The Spectral Ice Habit Prediction System (SHIPS). Part IV: Box Model Simulations of the Habit-Dependent Aggregation Process

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ABSTRACT

The purpose of this paper is to assess the prediction of particle properties of aggregates and particle size distributions with the Spectral Ice Habit Prediction System (SHIPS) and to investigate the effects of crystal habits on aggregation process. Aggregation processes of ice particles are critical to the understanding of precipitation and the radiative signatures of cloud systems. Conventional approaches taken in cloud-resolving models (CRMs) are not ideal to study the effects of crystal habits on aggregation processes because the properties of aggregates have to be assumed beforehand. As described in Part III, SHIPS solves the stochastic collection equation along with particle property variables that contain information about crystal habits and maximum dimensions of aggregates. This approach makes it possible to simulate properties of aggregates explicitly and continuously in CRMs according to the crystal habits.

The aggregation simulations were implemented in a simple model setup, assuming seven crystal habits and several initial particle size distributions (PSDs). The predicted PSDs showed good agreement with observations after rescaling except for the large-size end. The ice particle properties predicted by the model, such as the mass–dimensional \( m-D \) relationship and the relationship between diameter of aggregates and number of component crystals in an aggregate, were found to be quantitatively similar to those observed. Furthermore, these predictions were dependent on the initial PSDs and habits. A simple model for the growth of a particle’s maximum dimension was able to simulate the typically observed fractal dimension of aggregates when an observed value of the separation ratio of two particles was used. A detailed analysis of the collection kernel indicates that the \( m-D \) relationship unique to each crystal habit has a large impact on the growth rate of aggregates through the cross-sectional area or terminal velocity difference, depending on the initial equivalent particle distribution. A significant decrease in terminal velocity differences was found in the inertial flow regime for all the habits but the constant-density sphere. It led to formation of a local maximum in the collection kernel and, in turn, formed an identifiable mode in the PSDs. Remaining issues that must be addressed in order to improve the aggregation simulation with the quasi-stochastic model are discussed.

1. Introduction

Aggregation processes of ice particles in the atmosphere are important for understanding the precipitation process and the radiation of cloud systems. The aggregation process is one of the most efficient pathways for ice particles to grow into the precipitation-size particles in stratiform clouds (e.g., Field and Heymsfield 2003). Since the stratiform clouds typically cover larger areas than the convective portion of clouds, not only in middle latitudes but also in the tropics (Houze 1997), the aggregation process is critical to estimating the radiative forcing and emission into space by clouds. Physical understanding of the aggregation process has remained elusive because of the complicated hydrodynamics exhibited by ice crystals and aggregates of a variety of shapes, as well as a large dependence on the stickiness at the collision event (e.g., Pruppacher and Klett 1997, 607–610), itself critically dependent on the habit construction of the colliding hydrometeors. In cloud-resolving models (CRMs), particle properties of crystals and aggregates are typically assigned by categories, given the computational cost and the uncertainties associated with a lack of physical understanding. The variability of aggregation processes resulting from particle history or habits

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is directly related to the number of assumed categories. Thus, this approach is not ideal to study the variability.

The simulation of habits and sizes is particularly important in association with the aggregation process in order to obtain a realistic growth rate of mass and characteristic lengths. For the quasi-stochastic model (QSM) the collection rate of aggregation $K_{i,j}$ for a pair of particles $i$ and $j$ can be defined as

$$K_{i,j} = E_{\text{collection}}(i,j) \Delta V_{i,j}^V A_{i,j},$$

where $E_{\text{collection}}(i,j)$ is the collection efficiency (product of the collision and stickiness efficiency), $A_{i,j}$ is the cross-sectional area, and $\Delta V_{i,j}^V$ is the terminal velocity difference of the two colliding particles (e.g., Hashino and Tripoli 2011; i.e., Part III of this paper, hereafter HT11). The index $i$ and $j$ are omitted in the following. As discussed in HT11, $E_{\text{collection}}$ may depend on the habits as well as on temperature and humidity. The values of $A$ and $\Delta V$ are expected to be affected by the size and geometry of ice particles. The fractal dimension or power coefficient $b_1$ of the mass–dimensional $(m-D)$ relationship $m = a_1D^{b_1}$ of aggregates is known to be approximately 2 from observations (e.g., Kajikawa 1989, hereafter K89; Locatelli and Hobbs 1974, hereafter LH74; Mitchell 1996, hereafter, M96) and theories (Westbrook et al. 2004, hereafter W04). Westbrook et al. (2006) pointed out that the coefficient $a_1$ depends on the size of pristine crystals and habits. However, none of the QSMs in conventional microphysical schemes has been shown to simulate such dependence. It is critical for CRMs to represent habits and sizes of pristine crystals properly to simulate the aggregation process correctly.

The particle size distributions (PSDs) that result from the aggregation process are known to be approximately 2 from observations (e.g., W04; Field et al. 2007). It is crucial for cloud microphysical schemes in CRMs to be able to represent such a universal signature in aggregate-dominant clouds appropriately if the precipitation and radiative signatures from the clouds are to be improved. Recently, the parameterization of the aggregation process, based on the idea of a universal particle size distribution, has been developed and tested in a CRM (Thompson et al. 2008). The concept of the universal particle size distribution can also be used to validate aggregation simulations, as is done in this paper.

The Spectral Ice Habit Prediction System (SHIPS) has been developed to preserve the growth history of ice particles and evolve the particle properties, based on the history and local atmospheric conditions in CRMs (Hashino and Tripoli 2007, 2008; i.e., Parts I and II of this paper, hereafter HT07 and HT08, respectively). Ice crystal habits and particle types are diagnosed from particle property variables (PPVs) that are integrated according to growth history and local cloud microphysical processes and advected in the Eulerian dynamics framework. Categorization of ice particles and somewhat arbitrary physical links between them are not necessary for prediction purposes, and possible errors associated with them can be avoided. HT11 describes an enhanced QSM that solves the stochastic collection equation along with PPVs. The model calculates $A$ and $\Delta V$ based on the crystal habits and sizes explicitly. To predict the $m-D$ relationships of aggregates, SHIPS introduces another PPV related to the maximum dimension of aggregates. A simple parameterization of the growth of the maximum dimension and diagnoses of aspect ratio and porosity of the aggregate were proposed.

The purpose of this study is to validate the aggregation simulated by the enhanced QSM and to investigate the possible impacts of crystal habits and model parameters on the predicted PSD and ice particle properties. The aggregation process will be tested in a simple setup in which all particles stay in a unit volume and are well mixed (a box simulation), as done in aggregation simulations with pure stochastic (aggregation) models (PSMs) (W04; Maruyama and Fujiyoshi 2005, hereafter MF05). The box simulation setup is described in section 2. There are very few in situ observations or theoretical calculations available for the evolution of PSDs by collection processes with a specific ice crystal habit. Therefore, in section 3 the simulations are validated by means of a comparison with empirical relationships of ice particle properties, as well as a theoretical study with a PSM. Also, the formation of modes and the growth rate of the mean maximum dimension are discussed, with an analysis of collection kernels. The conclusions and directions for future research are given in section 4.

2. Experiment design

HT11 describes in detail a QSM with prediction of ice particle properties in the framework of SHIPS. The key prognostic variable for collision–coalescence processes is circumscribing sphere volume, which is used to obtain the particle sphere density based on a simple parameterization of the growth of maximum dimension and a circumscribing conceptual shape for each type of ice particles (ice particle model). The aspect ratio and porosity of aggregates, rimed aggregates, and graupel were parameterized based on predicted total mass, aggregation mass, and the ice crystal mass component. The growth model of maximum dimension for the aggregation process was introduced, based on a simple geometrical consideration of two colliding particles with two parameters: an angle between the two particles $\theta_0$ and a measure of
distance between the two (separation ratio) S. PPVs for ice crystals are averaged during the aggregation process, so that the PPVs indicate the average habit of the components of aggregates within a mass bin.

Box model simulations for the aggregation process are set up to validate a habit-dependent aggregation process and study the effects of habits. MF05 simulated PSDs and particle properties of aggregates with a similar setup. These included a prediction of the fractal feature of the aggregate shape made possible by implementing a PSM of aggregation with a Monte Carlo simulation. This study uses MF05’s results to validate the evolution of PSD simulated by SHIPS. Following MF05, the mass distributions of ice particles [or particle mass distributions (PMDs)] are initialized with an exponential distribution of mass. The exponential distribution of mass is derived from the observational study by Fujiiyoshi and Wakahama (1985) on the size distributions of snow particles comprising an aggregate:

\[ n(m) = \frac{N}{\bar{m}} \exp\left(\frac{-m}{\bar{m}}\right), \]  

where \( m \) is the mass of an ice crystal, \( N \) is the total number concentration, \( \bar{m} \) is the mean mass of the distribution, and \( n(m) dm \) gives the number concentration between \( m \) and \( dm \). The total mass content \( M \) is related to \( N \) and \( \bar{m} \) by \( M = N \bar{m} \), and the radius of the mean mass \( \tau \) is related to \( \bar{m} \) by \( \bar{m} = (4\pi/3)\rho_s \tau^3 \). In this study, \( M \) is set to \( 10^{-7} \) g cm\(^{-3} \), and the bulk sphere density \( \rho_s \) is set to 0.3 g cm\(^{-3} \), for all of the initialization. Three experiments were designed by changing \( \tau \) and equivalent distribution among different habits, as shown in Table 1. Here, \( \tau \) is set to either 2 \( \times 10^{-2} \) or 5 \( \times 10^{-2} \) cm. Experiments X2M04 and X5M04 have the same initial PMDs among the habits, whereas in X2S04 all habits have the PSD that the constant-density sphere has, based on the above PMD. Note that \( M \) for X2S04 varies among the habits because the mass of a crystal differs for a given diameter. The collection efficiency \( E_{\text{collection}} \) is assumed to be 0.4 in this study. It is expected that \( E_{\text{collection}} \) affects the evolution of PSDs as much as assumed habits. Since we focus on the habits, this sensitivity is beyond the scope of this paper and is not further discussed.

This study considers six nonspherical habits—plates, dendrites, columns, columnar polycrystals, planar polycrystals, and irregular polycrystals—with the axis ratios and aspect ratios shown in Table 2. Hexagonal monocrystals are initialized with the hexagonal crystal model defined in Fig. 3a of HT07. However, Eq. (10) of HT07 was mathematically incorrect for the assumed geometry of a hexagonal crystal. The correct mass–geometrical relationship is given by

\[ m = 3\sqrt{3}\phi(1 - \psi)a^3\rho_i^h, \]  

where \( m \) is the mass of a hexagonal crystal; \( a, c, \) and \( d \) are \( a \) and \( c \) axis lengths and dendritic arm length; \( \phi = c/a \) and \( \psi = d/a \) are the axis ratios; and \( \rho_i^h \) is the bulk density of the hexagonal crystal model (\( \rho_i^h \) is set to the bulk density of ice \( \rho_i = 0.91668 \) g cm\(^{-3} \)). Planar and irregular polycrystals are initialized with an \( m-D \) relationship for S1 and S3 as reported by Mitchell et al. (1990), respectively; whereas the columnar polycrystals are initialized with the bullet rosettes model of Chiruta and Wang (2003), which assumes four bullets. A mass bin whose representative hydrometeor\(^1\) is small (\( a \) or \( c \) axis lengths are less than 1 \( \mu m \)) is removed. In addition to the six habits, a sphere-shaped particle of the fixed bulk sphere density\(^2\) \( \rho_s = 0.3 \) g cm\(^{-3} \) is simulated as a reference. As shown in Table 2, \( \rho_s \) of the dendrite is the lowest, and one of the columnar polycrystals is the highest among the nonsphere habits; also, \( \rho_s \) of the planar and irregular polycrystal varies over the maximum dimension in a similar range. For the sphere case, \( \rho_s \) of the aggregate is kept constant.

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**Table 1.** Experiment setup. The parameter \( \tau \) is the radius of mean mass at the initial time, and \( E_{\text{collection}} \) is the constant collection efficiency assumed. PMD in the column of equivalent distribution (Equiv. dis.) indicates that the particle mass distribution is the same among different habits, while PSD indicates that the particle size distribution is the same; \( M \) is the mass content and \( N \) is the number concentration.

<table>
<thead>
<tr>
<th>( \tau ) (cm)</th>
<th>( E_{\text{collection}} )</th>
<th>Equiv. dis.</th>
<th>( M ) (g m(^{-3} ))</th>
<th>( N ) (L(^{-1} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>X2M04</td>
<td>2 ( \times 10^{-2} )</td>
<td>0.4</td>
<td>PMD</td>
<td>0.1</td>
</tr>
<tr>
<td>X2S04</td>
<td>2 ( \times 10^{-2} )</td>
<td>0.4</td>
<td>PSD</td>
<td>0.0015-0.1</td>
</tr>
<tr>
<td>X5M04</td>
<td>5 ( \times 10^{-2} )</td>
<td>0.4</td>
<td>PMD</td>
<td>0.1</td>
</tr>
</tbody>
</table>

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**Table 2.** Parameters for initialization of pristine ice crystals in box simulations. Axis ratios of ice crystals are defined as \( \phi = c/a \) and \( \psi = d/a \); \( \alpha \) is the aspect ratio of a cylinder fitted to the ice crystal and \( \rho_s \) is the bulk sphere density.

<table>
<thead>
<tr>
<th>Habit</th>
<th>( \phi )</th>
<th>( \psi )</th>
<th>( \alpha )</th>
<th>( \rho_s ) (g cm(^{-3} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plates</td>
<td>0.04</td>
<td>0.0</td>
<td>4.54 ( \times 10^{-2} )</td>
<td></td>
</tr>
<tr>
<td>Dendrites</td>
<td>0.04</td>
<td>0.9</td>
<td>4.54 ( \times 10^{-3} )</td>
<td></td>
</tr>
<tr>
<td>Columns</td>
<td>10.0</td>
<td>0.0</td>
<td>1.12 ( \times 10^{-2} )</td>
<td></td>
</tr>
<tr>
<td>Columnar polycrystals</td>
<td>10.0</td>
<td>0.0</td>
<td>1.34 ( \times 10^{-1} )</td>
<td></td>
</tr>
<tr>
<td>Planar polycrystals</td>
<td>0.04</td>
<td>0.25</td>
<td>3.14 ( \times 10^{-1} )-1.37 ( \times 10^{-2} )</td>
<td></td>
</tr>
<tr>
<td>Irregular polycrystals</td>
<td>0.04</td>
<td>0.25</td>
<td>3.14 ( \times 10^{-1} )-9.78 ( \times 10^{-3} )</td>
<td></td>
</tr>
</tbody>
</table>

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1 The representative hydrometeor of a mass bin is defined as having the mean mass of the bin.

2 The bulk sphere density is defined with a sphere circumscribing the particle.
and the maximum dimension is not predicted. The pressure and ambient temperature are set to 1000 hPa and \(-5^\circ\text{C}\). The mass spectrum starts from \(10^{-9}\) g, with the number of mass bins set to 100. The ratio of the adjacent mass bin boundaries is set to \(\sqrt{2}\).

3. Results and discussion

a. Evolution of aggregates of dendrites

In this section the predicted particle distributions and particle properties of aggregates of dendrites are discussed. The aggregation simulation was performed from the initial distribution with \(r = 2 \times 10^{-2}\) cm (X2M04). After rescaling, the particle size distributions predicted with SHIPS agree well with observations. Figures 1a–c show the evolution of particle mass distribution, particle size distribution, and rescaled PSD over 60 min of simulation, respectively. Rescaling of PSD was done following Field et al. (2007) where the rescaled PSD \(F_{23}(x)\) is calculated with the second and third moments \((M_2\) and \(M_3\), respectively) of PSD: \(F_{23}(x) = n(D)M_3^2/M_2^3\) and \(x = DM_2/M_3\). The mode of the PMD shifts from \(\sim 10^{-2}\) mg to \(\sim 1\) mg with time, which is related to a local peak in the collection kernel, as discussed later. If the gamma distribution is denoted as \(n(D) = D^\mu \exp(-\lambda D)\), the simulated PSDs at early times \(<20\) min seem to indicate \(\mu < 0\), whereas \(\mu > 0\) at later times. This is partly because the high concentration of the initial PSD with the mode at approximately 1.4 mm is depleted with time to create large particles, and partly because a local maximum of the collection kernel exists at the corresponding mass. A new mode emerges near 20 mm, which also corresponds to the local peak in the collection kernel. Figure 1c indicates that the rescaled PSDs agree well with the observation of tropical and midlatitude rescaled PSDs by Field et al. (2007). A large variability exists at \(x > 2\), where the simulated PSDs, except for 60 min, are approximately confined by the two observed PSDs.

Aggregates of dendrites predicted with SHIPS have properties that agree well with the observations available. The predicted particle properties are shown in Fig. 2: \(m-D\) relationship (Fig. 2a); mass–terminal velocity \((m-V_t)\) relationship (Fig. 2b); dimension–density \((D-\rho_s)\) relationship (Fig. 2c); the aspect ratio of the ice particle \(\alpha\) (Fig. 2d); the \(a\)-axis length of the average component crystal within an ice particle \(a\) (Fig. 2e); the relationship between the diameter of aggregates \(d_a\); and the number of component crystals in an ice particle \(N_{cc}\) (Fig. 2f). Each figure includes initialization (thin black line) and 60-min simulation (thick black line). It turned out that the properties of ice particles change little over time for a given mass. SHIPS predicts unique properties, given a mass point; and the properties are subject to the initial size distribution and habit. Figure 2a indicates that the slope of the predicted \(m-D\) relationship in logarithm changes with mass. The change of the slope at \(m = 1.1 \times 10^{-2}\) mg \((D = 1.6\) mm) corresponds to the change in the diagnosed type, from pristine crystal to aggregate. Another change of the slope is found at \(m = 1.8 \times 10^{-1}\) mg \((D \sim 7\) mm). Changes of the slope in \(m-V_t\) and \(D-\rho_s\) relationships exist at these mass points. The \(m-D\) power...
laws \( (m = a_1 D^{b_1}) \) were fit to the samples in \( D > 1.6 \) mm (for all the simulated aggregates) and \( 1.6 < D < 6.8 \) mm (for the range of aggregates of dendrites reported by K89). The resulting coefficients in CGS units were \( (a_1, b_1) = (4.27 \times 10^{-4}, 2.21) \) and \( (3.50 \times 10^{-4}, 1.87) \), respectively. The empirical \( m-D \) relationship for aggregates of dendrites from K89 is \( (a_1, b_1) = (4.82 \times 10^{-4}, 1.97) \). Thus, both of the fitted sets of \( (a_1, b_1) \) agree well with K89 AgP1e. The predicted \( m \) for a given \( D \) is 18% smaller than that of K89 AgP1e, on average. The LH74 AgD indicates an \( m-D \) relationship for aggregates of unrimed radiating assemblages of dendrites, which has larger (smaller) \( a_1 \) (\( b_1 \)) than those predicted. Figure 2b shows that the predicted terminal velocity for the aggregates is well confined by the \( m-V_t \) relationships for the aggregates of dendrites by K89 AgP1e and LH74 AgD. The bulk sphere density \( \rho_s \) predicted by SHIPS closely parallels the density of aggregates of planar crystals reported by Kajikawa (1982, hereafter K82) in the range of \( 1.6 < D < 7 \) mm. This is somewhat expected...
since predicted \( m-D \) relationships agree with an observational study, and \( D \) and \( \rho_s \) are related through \( \rho_s = m/(\pi/6)D^3 \). The slope of the \( D-\rho_s \) curve in \( D > 7 \) mm is larger than the slope of the smaller size, which corresponds to the abovementioned larger slope of the \( m-D \) curve in the range.

The simulated aggregates and component crystals in the aggregates exhibit the geometrical relationships similar to an observation. As shown in Figs. 2d–f, the diagnosed aspect ratio quickly reaches the mature aspect ratio \((a = \sqrt{\frac{2}{3}})\) at \( D \sim 5 \) mm where the average \( a \)-axis length and number of component crystals is \( a \sim 770 \mu m \) and \( N_{cc} \sim 10 \). As described in HT11, SHIPS calculates the average geometry of ice crystals within an ice particle by weighting the information by number concentration during the collection process. The average \( a \)-axis length reaches an asymptote of approximately 780 \( \mu m \) because the dendrites at the mode of the initial PMD have an \( a \)-axis length of 800 \( \mu m \) and the collection rate depends strongly on the number concentration. Figure 2f indicates an observed relationship between \( N_{cc} \) and snowflake diameter \( d_a \) by Kajikawa et al. (2002) for component crystals whose average diameter \( d_c \) is 0.5 to 3.5 mm. The value of \( N_{cc} \) for the simulation was obtained by dividing the total mass content component \( m \) by the crystal mass content components \( m_p \). The component crystals in this simulation have \( d_c = 1.56 \) mm. The simulated \( d_a-N_{cc} \) curve lies between the observed curves for the aggregates with \( 0.5 < d_c < 1.5 \) mm and \( 1.5 < d_c < 2.5 \) mm. Therefore, the simulated aggregates of dendrites have realistic and consistent physical properties when compared to observations.

We note that the initial \( m-D \) and \( m-V_t \) relationships differ from those of dendrites observed by Heymsfield and Kajikawa (1987, hereafter HK87). Another simulation with the same initial PMD was implemented by initializing \( D \) based on the \( m-D \) relationship for dendrites obtained by K82. The resultant properties are virtually identical to those for the dendrite model in SHIPS.

b. Aggregation simulations from the same initial PMD

Different habits of ice crystals can have different terminal velocity and cross-sectional area because of different geometrical representations. In this section, differences in the evolution of PSDs and PMDs due to differences in sweeping volume—namely, differences in terminal velocity and cross sections [see Eq. (1)]—are discussed. The aggregation simulations are started from the same initial PMD as the previous section, with \( \tau = 2 \times 10^{-2} \) cm \((X2M04)\). Note that initial PSDs for those cases are different because each habit has different \( m-D \) relationships, but the initial PMDs are identical. This experiment design can be considered as a situation where the PMD or size distribution in terms of melt mass and its moments, such as number concentration \( N \) and mass content \( M \), are known or predicted, but habit information is unknown.

For all the crystal habits, the aggregates indicate the exponential growth of a characteristic length. The uncertainty associated with habits in the PMD, PSD, and growth rate of the mean maximum dimension by aggregation can be significant, especially if the habit turns out to be dendrites. The PMDs and PSDs at 60 min are shown in Figs. 3a and 3b, and the time series of the mean maximum dimension is shown in Fig. 3c. The dendrites show the fastest evolution of PMD and PSD, followed by columns, plates, planar polycrystals, irregular polycrystals, and columnar polycrystals. Westbrook et al. (2007) argue that the exponential growth of the average particle size with fall time is a signature of aggregation, based on a theory and observations. Such exponential growth can be seen in Fig. 3c, although the growth rate varies among the habits, and especially dendrites show a decrease in the growth rate after 30 min or so. The growth rate of the melt diameter of mean mass (mean diameter) was calculated as the ratio of the melt diameter of mean mass (mean diameter) at initial time and that at 60 min of simulation, which is shown as Rat\( _m \) \((Rat_D)\) in Table 3. If one assumes dendrites, the Rat\( _m \) \((Rat_D)\) is 5.6 (11) times larger than that of columnar polycrystals after 1 h of aggregation.

The general features of the collection rate (or collection kernel) \( K \), cross-sectional area \( A \), and velocity difference \( \Delta V_t \) are described in the following. Figure 4 shows \( K \) with solid lines and \( \Delta V_t \) with color filled contours in the upper left of the diagonal, and \( A \) with color filled contours in the lower right for all the seven habits at 60 min of aggregation simulation. The vertical (horizontal) axis indicates the mass of a collector (collected) particle for \( K \) and \( \Delta V_t \), and the axis is opposite for \( A \). Six straight dashed–dotted lines indicate the pair of particles with the mass ratio of 0.1%, 5%, and 75% as indicated in Fig. 4g. For all the habits \( A \) is almost constant when a collector particle with constant \( m_1 \) collects particles with much smaller mass \( m_2 \). Only when \( m_2 \) is comparable to \( m_1 \) in magnitude can a rapid increase in \( A \) be seen. On the other hand, \( \Delta V_t \) decreases to zero as \( m_2 \) increases toward constant \( m_1 \). This has to occur if the particles at \( m_1 = m_2 \) have the same geometrical features. These two terms with opposite effects determine \( K \). Wherever the magnitude of decrease in \( \Delta V_t \) is much larger than that of increase in \( A \), \( K \) decreases. To facilitate quantitative comparisons among the habits, the pairs of particles with a fixed mass ratio, as indicated by the dashed–dotted lines in the upper left, are considered in the following.
The mean diameter \( \mu_D \) and standard deviation \( \sigma_D \) of PSDs at the initial time are well correlated with \( \text{Rat}_m \) and \( \text{Rat}_D \) (Table 3). As discussed below, this is because the \( \mu_D \) and \( \sigma_D \) at the initial time are the result of simply using the different \( m-D \) relationships among the habits, since the aggregation process did not change the order of habits in \( K \), given \( m \). Values of \( K \), \( A \), and \( D_{Vt} \) for an ice particle of \( m \), collecting another particle with 75\% of its mass at the initial time, are shown in Fig. 5. It is clear that dendrites already have the largest \( K \) among the habits, due to their largest \( A \) for a given \( m \), which is the result of the \( m-D \) relationship. Note that the increase in \( A \) with \( m \) is larger than the increase in \( D_{Vt} \) by a factor of 10. Figure 6 shows the same variables as Fig. 5, but at 60-min of simulation, corresponding to those on the 75\% dashed–dotted line in Fig. 4. Again, \( K \) for the aggregates of dendrites is the greatest over most \( m \) among the habits because of their largest \( A \) values, but not because of the \( \Delta V_i \). This relative magnitude of increases in \( A \) and \( \Delta V_i \) with \( m \) suggests that the definition or estimation of \( A \) largely affects the growth rate of aggregates, as emphasized by MF05. As shown later, \( m-D \) relationships \( (m = a_iD^{b_i}) \) of the aggregates predicted with SHIPS tend to have a similar slope coefficient \( b_1 \sim 2 \), which tends to retain the order of habits in the \( A \), given \( m \) from the initial condition. As \( A = (\pi/4)a_1^{z_1}(m_1^{1/2} + m_2^{1/2})^2 \) for the two planar crystals, the \( a_1 \) of the crystals is expected to play a role in determining the growth rate of the aggregates.

It turned out that the magnitudes of \( K \) are similar among the fixed-density sphere and nonspherical habits, except for the dendrites; but \( A \) and \( D_{Vt} \) between the fixed-density sphere and nonspherical habits are qualitatively different. The differences in the habits may lead to differences in \( A \) and \( \Delta V_i \) by a factor of 10–100 at a given \( m \).
The difference in the resulting $K$, however, may be up to a factor of 10. Compared with aggregates of a sphere, the other habits compensate for smaller $\Delta V_t$ by having a larger $A$ for a given $m$. The ice particles of all the nonspherical habits were diagnosed to be aggregates at $m$ larger than about $10^{-2}$ mg. It can be seen that the slope of $A$ increases from about that mass for the hexagonal crystals. This is the consequence of allowing the

**Fig. 4.** Comparison of $K$, $A$, and $\Delta V_t$ of an ice particle among the seven habits. See the caption of Fig. 3 for the notation of the seven habits. Note that $K$ and $\Delta V_t$ are shown in the upper left of the diagonal, whereas $A$ is in the lower right. The vertical (horizontal) axis indicates the mass of a collector (collected) particle for $K$ and $\Delta V_t$, and the axis is opposite for $A$. These three are symmetric about the diagonal, $\Delta V_t$ is indicated by the filled contours every 0.5 $[\log_{10} (\text{cm s}^{-1})]$ and $K$ is indicated by the solid lines (thin gray: $-12$, $-11$, $-10$ $[\log_{10} (\text{m}^3 \text{s}^{-1})]$; thin black: $-9$, $-8$, $-7$; thick gray: $-6$, $-5$, $-4$; thick black: $-3$, $-2$, $-1$); $A$ is indicated by the filled contours every 0.5 $[\log_{10} (\text{cm}^2)]$. Six straight dashed–dotted lines indicate the pair of particles with mass ratio 0.1%, 5%, and 75% as indicated in (g).
density of ice particles to decrease with \( m \) for the non-spherical habits.

The behavior of \( \Delta V_t \) plays a significant role in forming a mode in the PMD and PSD. The increase in \( \Delta V_t \) gets smaller with \( m \) because the increase in \( V_t \) for the aggregates declines with \( m \), as shown in Fig. 2b. Physically, the flow regime around the aggregates shifts from the intermediate to the inertia regime with \( m \) (cf. M96).

Because of the convergence to an almost constant \( V_t \) (at \( m \sim 2 \) mg for aggregates of dendrites), a dip of \( \Delta V_t \) exists at \( m \sim 2-100 \) mg for all the habits. This led to formation of the peak in \( \Delta V_t \) at \( m \sim 1 \) mg for aggregates of dendrites. The peak is reflected in \( K \) because of the smooth transition of \( A \). Those peaks, due to the convergence of \( V_t \) over \( m \), then approximately correspond to each mode in the PMD or PSD with the combination...
of the given concentration of collecting and collected particles (cf. the collection kernel of dendrite aggregates in Fig. 6a and Fig. 1a or 1b with Fig. 2a). In case of the dendrites, there are three more peaks in $K$. Among them, the peak at $m \sim 10^{-2} \text{mg}$ creates a mode at $D \sim 1.6 \text{mm}$ in PSD, as well as PMD. As mentioned above, the six dashed–dotted lines in Fig. 4 correspond to the pairs of particles whose mass ratio is 0.1%, 5%, and 75%. As also seen in Fig. 4, the aggregates of dendrites have a dip of $K$ on the line of 75% at $m \sim 1 \text{mg}$. However, those on the line of 5% do not create a dip in $K$ because the magnitude of decrease in $\Delta V_t$ at $m \sim 1 \text{mg}$ is not large enough to counteract that of increase in $A$. This is likely to be true for the other habits except for the constant-density sphere. Therefore, qualitatively speaking, the aggregation between ice particles with similar masses in the inertia flow regime can create a mode in the PMD and PSD because the magnitude of decrease in $\Delta V_t$ with $m$ is much larger than that of the increase in $A$.

It is interesting to note that columnar crystals show the two modes at $D = 0.67$ and 2.3 mm in the PSD at 60 min ($m = 1.7 \times 10^{-3}$ and $1.6 \times 10^{-1} \text{mg}$, respectively) as shown in Fig. 3. The smaller mode is simply the remaining concentration from the initial condition. The larger mode is associated with the large increase in $\Delta V_t$ and $K$ found at $m \sim 10^{-1} \text{mg}$. It corresponds to the fast increase in $V_t$, partly due to the rapid decrease of aspect ratio from a prolate to an oblate spheroid. The transition from prolate to oblate spheroids of the aggregates occurs as the aspect ratio is parameterized to reach $\frac{1}{3}$ [Eq. (15) of HT11]. Another factor is the increase in the bulk sphere density during an aspect ratio larger than 5, as formulated in HT11. These parameterizations of aggregates of needles or columns have to be validated.

c. Aggregation simulations from the same initial PSD

In this section, the aggregation simulations initialized with the same PSD among the habits (X2S04) are discussed. The $N$ and the moments of PSD are the same among the different habits, while $M$ and PMD are different. For instance, this corresponds to a situation where the PSD is known or measured, while the habit of the ice particles is not known or not observed.

The aggregation growth rates of the mean mass and mean maximum dimension are larger for denser particles when the initial PSDs are the same. The PMDs at 120 min and the time series of melt diameter at the mean mass are shown in Figs. 7a and 7c. In this setup, the fixed-density sphere evolves the fastest, followed by columnar polycrystals, irregular polycrystals, planar polycrystals, plates, columns, and dendrites; this is totally different from X2M04. It is interesting to note that PSDs at 120 min, shown in Fig. 7b, indicate that the columnar polycrystals, plates, irregular polycrystals, and planar polycrystals have higher concentrations of large particles ($D > 1 \text{mm}$). The differences in the $m$-$D$ relationships are reflected in the mean mass $\mu_m$, and the standard deviation $\sigma_m$, at the initial time (Table 3), where the spheres (dendrites) have the largest (smallest) $\mu_m$ and $M$. The measures of $\text{Rat}_m$ and $\text{Rat}_D$ are again well correlated with the habit-dependent $\mu_m$. Note that the $\text{Rat}_m$ and $\text{Rat}_D$ at 120 min, shown in Figure 7.
Table 3, are smaller than those for X2M04, which indicates a less active aggregation process in X2S04.

This experiment shows that, in the case of the same PSD, the denser particles can evolve faster because the $D\nu_t$ is larger for a given pair of $D$ at the initial time. Figure 8 shows the collection kernel $K$, cross section $A$, and velocity difference $\Delta V_t$ of a particle with the maximum dimension $D$ collecting the particle with 75% of $D$ at the initial times. The $A$, given $D$, at the initial time is the same among the habits (except for columns) because of the same initial PSD. However, the $\Delta V_t$ varies among the habits, and the sphere has the largest $\Delta V_t$ for a $D$, which is similar to X2M04 for a pair of $m$ (Fig. 5). As a result, the $K$ of the sphere is the largest, and that of dendrites is the second smallest among the habits. The reason why the spheres have the largest $\Delta V_t$ can be attributed to their largest $m$, given $D$, due to the high bulk sphere density. For a given $m$, an order of habits qualitatively similar to X2M04 can be found: dendrites and aggregates of dendrites have the largest $K$, given $m$, due to the largest $A$. However, the difference in $K$ at the initial time, given $D$, was significant enough for spheres to produce the aggregates with large mass whose $K$ is much larger than any of the dendrites at a later simulation time.

d. Comparison of predicted particle properties among habits

Predicted properties show good agreement with observations, and dependence on the habit used for initialization. The $m-D$, $m-V_t$, and $D-p_e$ relationships predicted at 60 min from X2M04 are shown in Figs. 9, 10, and 11, respectively, for all six habits. The simulated $m-D$ relationship shows good agreement with the empirical formula (Fig. 9). The empirical $m-D$ relationships for the aggregates that are denoted with “Ag” start between $\sim 10^{-3}$ and $\sim 10^{-1}$ mg. The simulated $m-D$ relationships for hexagonal monocrystals and columnar polycrystals show a clear change in slope at the above range of $m$, and the slope ($b_1 \sim 2$) becomes close to those of the empirical formulas for aggregates, namely K89 AgP1e, LH74 AgPSBC, and M96 AgS3 (see the caption of Fig. 9 for the notation). The change in slope corresponds to the change in diagnosis from pristine crystals to aggregates. Such clear change in slope, however, does not exist in the empirical formula, except that, for plates, a change in slope from HK87 P1a to LH74 AgPSBC can be seen. The planar and irregular polycrystals also show good agreement with M96 AgS1 and AgS3, respectively. The $m-D$ relationships simulated for these habits show subtle changes in slope because these $m-D$ relationships were initialized with the empirical formula whose $b_1$ is already close to 2. Note that the mass of the aggregates of dendrites with 3-mm diameter is about 10 times lower for X2M04 than that of LH74 AgD. Sensitivities of the $a_1$ of $m-D$ power laws for aggregates on the initial distributions are discussed later.

The simulated $m-V_t$ relationships are in moderate agreement with the empirical formula (Fig. 10). The simulated $m-V_t$ for the aggregates of plates and columns agree with LH74 AgPSBC. As discussed in 3b, the rapid
increase of $V_t$ with $m$ can be seen in the aggregates of columns. The simulated $m-V_t$ for the aggregates of columnar polycrystals show good agreement with M96 AgS3, while those of planar and irregular polycrystals are almost bounded by M96 AgS1, LH74 AgS1, and M96 AgS3. It is quite surprising to see such good agreement in the simulated $D$-$r_s$ relationships (Fig. 11). All the empirical relationships shown are those for aggregates. As discussed in $m$-$D$ relationships (Fig. 9), the changes in slope in the $D$-$r_s$ relationships start at $m$ diagnosed as aggregates, and then curve is almost parallel to the empirical curves for a range of $D$. The aggregates of plates clearly have higher density than those of dendrites for a given $D$. Note that the aggregates of columns show an increase in $r_s$ due to the assumption for long columns and the parameterization of aspect ratio.

e. Comparison with a pure stochastic model

This section compares the aggregation simulations of SHIPS with those of a PSM. The aggregation simulations are started from the initial distribution with $r = 5 \times 10^{-2}$ cm (X5M04) in order to compare them with the control experiment of MF05. Note that MF05 uses ice spheres as components of aggregates in a pure stochastic model. MF05 considered two versions of the stochastic model, model A and B. In MF05’s models, the chance of a close approach between two particles is determined by the sweep-out volume, which is the product of the terminal velocity difference and the circle area whose radius is the sum of the radii of the two particles. Model A determines a collision event by considering the overlap of the two circular cross sections of the approaching particles.
particles within the sweep-out volume. In model B, however, the collision event always occurs within the sweep-out volume. MF05 considers model B as the extreme case to maximize the shape effect on the collision. This study compares results with model B. Note that MF05 set the coalescence efficiency to unity.

The aggregates of some nonspherical habits predicted with SHIPS grow comparable to model B when $E_{\text{collection}} = 0.4$ is assumed. The mass density distributions for X5M04 at 90 min of simulation are shown in Fig. 12 in terms of maximum radius $R_{\text{max}}$ (Fig. 12a), and melted radius $R_{\text{melted}}$ (Fig. 12b). Aggregates of plates, columns, planar, and irregular polycrystals have a mass growth similar to model B. The aggregates of dendrites show the fastest evolution with the larger mode at $R_{\text{max}} = 20$ cm or $R_{\text{melted}} = 1$ cm. Although the mode is unrealistically large at 90 min, the rescaled PSDs of the dendrites still agree well with observed rescaled PSDs (not shown). It is noted that the aggregation period of 90 min is unlikely to occur for the dendrites in a real atmosphere. A control run of model B from MF05 is also shown in Fig. 12, which has a mode at $R_{\text{max}} \sim 1$ cm ($R_{\text{melted}} \sim 0.1$ cm). It can be seen that the aggregates of the nonspherical habits have a mode at $R_{\text{melted}} \sim 0.1$ cm similar to that of MF05. As discussed in section 3b, the mode is associated with the decrease in $D_{Vt}$ that forms a local maximum in $K$ at $R_{\text{melted}} \sim 0.1$ cm. Thus, it can be speculated that model B of MF05 had a similar convergence of $V_t$ at the mass causing the formulation of the mode.

The main difference between simulations of SHIPS and MF05’s pure Lagrangian stochastic models is that the mass density distributions simulated for all of the six habits showed two distinct modes in mass density distribution at certain times (Fig. 12). In SHIPS the two modes are an indication of the transition from the initial

![Fig. 10. Comparison of simulated $m-V_t$ relationships among habits. The thin black solid line indicates the initial condition, and the thick black solid line the prediction at 60 min. The references from which the empirical formulas were taken are shown in the caption of Fig. 9.](image-url)
PSD to the aggregation-dominated, universal PSDs, except that the dendrites at 90 min are already in the aggregation dominant state. Model B of MF05 shows only one large mode clearly at this time, although a leftover mode of the initial PSD can be seen.

The distinct and sharp modes in SHIPS may be attributed to the implicit mass sorting assumption in SHIPS. This assumption allows SHIPS to diagnose each mass bin as a group of a single habit and type of ice particle for every time step (see more discussion in HT07). In other words, the predicted properties in SHIPS are average quantities of the particles in a range of particle mass. This is likely to cause the distinct peak in $K$ due to the convergence of $V_t$. In reality, there is variability in $V_t$ for a mass $m$ (e.g., Sasyo and Matsuo 1980), which can form the convergence of $V_t$ over a range of $m$ of the collector particles. MF05 explicitly simulated the variation of those variables with their Monte Carlo method, and thus the less pronounced mode may be attributed to the variation in $\Delta V_t$. Passarelli and Srivastava (1979) incorporated the fluctuation of terminal velocity and density for a mass in the stochastic collection equation and demonstrated that including the spectrum of terminal velocity significantly enhances the aggregation process. Sasyo and Matsuo (1985) also included an observed variation of fall velocity into a new kernel and showed that it reduced the time required to reach specified snowfall intensity but did not increase snowfall intensity. Certainly, incorporation of the variability in fall velocity or cross-sectional area of ice particles has to be investigated in SHIPS in the future.

f. Effect of the initial PMD on predicted properties

SHIPS predicts ice particle properties that depend on initial PSDs and the size of component crystals.
g. Sensitivity of growth parameters and diagnosis

As described above, SHIPS simulates growth of ice particle properties by aggregation explicitly. To study the sensitivity of the aggregation process to the assumptions, the following 10 cases were simulated for aggregates of dendrites with the initial condition X2M04 (the parameters are explained in detail in HT11). Ncc3 assumes the number of crystal components \( N_{cc} = 10^3 \) to define the mass where the aspect ratio of aggregates \( \alpha \) becomes constant; alp25 sets the aspect ratio of mature aggregate \( \alpha_{lmt} = 0.25 \), alp50 sets \( \alpha_{lmt} = 0.5 \), th0 uses the angle between two aggregating particles \( \theta_0 = 0 \) for all conditions, and th45 uses \( \theta_0 = 45^\circ \) for all conditions. Another set of simulations use different values of the separation ratio \( S \)—that is, the normalized distance between the centers of two aggregating particles \( (S = 0 \text{; the centers of two particles overlap}; S = 1 \text{; two particles are connected at the edges}); S04, S05, and S07 assume \( S = 0.4, 0.5, \text{and } 0.7 \) for all mass ranges, respectively. Let \( \rho_{lmt,\text{min}} \) denote the minimum of the \( \rho_s \) of two aggregating particles. SLD models \( S \) to decrease from 0.6 at \( \rho_{lmt,\text{min}} = 10^{-3} \text{ g cm}^{-3} \) to 0 at \( \rho_{lmt,\text{min}} = 10^{-2} \text{ g cm}^{-3} \) as a linear function of logarithm of \( \rho_{lmt,\text{min}} \). KV uses the empirical \( m-D \) relationship of K89 AgP1e to obtain \( D \) of aggregates for \( m \) in a shifted bin, instead of using the growth model for maximum dimension and averaging [Eqs. (22)–(28) of HT11]. Ctr denotes the control case.

Comparison between the cases shows that the predicted properties are not strongly sensitive to the parameters \( N_{cc}, \alpha_{lmt,\text{min}} \), and \( \theta_0 \) but are strongly sensitive to the \( S \). Predicted \( m-D, m-V_r \), and \( D-\rho_s \) relationships at 60 min are shown respectively in panels (a), (b), and (c) of Figs. 14 and 15. The \( \rho_s \) indicates large sensitivity in \( D > 2 \) mm, where the particles are diagnosed as aggregates. Ncc3 in Figs. 14a and 14c shows that using a higher mass threshold for the mature aggregates \( m_{lmt} \) led to smaller \( D \) for an \( m \) and larger \( \rho_s \) for a given \( D \) than Ctr. Comparison of alp25 and alp50 illustrates that a larger...
aspect ratio of mature aggregates leads to less dense and larger aggregates. According to $\theta_0$ and $\theta_{45}$, a larger angle between aggregating two particles $\theta_0$ resulted in more dense and smaller aggregates. Figure 15a indicates that $S$ affects the power coefficient or fractal dimension $b_1$ of $m$-$D$ relationships significantly. A smaller $S$ resulted in a larger power coefficient because the two aggregates have a shorter distance between them and, therefore, higher density. This also means faster terminal velocity for a smaller $S$. The $b_1$ for $S = 0.4, 0.5, 0.6$ and $0.7$ fits in $1.6 < D < 6.8$ mm ($D > 1.6$ mm) are $2.58, 2.18, 1.87$, and $1.64, (2.64, 2.38, 2.21, \text{and}\ 2.19)$, respectively. W04 shows that $b_1$ becomes $2$ for the purely inertial regime, where the terminal velocity becomes constant over mass. Therefore, $S = 0.6$ seems to be an optimal choice, in terms of the fractal dimension, and is also the value most frequently observed by aircraft observations (HT11). The $\rho_s (D)$ of SLD is larger (smaller) than that of Ctr for $\rho_s > 10^{-3}$ g cm$^{-3}$, as expected from the decreased $S$ for the large aggregates. It is important to note that the aggregates of Ctr have similar properties to those of KV, which supports the growth model for maximum dimension and averaging used in SHIPS.

The growth rate of the mean maximum dimension $\mu_D$ predicted in SHIPS is largely controlled by $S$ and $\alpha$, because these are directly or indirectly related to $A$. As shown in Figs. 14d and 15d, time series of $\mu_D$ were calculated from the PSDs. The larger aspect ratio for matured aggregates (alp50) leads to faster evolution than Ctr and a smaller aspect ratio (alp25). The $D$ of alp50 is
larger than that of alp25 for a given \( m \), as shown in Fig. 14a, which results in a larger \( D \). This can be explained by increased growth of \( D \) over a time step where larger \( V_t \) for larger \( \theta_0 \) enhanced the collection rate. The case with the small angle between two particles (\( \theta_0 \)) resulted in larger \( D \) per aggregation; therefore, the \( \mu_D \) of \( \theta_0 \) is increased more than \( \theta_{45} \). Overall, the sensitivity of \( \mu_D \) to \( \theta_0 \) is smaller than the other parameters. The growth of \( D \) per aggregation is larger with a larger \( S \). Therefore, a larger \( S \) leads to faster evolution of PSDs, due to a larger \( \mu_D \). At 60 min, the \( \mu_D \) of S07 is about 3 times larger than in S04.

There are only limited observations available for \( S \) and \( \alpha \). The above simulations of dendrites create centimeter-size aggregates of dendrites within an hour. Collision or hydrodynamic breakup of large low-density aggregates can prevent aggregates from growing unrealistically large, if implemented in the model. But, decreasing \( S \) with the size of aggregates also makes it possible to have denser aggregates, as shown by SLD. In fact, Kajikawa et al. (2002) reported that the center of a third component dendritic crystal tends to attach to the connecting line between centers of two other dendritic crystals. Kajikawa and Heymsfield (1989) also observed that the \( S \) for thick plates decreases with the ratio of masses or sizes of two particles. In the case of aggregates of needles, \( S \) and \( \theta_0 \) were kept to zero for aggregates of needles in SHIPS until their aspect ratio became less than 5, in order to achieve larger density at the early stages of aggregation. These transitions of \( S \) would be related to the three-dimensional growth of aggregates, and the geometrical growth has to be consistent with the change of \( \alpha \). This study assumed a constant \( \alpha \) for the matured aggregates, following the finding of Korolev and Isaac (2003) with the cloud particle imager (CPI). In reality, it is possible for \( \alpha \) to be different in \( D > 1 \) mm from \( \alpha \), measured in the range \( (D < 1 \) mm) to which CPI is limited. Therefore, more observations and theoretical studies on \( S \) and \( \alpha \) are necessary to improve the aggregation simulation.

4. Summary and conclusions

To perform a focused study on the aggregation process, aggregation simulations were implemented using a simplified model configuration, whereby all hydrometeors
are assumed to be well mixed in a box (box model simulation). Seven different ice crystal habits are considered as the component of aggregates: plates, dendrites, columns, columnar polycrystals, planar polycrystals, irregular polycrystals, and constant-density spheres. The PSDs were initialized such that the particle mass distributions (PMDs) were the same among the habits or such that the PSDs were the same. A constant aggregation efficiency of 0.4 was used for all the habits.

The predicted PSDs with the enhanced quasi-stochastic model were rescaled and shown to match well with observation although there is a discrepancy at the largest particle sizes. The predicted aggregate properties include $m$-$D$ relationships, mass-terminal velocity relationships, and mean size and number of crystal components in an aggregate. These were found to be quantitatively similar to those observed. Furthermore, the predicted properties were shown to depend on crystal habits and initial PMD and PSD. For instance, the coefficient $a_1$ of the $m$-$D$ relationship $m = a_1 D^{a_2}$ for the aggregates depends on the mean size of component crystals. This suggests that using a fixed $a_1$ of the $m$-$D$ relationship for an aggregate category in conventional bulk microphysical parameterization or bin models may introduce errors in calculation of microphysical tendencies.

Values of $K$ for all the habits were decomposed into $A$ and $\Delta V_t$ according to Eq. (1) and analyzed along with time series of mean maximum dimension and mean mass. It turned out that crystal habit for a given initial PMD or PSD has a significant impact on the growth rate of aggregates. The aggregates of nonspherical habits evolve faster than those of constant-density spheres if all the habits are initialized with the same PMD, but different PSD, according to the $m$-$D$ relationships. For instance, the growth rate of the aggregates of dendrites is the largest due to the largest $A$ even though the shape is not reflected in the aggregation efficiency. On the other hand, the aggregates of ice spheres with a constant density actually evolve fastest if these habits are initialized with the same PSD. Comparison of these experiments indicates that the initial conditions, specified by the different $m$-$D$ relationships of crystals, mostly determine the growth rate of mean maximum dimension and mean mass. The determining term is $A$ for the experiment with the same PMD and $\Delta V_t$ for one with the same PSD. Furthermore, it was shown that the aggregation between particles with similar masses in the inertial flow regime can form a major mode in the PMDs and PSDs. This is because for the pair of particles with a fixed mass ratio ($\sim 75\%$) the magnitude of decrease in $\Delta V_t$ with mass can
be much larger than that of increase in $A$. In turn, it can lead to the decrease in $K$, which forms a local maximum in $K$ at the smaller mass, and then form the mode.

The PSDs were compared with the pure-stochastic aggregation model of MF05. Both the models show a similar evolution of the distributions when collection efficiency of 0.4 is assumed. The aggregates of the non-spherical habits simulate a mode at the melted radius ($\sim 0.1$ cm) similar to that of MF05, which was associated with a local maximum in $K$, due to the decrease in $\Delta V_t$. The spread of PSDs was narrower for SHIPS than for MF05, which is possibly related to the simulated variability in $\Delta V_t$ and $A$.

Sensitivity tests of the aggregation simulations were implemented on the parameters introduced in this study. Comparison between the tests shows that the predicted properties are strongly sensitive to the separation ratio of the two aggregating particles $S$ used in the growth model of the maximum dimension. The optimal value of $S$ turned out to be a typically observed value, which supports formulation of the growth model. The growth rate of PMD and PSD turned out to be largely controlled by $S$ and aspect ratio of aggregates because these are directly or indirectly related to $A$.

This research raised key questions regarding to the fundamental understanding of the aggregation process and showed the possibility of simulating a habit-dependent aggregation process with the QSM in a CRM. SHIPS was able to competently reproduce relationships between ice particle properties, based on a simple growth model of maximum dimension. However, the simulated growth of PSD and the dependence on habits have to be validated against more appropriate theoretical models (PSMs) or observational studies. Certainly, the variability in the fall velocity difference or the cross-sectional area of ice particles that affect the growth rate of aggregates and formation of modes should be studied further. Also, it is not clear to what degree the equivalence of the PSMs and QSMs (cf. Pruppacher and Klett 1997, 628–630) holds for particles whose terminal velocities and dimensions vary at a mass. The parameterizations sensitive to the prediction of particle properties and evolution of PSD (i.e., those for separation ratio and aspect ratio of aggregates) should be investigated observationally to reduce the uncertainty inherent in the modeling approach taken in this paper.

This paper clearly shows that the aggregation process depends on the habit and size assumed at the initiation, which is typically missed by conventional CRMs that cannot explicitly evolve habits. The authors believe that the approach taken with SHIPS can improve the simulation of aggregation and, therefore, precipitation from cloud systems, dynamic feedback, latent heat release, and the radiative signatures associated with them. In addition, the increased understanding of the physics of aggregation gained by this detailed look at the process will help in the design of low-order representations of the aggregation process appropriate for simplified approaches, such as bulk schemes.

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