Equatorial Waves in the Upper Troposphere and Lower Stratosphere Forced by Latent Heating Estimated from TRMM Rain Rates

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ABSTRACT

Equatorial atmospheric waves in the upper troposphere and lower stratosphere (UTLS), excited by latent heating, are investigated by using a global spectral model. The latent heating profiles are derived from the 3-hourly Tropical Rainfall Measuring Mission (TRMM) rain rates, which include both convective- and stratiform-type profiles. The type of heating profile is determined based on an intensity of the surface rain rate. Latent heating profiles over stratiform rain regions, estimated from the TRMM Precipitation Radar (PR) product, are applied to derive the stratiform-type latent heating profiles from the gridded rain rate data. Monthly zonal-mean latent heating profiles derived from the rain rates appear to be reasonably comparable with the TRMM convective/stratiform heating product.

A broad spectrum of Kelvin, mixed Rossby–gravity (MRG), equatorial Rossby (ER), and inertia–gravity waves are generated in the model. Particularly, equatorial waves (Kelvin, ER, and MRG waves) of zonal wavenumbers 1–5 appear to be dominant in the UTLS. In the wavenumber–frequency domain, the equatorial waves have prominent spectral peaks in the range of 12–200 m of the equivalent depth, while the spectral peaks of the equatorial waves having shallower equivalent depth (<50 m) increase in the case where stratiform-type heating is included. These results imply that the stratiform-type heating might be relevant for the shallower equivalent depth of the observed convectively coupled equatorial waves.

The horizontal and vertical structures of the simulated equatorial waves (Kelvin, ER, and MRG waves) are in a good agreement with the equatorial wave theory and observed wave structure. In particular, comparisons of the simulated Kelvin waves and the High Resolution Dynamics Limb Sounder (HIRDLS) satellite observation are discussed.

1. Introduction

Equatorial atmospheric waves, vertically propagating from the troposphere, play an important role in the low-frequency oscillations in the lower and middle stratosphere such as the quasi-biennial oscillation (QBO; Baldwin et al. 2001). The QBO, dominating the tropical variability in the stratosphere, influences global-scale circulations in both the troposphere and the stratosphere such as the El Niño–Southern Oscillation (ENSO) teleconnection and stratospheric sudden warming (Garfinkel and Hartmann 2008; Holton and Tan 1980). Since equatorial atmospheric waves provide the forcing of the QBO (Holton and Lindzen 1972; Dunkerton 1997), more realistic simulation of the equatorial waves might be needed for better simulation of the global circulation.

The equatorial waves such as Kelvin and Rossby waves have been also suggested to be involved in the dynamics of the tropical tropopause layer (TTL), which is a transition layer between the troposphere and the stratosphere (Fueglistaler et al. 2009). Convectively driven Kelvin wave temperature anomalies contribute to the coldest temperatures at the cold-point tropopause (Tsuda et al. 1994; Zhou and Holton 2002; Randel and Wu 2005; Ryu et al. 2008) and likely influence formation of the “cold trap” (i.e., the coldest region in the TTL occurring over the western tropical Pacific). In particular, Ryu et al. (2008) showed that the Kelvin wave modulation due to...
background flow can explain the existence of the cold trap, which presumably causes dehydration of tropospheric air entering the stratosphere. In addition, modeling studies (Boehm and Lee 2003; Norton 2006; Ryu and Lee 2010) imply that the Rossby wave response to tropical convection can contribute to a large-scale ascent in the TTL. TTL upwelling is an important process for troposphere-to-stratosphere transport of water vapor. Moreover, a possible linkage of stratospheric water vapor to global surface climate change on a decadal scale (Solomon et al. 2010) underscores the importance of mechanisms for formation of the cold trap and the TTL upwelling.

Observational studies on the equatorial waves (Wheeler and Kiladis 1999; Kiladis et al. 2009) have suggested that variability in tropical convection is closely linked to equatorial waves having characteristics of the classical wave theory of Matsuno (1966), referred to as convectively coupled equatorial waves. The first convincing analysis for the existence of Matsuno’s equatorially trapped waves in cloudiness data was provided by Takayabu (1994). Model studies imply that simulated convectively coupled equatorial waves are highly dependent on the convective parameterization (Frierson 2007; Lin et al. 2008); moreover, the variability in tropical convection such as the Madden–Julian oscillation (MJO; Madden and Julian 1971) and the equatorial wave signals are poorly simulated in a majority of global circulation models (Slingo et al. 1996; Lin et al. 2006).

On the other hand, Ricciardulli and Garcia (2000) suggest that the variability of tropical heating is an important factor in the excitation of equatorial waves and therefore accurate simulation of the tropical circulation.

Latent heating is a dominant component of total diabatic heating associated with a convective system, which excites the atmospheric waves in the tropics. In particular, the latent heating profile associated with stratiform precipitation has been proposed to play a role in setting the tropospheric circulation in the tropics (Lin et al. 2004; Schumacher et al. 2004). Moreover, stratiform-type heating has been suggested as likely to be involved in observed convectively coupled equatorial waves. According to observational studies (Wheeler and Kiladis 1999; Kiladis et al. 2009), convectively coupled equatorial waves have a smaller vertical wavelength (i.e., 12–50 m of the equivalent depth) than the vertical wavelength (corresponding to $h = 200$ m) of the peak projection response to a typical deep convective heating (with a depth of about 14 km; Frank and McBride 1989). As one possible mechanism for the smaller vertical wavelength, they suggested a shallow upper-level heating with cooling below associated with stratiform precipitation. While previous studies have mostly focused on the tropospheric circulation or tropospheric features of convectively coupled equatorial waves, this study attempts to examine the contribution of tropical heating, and stratiform heating in particular, to equatorial waves in the upper troposphere and lower stratosphere (UTLS). As mentioned in previous studies (Wheeler and Kiladis 1999; Kiladis et al. 2009), it is implicitly expected that the stratiform-type heating will be associated with shallower vertical wavelength equatorial waves than those forced by the convective-type heating. Thus, we also examine whether/how the stratiform-type heating can contribute to a realization of the equatorial waves with vertical wavelengths comparable to those of the observed convectively coupled equatorial waves.

We utilize a global primitive equation model to simulate equatorial waves. Because of a sensitivity of latent heating variability to the convective parameterization, which is one of the most uncertain components in GCMs, we prescribe a gridded space–time variable heating derived from observed surface rain rates instead of parameterizing the heating with the convective parameterization. In this way, we can simulate the equatorial waves forced by latent heating with more realistic spatial and temporal variability. The latent heating profile is derived from surface rain rates assuming that the latent heating/cooling results from condensation/evaporation of precipitation falling onto the surface. For the surface rain rate, we use the Tropical Rainfall Measuring Mission (TRMM) 3B42 rain rate data, having the highest resolution in both time and space among global gridded data. This method ensures that the variability of the heating is realistic, and this variability is key to the wave forcing. The TRMM data are described below in section 2. Section 2 describes how we derive the convective/stratiform heating (CSH) profiles from the observed surface rain rates. A brief description of the model and experiment design will be presented in section 3. Relationships between convection and equatorial waves in the UTLS are analyzed in section 4. Kelvin waves in the model are compared with the High Resolution Dynamics Limb Sounder (HIRDLS) satellite data in section 5. Conclusions are drawn in section 6.

2. Convective/stratiform heating

The vertical scale of the heating is one of the important components characterizing the wave response although the wave response is relatively insensitive to variations in the details of the vertical distribution of the heating (Salby and Garcia 1987). On the other hand, since Salby and Garcia (1987) used only positive heating profiles within a confined layer, their result does not imply that a full-sine shape such as what we will use for
stratiform heating in this study will have a similar response to a half-sine shape with the same depth. Accordingly, we considered a stratiform-type heating profile as well as a convective-type heating profile, which are differentiable in terms of the vertical scale and are also two dominant modes in heating profiles observed in tropical precipitation systems (Houze 1989; Yang and Smith 1999). This section describes how we determine the heating type and profile (convective or stratiform) from the surface rain rate (TRMM 3B42). The TRMM is mainly monitoring rainfall by using the TRMM Microwave Imager (TMI) and spaceborne Precipitation Radar (PR) visible and infrared scanner in tropical and subtropical regions since its launch in 1997 (Kummerow et al. 2000). The TRMM orbits at an altitude of 350 km with an inclination of 35°. The 3B42 product is derived from geostationary precipitation data with calibration by these TRMM data. The TRMM satellite rainfall product has been validated by comparing with ground observations and results have shown that it can be accepted as reliable (Wolff et al. 2005; Ji 2006; Yang et al. 2006). After we derive a heating profile at each grid point of the rain rate data (i.e., with 0.25° resolution in longitude and latitude), the heating profiles within each grid box of the model are averaged. In this study, our model has roughly 2.8° horizontal resolution. As a result, the averaged heating profile at each model grid consists of convective or/and stratiform heating, hereafter referred to as convective/stratiform heating.

Convective heating is characterized by warming throughout the profile, while stratiform heating is characterized by warming above the melting level and cooling below it based on observations (Houze 1982, 1989; Yang and Smith 1999). For the convective heating, we assume a positive half-sine profile from the surface to the precipitation-top height (PTH; i.e., cloud-top height) similar to earlier studies (Pandya et al. 1993; Shige et al. 2004; Grimsdell et al. 2010). The magnitude of the heating profile is dependent on the surface rain rate and PTH by a constraint that the vertically integrated heating from the surface to the PTH equals the latent heating due to condensation that results in precipitation at melting level. Below the melting level, a negative half-sine profile is assumed for the cooling due to evaporation that corresponds to the difference of the surface rain rate from the precipitation at the melting level. It implies that for the stratiform heating profile, additional information including the altitude and precipitation at the melting level is needed in addition to the surface rain rate and PTH needed for the convective heating profile. While the TRMM 3B42 product provides only the surface rain rate, the TRMM PR data provide the surface rain rate, melting level, precipitation at the melting level, and the PTH (2A25), including the classification of convective and stratiform rain type derived from PR reflectivities (TRMM 2A23). The TRMM PR data has 4.3-km horizontal and 250-m vertical resolution over a 220-km orbital swath (scan width). The TRMM PR orbits about 16 times per day between 38°S and 38°N and there are generally 9150 scans along the orbit. However, the temporal resolution of the PR data at a given grid point is not sufficient for forcing atmospheric waves in a global model. Therefore, we first estimate stratiform heating profiles from the TRMM PR data and then examine whether the melting level and the precipitation at the melting level can be parameterized as a function of the surface rain rate and the PTH (section 2a). In other words, using characteristics of the stratiform heating profile derived from the TRMM PR data, we attempt to derive the stratiform heating profile from the TRMM 3B42 surface rain rate (section 2c).

### a. Estimation of stratiform heating profile from the TRMM PR rainfall profile

For the stratiform heating, a positive half-sine profile is assumed for the warming above melting level due to condensation that results in precipitation at melting level. Below the melting level, a negative half-sine profile is assumed for the cooling due to evaporation that corresponds to the difference of the surface rain rate from the precipitation at the melting level. It implies that for the stratiform heating profile, additional information including the altitude and precipitation at the melting level is needed in addition to the surface rain rate and PTH needed for the convective heating profile. While the TRMM 3B42 product provides only the surface rain rate, the TRMM PR data provide the surface rain rate, melting level, precipitation at the melting level, and the PTH (2A25), including the classification of convective and stratiform rain type derived from PR reflectivities (TRMM 2A23). The TRMM PR data has 4.3-km horizontal and 250-m vertical resolution over a 220-km orbital swath (scan width). The TRMM PR orbits about 16 times per day between 38°S and 38°N and there are generally 9150 scans along the orbit. However, the temporal resolution of the PR data at a given grid point is not sufficient for forcing atmospheric waves in a global model. Therefore, we first estimate stratiform heating profiles from the TRMM PR data and then examine whether the melting level and the precipitation at the melting level can be parameterized as a function of the surface rain rate and the PTH (section 2a). In other words, using characteristics of the stratiform heating profile derived from the TRMM PR data, we attempt to derive the stratiform heating profile from the TRMM 3B42 surface rain rate (section 2c).

We estimate stratiform-type heating profiles from the TRMM PR rainfall profile as well as the melting level and PTH from the TRMM 2A25 product and the classification of convective and stratiform rain type derived from PR reflectivities (2A23). First, the PR data are collected in cases where the rain type flag is certain stratiform (rain type 100 or 110 in product 2A23) for 5 days from 1 to 5 January 2006, which include about 35 900 cases. Figure 1 shows the TRMM stratiform-type rainfall profile averaged as a function of PTH in the tropics (10°S–10°N). The percentage on the x axis represents a cumulative frequency of the averaged rainfall profile having a corresponding PTH. The result shows that the intensity of the precipitation increases with PTH up to around 4.2 km. Below the melting level (about 4 km), the precipitation decreases approaching the surface because of evaporation. These characteristics and the amplitude derived from 5 days of stratiform rainfall profiles correspond well with
results from 3 yr of TRMM PR rainfall profiles shown by Shige et al. (2004). It implies that the rainfall profiles from these 5 days can be used to study the general characteristics of the tropical stratiform rainfall profile.

Figure 2 illustrates how we determine the stratiform latent heating profile from the TRMM PR information [i.e., PTH, melting level, precipitation at melting level (Pm), and surface rain rate (Psfc)]. Water vapor budget analyses from observations (Houze et al. 1980; Gamache and Houze 1983; Chong and Hauser 1989) show that precipitation falling into the melting level from the stratiform anvil includes both condensates advected from the convective region and condensates generated in the stratiform anvil above the melting level. The fraction of the precipitation at melting level that is contributed by condensates generated in the stratiform anvil [called $R_{\text{anvil}}$ in Shige et al. (2004)] is taken into account in our calculation based on the results that Shige et al. (2004) reported from cloud-resolving model (CRM) simulations. The results of Shige et al. (2004) show that $R_{\text{anvil}}$ is less than 1 and tends to decrease with precipitation at melting level, Pm (cf. Fig. 7 of Shige et al. 2004). In other words, $1 - R_{\text{anvil}}$ is assumed to be a fraction of condensates carried into the stratiform anvil from the convective region, and $1 - R_{\text{anvil}}$ increases with Pm.\(^1\) The interpretation is that more condensates are carried from the convective region into the stratiform anvil closer to the convective region where more Pm is expected. Accordingly, the heating profile above the melting level up to the precipitation height top is determined to be proportional to Pm multiplied by the factor $R_{\text{anvil}}$. Similar to Fig. 7 of Shige et al. (2004), we assign the fraction $R_{\text{anvil}}$ values ranging from 0.9 to 0.42 depending on Pm.

The stratiform heating profiles estimated from the TRMM PR product in the tropics (10°S–10°N) for 5 days (1–5 January 2006) are next binned as a function of surface rain rate and precipitation-top height in Fig. 3. It turns out that both heating above the melting level and cooling below it monotonically increase with surface heating $\alpha \ P_m \times R_{\text{anvil}}$

$\text{Cooling} \alpha \ P_m - P_{sfc}$

$\text{Precipitation-top height}$

\(1\) Consistently, a ratio of condensates generated in the convective region to precipitation falling onto the surface of the convective region is also considered in deriving the convective-type heating profiles, which will be explained in section 2c.
rain rate (Fig. 3a). This means the intensity of stratiform heating is highly related to the surface rain rate. Each percentage on the top of the figure indicates a cumulative frequency of a corresponding surface rain rate (i.e., the frequency of occurrence of rain rates lower than the value). Also, melting level tends to increase with precipitation-top height. The melting level ranges from 3.9 to 4.5 km in the tropics (Fig. 3b), while the melting level is lower in the subtropics (>10°) where it is between 2.9 and 3.3 km (not shown). According to this result, the
melting level and peak cooling rate below the melting level can be determined by the precipitation-top height and surface rain rate, respectively. A lookup table of the melting level depending on the PTH was prepared for both tropics and subtropics, separately.\(^2\)

A lookup table of the peak cooling rate was made as a function of the cumulative frequency of the surface rain rate (the top x axis of Fig. 3a) instead of the rain rate itself. The reason is that this table will be also applied for the TRMM 3B42 rain rate data, which has coarser resolution than the TRMM PR data, to make a gridded heating dataset to force the model. Because the rain rate (mm h\(^{-1}\)) is an average over a grid square, when we average over regions without rain the average rate goes down. Likewise, since the range of the rain rate will be changed depending on its spatial resolution, the cumulative frequency of the surface rain rate rather than the rain rate itself is used as an independent parameter determining the peak of the cooling rate. As a result, the distribution of the cooling profile below the melting level as a function of the cumulative frequency of the surface rain rate (Fig. 3a) will remain the same even though the range of the surface rain rate will change with spatial resolution.

To check the reliability of these parameters, stratiform heating profiles are reconstructed using the surface rain rate and the PTH from the same TRMM PR product. Instead of using the data of the melting level and the precipitation at the melting level from the TRMM PR product, they are read from the lookup table, depending on the PTH and surface rain rate, made from Fig. 3. The reconstructed stratiform heating profile (Fig. 4) shows very similar features to the original (Fig. 3b) in terms of both shape and amplitude. Based on this result, we estimate the stratiform heating profile from the surface rain rate and the PTH.

b. A criterion for determining the rain type:

Surface rain rate

In this section, we explain how to determine the rain type (convective or stratiform) from the surface rain rate before deriving the heating profile. Because convective areas in the tropics have intense rainfall rates while stratiform areas spread with weak rainfall (Schumacher and Houze 2003), the two rain types might be differentiated by the intensity of rain rate. While the TRMM 3B42 data provide only the surface rain rate, the TRMM PR data provides the surface rain rate as well as the rain type (i.e., convective versus stratiform). Accordingly, we examine a characteristic of the surface rain rate data depending on the rain type using the TRMM

\(^2\) It should be noted that the vertical resolution of the model we used in this study is 0.75 km, which is comparable to the range of the melting level (about 0.6 km). Below 4.5 km the model vertical levels are 0.365, 1.114, 1.864, 2.615, 3.364, 4.115, and 4.865 km. By selecting the model vertical level, which is the nearest level to the melting level read from the lookup table, it turns out that the melting level is set to be 4.115 km in the tropics and 2.615 or 3.364 km in the subtropics.
PR data before we decide the criterion for the rain type to be applied to the TRMM 3B42 data. From the TRMM PR data, the rain distributions of the convective and stratiform areas between 38°S and 38°N are compared in Fig. 5. Each bar indicates a frequency of corresponding rain rate in each range and a solid line shows cumulative frequency (i.e., total frequency below that rain rate; see Fig. 5). The convective rain shows a long-tailed distribution and the stratiform rain is dominated by weak rain. For instance, 80% of the convective rain is more than 5 mm h⁻¹, while 85% of the stratiform rain is smaller than 5 mm h⁻¹. In other words, the probability that the stratiform rain rate falls in the range where the rain rate is smaller than 5 mm h⁻¹ is 85% even though there is a 20% chance of convective rain. According to this result, the intensity of the surface rain rate might be a reliable indicator to determine if the rain is convective or stratiform.

On the other hand, Schumacher and Houze (2003) show that stratiform rain covers about 73% of rain area in the tropics using the TRMM PR data over a 3-yr period (1998–2000). Based on their result, a threshold rain rate for the stratiform-type rain is selected to meet the condition that about 70% of the total rain area has a smaller rain rate than the threshold rain rate. As mentioned in the previous section, the TRMM 3B42 rain rate data are used to derive a gridded heating product. We examined the distribution of the TRMM 3B42 surface rain rate for 12 months (1 April 2006–31 March 2007), which includes the time period of the model simulation that will be described later in this paper (Fig. 6). Because latent heating in the tropics is an important forcing for equatorial atmospheric waves, we show the result using rain rate data between 20°S and 20°N where the heating is dominant. However, the results from different latitude bands (e.g., 10°S–10°N and 40°S–40°N) indicate a similar distribution (not shown). Because the TRMM 3B42 data have coarser resolution (0.25° in both longitude and latitude) than the TRMM PR data (4 km), the overall rain rate is smaller compared with Fig. 5. Figure 6 shows that about 70% of rain has a rate less than 1.6 mm h⁻¹, which means that 70% of the total rain area has smaller than 1.6 mm h⁻¹ rain rate. Thus, we choose a rain rate of 1.6 mm h⁻¹ as a threshold to assume the rain type for the TRMM 3B42 product. In other words, if the surface rain rate is smaller than 1.6 mm h⁻¹, we specified the rain as stratiform rain, whereas if it is larger than 1.6 mm h⁻¹, it is specified as convective.

c. CSH profiles derived from the TRMM 3B42 rain rate

The convective/stratiform heating profiles are derived from the 3-hourly TRMM gridded rain rate (3B42) for 120 days from 1 April to 30 July 2006, which is our model simulation period. Based on the criteria for rain type (from surface rain rate) described in section 3b, a type of heating profile (convective or stratiform) is selected at each grid point of the rain rate data. The convective- or stratiform-type heating profile is then derived from the
surface rain rate and the PTH by the method explained in section 3a. In the calculation of the stratiform heating profile, a fraction of precipitation at melting level \((1 - R_{\text{anvil}})\) is considered to be condensates carried into the stratiform anvil from the convective region. To be consistent with that, total condensates generated in the convective region should be larger than precipitation falling into the surface over the convective region. In other words, a ratio of the condensates generated in the convective region to precipitation falling over the region \((R_{\text{conv}}\) in Shige et al. 2004) is greater than unity and this ratio is also considered in the calculation of the convective heating profile. The CRM simulation result of Shige et al. (2004) shows that \(R_{\text{conv}}\) tends to decrease with PTH. We take a smoothed curve of \(R_{\text{conv}}\) in Fig. 7 of Shige et al. (2004) in which \(R_{\text{conv}}\) monotonically decreases from 1.44 to 1.19 with the PTH between 2 and 7 km and from 1.19 to 1.14 with the PTH between 7 and 16 km. According to our sensitivity test (not shown) using several various curves of \(R_{\text{conv}}\) based on the curve in Fig. 7 of Shige et al. (2004), a difference among the monthly mean heating distributions derived from these various curves of \(R_{\text{conv}}\) is not significant. Accordingly, the convective heating profile is assumed to be in a half-sine shape from the surface to the PTH and the maximum of the heating is determined such that total column heating is proportional to the surface rain rate multiplied by \(R_{\text{conv}}\). However, it should be noted that the loss of condensates from convective regions \((R_{\text{conv}} - 1) \times P_{\text{sfc}}\) is not exactly compensated by the gain of condensates over stratiform regions \((1 - R_{\text{anvil}}) \times P_{\text{m}}\), because they are independently calculated. Nevertheless, a total difference of the condensates accumulated for 120 days of the model simulation period is small, less than 5% of total condensates.

After the convective/stratiform heating profiles are derived from the TRMM 3B42 rain rate data (with 0.25° resolution in longitude and latitude), they are averaged over the model grid (about 2.8° resolution) to force the model, which was referred as the CSH profiles in the previous section. In addition to the CSH product, a convective heating–only dataset is also derived to examine the effect of the stratiform-type heating. For the convective heating–only product, all grid points are assumed to have a convective-type vertical profile of the heating. The two heating products derived from the TRMM 3B42 rain rate are compared with the monthly-mean TRMM CSH product in Fig. 7. The TRMM CSH product has a spatial resolution of 0.5° longitude \(\times\) 0.5° latitude (http://disc.sci.gsfc.nasa.gov/precipitation/documentation/TRMM_README/TRMM_CSH_readme.shtml). Since the TRMM CSH product is a monthly product, it is not appropriate to be used as a forcing for the waves. However, it is a useful dataset to compare with our derived heating products. Figure 7 shows the zonal-mean heating distributions for June 2006. First, the (latent) heating profiles derived from the TRMM 3B42 rain rate (Figs. 7b,c) show a larger amplitude than the TRMM CSH product (Fig. 7a) in the tropics. Intercomparisons of diabatic heating diagnosed from the 40-yr European Centre for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-40) and NCEP
reanalysis with TRMM CSH have shown that the TRMM CSH is generally too weak by a factor of 2 (Chan and Nigam 2009). More recently, a new TRMM CSH algorithm–derived heating shows an increase in an amount of low- and midlevel heating and a lowering of a peak of the heating by about 1 km (Tao et al. 2010). Considering the underestimation of the TRMM CSH and the changes of the new TRMM CSH, the overall values of the derived heating (Figs. 7b,c) from the TRMM 3B42 seems to be reasonable. Moreover, the amplitude of the CSH (Fig. 7c) in the tropics appears to be similar to the annual mean latent heating estimated from TRMM PR by Schumacher et al. (2004). For example, the maximum of the heating over the western Pacific (120°E–135°E) at 10°N is about 3.5 K day$^{-1}$ (not shown), which is comparable to their result (see Fig. 4 of Schumacher et al. 2004).

Month-to-month variations of the zonal-mean distributions of the derived heating for the 3 months April–June 2006 also show very similar agreement with the TRMM CSH product (not shown). The heating is dominant in the Southern Hemisphere (SH) during April, while the dominant heating moves to the Northern Hemisphere (NH) during June, as can be seen in Fig. 7. Both the convective-only heating (Fig. 7b) and the CSH (Fig. 7c) products show that the largest heating is located off the equator in the NH (around 7°N), which corresponds well to the TRMM CSH product (Fig. 7a). On the other hand, our heating including stratiform-type heating (i.e., CSH in Fig. 7c) looks more similar to the TRMM CSH product (Fig. 7a) than the convective-only heating (Fig. 7b). The CSH (Fig. 7c) shows larger heating above 4 km than the convective-only heating (Fig. 7b), which can be attributed to the stratiform-type heating profile. The stratiform heating profile is characterized by a positive (warming) half-sine profile in the upper level and a negative (cooling) half-sine profile below it, while a convective heating profile is characterized by a positive half-sine profile throughout the level from surface to precipitation-top height (see Fig. 2). Provided that the vertically integrated condensates are the same, the stratiform-type profile results in a stronger latent heating, especially in the upper level, than the convective-type heating profile because of less density in the upper level than in the lower level. As a result, the level of the maximum heating in the tropics rises up to 6 km in the CSH (Fig. 7c) compared to 4 km in the convective-only heating (Fig. 7b). The peak is closer to 7 km in the TRMM CSH$^3$ (Fig. 7a). The response of atmospheric waves to the different heating profiles will be discussed later in this paper. The design of model experiments for this will be described in the following section.

3. Model and experiment design

A global spectral model is employed to simulate the equatorial waves generated by space–time variable latent heating. The model is a nonlinear primitive equation model, originally developed by Saravanan and McWilliams (1995). It uses a semi-implicit time-stepping scheme to compute the evolution of the vorticity, divergence, and temperature fields. All dynamic fields are represented on a horizontal grid in terms of a spherical harmonic expansion. The simulations use a triangular truncation with 40 zonal wavenumbers (T40) and a vertical grid with 80 sigma levels that are spaced at intervals of 0.75 km from the surface to 60 km. The model includes radiative damping parameterized by a Newtonian cooling and the damping rate has a value of 1/20 day$^{-1}$ throughout the model domain, except that it is increased near the surface and approaching the model top (Holton and Wehrbein 1980). The winds are relaxed to the initial zonal-mean state above 50 km. This serves as a sponge layer to prevent waves from reflecting at the model top. An eighth-order horizontal hyperdiffusion is used to prevent energy from accumulating at the smallest model scales. It is set to damp the smallest scale at a rate of 1 day$^{-1}$.

Instead of convective parameterization, the latent heating is prescribed from the TRMM 3B42 rain rate and global-merged IR brightness temperature every 3 h during 120 days from 1 April to 29 July 2006. The 3-hourly latent heating data are interpolated to match the model time step (i.e., 7.5 min) to force the model. Our sensitivity test shows that it takes about 60 days (45 days) for most vertically propagating waves, to pass 40 km (30 km). That is why the latent heating was started about 60 days ahead of our target time period (June 2006). As described in the previous section, two different forms of latent heating data are used to force the model. One includes only convective-type heating profile (hereafter “CV-only”) and the other includes both convective and stratiform-type heating profile (hereafter “CV+ST”). Therefore, differences in the two simulation results can be considered as a stratiform-type profile effect.

The model is initialized with zonal-mean zonal winds and temperatures for June 2006 obtained from data assimilation products, namely the NCEP–NCAR reanalysis data and the climatological Committee on Space Research (COSPAR) International Reference Atmosphere (CIRA) data (Fleming et al. 1990). Because the

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$^3$ It should be noted that the peak of 10 yr of heating data derived by the new TRMM CSH algorithm occurs around 6 km (Tao et al. 2010).
NCEP–NCAR reanalysis data are available up to 10 hPa, the NCEP–NCAR data were smoothly merged with the CIRA climatology above 10 hPa. Since the static stability and zonal wind play an important role in the vertical propagation of the convectively driven waves, the zonal-mean zonal wind and temperature are relaxed to the observed zonal-mean values for June 2006 with a 1-day time scale. However, it should be noted that the space–time varying heating sources allow an evolution of a zonally asymmetric dynamic field (i.e., zonal wind) although the zonal-mean zonal wind and temperature remain close to the observed zonal-mean distribution. Figure 8 displays the longitude–height cross section of the zonal wind averaged between 10°S and 10°N for June 2006 from (a) the NCEP–NCAR reanalysis and (b) the model simulation of the CV+ST case. The model result shows the baroclinic structure of the tropical troposphere, which is consistent with the observation. Particularly, the Eastern Hemisphere is characterized by easterly vertical shear [i.e., westerlies in the lower troposphere (below 5 km) and easterlies in the upper troposphere]. The Western Hemisphere is characterized by westerly vertical shear, which appears to be stronger in the model result than the observations. The simulated zonal wind in the upper troposphere is mostly confined below about 15 km, while the observed wind reaches up to 20 km. It may be attributable to the heating profiles that force the model (Figs. 7b,c), for which the tops are on average lower than the TRMM CSH profiles (Fig. 7a). Except for that, the overall structure of the simulated asymmetric zonal wind well matches the observation.

In the next section, to analyze the equatorial waves generated by the latent heating, we use perturbations (or anomalies) of dynamic fields, which are defined as a deviation from the initial state (or observed zonal-mean distribution for June 2006). Space–time spectral analysis is used to identify the equatorial waves simulated in the model, which is motivated by Wheeler and Kiladis (1999). Among the equatorial waves, we particularly analyzed three dominant waves [i.e., the Kelvin, mixed Rossby–gravity (MRG), and equatorial Rossby (ER) waves]. The spectral analysis is applied on symmetric and antisymmetric components about the equator of 3-hourly model output data for 60 days (17 May–15 July 2006). This 60-day period is selected to cover 15 days before and after the target period (i.e., June 2006) and also 45 days later than the heating started to allow the model to ramp up the waves in the UTLS. As stated previously, the zonal-mean zonal wind and temperature are forced to remain the same as the observed zonal mean fields for June 2006. Because the static stability and the zonal wind can influence the vertical propagation of the waves, we attempt to sample the data for the period not far from June 2006. On the other hand, it should be noted that the 60-day sample studied is not long enough to properly characterize observed ER waves, and therefore the ER waves analyzed in this study instead represent “transient” ER waves.

We first decomposed the data into symmetric and antisymmetric components. A field $F(\phi)$ can be written as $F(\phi) = F_{\text{sym}}(\phi) + F_{\text{antisym}}(\phi)$, where $\phi$ is latitude, $F_{\text{sym}}(\phi) = [F(\phi) + F(-\phi)]/2$, and $F_{\text{antisym}}(\phi) = [F(\phi) - F(-\phi)]/2$. Using both a 60-day sample and a 90-day sample of the TRMM 3B42 rain rate, we filtered the equatorial Rossby wave (isolated with the same spectral domain used in Fig. 9) and compared phases of two filtered waves. The result shows that the phases of the ER waves are similar to each other (not shown).
The space–time spectrum is calculated using a discrete Fourier transform for each latitude and height. It should be noted that there is simply no background spectrum that needs removing in our case, unlike previous studies using observation data or climate simulations (Wheeler and Kiladis 1999; Cho et al. 2004; Lin et al. 2006). To filter signals corresponding to each wave band in the spectral domain, an inverse space–time Fourier transform is used. Dynamical structure of the filtered waves in the UTLS and its relationship to the tropical convection (i.e., tropical heating) will be examined by comparing with observations and with equatorial wave theory in the next section.

4. Relationship between convection and waves in the UTLS

Figure 9 presents the zonal wavenumber–frequency spectrum of temperature averaged between 16 and 20 km, over 15°S–15°N latitude; 60 days of data (17 May–15 Jul 2006) are used. Results are shown for the (left) CV-only and (right) CV+ST cases, for the (top) symmetric and (bottom) antisymmetric part. The dispersion curves of the equatorial waves (Kelvin, ER, and MRG waves) for the three equivalent depths of $h = 12, 50,$ and 200 m are superimposed (solid lines).
20 km and between 15°S and 15°N, which represents the signal from the lower stratosphere exclusively. Three-hourly temperature data for 60 days (17 May–15 July 2006) are used. The superimposed lines represent the dispersion curves of the equatorial waves (Kelvin, equatorial Rossby, and mixed Rossby–gravity waves) for three equivalent depths of \( h = 12, 50, \) and 200 m. The left column shows results from the CV-only case and the right column is from the CV+ST case. A broad spectrum of Kelvin, MRG, ER, and inertia–gravity (IG) waves are generated in the model, while the inertia–gravity waves appear to be dominant at high frequency \([i.e., >0.5 \text{ cpd (not shown)}]\). For the static stability of the tropical troposphere, the equivalent depths of \( h = 200 \) and 50 m correspond to the theoretically determined vertical wavelength scales of about \( 28 \) and 14 km from the peak projection response to the deep convective heating with 14- and 7-km depth, respectively (Wheeler and Kiladis 1999; Wheeler et al. 2000). In the stratosphere, the vertical wavelength of the waves reduces to about half of the tropospheric value because of the strong static stability in stratosphere. For the static stability of a mean value of tropospheric and stratospheric value \((about 0.015 \text{ s}^{-1})\), the equivalent depths of 12, 50, and 200 m roughly correspond to 4.5, 9.3, and 18.5 km. The equatorial waves, captured in observed dynamical fields, have prominent spectral peaks in the range of \( h = 50–200 \) m, while the outgoing longwave radiation (OLR) has spectral peaks of the equatorial waves at shallower equivalent depths of \( h = 12–50 \) m (Wheeler et al. 2000; Hendon and Wheeler 2008). Wheeler et al. (2000) attributed stratiform precipitation as one of the possible mechanisms causing smaller equivalent depth than the peak projection response. The shallower heating, associated with the stratiform precipitation, might be related to the equatorial waves having smaller equivalent depth and therefore slower phase speed.

Consistently, our results show that the spectral peaks of the equatorial waves having shallower equivalent depth (smaller than 50 m) increase in the case where stratiform heating is included (right column in Fig. 9). On the other hand, strong spectral peaks dominantly appear in the range of 50–200 m of the equivalent depth in the case of convective heating (left column). In other words, in the range of \( h = 12–50 \) m, more yellow-to-red color can be seen in the CV+ST case (Fig. 9, right) rather than in the CV-only case (Fig. 9, left), especially for the Kelvin and MRG waves. In the symmetric part, the spectral peak at zonal wavenumber \( = 1 \) and period \( = 30 \) days represents the eastward propagating MJO. This MJO signal is more prominent in the lower levels (below 16 km) and decreases with height (not shown). Strong spectral peaks of the Kelvin wave are dominant in the range of zonal wavenumber between 1 and 5 and period between 5 and 20 days, which is similar to the result of Hendon and Wheeler (2008). For the ER wave, prominent peaks appear at westward zonal wavenumber 1 with 30-day period and wavenumber 2 with 20-day period. In the antisymmetric part, it can be seen that mixed Rossby–gravity waves having shallower equivalent depth (smaller than 50 m) are enhanced in the case where stratiform-type heating is included.

To examine the prominence of the Kelvin, MRG, and ER waves as well as the MJO signal in observations during the study period \((i.e., 16 \text{ May–15 July 2006})\), we applied wave filters to the TRMM 3B42 rain rate data for 365 days from March 2006 to February 2007, isolating the wave signals in each of the wave bands drawn in Fig. 9. For the MJO filtering, the wavenumber–frequency domain corresponding to a zonal wavenumber of 1–2 and period of 25–70 days was used. The result (not shown) gives evidence that the Kelvin, ER, and MRG waves are active during the study period, and the MJO signal is also detected during the period although the strongest MJO activity occurred during the boreal wintertime. These clear signals in the rain rates are expected because there would be no atmospheric Kelvin wave (or other wave) responses in Fig. 9 unless there was a prominent projection of the heating \((i.e., \text{rain rate})\) spectrum onto these particular spectral windows.

Similar to Fig. 9, spectral peaks of Kelvin, ER, and MRG waves are also found in the zonal wavenumber–frequency distribution of vertical flux of zonal momentum (not shown). Spectral peaks of the inertia–gravity wave dominantly appear at high frequency \((\approx 0.5 \text{ cpd})\) particularly in stratosphere, which can be expected because the contribution of the IG waves to oscillations of the tropical zonal wind \((e.g., \text{QBO and semianual oscillation})\) is substantial. On the other hand, the signal of IG waves is relatively weak compared to the Kelvin, ER, and MRG waves in the UTLS.

In the next section, the composite structure of dynamic fields associated with the spectral peaks of three dominant equatorial waves will be examined. Characteristics of the equatorial waves are extracted from the total dynamic fields \((i.e., \text{temperature, zonal, meridional and vertical wind, and geopotential height})\) by filtering in the wavenumber–frequency domain. The defined regions of each wave band are displayed in Fig. 9. The rain rate data are also filtered in the same way and averaged over a latitude band between the equator and 15°N. For the compositing, we chose cases when the filtered wave signals of rain rate are strong \((based \text{on Hovmøller diagrams for each wave})\). The total number of realizations used for the composites are 144, 184, and 104 for the
Kelvin, ER, and MRG waves, respectively. Next, the filtered dynamic fields are composited by placing the longitude where the filtered rain rate is the strongest at 180° at a given latitude, height, and time.

a. Kelvin wave

Figure 10 displays the longitude–height cross section of the Kelvin wave–filtered temperature (shading), geopotential height (contours), and zonal vertical wind (vectors) anomalies, composited by placing the longitude where the Kelvin wave–filtered rain rate is the strongest at 180°.

FIG. 10. Longitude–height cross section of the Kelvin wave–filtered temperature (shading), geopotential height (contour), and zonal vertical wind (vectors) anomalies, composited by placing the longitude where the Kelvin wave–filtered rain rate is the strongest at 180°.

peaks at zonal wavenumber 2 and period 20 days and zonal wavenumber 3 and period 8.6 days (right column of Fig. 9).

As expected from Fig. 9, dynamic anomalies filtered with the Kelvin wave band roughly have a horizontal scale corresponding to zonal wavenumber-3 wave in the UTLS (between 16 and 20 km). Moreover, the eastward tilted wave structure of temperature, zonal and vertical velocity, and geopotential height in the stratosphere well corresponds to the theoretical phase relationships among those dynamic fields for the vertically propagating Kelvin wave (Wallace and Kousky 1968). Particularly, the eastward/upward motion of wave packets implies an upward transport of westerly (eastward) momentum. The westerly acceleration of the QBO due to the Kelvin wave will be discussed later in this paper. The strong Kelvin wave signal in the stratosphere (i.e., an upward eastward tilted temperature signal) seems to be directly connected to the strong tropospheric vertical motion collocated around 180°, which is presumably driven by the latent heating associated with the tropical convection. It implies that the Kelvin wave signal centered around 180° represents a convectively forced Kelvin wave. The Kelvin wave signal is more robust in the case when stratiform heating is included (i.e., the CV+ST case; Fig. 10a) than when convective heating only is included (CV-only case; Fig. 10b).

Tropospheric responses to the heating also appear to be different depending on the heating profile. First of all, tropospheric temperature anomalies around 180° reflect the combined impact of latent heating, radiation, and adiabatic effects. As expected, the tropospheric warm temperature perturbation is dominant in the middle level around 5 km in the CV-only case, while strong warm temperature perturbation appears in the upper tropospheric and cold temperature perturbation appears below 5 km in the CV+ST case. Likewise, vertical motion is induced throughout the troposphere extending from the surface up to 13 km in the CV-only case, while stronger vertical motion is induced in the upper troposphere in the CV+ST case. As a result, the stronger and shallower heating causes the stronger wave signal in the UTLS in the CV+ST case. On the other hand, the deeper heating in the CV-only case presumably excites waves with larger vertical wavelength (i.e., larger group velocity in both zonal and vertical directions), which consequently causes weaker wave signals in the UTLS and also causes the stratospheric wave signal to be shifted eastward in Fig. 10b compared to Fig. 10a. Although an interaction between convectively driven waves and convection is not taken into account in this study, it might be meaningful to compare tropospheric structures of convectively driven waves from the two different heating cases. In the real atmosphere, the
tropospheric structure associated with the wave affects
initiation of subsequent convection. The stratiform in-
stability mechanism for wave amplification of the Kelvin
wave (Mapes 2000; Straub and Kiladis 2003) can be in-
directly diagnosed by comparing results from CV-only
case and CV+ST case. Interestingly, the tropospheric
structure of the convectively driven Kelvin wave in the
CV+ST case resembles the observed Kelvin wave
structure obtained by regressing the ECMWF reanalysis
data (temperature and zonal and vertical velocity)
against the Kelvin wave–filtered OLR (Fig. 3a of Straub
and Kiladis 2003). For example, upward motion is strong
in the upper troposphere where warm anomalies appear,
and weak westward-tilted upward motion collocates
with cold anomalies in the lower troposphere (below
5 km). Downward motion, located to the east of the
heating (OLR), collocates with cold anomalies in the
upper troposphere and warm anomalies in the lower
troposphere. This baroclinic structure of temperature
did not appear in the CV-only case (Fig. 10b). Particu-
larly, Straub and Kiladis (2003) pointed out that the
baroclinic mode of a heat source, associated with strat-
iform precipitation, is an important component of wave
instability theories for Kelvin waves. Likewise, our re-
sult also implies that the stratiform-type heating is im-
portant for the observed Kelvin wave structure. On the
other hand, an absence of the temperature signal near
the surface in our result seems to be due to the simple
representation of surface processes in our model. Except
for the signal near the surface [i.e., the “cold pool”
signal at the lowest levels in Fig. 8b of Kiladis et al.
(2009)], the overall structure of the simulated Kelvin
wave from the CV+ST case also reasonably corre-
ponds to the observed Kelvin wave structure by Kiladis
et al. (2009). Particularly, the westward tilted zonal wind
signal in the lower troposphere is consistent with our
result from the CV+ST case, which is absent in the re-
sult from the CV-only case.

The horizontal distribution of the composite Kelvin
waves at 15 km is displayed in Fig. 11. Although the
Kelvin wave filtering was applied to total dynamic fields
(symmetric + antisymmetric), the horizontal structure is
almost symmetric across equator. Westerly anomalies
appear to the east of 180° (heating) and are in phase with
positive geopotential height anomalies (high pressure),
while easterly anomalies, collocating with negative geo-
potential height anomalies (low pressure), appear to
the west of 180°. Because the westerly anomalies are in
phase with the upward motion as seen in the vertical
structure (Fig. 10), the westerly anomalies (upward mo-
tion) lead cold anomalies by one-quarter of a cycle via
adiabatic cooling, which well corresponds to the theo-
retical phase relationships of the Kelvin wave (Wallace
and Kousky 1968). Likewise, warm anomalies follow
easterly (downward) anomalies to the west of 180°. The
Kelvin wave signal is overall stronger in the CV+ST case
than the CV-only case, as expected from Fig. 10.

b. Mixed Rossby–gravity waves

Figure 12 presents the vertical structure of the west-
ward propagating mixed Rossby–gravity wave averaged
between 3° and 15°N. In accord with Fig. 9, waves with
zonal wavenumbers of about 3 or 4 dominantly appear in
the lower stratosphere, but the wave amplitude is much
weaker than the Kelvin wave. Westward tilted MRG
waves appear above a strong tropospheric vertical motion
around 180°. The connection between the tropospheric
vertical motion (driven by latent heating) and the stratospheric MRG waves is more obvious in the CV+ST case. The stratospheric structure of the temperature, wind, and geopotential height (pressure) anomalies can be compared to the theoretical MRG wave (Andrews et al. 1987; Holton 1992). The eastward and upward motion coincides with the high pressure (solid line), while the westward and downward motion goes with the low pressure (dashed line), particularly in the stratosphere. The zonal and vertical wind anomalies are toward cold anomalies and away from warm anomalies. This vertical structure of the MRG wave is consistent with phase relationships in the theoretical MRG wave. In addition, the vertical structure of temperature perturbations in the MRG wave, especially forced by the CV+ST heating (Fig. 12a), seems to be comparable to the MRG wave appearing in observations (see Fig. 14 of Kiladis et al. 2009).

Horizontal cross sections of the MRG waves at 15 km are presented in Fig. 13. As expected from Fig. 12, the MRG wave signals are more obvious in the CV+ST case than the CV-only case. A counterclockwise gyre is symmetric about the equator east of 180° (i.e., convective region), while a clockwise gyre is west of 180°. Temperature anomalies are antisymmetric about the equator. Cold anomalies coincide with equatorward wind anomalies and warm anomalies are in phase with poleward wind anomalies in both the NH and the SH. Moreover, the cold anomalies (equatorward wind anomalies) located around 180° above the convective heating region in the NH are consistent with the horizontal structure of the theoretical MRG wave showing that equatorward wind collocates with divergence. Overall the horizontal structure of the convectively coupled MRG waves is in good agreement with the theoretical MRG wave structure (Andrews et al. 1987; Holton 1992). In addition, it is worth noting that the MRG gyres move off the equator west of 120°E, which is more clearly seen in the CV+ST case. Such off-equatorial MRG gyres have been also detected in the observed OLR and wind data.
Frank and Roundy 2006; Kiladis et al. 2009) and they have been suggested to be associated with tropical cyclone formation from observational evidence (Frank and Roundy 2006; Bessafi and Wheeler 2006; Chen and Huang 2009). In addition, the vertical structure of the MRG wave temperature perturbation, especially in the CV+ST heating case (Fig. 12a), seems to be comparable to the MRG wave appearing in observations (Fig. 14 of Kiladis et al. 2009).

c. Equatorial Rossby wave

Figure 14 displays the vertical structure of the composite ER wave averaged between the equator and 16°N. It should be noted that for the space–time spectral analysis, only 60 days of data are used and the time mean is removed, so very slow ER waves (having a period of longer than 30 days) are not included in this composite structure. In other words, this composite ER wave structure represents transient ER waves with a period of 30 days or less. The positive temperature anomalies in the upper troposphere are collocated with the strong upward motion around 180°. The positive geopotential height (pressure) anomalies above the warm anomalies and negative geopotential height anomalies below it show the baroclinic structure of the ER wave in the troposphere. The ER waves in the upper troposphere and lower stratosphere are westward tilted and the wave amplitude is stronger in the CV+ST case. The westward tilted ER wave is substantially reduced above 20 km where the static stability is large. To the west of the convective region (i.e., warm anomalies around 180°), westerly anomalies in the lower troposphere and easterly anomalies in the upper troposphere are consistent with the observed ER wave structure (Wheeler et al. 2000; Kiladis et al. 2009).

Our composite ER waves (Fig. 14) are constructed by following the strongest signal of the ER wave–filtered rain rate for 17 days (10–26 June 2006). During this period, the ER wave propagates westward between 120°W and 140°E, which corresponds to a longitude zone with westerly vertical shear in the zonal wind (i.e., westerly in the upper troposphere and easterly in the lower troposphere; Fig. 8). In other words, the composite structure around 180° in Fig. 14 presumably represents the ER waves propagating through the westerly shear zone. The horizontal distribution of the composite ER wave at 14 km (Fig. 15) seems to less resemble the observed structure (Fig. 17 of Kiladis et al. 2009) compared with the vertical structure. However, key features are still comparable with the observed structure, such as the westerlies straddling the equator west of 180°, which are associated with the anticyclonic circulation in both hemispheres (Fig. 15), and the anticyclonic cell collocating with the cold anomalies, particularly in the SH (Fig. 15a).

5. Comparison with the HIRDLS satellite data:
Kelvin wave

The previous section shows that the Kelvin wave is the most prominent global-scale equatorially trapped wave (e.g., Fig. 9). Moreover, the Kelvin wave significantly contributes to forcing the QBO in zonal wind, corresponding to about 20%–70% of the total wave forcing (Hitchman and Leovy 1988; Canziani and Holton 1998; Tindall et al. 2006; Ern and Preusse 2009a,b). In this section, comparisons of the simulated Kelvin waves with the Kelvin waves seen in the HIRDLS satellite data are discussed.

The rain rate data, filtered with the Kelvin wave band (see Fig. 9), has a maximum amplitude around 180° on 15 June 2006 (not shown). Accordingly, longitude–height cross sections of the Kelvin wave–filtered dynamic fields on 15 June 2006 and 5 days ahead of it (10 June 2006) are displayed in Fig. 16. HIRDLS temperatures are spectrally analyzed and filtered with the Kelvin wave band as described in Alexander and Orland (2010) and shown in the top panels. HIRDLS is a limb-scanning infrared instrument with high vertical resolution of about 1 km (Gille et al. 2008). The Kelvin wave signal is absent below about 15 km
in the HIRDLS satellite data because of the presence of deep clouds. Eastward tilted Kelvin waves with zonal wavenumber of 3 appear to be dominant in the lower stratosphere from both the HIRDLS data and the model results. On 10 June 2006, the observed Kelvin waves have a strong amplitude between 60°W and 60°E and between 150°E and 120°W. Moreover, the model results show that the Kelvin waves are connected to the strong vertical velocity perturbations in the troposphere, centered around 0° and 120°E. Black solid lines marking the 20-km altitude level and locations of dominant wave packets are overplotted on each panel for reference. On 15 June 2006, the phase of the Kelvin waves has moved eastward. The simulated Kelvin waves in the upper troposphere and lower stratosphere between 120°E and 120°W also seem to be associated with the strong vertical velocity around 180°. The phase of the observed Kelvin waves matches reasonably well with the simulated Kelvin waves, particularly in the CV+ST case. In the CV-only case, the phase of the Kelvin waves appears farther to the east, compared with the CV+ST case and to the observation. It indicates that the CV-only case tends to generate Kelvin waves with the faster phase speed and group velocity (i.e., larger vertical wavelength) than the CV+ST case. For instance, the Kelvin waves around 180° from the CV+ST case have a smaller vertical wavelength of about 5–8 km, while in the CV-only case it is about 10 km.

The period of June 2006 corresponds to the westerly phase of the QBO and the peak of the westerly located around 26 km (Fig. 17, right). To estimate zonal wind forcing due to the Kelvin wave, the vertical convergence of the density-weighted zonal momentum flux is calculated from cross spectra as

\[ \frac{1}{\rho} \frac{\partial}{\partial z} \text{Re}[\rho \hat{u}(\kappa, \omega) \hat{w}^*(\kappa, \omega)]_{KW}. \] (1)

Here, \( \hat{u} \) and \( \hat{w} \) denote the Fourier coefficients of zonal and vertical velocities and an asterisk is the complex conjugate; \( \kappa \) is zonal wavenumber, \( \omega \) is frequency, and \( \rho \) is density; and KW indicates the wavenumber–frequency range of the filtered Kelvin wave: \( 1 \leq \kappa \leq 5, 0.05 < \omega < 0.3 \text{ cpd, and } 7 \leq \text{phase speed} \leq 75 \text{ m s}^{-1}. \) In other words, 2 times the cospectra of \( u \) and \( w \) is summed over the spectral elements corresponding to \( (\kappa, \omega) \), where \( \omega > 0 \). This Kelvin wave band is the same as in Fig. 9 except for an extended phase speed (or equivalent depth) range. Figure 17 (left panel) displays the vertical convergence of the density-weighted zonal momentum flux due to the Kelvin wave for June 2006 from the HIRDLS satellite data (thick solid) and model results from the CV+ST case (solid) and the CV-only case (dashed).

First of all, the HIRDLS estimation shows a pronounced westerly acceleration below about 26 km (strong westerly shear zone), which similarly appears in the model result from the CV+ST case. Averages over a 3-km layer (21–24 km) centered about this peak of westerly acceleration are 0.062 m s\(^{-1}\) day\(^{-1}\) from the HIRDLS data and 0.052 and 0.045 m s\(^{-1}\) day\(^{-1}\) from the CV+ST and CV-only cases, respectively. The zonal wind forcing due to the Kelvin wave from the HIRDLS data and the CV+ST case roughly corresponds to the estimation from another satellite observation, the Sounding of the Atmosphere Using Broadband Emission Radiometry (SABER) instrument of the Thermosphere Ionosphere Mesosphere Energetics and Dynamics (TIMED) satellite [about 0.05–0.06 m s\(^{-1}\) day\(^{-1}\) per Fig. 9 of Ern and Preusse (2009b)]. Moreover, a contribution of the Kelvin wave to the total zonal wind forcing between 21 and 24 km for June 2006 is about 47% in the CV+ST case simulation. The Kelvin wave contribution to the total zonal wind forcing is somewhat larger than an estimate from the SABER satellite measurement, which is about 20%–35% (Ern and Preusse 2009a). Another zonal wind
acceleration occurs below 20 km in both the HIRDLS data and model results although the HIRDLS data show a weaker acceleration that might be related to the presence of clouds below 18 km, both penetrating convective clouds and thin cirrus, which cause a decreasing number of HIRDLS measurements (and corresponding increase in noise in the Kelvin wave signals).

The zonal wind acceleration in the westerly shear zone appears to be due to waves with the phase speed close to the zonal wind $u$, while the acceleration below

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![Figure 16](image-url)

**Fig. 16.** Longitude–height cross section of the Kelvin wave–filtered temperature (shading) and zonal vertical wind (vectors) anomalies at days 10 and 15 from the model simulation of (top) the CV+ST case, (middle) the CV-only case, and (bottom) HIRDLS observations. The data are averaged between 6.3°S and 6.3°N for the HIRDLS data and between 7.3°S and 7.3°N for the model results.
20 km appears to be related to the waves with lower phase speed (not shown). The dispersion relation of the Kelvin wave is as follows:

\[ \omega = -Nk/m, \]  

where \( \omega \) is the intrinsic frequency, \( m \) is the vertical wavenumber, and \( N \) is the buoyancy frequency (Andrews et al. 1987). In case of nonzero background (zonal) wind, the frequency is Doppler shifted as follows:

\[ \omega = \omega - ku, \]  

where \( \omega \) is the ground-based (or absolute) frequency. From Eqs. (2) and (3), \( m \) and the vertical group velocity \( C_{gz} \) take the following form:

\[ m = -N/(c - u), \]  

\[ C_{gz} = \frac{\partial \omega}{\partial m} = Nk/m^2, \]  

where \( c = \omega/\kappa \) is the phase speed of the waves. In the westerly shear zone (20–26 km), the waves with phase speed close to \( u \) have a very large vertical wavenumber [Eq. (4)]. In other words, the vertical group velocity approaches to zero [Eq. (5)], which results in the vertical convergence of the westerly momentum flux due to the dissipation of the vertically propagating Kelvin wave.

On the other hand, the vertical convergence of the westerly momentum flux below 20 km seems to be due to a sharp vertical gradient of the buoyancy frequency (or static stability), filtering the relatively slow Kelvin waves (not shown). Below 20 km the background zonal wind is a weak easterly and the vertical gradient of the zonal wind is small (Fig. 17, right). From Eq. (4), we can get the following:

\[ \frac{\partial m}{\partial z} \approx \frac{1}{c - u} \frac{\partial N}{\partial z}. \]  

Particularly, in the case of waves with lower phase speed [i.e., \( 1/(c - u) \) is large], \( m \) grows large with height where the buoyancy frequency sharply increases with height near and below 20 km (not shown), which makes the waves more susceptible to damping. Accordingly, as the Kelvin waves with lower phase speed approach 20 km, the vertical group velocity is substantially reduced (i.e., dissipation of the vertically propagating Kelvin waves), resulting in the convergence of the westerly momentum flux.

6. Conclusions

Equatorial atmospheric waves in the UTLS forced by two types of latent heating derived from the TRMM rain rate and the IR brightness temperature are investigated using a global spectral model. One type of the latent
heating data includes only convective-type profiles, while the other includes both convective- and stratiform-type profiles. How we derive the latent heating profiles from the rain rate and the IR brightness temperature is explained in section 2. In the case where the stratiform-type profile is included, the zonal-mean heating distribution for June 2006 is reasonably comparable to the monthly mean TRMM CSH distribution except for its stronger amplitude than the TRMM CSH. However, considering that the TRMM CSH is chronically weak by a factor of 2 (Chan and Nigam 2009) and recalling reported changes for a new TRMM CSH algorithm–derived heating distribution (Tao et al. 2010), the amplitude of the heating derived from the TRMM 3B42 rain rate is in good agreement with the TRMM CSH. While the zonal-mean zonal wind and temperature are relaxed to the observed zonal-mean distribution for June 2006 during the model simulation, the space–time variable heating allows an evolution of zonally asymmetric dynamic fields (e.g., zonal wind). Interestingly, the longitudinal distribution of the tropical zonal wind, simulated by the latent heating, appears to be similar to observations.

A broad spectrum of Kelvin, mixed Rossby–gravity, equatorial Rossby, and inertia–gravity waves are generated in the model. Particularly, the equatorial waves (Kelvin, ER, and MRG waves) of zonal wavenumber 1–5 appear to be dominant in the UTLS. In the wavenumber–frequency domain, the equatorial waves have prominent spectral peaks in the range of 12–200-m equivalent depth, while the spectral peaks of the equatorial waves, having shallower equivalent depth (<50 m), increase in the case where stratiform-type heating is included. The shallower heating, associated with the stratiform precipitation, is likely related to the equatorial waves having smaller equivalent depth (i.e., vertical wavelength) and therefore slower phase speed. The spectral peaks in the range of 12–50-m equivalent depth (CV+ST case) correspond to those of convectively coupled equatorial waves in observed OLR data (Wheeler and Kiladis 1999; Kiladis et al. 2009). The theoretically determined equivalent depth of the equatorial waves based on a typical deep convective heating (7–14 km) is about 50–200 m. Accordingly, our results imply that the stratiform-type heating might be relevant for the shallower equivalent depth of the observed convectively coupled equatorial waves.

The horizontal and vertical structures of the simulated equatorial waves (Kelvin, ER, and MRG waves) are in a good agreement with equatorial wave theory (Matsuno 1966; Andrews et al. 1987; Holton 1992) and observed wave structure (Wheeler et al. 2000; Straub and Kiladis 2003; Kiladis et al. 2009). In particular, the simulated Kelvin waves are compared with those observed by the HIRDLS satellite. A snapshot of the vertical wave structure at a specific time shows that the phase of the simulated Kelvin wave in the lower stratosphere well coincides with the HIRDLS observation. On the other hand, the model simulation period corresponds to the westerly phase of the QBO (June 2006) having the peak of the westerly around 26 km. The zonal wind forcing due to the simulated Kelvin wave in the westerly shear zone (averaged between 21 and 24 km) is about 0.052 m s$^{-1}$ day$^{-1}$ (CV+ST case), which is comparable to 0.062 m s$^{-1}$ day$^{-1}$ estimated from the HIRDLS observation. Moreover, the Kelvin wave contribution to the total zonal wind forcing is about 47% in the model simulation (CV+ST case). Another peak in zonal wind forcing appears below 20 km, which seems to be associated with dissipation of the slower Kelvin waves due to the presence of a strong vertical gradient in static stability.

In this study, the equatorial waves generated by latent heating during the early NH boreal summer (June 2006) have been investigated. Since the equatorial waves, forced in the troposphere, hinge on a background flow with seasonal and interannual (including QBO) variations as well as the variability in the heating itself, the equatorial waves forced by heating in different seasons and in different background flows might need to be explored.

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