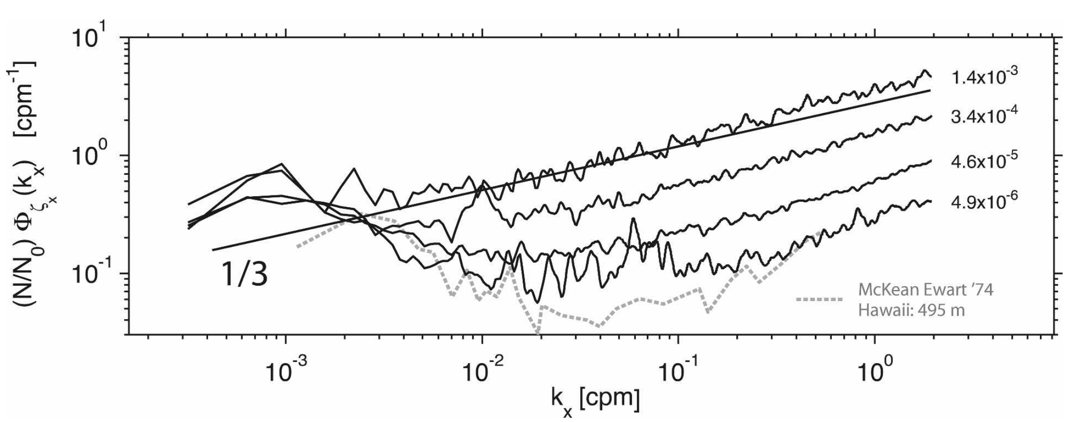

FIG. 4. Average isopycnal slope spectra, binned in one-decade bins of the turbulent diffusivity $K_\rho = 0.2\epsilon/N^2$ and normalized by (N_0/N) (from Klymak and Moum 2007b). A line with slope $k_x^{1/3}$ has been added for comparison.

(1981) and discussed in section 2. Therefore, following Klymak and Moum (2007b), it is tempting to speculate that, for their case as well, the vertical spectra of the vertical displacement gradient (strain) should also follow a k_v^{-1} slope below a wavenumber of about 1 cpm. If the interpretation of the spectra to the left of the Ozmidov wavenumber in terms of an Obukhov/Corrsin inertial-convective subrange is correct, then for a given ocean measurement case, the vertical and horizontal spectra should coincide. Assuming similar dynamics are active for the experiments of Gargett et al. and of Klymak and Moum, the spectra would not be even nearly isotropic, invalidating another assumption of Obukhov and Corrsin.

Although a $k_h^{1/3}$ horizontal spectral range together with a k_v^{-1} vertical spectral range, both below the Ozmidov wavenumber, are inconsistent with the hypothesis of a classical inertial-convective subrange, they are consistent with the hypothesis of stratified turbulence. [Note that, using scaling arguments similar to those presented in section 2, Lindborg (2006) has argued that the vertical spectra must behave as k_v^{-1} in stratified turbulence.] In addition, using Eq. (6) and the value of C_P of about 0.5 found from the numerical simulations, values for the dissipation rates obtained from fitting the data to the inertial range power-law curves are within about 20% of the directly measured values (J. M. Klymak 2006, personal communication).

Recently published horizontal temperature spectra, inferred by seismic reflection transects (Holbrook and Fer 2005), are consistent with these results of Klymak and Moum (2007b) and with the hypothesis of stratified turbulence. Holbrook and Fer obtain horizontal temperature spectra from about 10 km down to about 30 m (see Fig. 5). They noted that, for horizontal scales below about 300 m, "the slope is close to that expected in the inertial-subrange of turbulence." As pointed out above, however, these results are probably not consistent with the inertial-convective subrange, but are consistent with a stratified turbulence inertial range.

Dissipation rates were not measured in this study. Assuming, however, that the inertial range relation as given by Eq. (6) for the potential energy spectrum is valid, using the value $C_P = 0.5$ (Lindborg 2006), and using the value of 0.4 for the mixing efficiency ϵ_P/ϵ_K as obtained in the direct numerical simulations of Riley and deBruynKops (2003), from the measured spectrum we estimate the dissipation rate to be $\epsilon_K \approx 3 \times 10^{-8}$ m^2 s^3. In these experiments the buoyancy frequency was measured to be $N \approx 4 \times 10^{-3}$ s^{-1}. With these values of ϵ_K and N, we obtain the value of 0.7 m for the Ozmidov length scale. Again we find that the inertial range extends to considerably larger scales than ℓ_O. In addition, estimating the local Froude number at horizontal scale ℓ as $(\ell_O/\ell)^{2/3}$, we conclude that the Froude number is small in this inertial range, again consistent with the arguments for a stratified turbulence inertial range.

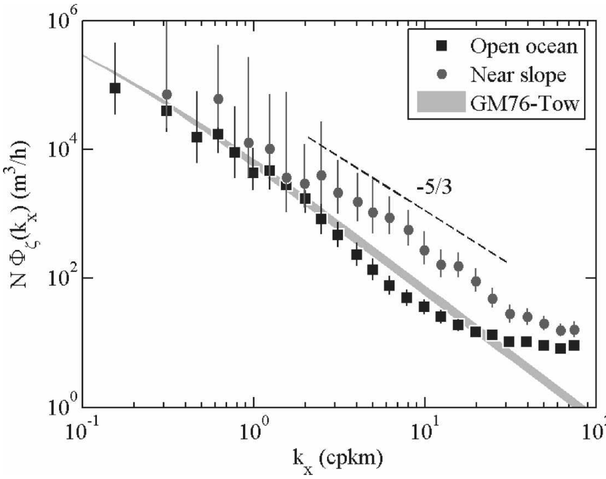

FIG. 5. Horizontal wavenumber spectra of vertical displacement, scaled by the average buoyancy frequency. Vertical bars are 95% confidence intervals. The dashed line shows the −5/3 slope (from Holbrook and Fer 2005).

Data from a much earlier field experiment also show consistency with a stratified turbulence interpretation and in a similar range as the above-mentioned experiments. Ewart (1976) obtained horizontal temperature spectra along isobaric trajectories at various depths and geographic locations using the self-propelled underwater research vehicle (SPURV). The SPURV data were recorded at very high frequencies (corresponding to data samples at every 0.185 m) and with high temperature accuracy. A typical dataset is shown in Fig. 6, for this case including data taken at depths of 300, 650, 1000, and 1600 m off the coast of San Diego (at 30°N, 124°W). [Ewart also presented data from experiments near the Cobb Seamount (at 47°N, 131°W), off the coast of Mexico (at 21°N, 110°W), and off the Hawaiian islands (at 20°N, 156°W). Out of the 14 datasets presented, 12 are consistent with the following interpretation.] A solid line at a slope of $k_h^{-5/3}$ has been added for reference. Below horizontal wavenumbers of 10^{-2} cpm, the data appear to correspond to the Garrett–Munk internal wave scaling of k_h^{-2} (Garrett and Munk 1979). Above 10^{-2} cpm, however, a fairly consistent $k_h^{-5/3}$ spectral range is found, compatible with the stratified turbulence hypothesis. Ewart remarks on this high-wavenumber range behavior, noting that the data records show "intermittent patches of intense fluctuations," and he suggests it is due to intermittent turbulence. This is also consistent with the behavior of strati-

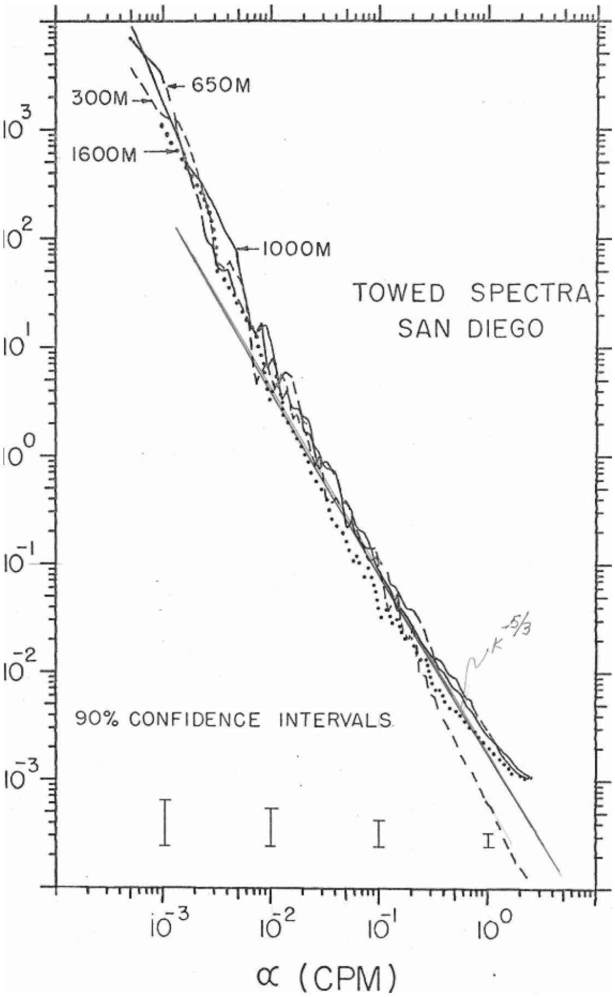

FIG. 6. Power spectra of temperature vs horizontal wavenumber α computed from runs off the coast of San Diego (30°N, 124°W). The spectra are normalized by N/N_0G, where G is the temperature gradient averaged over several measurements, and the buoyancy frequency N has been fitted to the exponential profile $N = N_0\,e^{-z/b}$. The units of the spectra are m^2 cpm^{-1}. Reproduced with written permission from Ewart (1976).

fied turbulence, not only in the spectral behavior but in that it consists of patches of intermittent, three-dimensional turbulence (Riley and deBruynKops 2003; Lindborg 2006).

High-frequency, small-scale turbulence data, which can be used to test the hypothesis of stratified turbulence, are now becoming available from the atmosphere as well. Frehlich et al. (2008) present velocity and temperature data taken in a stable, nocturnal boundary layer. Their data discussed below were taken from in situ, fast response turbulence sensors on a tethered lifting system. The high-frequency velocity data were taken with fine hot-wire sensors, and the corresponding temperature data were taken with fine cold-

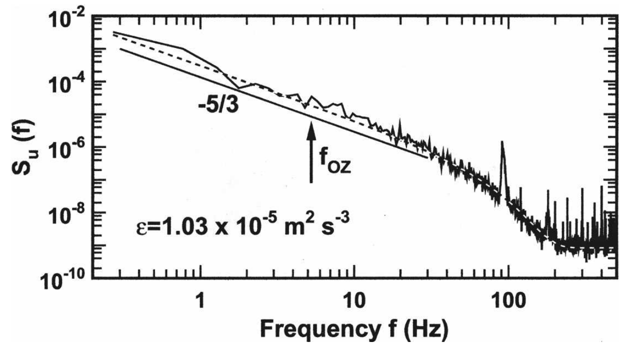

FIG. 7. Frequency spectrum of horizontal velocity measured in a stable, nocturnal boundary layer. Both a line with a $-5/3$ slope and the Ozmidov frequency are indicated on the plot (from Frehlich et al. 2008). The mean horizontal wind speed was about 9.2 m s^{-1}.

wire sensors. The sensors had sampling rates of 1 KHz. In Fig. 7 the frequency spectrum of the horizontal velocity is presented, while in Fig. 8 the corresponding spectrum for temperature is given. The Ozmidov frequency U/ℓ_O, where U is the local mean horizontal velocity, is indicated on the plots. It has a value of approximately 5.26 Hz, corresponding to an Ozmidov scale of 1.73 m. Also, a line with a slope of $-5/3$ is included on each plot. Using the horizontal scale $\mathcal{L}_L$ estimated from $\mathcal{L}_L = \sigma_u^3/\epsilon_K \approx 12.8$ m [see Eq. (3)], where σ_u is the rms horizontal velocity, they estimate a Froude number of $F_h = \sigma_u/N\mathcal{L}_L = 0.28$, in the same range as for the simulations of Riley and deBruynKops (2003). Consistent with the ocean data and with the numerical simulations, and compatible with the stratified turbulence hypothesis, inertial range behavior is evident for both the velocity and temperature fields, and extends to well above the Ozmidov scale.

4. Conclusions

Stratified turbulence dynamics occur in strongly stratified flows, consist of both internal waves and

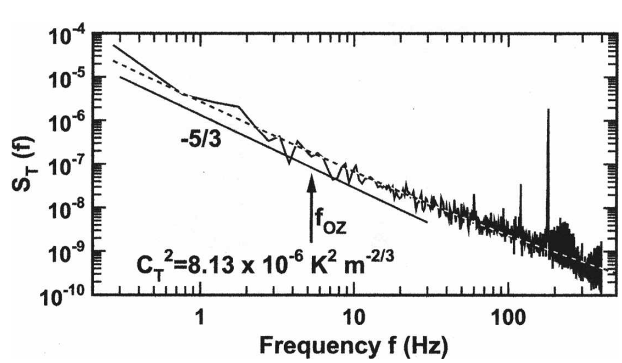

FIG. 8. As in Fig. 7, but for temperature.

quasi-horizontal meandering motions, and for large enough Reynolds numbers, possess a strong downscale cascade of energy and the development of inertial ranges in horizontal spectra (Riley and deBruynKops 2003; Lindborg 2005, 2006). There are several sets of horizontal spectra from ocean experiments that display what has been interpreted as Kolmogorov–Obukhov–Corrsin spectral inertial range behavior; however, the hypotheses made in these inertial range theories clearly do not apply. It is hypothesized here that these data as well as some atmospheric data can be explained by the dynamics of stratified turbulence, and the data are shown to be consistent with this hypothesis.

These results suggest that the dynamics of stratified turbulence might also explain strongly stratified flows in other geophysical situations, specifically when the effects of the earth's rotation are not strong, but when the Froude number F_h is small. This could have important implications. For example, it would indicate that, even though the effects of buoyancy are strong, in such a flow there will be a vigorous transfer of energy downscale. This would provide a pathway for energy transfer from horizontal scales of a few hundred meters or more, down to the Ozmidov scale and below, where turbulence will behave more classically. In addition, stratified turbulence may help explain the horizontal dispersion of, for example, plumes in the atmosphere and oceans, which have been observed to meander considerably in the horizontal under strongly stable conditions. And stratified turbulence may suggest better parameterizations of the subgrid-scale motions in larger-scale models.

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APPENDIX

The Ozmidov Length Scale

The Ozmidov scale ℓ_O, defined as

$$\ell_O = \left(\frac{\epsilon_K}{N^3}\right)^{1/2},$$

is generally interpreted as, for a given flow, the largest horizontal scale that can overturn or, similarly, the scale at which buoyancy forces and inertial forces are comparable (see Lesieur 1997). These interpretations lead to the conclusion that buoyancy has a gradually weaker effect at scales smaller than ℓ_O, while it becomes dominant at scales larger than ℓ_O.

To develop the expression for the Ozmidov scale, the existence of a stratified turbulence inertial range is very useful. Consider such an inertial range with the horizontal spectrum of the horizontal kinetic energy given as, in Eq. (5),

$$E_K(k_h) = C_K \epsilon_K^{2/3} k_h^{-5/3}.$$

Associated with horizontal wavenumber k_h is the horizontal length $\ell_{k_h} = 1/k_h$ and the kinetic energy (Onsager 1949)

$$K_{k_h} \sim u_{k_h}'^2 \sim E_K(k_h)k_h = C_K \epsilon_K^{2/3} k_h^{-2/3} = C_K \epsilon_K^{2/3} \ell_{k_h}^{2/3}.$$

(Note that, alternatively, realizing that ϵ_K is constant in this inertial range, independent of k_h, then at a given wavenumber k_h, a velocity u'_{k_h} can be defined using $\epsilon_K = u_{k_h}'^3/\ell_{k_h}$.) Length scales of motion that can overturn will have potential energy

$$P_h \sim \ell_{k_h}^2 N^2 \sim \frac{N^2}{k_h^2}.$$

For the flow at the scale ℓ_{k_h} to be able to overturn, $K_h > P_h$ or, disregarding the constants of order 1,

$$\epsilon_K^{2/3} k_h^{-2/3} > \frac{N^2}{k_h^2}, \quad \text{or} \quad \epsilon_K^{2/3} > N^2 \ell_h^{4/3}.$$

The Ozmidov scale, the largest scale for overturning, occurs when $\epsilon_K^{2/3} = N^2 \ell_h^{4/3}$, giving (again neglecting constants of order 1)

$$\ell_O = \left(\frac{\epsilon_K}{N^3}\right)^{1/2}.$$

There are a couple of important ramifications from this argument. First, note that the local Froude number at the Ozmidov scale is

$$F_O = \frac{u'_O}{N\ell_O} = \frac{(\epsilon_K \ell_O)^{1/3}}{N\ell_O} = 1,$$

consistent with the interpretation that the inertial and buoyancy forces are comparable at the Ozmidov scale. Second, the ratio of the Ozmidov scale to the local scale ℓ_{k_h} is

$$\frac{\ell_O}{\ell_{k_h}} = \left(\frac{\epsilon_K}{N^3}\right)^{1/2} \left(\frac{\epsilon_K}{u_{k_h}'^3}\right) = \left(\frac{u_{k_h}'^3}{N^3 \ell_{k_h}^3}\right)^{1/2} = F_{k_h}^{3/2}, \quad \text{or}$$

$$F_{k_h} = \left(\frac{\ell_O}{\ell_{k_h}}\right)^{2/3},$$

where $F_{k_h} = u'_{k_h}/N\ell_{k_h}$ is the local Froude number at the scale ℓ_{k_h}. Therefore, as scales become larger than ℓ_O, the local Froude number decreases, and the effects of buoyancy increase, and vice versa, another interpretation of the Ozmidov scale.

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