How Does the Vertical Profile of Baroclinicity Affect the Wave Instability?

TOSHIKI IWASAKI
Graduate School of Science, Tohoku University, Aramaki, Japan

CHIHIRO KODAMA
Research Institute for Global Change, Japan Agency for Marine-Earth Science and Technology, Yokosuka, Japan

(Manuscript received 28 July 2010, in final form 4 December 2010)

ABSTRACT

The growth rate of baroclinic instability waves is generalized in terms of wave–mean flow interactions, with an emphasis on the influence of the vertical profile of baroclinicity. The wave energy is converted from the zonal mean kinetic energy and the growth rate is proportional to the mean zonal flow difference between the Eliassen–Palm (E-P) flux convergence and divergence areas. Mass-weighted isentropic zonal means facilitate the expression of the lower boundary conditions for the mass streamfunctions and E-P flux.

For Eady waves, intersections of isentropes with lower/upper boundaries induce the E-P flux divergence/convergence. The growth rate is proportional to the mean zonal flow difference between the two boundaries, indicating that baroclinicity at each level contributes evenly to the instability. The reduced zonal mean kinetic energy is compensated by a conversion from the zonal mean available potential energy.

Aquaplanet experiments are carried out to investigate the actual characteristics of baroclinic instability waves. The wave activity is shown to be sensitive to the upper-tropospheric baroclinicity, though it may be most sensitive to baroclinicity near 800 hPa, which is the maximal level of the E-P flux. The local wave energy generation rate suggests that the increased upper-tropospheric zonal flow directly enhances the upper-tropospheric wave energy at the midlatitudes. Note that the actual baroclinic instability waves accompany a considerable amount of the equatorward E-P flux, which causes extinction of wave energy in the subtropical upper troposphere.

1. Introduction

The growth rate of baroclinic instability waves has been widely used in the diagnosis of the storm-track activity. The growth rate of Eady baroclinic instability waves is proportional to the vertical wind shear and the inverse of the static stability of the basic state (Eady 1949). Lindzen and Farrell (1980) derived the growth rate of Charney waves in a form similar to that of Eady waves. Hoskins and Valdes (1990) showed the coincidence of the growth rate with the storm track in the Northern Hemispheric winter. Recently, many authors have tried to explain the global warming–induced change in storm track by means of the growth rate of Eady waves (e.g., Hall et al. 1994; Lunkeit et al. 1998; Geng and Sugi 2003; Yin 2005).

A question that has arisen is how the vertical profile of baroclinicity affects the wave activity. Global warming is projected to decrease the vertical wind shear in the lower troposphere but increase it in the upper troposphere and lower stratosphere (UTLS) (Solomon et al. 2007). The sea surface temperature gradients are important for baroclinic instability, as was mentioned by Lunkeit et al. (1998) in their simplified GCM. However, Yin (2005) stated in his multimodel ensemble analyses of the global warming that the poleward shift of the storm track is caused mainly by changes in the upper-tropospheric baroclinicity. The aquaplanet experiment by Kodama and Iwasaki (2009) showed that a polar SST rise reduces the baroclinic instability wave activity as expected, while a uniform SST rise enhances static stability in the subtropics and wind shear in the midlatitude upper troposphere and then shifts the storm track in the poleward direction.

The wave activity is understood in terms of energy conversion rates in the mass-weighted isentropic zonal...
means (MIM). MIM facilitates presenting Lagrangian-mean meridional circulations and wave–mean flow interactions (Gallimore and Johnson 1981; Andrews 1983; Townsend and Johnson 1985; Tung 1986; Iwasaki 1989) and exactly treats intersections of isentropes with the lower boundary, which is essential to the baroclinic instability (Iwasaki 1990). The basic part of the diagnosis package is available online (at http://wind.gp.tohoku.ac.jp/mim/).

MIM leads us to a simple cascade-type energy conversion diagram (Iwasaki 2001; Uno and Iwasaki 2006). The wave energy is generated from the zonal mean kinetic energy through the wave–mean flow interactions. In this work, the growth rate for instability wave is generalized, where the energy conversion rate need to be integrated over the certain domain. At first, it applies to Eady waves as a lesson for a conceptual understanding of the relationship between the energy conversion and the growth rate. Actual baroclinic instability waves are studied using aquaplanet simulation experiments. In particular, solar conditions are modified to study the sensitivity of wave activity to the UTLS vertical shear.

2. Generalized growth rate of baroclinic instability waves

The growth rate of baroclinic instability waves is generalized on the basis of wave energy equation in MIM. Isentropic zonal means are expressed in the Cartesian form as

$$\overline{A} = \frac{1}{L_x} \int A(x, y, \theta, t) \, dx. \tag{1}$$

For isentropes intersecting the ground, the surface values $A(x, y, \theta, t) = A(x, y, \theta_s, t)$ are taken when $\theta < \theta_s(x)$. Isentropic zonal mean pressure is used for vertical coordinates

$$p_1 = \bar{p}. \tag{2}$$

The essential features of MIM are unchanged even when isentropic coordinates are converted into $p_1$ coordinates. The use of $p_1$ coordinates makes it easy to consider the compatibility with the transformed Eulerian mean (TEM) diagnosis by Andrews and McIntyre (1976), except for the lower boundary condition. In both isentropic coordinates and $p_1$ coordinates, the essential features of MIM come from the following definitions of zonal means:

$$\overline{A^p} = A \left( \frac{\partial p}{\partial \theta} \right) \left( \frac{\partial p}{\partial \theta} \right)^{-1} = A \frac{\partial \bar{p}}{\partial \bar{p}}, \tag{3}$$

where asterisks indicate the local mass weight ($\partial \bar{p}/\partial \bar{p}$). The mass weight becomes zero when $\theta < \theta_s(x)$. Eddies are defined as

$$A' = A - \overline{A^p}. \tag{4}$$

Then, their correlations are given by $(A'B')^p = (AB)^p - A\overline{B^p}$. The mass weighting is needed to present conservation principles with complete treatment of the lower boundary conditions. In MIM, the mass streamfunctions represent air mass conservation completely. The lower boundary of Eliassen–Palm (E-P) flux becomes the form drag over mountains and exchanges the atmospheric angular momentum with the solid earth (Tanaka et al. 2004). These are great advantages to completely computing the energetic of the global atmosphere, since the lower boundary plays an important role in the energy conversions.

The wave energy is defined as the sum of eddy available potential energy $A_E$ and eddy kinetic energy $K_E$

$$W = A_E + K_E. \tag{5}$$

Expressions of $A_E$ and $K_E$ in MIM are found in appendix A. The zonal mean system can easily separate the wave energy from the basic zonal mean state energy and diagnose the dynamic wave energy generation rate in the context of wave–zonal mean interactions. According to Iwasaki (2001) and Uno and Iwasaki (2006), globally integrated energy equations can be derived for the zonal mean available potential energy $A_Z$, zonal mean kinetic energy $K_Z$, and wave energy:

$$\frac{\partial}{\partial t} \langle A_Z \rangle = \langle Q_Z \rangle - \langle C(A_Z, K_Z) \rangle, \tag{6}$$

$$\frac{\partial}{\partial t} \langle K_Z \rangle = \langle C(A_Z, K_Z) \rangle - \langle C(K_Z, W) \rangle - \langle \delta_Z \rangle, \tag{7}$$

$$\frac{\partial}{\partial t} \langle W \rangle = \langle C_W(K_Z, W) \rangle + \langle Q_E \rangle - \langle \delta_E \rangle, \tag{8}$$

where angle brackets denote global integrals for the whole atmosphere; $Q_Z$ and $Q_E$ are diabatic generation rates of the zonal mean and eddy available potential energies, respectively, and $\delta_Z$ and $\delta_E$ are the zonal mean and eddy dissipations, respectively. The definitions of parameters are shown in appendix D. In appendix A, we derive an important relationship of conversion rates,

$$\langle C_W(K_Z, W) \rangle = \langle C(K_Z, W) \rangle, \tag{9}$$

which guarantees the energy conservation principle for adiabatic processes. Thus, the energy Eqs. (6)–(9) lead
The spatial variance of the mean zonal flow. Equation (12) is the local dynamic wave energy generation rate. The first and second terms are the shear production of wave energy and its vertical relocation, respectively. Spatial distributions of $C(K_Z, W)$ in a meridional plane are considerably different from those of $C^\alpha(K_Z, W)$, although their global integrals are exactly equal as mentioned above. Waves are generated in a different place from where the zonal mean kinetic energy is lost through wave–mean flow interactions. Detailed discussions regarding local conversion rates are provided in section 5. In Eqs. (11) and (12), $\varepsilon$ and $\varepsilon'$ are the energy conversion terms that are associated with mean meridional flow, and they are much smaller than the energy conversion terms of the zonal wave–mean flow interactions.

According to Eq. (8), the dynamic increasing rate of the wave energy becomes $\langle C^\alpha(K_Z, W) \rangle / \langle W \rangle$ if we neglect diabatic effects and dissipative processes. Thus, growth rate of instability waves can be generalized as

$$\alpha = 0.5 \langle C^\alpha(K_Z, W) \rangle / \langle W \rangle \approx 0.5 \left( \frac{\mathbf{F} \cdot \mathbf{V}^\alpha}{\rho_0} \right) / \langle W \rangle, \quad (13)$$

where the reason for the factor of 0.5 is that the wave energy is the second order of the perturbation geopotential height, as shown in appendix B. The leading term of the wave energy generation rate is the inner product of the E–P flux and shear vector of mean zonal wind. In this definition, the growth rate needs information about the wave structures. If the profile of the E–P flux is given previously, the growth rate can be estimated only from zonal mean parameters, as the growth rate of the Eady waves can. However, zonal mean parameters may affect the profile of the E–P flux, so that the generalized growth rate still contains information about wave structures. The growth rate is not specific to baroclinic instability waves but rather is applicable to all waves generated from wave–mean flow interactions. It is noted that the growth rate should be estimated from integrating the conversion rate and wave energy within a certain domain because waves have spatially coherent structure. It takes into account the baroclinicity over the particular domain rather than the baroclinicity at a single level. As for baroclinic instability waves and planetary waves, the integration area needs a hemisphere or more. In a realistic atmosphere, the diagnoses may provide the statistical average of many existing waves. The relationship (9) leads us to another expression of the growth rate as follows:

$$\alpha = 0.5 \langle C(K_Z, W) \rangle / \langle W \rangle \approx 0.5 \left( \frac{\mathbf{u}^\alpha \mathbf{V} \cdot \mathbf{F}}{\rho_0} \right) / \langle W \rangle, \quad (14)$$

where $u$, $v$, $\rho_0$, $\Phi$, $\Phi_1$, and $\mathbf{F}$ are the zonal flow, meridional flow, reference density, geopotential height, geopotential height for the zonal ground state, and E–P flux, respectively. Here, the log pressure coordinate is defined as $z_1 = -H \log \rho / \rho_0$. In $C(A_Z, K_Z)$, the mean-meridional direct circulations convert the zonal mean available potential energy into the zonal mean kinetic energy. In $C(K_Z, W)$, the zonal wave–mean flow interactions convert the zonal mean kinetic energy into the wave energy. In general, since the E–P flux exchanges negligibly small angular momentum with the solid earth, it mainly redistributes the angular momentum in the atmosphere while conserving its total value (Tanaka et al. 2004). Equation (11) indicates that the zonal mean kinetic energy is converted into the wave energy, when the E–P flux reduces

$$\begin{align*}
A_Z & \rightarrow K_Z \\
K_Z & \rightarrow W
\end{align*}$$

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**Fig. 1.** Energy conversion diagram in the MIM system (Uno and Iwasaki 2006). Here $A_Z$, $K_Z$, and $W$ are the zonal mean available potential energy, zonal mean kinetic energy, and wave energy, respectively; $Q_Z$ and $Q_E$ are the diabatic generation rate of the zonal mean and eddy available potential energies, and $\delta_Z$ and $\delta_E$ are the dissipation rates of the zonal mean and eddy kinetic energy.
where $C(K_Z, W)$ is the sink of zonal mean kinetic energy, which is converted into the wave energy. This expression is convenient for discussing the wave energy source. The energy conversion rate is proportional to the mean zonal wind difference between the E-P flux convergence and divergence areas.

3. Energetics of Eady waves

Let us first consider the Eady problem (Eady 1949) to understand the essence of the baroclinic instability in the context of wave–mean flow interactions. MIM enables us to explicitly express the E-P flux divergence/conversion and mean-meridional flow in the Eady waves, both of which are caused by intersections of isentropes with lower/upper boundaries (Iwasaki 1990). The generalized growth rate defined above is obtained for Eady waves and compared with its conventional value.

Energy conversion mechanisms are decomposed into several steps as shown in Fig. 2. Assuming the existence of initial waves, the normal modes exponentially grow as follows. (a) The E-P flux of Eady waves, which is composed only of vertical components, transfers the westward momentum upward. (b) The E-P flux causes divergence (convergence) near the lower (upper) boundaries and redistributes the momentum. It tends to reduce the zonal mean kinetic energy, as discussed by Takayabu (1993). The reduced zonal mean kinetic energy is converted into the wave energy through wave–mean flow interactions. (c) The intersections of isentropes also cause mean-meridional flow bounded to the boundaries. Iwasaki (1990) showed that the Eady solution satisfies the relationship of extratropical pumping

$$\overline{v^e} = -\frac{1}{f} \mathbf{V} \cdot \mathbf{F}. \quad (15)$$

Note the quasigeostrophic balance relationship (Uno and Iwasaki 2006)

$$C(A_Z, K_Z) \approx C(K_Z, W). \quad (16)$$

The mean-meridional flow bounded to the boundaries converts the zonal mean available potential energy to the zonal mean kinetic energy through $C(A_Z, K_Z)$ in compensation for the loss of zonal mean kinetic energy through $C(K_Z, W)$. (d) The Coriolis force for mean-meridional flow accelerates (decelerates) the mean zonal flow near the upper (lower) boundary. It recovers the geostrophic balance of the Coriolis forces of the mean zonal flow with the meridional pressure gradient. These four steps [(a)–(d)] proceed simultaneously and continuously.

In the isobaric zonal mean system, the eddy heat transport is essential for baroclinic instability and conducts mean-eddy energy conversions. On the other hand, in the MIM without any eddy heat flux, the mean meridional flows, which are bounded to the upper or lower boundaries, conduct meridional heat transport and the energy conversion from the zonal mean available potential energy to the zonal mean kinetic energy.

In the Eady waves, the vertical component of the E-P flux is vertically constant in the free atmosphere and allows divergence (convergence) in the vicinity of the lower (upper) boundaries. Assuming the meridional constancy for simplicity, the energy conversion rate $C(K_Z, W)$ can be approximated by the following formula:

$$\langle C(K_Z, W) \rangle = (U_T - U_B) F_z L_y, \quad (17)$$

where $U_T$ and $U_B$ are basic zonal velocities at the top and bottom of the domain, $F_z$ is the vertical E-P flux in the free atmosphere, and $L_y$ is the meridional length. In
appendix C, the vertical component of the E-P flux is derived as

$$F_z = \frac{2c_k}{\lambda T_y}(W),$$  \hspace{1cm} (18)

where $k$, $c_i$, and $\lambda (= U_T - U_B)$ are respectively the zonal wavenumber, imaginary phase velocity, and vertical shear per nondimensional vertical coordinates. Note that $c_k$ is the growth rate of the perturbation geopotential height of Eady waves. Substituting Eqs. (17) and (18) into Eq. (14), we have

$$\alpha = 0.5\langle C(K_Z, W) \rangle / \langle W \rangle = c_k.$$  \hspace{1cm} (19)

The energy conversion from the zonal mean kinetic energy is consistent with the growth rate of the Eady waves obtained in the conventional manner.

As is well known, the maximum of the growth rate becomes

$$c_k \approx 0.31\lambda / N.$$  \hspace{1cm} (20)

In Eq. (20), the dependence of the growth rate on the static stability is related to the E-P flux. When the static stability is greater, the temperature perturbation corresponds to smaller amplitudes of isentropic surfaces with lesser form drag over the isentropes; that is, the E-P flux is smaller. On the other hand, the dependency on the vertical shear is related to the mechanical efficiency for momentum exchange. In Eq. (11), the energy conversion rate is proportional to the zonal flow difference between the upper and lower boundaries for constant E-P flux. The flow difference is the integration of vertical shear from the bottom to the top, so that the vertical shear at each level can be said to contribute evenly to the energy conversion rate in the Eady waves. The Eady growth rate is regarded as a special case of the generalized growth rate for constant vertical shear.

### 4. An aquaplanet experiment on the sensitivity to upper-level baroclinicity

As mentioned in the previous section, the intersections of isentropes with the lower (upper) boundaries play essential roles in the E-P flux divergence (convergence) and equatorward (poleward) mean flow in the Eady waves. In the actual atmosphere, the isentropes intersect the lower boundary as assumed in the Eady waves. However, the actual atmosphere does not have any rigid upper boundary, which is very different from the Eady waves. To see more realistic baroclinic instability waves, a diagnosis is made of an aquaplanet experiment for a perpetual equinox 10-yr simulation after a 1-yr spinup with a zonally symmetric sea surface temperature (Neale and Hoskins 2000). The model used is a Meteorological Research Institute (MRI)-Japan Meteorological Agency (JMA) global spectral model (GSM) (Shibata...
et al. 1999; Yukimoto et al. 2006). The model has a horizontal truncation wavenumber at triangular 63 and 45 vertical levels. The detailed setting of the control equinox experiment is found in Kodama and Iwasaki (2009).

Figure 3 shows vertical profiles of the vertical component of the E-P flux and its divergence at 45°N. The E-P flux has a maximum near 800 hPa and diverges (converges) below (above) this level. This is very different from Eady waves, whose E-P flux is vertically constant, except near the upper or lower boundaries. The leading term of the growth rate equation [Eq. (13)] is \( \alpha \approx 0.5/(F/\rho_0) \cdot \nabla^2 \phi \approx (F / \rho_0 \partial u / \partial z) / \langle W \rangle \). The numerator is the vertical shear averaged over the domain with the weight of vertical E-P flux. In this sense, the growth rate must be most sensitive to the baroclinicity near 800 hPa. In reality, it was confirmed in the similar aquaplanet experiment that the baroclinic instability wave activity is enhanced by the increased meridional gradient of SST (Kodama and Iwasaki 2009).

The baroclinicity at single level, however, is not enough to estimate the growth rate in Eq. (13). In Eq. (14), \( \langle C(K_z, W) \rangle \) is proportional to the zonal flow difference between the convergence and divergence areas. In this experiment, the convergence and divergence centers are located near 400 hPa and near 900 hPa, respectively, and their flow difference may cause the wave instability. How is the wave instability affected by the baroclinicity in the UTLS? A sensitivity experiment is designed to answer this question, where the solar condition is changed to the solstice without any change in the SST or zonal mean ozone distributions. Even though the SST is symmetric between the two hemispheres, the asymmetric solar heating due to ozone and water vapor makes a significant difference in the zonal mean temperature, particularly in the UTLS, as shown in Fig. 4. Thus, in midlatitudes, the UTLS meridional temperature gradient is much greater in the winter hemisphere than in the summer hemisphere. As a result, the UTLS vertical wind shear in midlatitudes is enhanced (reduced) in the winter (summer) hemisphere. It is somewhat strange that the subtropical jet stream is weakened (strengthened) in the winter (summer) hemisphere, in geostrophic balance with cooling (warming) in the subtropical upper troposphere of the summer (winter) hemisphere. This may result from the mismatch of the equinox SST under the solstice solar condition and be associated with eddy angular momentum transport (Held and Hou 1980). Such subtropical changes in the westerlies, however, seem to be smaller and to have much less impact on the baroclinic wave activity than the extratropical changes.

Figure 5 shows the latitudinal profiles of the vertically integrated wave energy, \( C(W, K_z) \), and \( C(K_z, W) \). The solstice solar condition significantly enhances (reduces) the wave energy and its generation rate in midlatitudes of the winter (summer) hemisphere. The generalized growth rate is estimated from integrating the wave energy and its generation rate over each of hemispheres at about 0.065, 0.054, and 0.043 day\(^{-1}\) for the solstice winter, equinox, and solstice summer, respectively. This is consistent with the fact that the integrated wave energy is greater in the winter hemisphere than in the summer hemisphere. Thus, the UTLS baroclinicity is considered to enhance the wave instability as well as the low-level baroclinicity. In this case, the generalized growth rate may indicate the statistical average of many waves over the hemisphere. Attention must be paid to the poleward (equatorward) shift of peak positions of wave energy and wave energy generation rates in the winter (summer) hemisphere under the solstice solar condition (Fig. 5). This problem will be discussed in the next section. The above estimate of the growth rate suggests that the wave energy generation rate must be a higher-order function of baroclinicity. In the analogy with the Eady waves, the E-P flux is expected to be proportional to wave energy. In reality, the E-P flux is greatly impacted by the change in solar condition, as shown in Fig. 4. Thus, the wave energy generation rate is expected to be of a higher order of baroclinicity than the wave energy, since it is the integration of inner product of E-P flux and shear vector of mean-zonal wind.

5. Local wave energy generation rates

Where is the wave energy generated from in the latitude–height section? Global warming has been projected to increase the upper-tropospheric eddy kinetic...
energy (Yin 2005). Similar changes were found in the aquaplanet experiment with a uniform SST rise (Kodama and Iwasaki 2009). In this section, discussions are extended to latitude–height distributions of the wave energy generation rate and its sensitivity to the vertical profile of baroclinicity in the above-mentioned aquaplanet experiment.

Figure 6 shows the meridional cross sections of wave energy, \( C^W(K_z, W) \), and \( C(K_z, W) \). \( C^W(K_z, W) \) has a temporal change term of geopotential height, which is the second term on the right-hand side of Eq. (12). This term is estimated from 3-hourly model outputs with the centered finite time difference. The wave energy has a local maximum in the midlatitude upper troposphere and increases along the polar night jet stream in the winter stratosphere. Roughly speaking, these overall meridional distributions of wave energy are similar to those of the wave energy generation rate, although the former is relatively greater than the latter in the stratosphere. Detailed differences between these distributions may come from the propagation, deformation, and dissipation of waves. The change in solar condition enhances (reduces) the wave energy generation rate and accordingly the wave energy in the midlatitude upper troposphere of the winter (summer) hemisphere. This is followed by the change in the wave energy, although the wave energy has relatively stronger impact in the stratosphere.

Of interest are the relative contributions of meridional and vertical E-P flux to the local wave energy generations, since the meridional and vertical terms are associated with barotropic and baroclinic processes, respectively. The vertical E-P flux is continuously diffracted into the equatorward E-P flux in the upper troposphere. Actual instability waves accompany the equatorward E-P flux in the upper troposphere as shown in Fig. 4e, although the Eady waves do not have any meridional E-P flux. Here, the wave energy generation rate is decomposed into the meridional and vertical E-P flux terms. The temporal term in Eq. (12)—which is important for traveling waves, including baroclinic instability waves—is regarded as the vertical E-P flux term. Assuming a perturbation function of geopotential height as \( \Phi = \Phi_0 \exp[i(k(x - ct)) ] \), we have the relation \( \partial \Phi / \partial y \big|_z = -c (\partial \Phi / \partial x) \big|_z \). Using this relation, leading terms of local wave energy generation rate can be approximated as

\[
C^W(K_z, W) \approx \frac{F_y}{\rho_0} \left( \frac{\partial \bar{u}^2}{\partial y} \right) \big|_{z_i} - \frac{1}{\rho_0 \partial z_i} g \left( \frac{\partial \Phi}{\partial t} \right) \big|_{z_i} - \frac{\bar{u}^*}{\rho_0} \frac{\partial F}{\partial z_i} \big|_{z_i} - \frac{c - \bar{u}^*}{\rho_0} \frac{\partial F}{\partial z_i} \big|_{z_i} \approx \frac{F_y}{\rho_0} \left( \frac{\partial \bar{u}^2}{\partial y} \right) \big|_{z_i} + \frac{c - \bar{u}^*}{\rho_0} \frac{\partial F}{\partial z_i} \big|_{z_i}. \tag{21}
\]

In Eq. (21), the first and second terms are associated with the meridional and vertical E-P flux, respectively. Figure 7 shows contributions of these two terms to the wave energy generation rate. It is confirmed from the similarity between Figs. 7e and 6c that leading terms dominate the wave energy generation rate. The meridional term makes weakly negative contributions in the subtropical upper troposphere as shown in Fig. 7a. The wave energy is reduced during the equatorward propagation in the direction of the downgradient of mean zonal wind. This causes a distinct subtropical wave energy loss, as shown in Fig. 5 of the vertically integrated wave energy generation rate. The vertical E-P flux term...
primarily generates the wave energy in the midlatitude upper troposphere (Fig. 7c). Equation (21) suggests that the enhanced mean zonal flow in the upper troposphere is expected to directly increase the wave energy generation rate in the region of vertical E-P flux convergence.

In Fig. 5, vertically integrated wave energy generation rates are distinctly shifted poleward (equatorward) in the winter (summer) hemisphere under the solstice solar conditions. Such shifts are mainly caused by the changes in the vertical term (Fig. 7d).

The relationship between \( C^W(K_Z, W) \) and \( C(K_Z, W) \) (dynamic loss of zonal mean kinetic energy) is now further discussed. Equation (21) can be rewritten as

\[
C^W(K_Z, W) \approx C(K_Z, W) + \frac{1}{\rho_0} \left( \frac{\partial \overline{uu} F_y}{\partial y} \right)_{z_i} + \frac{c}{\rho_0} \frac{\partial F_z}{\partial z_i}.
\]

(22)

\( C^W(K_Z, W) \) adds two displacement terms to \( C(K_Z, W) \) on the right-hand side of Eq. (22). Vertically integrated
$C(K_Z, W)$ has double maxima in the subtropics and midlatitudes, as shown in Fig. 5. The subtropical maximum is largely caused by the meridional convergence of E-P flux. Possibly, waves propagating in the direction of the downgradient of mean zonal flow are absorbed by the mean flow near their critical latitudes ($\frac{u^*}{C_0} c' \approx 0$). Released angular momentum from waves reduces mean zonal flow and zonal mean kinetic energy in the subtropics. The second term of Eq. (22) makes its divergence (convergence) in the midlatitudes (subtropics) and transfers the energy from subtropics to midlatitudes. As a result, the zonal mean kinetic energy in the subtropics changes to the wave energy in midlatitudes. The last term of Eq. (22) vanishes when it is vertically integrated. It redistributes the wave energy generation rate from the upper troposphere to the lower troposphere, since phase speeds of traveling cyclones are mostly positive.

Finally, some comments are added with regard to the difference in the spatial distributions between $C^W(K_Z, W)$ and $C(K_Z, W)$. This is partly related to the invariance for the Galilean transformation of any constant zonal velocity. The zonal mean kinetic energy is not invariant for the Galilean transformation, so that $C(K_Z, W)$ is not invariant. On the other hand, the wave energy is
invariant for the Galilean transformation. Equation (12) shows that all the terms of \( C^W(K_Z, W) \) are invariant for the transformation. In particular, the temporal term \( (\partial \Phi / \partial t) \), in Eq. (12) alters the zonal advection \( \bar{u} \bar{v} \) into an invariant form of \( (u^2 - c) \) in Eq. (21).

6. Conclusions

The baroclinic instability wave activity was studied in terms of wave–mean flow interactions, with a focus on the sensitivity to vertical profile of baroclinicity. The growth rate can be generalized as

\[
\alpha = 0.5 \langle C^W(K_Z, W) \rangle / \langle W \rangle = 0.5 \langle C(K_Z, W) \rangle / \langle W \rangle,
\]

development of the zonal mean kinetic energy from the zonal mean available potential energy into zonal mean kinetic energy in compensation for its loss due to wave energy generation.

In the Eady waves, the generalized growth rate becomes consistent with the conventional value of baroclinic instability. The E-P flux is vertically constant except that its divergence and convergence are caused by intersections of isentropes with the lower and upper boundaries. The energy conversion rate from the zonal mean kinetic energy to the wave energy is proportional to the difference in mean zonal flow between upper and lower boundaries. This means that the vertical shear at each level evenly contributes to the instability. The mean-meridional flow caused by the intersections of isentropes with the upper and lower boundaries converts the zonal mean available potential energy into zonal mean kinetic energy in compensation for its loss due to wave energy generation.

Realistic baroclinic instability waves were studied by using aquaplanet experiments. The vertical component of the E-P flux is shown to have a maximum near 800 hPa, in contrast to the vertical constancy of the E-P flux in the Eady waves. The wave energy generation rate may be most sensitive to the baroclinicity at the maximal level of the E-P flux. The sensitivity of the wave activity to the low-level baroclinicity was confirmed in the authors’ previous impact study of a polar SST rise on the wave activity in aquaplanet experiments (Kodama and Iwasaki 2009). The generalized growth rate, however, integrates the baroclinicity over the certain domain and probably has an impact from the spatial distribution of baroclinicity. In this study, our attention is focused on the sensitivity to the upper-tropospheric and lower-stratospheric (UTLS) baroclinicity, where only the solar condition is changed from the equinox to the solstice without changing the SST. The generalized growth rates are shown to be sensitive to upper-tropospheric baroclinicity and causes differences in the hemispheric integration of wave energy.

Local wave energy is considered according to the wave energy generation rate, rather than the growth rate. The winter solstice condition increases the wave energy generation rate due to the enhanced UTLS vertical shear and accordingly increases wave energy in the midlatitude UTLS. Actual baroclinic instability waves considerably accompany the equatorward E-P flux in the UTLS, which plays important roles in subtropics–extratropics interactions in the upper troposphere. The equatorward flux causes extinction of wave energy through the processes of downgradient propagation of mean zonal flow in the subtropical upper troposphere.

The above considerations on the sensitivity of wave instability to the spatial profile of baroclinicity have implications for climate change due to global warming. Climate models tend to reduce the low-level baroclinicity but enhance the UTLS baroclinicity. These two changes are expected to have opposite effects on baroclinic wave activity. However, the relative magnitude of these two changes is dependent on the models. Their combined effects on the instability are so complicated that models may give a diversity of future projections for baroclinic wave activity.

Acknowledgments. The atmospheric GCM used in this experiment was developed at the Meteorological Research Institute and the Numerical Prediction Division of the Japan Meteorological Agency. Parts of numerical experiments were performed at the Cyber-science Center, Tohoku University. This study is supported in part by the Japanese Ministry of Education, Culture, Sports, Science and Technology through a Grant-in-Aid for Scientific Research in Innovative Areas (2205).

APPENDIX A

Local Energy Conversion Rates

The authors’ previous works mainly focused on the globally integrated energies (Iwasaki 2001; Uno and Iwasaki 2006). This work deals with local energetics in latitude–height distributions, which is slightly complicated as compared with global integrals because the loss of the zonal mean kinetic energy \( C(K_Z, W) \) has a different spatial distribution from that of the wave energy generation rate \( C^W(K_Z, W) \). Here, the governing equations of local energetics are derived briefly.

The atmospheric potential energy of unit mass is given by

\[
P = U + \Phi. \tag{A1}\]

The tendency of potential energy is written as
\[
\frac{d}{dt} P = \frac{d}{dt} (C_p T + \Phi - RT) = Q + \left( \frac{\partial \Phi}{\partial t} \right)_p + \mathbf{v} \cdot \nabla_p \Phi - \frac{d}{dt}(p/\rho). \tag{A2}
\]

Taking the mass-weighted isentropic zonal means of Eq. (A2), the tendency of the total potential energy at an isentropic zonal mean pressure becomes

\[
\frac{\partial P^\|}{\partial t} + \frac{\partial}{\partial y} (P^\| + p/\rho) \bar{\mathbf{w}}^\| + \frac{\partial}{\partial \rho_0} \rho_0 (P^\| + p/\rho) \bar{w}^\|_y = Q^\| + \left( \frac{\partial \Phi}{\partial t} \right)_p + (\mathbf{v} \cdot \nabla_p \Phi)^\| - \frac{\partial (p/\rho)^\|}{\partial t}.
\tag{A3}
\]

The zonal and global ground state energies can be written as

\[
P_Z = C_p \theta (p_1/p_0) \bar{\kappa} + \Phi_1 \quad \text{and} \quad (A4)
\]

\[
P_G = C_p \theta (p_{11}/p_0) \bar{\kappa} + \Phi_{11}, \tag{A5}
\]

where

\[
\frac{\partial \Phi_1}{\partial \rho_1} = -R \theta p_1^{\kappa - 1} p_0^{-\kappa} \quad \text{and} \quad (A6)
\]

\[
\frac{\partial \Phi_{11}}{\partial \rho_{11}} = -R \theta p_1^{\kappa - 1} p_0^{-\kappa}. \tag{A7}
\]

Here \( p_1 \) and \( p_{11} \), which are zonal and global means over isentropic surfaces, are used as the vertical coordinates for convenience. According to similar derivations, we have tendency equations of zonal and global ground-state potential energies as follows:

\[
\frac{\partial P_Z}{\partial t} + \frac{\partial}{\partial y} (P_Z + p_1/p_0) \bar{w}^\| + \frac{\partial}{\partial \rho_0} \rho_0 (P_Z + p_1/p_0) \bar{w}^\|_y = \Pi(p_1) \left[ \frac{Q}{\Pi(p)} \right]^\| - \frac{\partial p_1}{\partial \rho_0} \rho_0^\| \frac{\partial \Phi_1}{\partial t} \right]_{z_1} + \bar{w}^\| \left( \frac{\partial \Phi_1}{\partial y} \right)_{z_1} \tag{A8}
\]

and

\[
\frac{\partial P_G}{\partial t} = \Pi(p_{11}) \left[ \frac{Q}{\Pi(p)} \right] \frac{\partial p}{\partial \rho_{11}} - \frac{\partial p_{11}}{\partial \rho_0} \rho_{11} \frac{\partial \Phi_{11}}{\partial t} \right]_{z_{11}}. \tag{A9}
\]

Subtracting Eq. (A9) from Eq. (A8), the tendency equation of zonal mean available potential energy \( A_Z (= P_Z - P_G) \) becomes

\[
\frac{\partial A_Z}{\partial t} + \frac{\partial}{\partial y} (A_Z + p_1/p_0) \bar{w}^\| + \frac{\partial}{\partial \rho_0} \rho_0 (A_Z + p_1/p_0) \bar{w}^\|_y = Q_Z - C^A(A_Z, K_Z). \tag{A10}
\]

Note that \( A_Z \) is diabatically generated by meridional differential heating.

\[
Q_Z = \Pi(p_1) \left[ \frac{Q}{\Pi(p)} \right]^\| - \Pi(p_{11}) \left[ \frac{Q}{\Pi(p)} \right] \frac{\partial p}{\partial \rho_{11}}. \tag{A11}
\]

The dynamic energy sink of local \( A_Z \) has three terms:

\[
C^A(A_Z, K_Z) = -\bar{w}^\| \left( \frac{\partial \Phi_1}{\partial y} \right)_{z_1} + \frac{1}{\rho_0} \frac{\partial p_1}{\partial \rho_0} \left( \frac{\partial \Phi_1}{\partial t} \right)_{z_1} - \frac{1}{\rho_0} \frac{\partial p_{11}}{\partial \rho_0} \left( \frac{\partial \Phi_{11}}{\partial t} \right)_{z_{11}}. \tag{A12}
\]

The first term conducts energy conversion from the zonal mean potential energy to the zonal mean kinetic energy through mean-meridional circulations. The second and third terms are the vertical redistribution of the available potential energy due to work.

Subtracting Eq. (A8) from Eq. (A3) leads to the tendency of the eddy available potential energy \( A_E (= P - P_Z) \) as follows:

\[
\frac{\partial A_E}{\partial t} + \frac{\partial}{\partial y} (A_E + p/p_0 - p_1/p_0) \bar{w}^\| + \frac{\partial}{\partial \rho_0} \rho_0 (A_E + p/p_0 - p_1/p_0) \bar{w}^\|_y = \Pi(p_1) \left[ \frac{Q}{\Pi(p)} \right]^\| - \frac{\partial p_1}{\partial \rho_0} \rho_0^\| \frac{\partial \Phi_1}{\partial t} \right]_{z_1} + \bar{w}^\| \left( \frac{\partial \Phi_1}{\partial y} \right)_{z_1} + \bar{w}^\| \left( \frac{\partial \Phi_1}{\partial y} \right)_{z_1} + (\mathbf{v} \cdot \nabla_p \Phi_1)^\|. \tag{A13}
\]
The zonal mean kinetic energy for unit mass is defined as

$$K_Z = \frac{1}{2} (\bar{u}^2 + \bar{v}^2).$$ \hspace{1cm} (A14)

Because the derivation of the local tendencies of kinetic energy is almost similar to the previous work (Iwasaki 2001), only the result is shown below:

$$\frac{\partial K_Z}{\partial t} + \frac{\partial}{\partial y} K_Z \bar{v}^p + \frac{1}{\rho_0} \frac{\partial}{\partial z_i} \rho_0 K_Z \bar{w}^p = C(A_Z, K_Z) - C(K_Z, W) - \delta_{E}. \hspace{1cm} (A15)$$

The energy conversion from $A_Z$ to $K_Z$ is conducted by the mean-meridional flow along the pressure gradient force as

$$C(A_Z, K_Z) = -\bar{v}^p \frac{\partial \Phi_0}{\partial y} \bigg|_{z_i}. \hspace{1cm} (A16)$$

The vertical integration of Eq. (A16) is exactly equal to that of the dynamic loss of $A_Z$ in Eq. (A12), although their vertical distributions are different. Energy conversion from $K_Z$ to $W$ is conducted by the deceleration of westerlies through wave–mean flow interactions as

$$C(K_Z, W) = -\bar{w}^p \frac{\bar{\mathbf{F}} \cdot \mathbf{F}}{\rho_0} + \varepsilon, \hspace{1cm} (A17)$$

where the E-P flux and $\varepsilon$ are written as follows:

$$\mathbf{F} = \rho_0 \left[ -(u'v')^p - (u'w')^p + \frac{\bar{p}}{g} \frac{\partial \Phi}{\partial x} \bigg|_{z_i} \right] \hspace{1cm} \text{and} \hspace{1cm} (A18)$$

$$\varepsilon = -\bar{v}^p \left[ \frac{\partial \Phi}{\partial y} \bigg|_{z_i} - \frac{\partial \Phi}{\partial y} \bigg|_p \right] + \bar{v}^p \left[ \frac{\partial}{\partial y} (v')^q + \frac{1}{\rho_0} \frac{\partial}{\partial z_i} \rho_0 (w')^p \right]. \hspace{1cm} (A19)$$

Note that $\varepsilon$ is the work due to the eddy forcing in the meridional direction; it is negligibly small as compared with that in the zonal direction.

The eddy kinetic energy of unit mass is defined as

$$K_E = \frac{1}{2} [(u')^2 + (v')^2]. \hspace{1cm} (A20)$$

The tendency of the mass-weighted isentropic zonal mean of the eddy kinetic energy is given by

$$\frac{\partial K_E}{\partial t} + \frac{\partial}{\partial y} K_E \bar{v}^p + \frac{1}{\rho_0} \frac{\partial}{\partial z_i} \rho_0 K_E \bar{w}^p = -(u'u')^p \frac{\partial u'}{\partial y} - (u'w')^p \frac{\partial w'}{\partial z_i} - (v')^q \frac{\partial v'}{\partial y} - (v')^q \frac{\partial w'}{\partial z_i} \hspace{1cm} (A21)$$

where $\delta_E$ is the frictional loss of the eddy kinetic energy. Uno and Iwasaki (2006) defined the wave energy as the eddy available potential energy and the eddy kinetic energy. Its tendency is given by

$$\frac{\partial W^p}{\partial t} + \frac{\partial}{\partial y} (W + p/\rho_0 - p/\rho_1) \bar{v}^p + \frac{1}{\rho_0} \frac{\partial}{\partial z_i} \rho_0 (W + p/\rho_0 - p/\rho_1) \bar{w}^p = C(W, K_Z, W) + Q_{E} - \delta_{E}. \hspace{1cm} (A22)$$

The diabatic wave generation becomes

$$Q_{E} = \bar{Q}^p - \Pi(p) \frac{\overline{Q^p}}{\Pi(p)} \hspace{1cm} (A23)$$

The dynamic wave energy generation rate is written as follows:

$$C^W(K_Z, W) = \frac{\mathbf{F} \cdot \bar{W}^p}{\rho_0} - \frac{1}{\rho_0} \frac{\partial}{\partial z_i} \left[ \rho_0 \frac{\partial \Phi}{\partial t} \bigg|_{z_i} + \bar{u}^p \frac{\partial \Phi}{\partial x} \bigg|_{z_i} \right] + \frac{1}{\rho_0 g} \frac{\partial}{\partial z_i} \rho_1 \frac{\partial \Phi}{\partial t} + \varepsilon', \hspace{1cm} (A24)$$

where

$$\varepsilon' = -\bar{v}^p \left[ \frac{\partial \Phi}{\partial y} \bigg|_{z_i} - \frac{\partial \Phi}{\partial y} \bigg|_p \right] - (v')^q \frac{\partial v'}{\partial y} - (v')^q \frac{\partial w'}{\partial z_i}. \hspace{1cm} (A25)$$

Note that the last two terms are rewritten as

$$\varepsilon' - \varepsilon = -\frac{\partial}{\partial y} \left( (v')^q \bar{v}^p - \frac{1}{\rho_0} \frac{\partial}{\partial z_i} \rho_0 (v'w')^p \bar{v}^p \right). \hspace{1cm} (A26)$$

Its global integral vanishes and we obtain the exact conservation relationship

$$\langle C^W(K_Z, W) \rangle = \langle C(K_Z, W) \rangle. \hspace{1cm} (A27)$$
Another important point is that the distribution of
C^W(K_Z, W) is invariant for a Galilean transformation for
any constant zonal velocity but the distribution of C(K_Z, W)
is not invariant, as discussed in section 5.

APPENDIX B

Local Wave Energy and Its Quasigeostrophic
Expression

For the global energetic, an important difference of
MIM from the transformed Eulerian mean (TEM) lies
in the treatment of the lower boundary. Otherwise, the
basic characteristics are similar to the TEM, particularly
in isentropic zonal mean pressure coordinates. Except
near the lower boundary, local wave energy can be sim-
plified in the isobaric coordinates as follows. The eddy
available potential energy, which is defined as the dif-
ference between Eqs. (A1) and (A4) is approximated as

\[ \Delta E (p) \approx \frac{C_p \rho_0^2}{(\kappa + 1)} \left( p^{\kappa+1} - p_i^{\kappa+1} \right) \frac{\partial \theta}{\partial p_i} \]

\[ \approx \frac{1}{2 \rho_0 \theta} \left( p - p_i \right)^{\kappa} \frac{\partial \theta}{\partial p_i}. \]  

(B1)

With the aid of \( p(x, \theta) - p_i \approx -[\theta(x, p = p_i) - \theta(p_i)] \partial \theta / \partial p_i \)
and \( N^2 = (g / \theta)(\partial \theta / \partial z) = -(\rho_0 g^2 / \theta)(\partial \theta / \partial p_i) \), it is approxi-
mated by isobaric zonal means as

\[ \Delta E \approx \frac{1}{2 N^2} \left( \frac{\theta'}{\theta} \right)^{\kappa} \]  

where overbars and eddies indicate isobaric zonal means
and eddies. Thus, the wave energy can be approximated by

\[ W^* = \Delta E + K^*_E = \frac{1}{2 N^2} \left( \frac{\theta'}{\theta} \right)^{\kappa} + \frac{1}{2} \left( \frac{v}{v} \right)^{\kappa}. \]  

(B3)

Under the quasigeostrophic approximations, we have

\[ W^* \approx \frac{1}{2 N^2} \left( \frac{\partial \Phi}{\partial p} \right)^{\kappa} + \frac{1}{2} \left( \frac{\phi}{\phi} \right)^{\kappa}. \]  

(B4)

APPENDIX C

Energy of Eady Waves

The wave energy of Eady waves is examined, assum-
ing that the solution is uniform in the meridional direction
for simplicity. The perturbation geopotential height is given by

\[ \Phi' = \text{Re} \left\{ B \left[ -\frac{1}{2} + \frac{c_i}{A} \right] \mu \cosh \mu z + \sinh \mu z \right\} \times e^{-i \omega t + ikx \varphi}. \]  

(C1)

The vertical coordinate, vertical scaling, vertical profile of
zonal flow, and steering flow are given by \( z = (p_s - p_i) / \rho_i \mu \), \( \mu = N \rho_0 k / f \rho_i g \), \( U(z) = \lambda z + U_B \), and \( U = U(0.5) \). Then, the wave energy becomes

\[ W^* = \frac{1}{4} \left( \frac{k^2 B^2 p_s}{f_0 g} \right) e^{2k z}. \]  

(C2)

The global integral of the wave energy is given by

\[ \langle W \rangle = \int_0^1 \frac{L_y \rho_s}{g} \frac{1}{\lambda L_y} \]  

(C3)

Considering Eqs. (C1) and (C3), the E-P flux is related to
the wave energy as

\[ F_z = \rho_0 f_0 \frac{\partial u^p}{\partial z} = \frac{c_i k}{2 \lambda f_0 g} \beta^2 \frac{\kappa}{\lambda} \approx \frac{2 c_i k}{\lambda} \langle W \rangle. \]  

(C4)

Thus, the wave energy equation without the dissipation
term becomes

\[ \frac{\partial}{\partial t} \langle W \rangle = \langle C(K_Z, W) \rangle \approx \langle U_T - U_B \rangle F_z, L_y \]

\[ \approx \langle U_T - U_B \rangle \frac{2 c_i k}{\lambda} \langle W \rangle = 2 c_i k \langle W \rangle. \]  

(C5)

APPENDIX D

List of Symbols

\begin{align*}
\begin{array}{ll}
u, \nu & \text{Zonal and meridional velocities} \\
T & \text{Temperature} \\
\theta & \text{Potential temperature} \\
p & \text{Pressure} \\
\Phi & \text{Geopotential height} \\
F & \text{Eliassen–Palm flux} \\
t & \text{Time} \\
x, y & \text{East–west and north–south distances} \\
\overline{A}, \overline{A} & \text{Zonal and global means on isentropic surfaces} \\
\overline{A^*} & \text{Mass-weighted isentropic zonal mean} \\
\end{array}
\end{align*}
\( p_1, p_{1|} \) Isentropic zonal and global mean pressures \((p_1 = \pi, p_{1|} = \overline{\pi})\)

\( z_1 \) Logarithmic pressure coordinates \([= -H \log(p_1/p_0)]\)

\( w_1 \) Vertical velocity \((=dz_1/dt)\)

\( \rho \) Air density

\( \rho_0 \) Reference air density \([=p_1/(RT)]\)

\( \Phi_1, \Phi_{1|} \) Zonal and global ground-state geopotential heights \((\Phi_1 = \int RT_1 \, dp_1, \Phi_{1|} = \int RT_{1|} \, dp_{1|})\)

\( A_Z, A_E \) Zonal mean and eddy available potential energies

\( K_Z, K_E \) Zonal mean and eddy kinetic energies

\( W \) Wave energy

\( C(A, B) \) Energy conversion rate from \( A \) to \( B \)

\( Q_Z, Q_E \) Generation rates of zonal mean and eddy available potential energies

\( \delta Z, \delta E \) Dissipation rates of zonal mean and eddy kinetic energies

\( e, e' \) Energy conversion \( C(K_Z, W), C^W(K_Z, W) \) associated with \( \bar{v^2} \)

\( \alpha \) Growth rate of wave

\( f \) Coriolis parameter

\( g \) Gravitational constant

\( R \) Gas constant

\( C_v, C_p \) Heat capacities at constant volume and at constant pressure

\( \Pi \) Exner function \([\Pi(p) = C_p p^\kappa \rho_0^\kappa]\)

\( \rho_0 \) Reference pressure

\( \kappa = R/C_p \)

\( N \) Brunt–Väisälä frequency

\( U, U_T, U_B \) Zonal mean flow at each level, top level, and bottom level in Eady problem

\( \lambda \) Vertical shear of the Eady problem \((=U_T - U_B)\)

\( c_i \) Imaginary phase speed of the Eady wave

\( k \) Zonal wave number of the Eady wave

\( \mu \) Vertical exponent of the Eady wave

\( \overline{\Lambda} \) Isobaric zonal mean

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