A Systematic Error in Plane-Parallel Radiative Transfer Calculations

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ABSTRACT

Systematic errors in plane-parallel radiative transfer calculations are examined in the context of limb radiance. Calculation of the multiple-scattering component of limb radiance using plane-parallel geometry can lead to a systematic overestimation that increases with surface albedo and tangent height. It is demonstrated that the cause of this is the overestimate of the surface-reflected radiance component. A simple yet effective correction is introduced that reduces these errors from 4%–8% to <2%. This error source may manifest itself in any application of radiative transfer calculations employing plane-parallel geometry.

1. Introduction

Models that compute the scattered, or diffuse, radiation field often solve the equation of radiative transfer in plane-parallel geometry. A plane-parallel, or flat, atmosphere is one-dimensional and so extends to infinity in the horizontal with the only variation in the vertical. The primary reasons for making such an approximation are the simplified solution and greatly reduced computation requirements over spherical models. For many applications (e.g., simulating nadir radiances) this approximation introduces small errors.

With the proliferation of instruments such as the Optical Spectrograph and Infrared Imager System (OSIRIS; Llewellyn et al. 2004) and the Scanning Imaging Absorption Spectrometer for Atmospheric Chartography (SCIAMACHY; Bovensmann et al. 1999) that measure sunlight scattered from the earth's limb, a number of pseudospherical radiative transfer models have been developed (Griffioen and Oikarinen 2000; McLinden et al. 2002; Berk et al. 2005) to simulate this geometry. Such models calculate the limb radiance from multiple solutions of the radiative transfer equation in plane-parallel geometry. The multiple solutions are necessary to account for the variation in geometry [e.g., solar zenith angle (SZA)] along the tangent path. The tangent path defines the path of the limb ray through the atmosphere and is quantified by the tangent height, the lowest altitude along the tangent path.

This note discusses a systematic error that may result from the use of plane-parallel geometry. These errors are examined in the context of limb radiances but are equally relevant to other applications.

2. A systematic error in plane-parallel geometry

a. Nature of the problem

Loughman et al. (2004) compared limb radiances calculated using pseudospherical radiative transfer models to fully spherical models and found, in general, good agreement. For pure Rayleigh atmospheres, differences in radiance were typically <4% increasing to <6% when aerosols were considered. One systematic effect that was observed, however, was an increase in the pseudospherical calculated radiance, relative to the spherical, for increasing tangent heights and surface albedo. It was suggested that this was due to the absence of limb darkening in the pseudospherical models although no definitive explanation was given. A more recent study (Doicu and Trautmann 2009) documented similar results but attributed the increasing error with tangent height to smaller absolute radiances.

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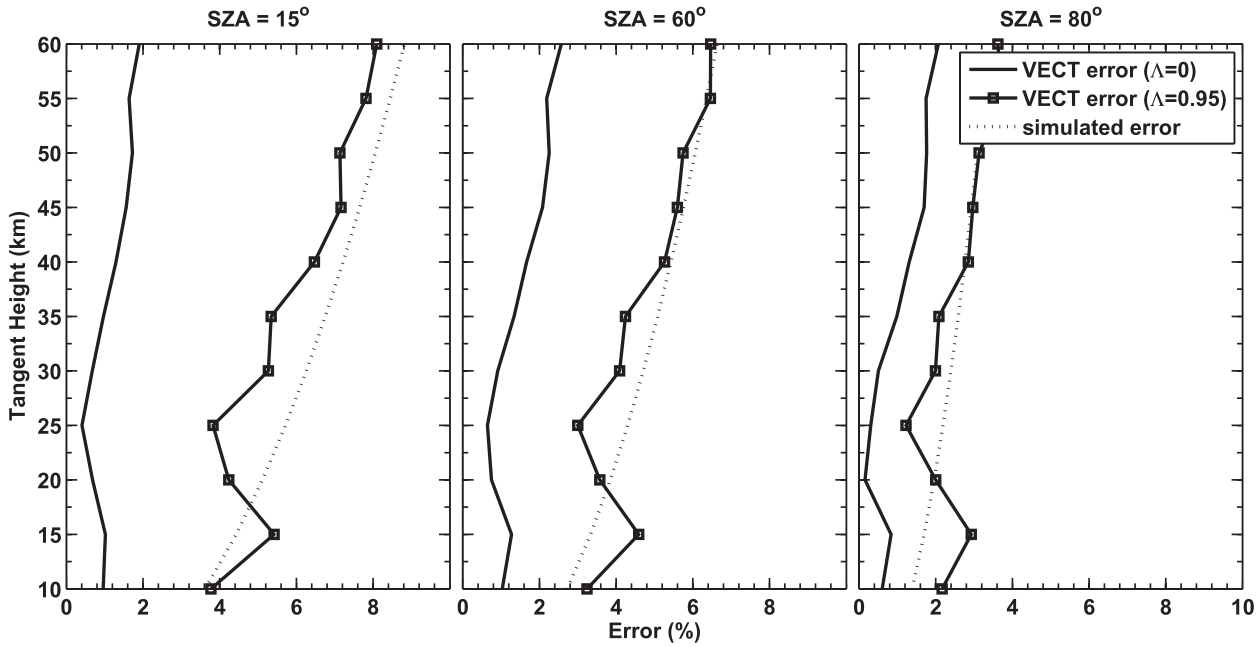

FIG. 1. Percent error in VECTOR limb radiance, defined as (VECTOR − Siro)/Siro, at 600 nm and for SZAs of (left) 15°, (middle) 60°, and (right) 80° for surface albedos (Λ) of 0 and 0.95. Also shown is the simulated error estimate using Eq. (4).

The Loughman et al. (2004) study was repeated here using simulations from the Vector Orders-of-Scattering Radiative Transfer (VECTOR) model (McLinden et al. 2002, 2006), a pseudospherical code, and the SASKTRAN spherical radiative transfer model (Bourassa et al. 2008). These were compared to calculations from the spherical Siro Monte Carlo model (Oikarinen et al. 1999), which was used as the reference in the original Loughman et al. (2004) study. All models were run in scalar mode (neglecting polarization) using identical background atmosphere, ozone, and aerosol profiles, and Rayleigh and ozone cross-sectional data as described in Loughman et al. (2004). Relative differences in limb radiance between VECTOR and Siro are examined first, defined as (VECTOR − Siro)/Siro, and are shown in Fig. 1. Results are presented for 600 nm, solar zenith angles of 15°, 60°, and 80°, a change in azimuthal angle of 90°, and surface albedos of 0 and 0.95. These results are virtually identical to pseudospherical/spherical comparisons from the original study (Loughman et al. 2004, their Figs. 6–8). Errors are small (1%–2%) when the albedo is 0, but for an albedo of 0.95 errors are seen to increase with tangent height, an effect that becomes more pronounced with high sun. The "kink" at lower tangent heights was also seen in the original comparison and its origin is unclear.

In a plane-parallel atmosphere, the basis of the multiple scattering in pseudospherical models, the contribution from surface-reflected light is based on integration over a solid angle of 2π as the surface extends to infinity. Note that "surface" can mean a scattering layer in addition to the physical surface. In a spherical atmosphere, a given point in the atmosphere receives photons emanating from, effectively, a smaller surface, one that subtends an angle $2\pi(1 - \cos\theta^*)$, where θ^* is the surface–atmosphere cutoff zenith angle at the horizon. For the physical surface it is $\theta^* = \sin^{-1}[r_E/(r_E + z)]$, where z is tangent height and $r_E = 6378$ km is the radius of the earth. It is shown, below, that this extra contribution leads to the overestimate in the plane-parallel geometry. Based on this hypothesis, this error will increase with surface albedo and tangent height and decrease with SZA. This is shown schematically in Fig. 2.

b. A simple model

Consider the radiance at a point in the atmosphere integrated over all directions,

$$L = \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} I(\theta', \phi') \sin\theta' \, d\theta' \, d\phi', \tag{1}$$

where I is the radiance as a function of zenith angle θ and azimuthal angle ϕ, and where L, to within a constant, is the mean radiance or intensity (sometimes called flux density or actinic flux). This important quantity describes the energy available to drive photochemical reactions.

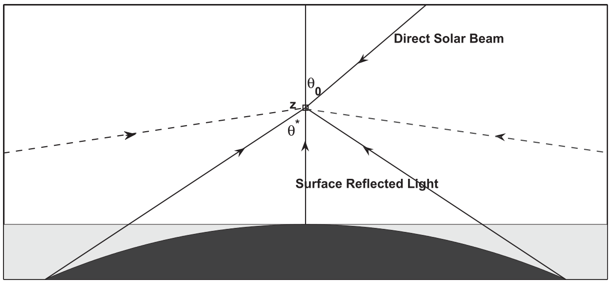

FIG. 2. Depiction of the surface contribution to the total radiance at altitude z in a plane-parallel (gray) and spherical (black) atmosphere. The dashed lines show an additional contribution to a point at altitude z from the plane-parallel atmosphere. Here θ^* represents the zenith angle at the horizon for a point at z and is defined as $\sin\theta^* = r_E/(r_E + z)$, where r_E is the radius of the earth.

Likewise, in a conservative, isotropically scattering medium, it is proportional to the multiple-scattering source function.

To quantify this error source, consider the case of a transparent, conservative atmosphere with an underlying Lambertian surface of albedo Λ. Under these conditions, the mean radiance is the sum of two components: the direct solar beam and the surface reflected light, with the only energy loss arising from absorption at the surface. In a plane-parallel atmosphere the mean radiance has the form

$$L_{\rm pp} = F_0(1 + 2\mu_0\Lambda), \tag{2}$$

where F_0 is the extraterrestrial solar flux and μ_0 is the cosine of the SZA. The second term in brackets represents the enhancement due to surface reflection and can lead to a threefold increase over the case of no surface reflection, a well-known effect (Kaminski and McConnell 1991). In deriving this expression, the surface contribution was integrated over a solid angle of 2π sr. In an analogous spherical atmosphere, the mean radiance becomes

$$L_{\rm sph} = F_0[1 + 2\mu_0\Lambda(1 - \cos\theta^*)], \tag{3}$$

where the surface now contributes only over a solid angle of $2\pi(1 - \cos\theta^*)$ sr. The relative error in mean radiance for plane-parallel geometry, $(L_{\rm pp} - L_{\rm sph})/L_{\rm sph}$, is then

$$\epsilon = \frac{2\mu_0\Lambda\cos\theta^*}{1 + 2\mu_0\Lambda(1 - \cos\theta^*)}, \tag{4}$$

which is a function of altitude (through θ^*), albedo, and SZA. The value of ϵ is 0 for $z = 0$, $\mu_0 = 0$, and/or $\Lambda = 0$ and increases with these three variables. At $z = 60$ km, $\mu_0 = 1$, and $\Lambda = 1$, ϵ has a value of 0.1.

This framework can be related to limb radiance by recognizing that the expression for mean radiance, Eq. (1), is proportional to the multiple-scattering source function S

$$S(\theta, \phi) = \frac{\tilde{\omega}}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} P(\theta, \phi; \theta', \phi')I(\theta', \phi')\sin\theta'\, d\theta'\, d\phi' \tag{5}$$

if the atmosphere is conservative (or the single-scattering albedo $\tilde{\omega} = 1$) and composed of isotropic scatterers [or the phase function $P(\mu, \phi; \mu', \phi') = 1$]:

$$S = \frac{1}{4\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} I(\theta', \phi')\sin\theta'\, d\theta'\, d\phi'. \tag{6}$$

Because limb radiance is heavily weighted toward the scattering source at the tangent height, it suffices to use altitude and tangent height interchangeably. Equation (4) has been evaluated for an albedo of 0.95 for the three SZAs considered in Fig. 1 and is shown in that plot. The plane-parallel limb radiance errors appear to be well represented by this simple expression, thereby confirming the nature of this error source as an overestimate of the surface contribution. While appreciable scattering and some absorption by ozone occurs at 600 nm, the limb radiance remains composed primarily of singly scattered light. This, together with the fact that relative (rather than absolute) error is being modeled, explains why such a simple expression can quantitatively predict the plane-parallel error. Once multiple scattering becomes important Eq. (4) is no longer adequate.

Wavelengths not considered in the Loughman et al. (2004) study can be examined by using the SASKTRAN radiative transfer code (Bourassa et al. 2008). This is a spherical code that can alternatively be run in pseudospherical mode by increasing the radius of the earth by a factor of 1000 for the multiple-scattered component. The benefit of using the same code for both geometries is that the numerical schemes and approximations and grid discretizations are identical. The SASKTRAN simulations use the same atmosphere, parameters, and simulation geometries as Loughman et al. (2004) but without aerosols. The pseudospherical limb radiance errors are now defined as (ps − sph)/sph, where ps refers to pseudospherical radiances and sph to spherical ones. These are shown in Fig. 3 for wavelengths of 600 and 750 nm for albedo values of 0 and 0.95 along with the predicted error using Eq. (4). Again errors are small ($<2\%$) for a surface albedo of 0. A small increase with tangent height is evident because of a weak multiple-scattering effective surface. The errors for an albedo of 0.95 display the anticipated increase with tangent height and higher sun. The absolute

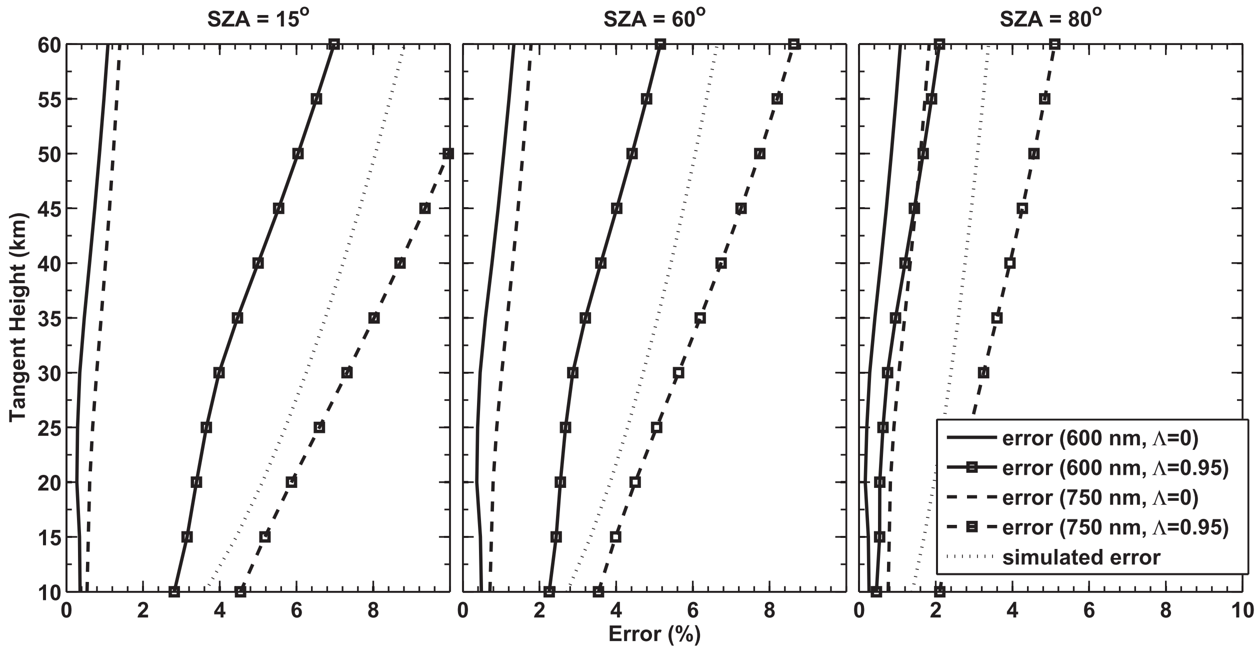

FIG. 3. Percent error in SASKTRAN pseudospherical (ps) limb radiance, relative to SASKTRAN spherical (sph) limb radiance, defined as (ps − sph)/sph, at 600 and 750 nm for SZAs of (left) 15°, (middle) 60°, and (right) 80°. Also shown is the simulated error estimate using Eq. (4).

value of the simulated error falls between the 600- and 750-nm model values.

c. A simple correction

Comparing Eqs. (2) and (3), it can be seen that if an effective albedo, defined as $\Lambda' = \Lambda(1 - \cos\theta^*)$, is used in Eq. (2), these expressions become identical. In other words, scaling the albedo in a plane-parallel atmosphere by $1 - \cos\theta^*$ eliminates the systematic error. A practical drawback of implementing this is that the albedo is now a function of altitude and thus requires an evaluation of the model for each altitude being simulated. An alternative is to carry out two simulations, one using Λ and one with an albedo of 0. The total radiance can be decomposed into a component that consists solely of atmospheric scattering, $I(\Lambda = 0)$ or $I(0)$, and a component that has been reflected off the surface at least once, $I(\Lambda) - I(0)$. With the radiance decomposed in this way, the error source can be removed by applying the scaling $\Lambda'/\Lambda = 1 - \cos\theta^*$ only to the surface-reflected component as follows:

$$I_c(\Lambda) = I(0) + [I(\Lambda) - I(0)](1 - \cos\theta^*), \qquad (7)$$

where I_c represents the corrected limb radiance (shown only as a function of albedo). This has been done for the SASKTRAN pseudospherical limb radiances at an albedo of 0.95 for wavelengths of 325, 345, 470, 600, and 750 nm. Results are shown in Fig. 4 for errors before and after correction using Eq. (7). Postcorrection errors are now much smaller, less than 2%, for all wavelengths, SZAs, and tangent heights. At 345 and 450 nm there remains some tangent height dependence because in this portion of the spectrum there is considerable multiple scattering and virtually no absorption. The increase with tangent height is due to the presence of an effective scattering surface that behaves similarly to a physical surface that cannot be easily corrected. In the corrected plane-parallel simulation, the surface-reflected radiance, while downscaled, is still distributed over 2π sr and these "edge" rays traverse an artificially long path (see Fig. 1).

This type of correction is more robust than what might be expected, given the approximations made when deriving it, since it is simply an adjustment of the albedo; multiple scattering and absorption are still accounted for.

3. Conclusions

Systematic errors in plane-parallel radiative transfer calculations have been examined in the context of limb radiance. Calculation of the multiple-scattering component of limb radiance using plane-parallel geometry was shown to lead to a systematic overestimation, a result that increases with surface albedo and tangent height. It was demonstrated that the cause of this is an overestimate of the surface-reflected radiance component. Plane-parallel atmospheres have a surface contribution encompassing

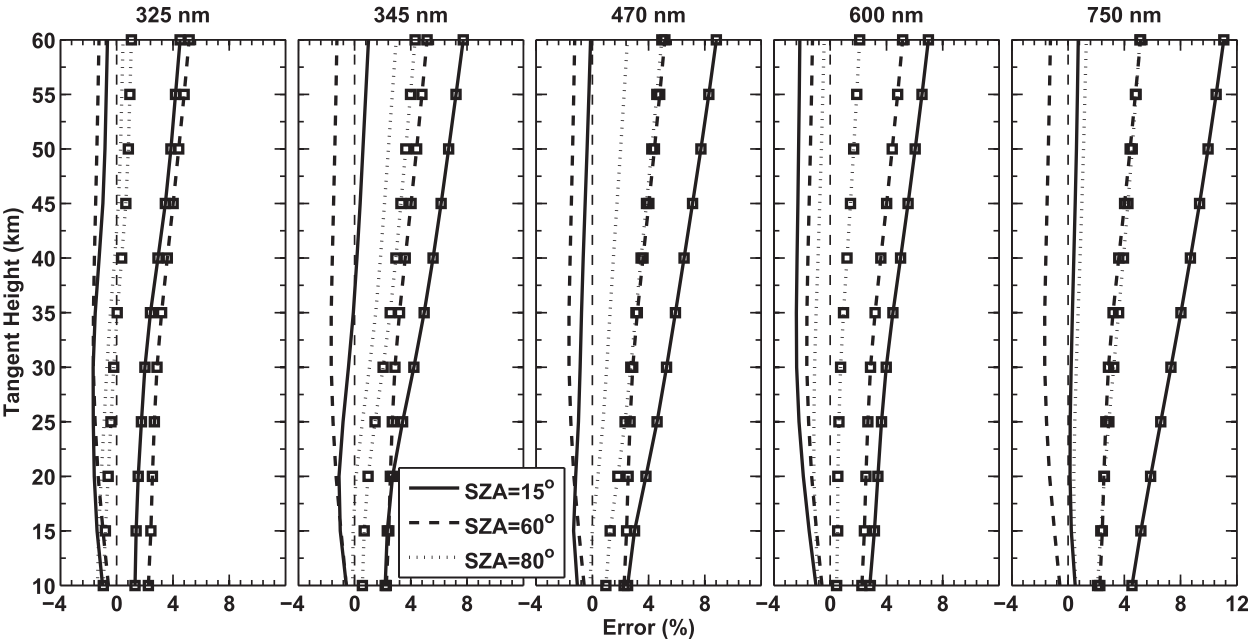

FIG. 4. Percent error in SASKTRAN pseudospherical limb radiance for an albedo of 0.95 before (with squares) and after (without squares) correction using Eq. (7). Error is defined as (ps − sph)/sph and (ps_{corr} − sph)/sph.

2π sr whereas spherical atmospheres have a smaller contribution that depends on altitude. A simple yet effective correction is introduced that reduces these errors from 4%–8% to <2%.

This error source may lead to systematic errors in trace gas or aerosol retrievals whenever limb radiance profiles must be simulated or the ratio of radiances relative to that at a reference altitude is used. Additionally, while this error source has been discussed in the context of limb radiances, it is relevant to any application of plane-parallel geometry. This includes the calculation of photodissociation rates in photochemical models, which are almost exclusively calculated in plane-parallel geometry. Likewise, it is also applicable to the modeling of multiply scattered light in planetary atmospheres such as Mars, which is smaller than Earth and hence leads to larger errors and at times possesses an optically thin atmosphere in the visible.

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