On the Warm Core of a Tropical Cyclone Formed near the Tropopause

TOMOKI OHNO AND MASAKI SATOH
Atmosphere and Ocean Research Institute, University of Tokyo, Kashiwa, Chiba, Japan

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ABSTRACT

On the basis of numerical results of a three-dimensional model diagnosed using balance dynamics, a mechanism by which the upper-level warm core of tropical cyclones (TCs) forms is proposed. The numerical results reveal that an upper-level warm core develops when TCs intensify just prior to reaching the mature stage. Potential temperature budget analysis reveals that for the tendency of potential temperature, the azimuthal-mean component of advection is dominant at the upper level of the eye at the mature stage. Sawyer–Eliassen diagnosis shows that tendencies due to forced flow by diabatic heating and diffusion of tangential wind are dominant in the eye and are negatively correlated to each other. The distributions of the diabatic heating in the simulated TC are not peculiar. Therefore, it is unlikely that the heating distribution itself is the primary cause of the flow from the lower stratosphere. The analyses of forced circulations of idealized vortices show that the upper-level subsidence is enhanced in the eye when the vortex is sufficiently tall to penetrate the statically stable stratosphere. This result is deduced because the stronger inertial stability extends the response to the heating of the lower stratosphere and causes upper-level adiabatic warming. Therefore, the upper-level warm core emerges if angular momentum is transported into the lower stratosphere due to processes such as convective bursts. The present analysis suggests that TCs can be even stronger than that expected by theories in which the TC vortex is confined in the troposphere.

1. Introduction

Tropical cyclones (TCs) are characterized by warm-core structures. The intensity and height of the warm core are related to surface pressure in the eye of the TC through hydrostatic and gradient wind balances; thus, it is important to determine the mechanism controlling the warm-core structure. However, there is a large amount of variation among profiles (Durden 2013), and the controlling factor for the vertical structure of the warm core is not fully understood (Stern and Nolan 2012). The center of a warm core can be located at either the middle or upper troposphere (e.g., Hawkins and Rubsam 1968; Halverson et al. 2006); however, the relative frequency is unclear. A warm core can have multiple maxima in the vertical direction (Hawkins and Imbembo 1976). Moreover, the dynamics of the warm core are still not fully understood (i.e., it is unknown whether the height of the warm-core maximum is related to the intensity of TCs). Although the warm-core structures are observed in the troposphere in many studies, several cases of high-level warm core near the tropopause have been reported in some studies on the basis of numerical simulations (e.g., Persing and Montgomery 2003; Chen and Zhang 2013; Wang and Wang 2014).

Zhang and Chen (2012) and Chen and Zhang (2013) investigated Hurricane Wilma (2005), which reached an extreme intensity of 882 hPa. They showed that Wilma had an upper-level warm-core structure, in which the upper-level warming was above the $\theta = 380$-K surface, and that the central surface pressure abruptly fell with the upper-level warming using numerical simulations. They suggested that convective bursts (CBs) in the inner-core regions contribute to the formation of the upper-level warm core, and they noted the importance of upper-level processes.

Conversely, Wang and Wang (2014) argued that, although the warming associated with intensification leads...
to a reduction of convection (Van Sang et al. 2008), further warming is possible despite this reduction of convection due to the increase of inertial stability. The relation between the distributions of inertial stability and diabatic heating was also investigated by Schubert and Hack (1982) and Vigh and Schubert (2009). They used a simple diagnostic analysis to show that local warming is considerably more efficient if the diabatic heating is confined to a region of relatively high inertial stability. These studies imply that the upper-level warm-core process is related to the local Rossby radius of deformation, which depends on inertial and static stability (Shapiro and Willoughby 1982). However, the previous studies focus mainly on only the inertial stability itself; little attention has been paid to relative roles of inertial and static stabilities.

The present study aims to investigate the upper-level warming process and to clarify factors controlling the warm-core structure. First, a TC simulation is conducted under ideal conditions using a three-dimensional non-hydrostatic numerical model; the simulated structure of the eye is analyzed in detail. In section 2, the model settings are described, and in section 3, the experimental results are shown and discussed. A possible mechanism of the upper-level warm core is discussed in section 4 by focusing on balance dynamics and analysis of the linear forced theory. In section 5, the robustness of the mechanism discussed is verified via sensitivity experiments, and conclusions are given in section 6.

2. Numerical model and experimental design

The TC simulations were conducted using a simplified version of the Nonhydrostatic Icosahedral Atmospheric Model (NICAM; Tomita and Satoh 2004; Satoh et al. 2008). In this study, NICAM was modified to be run on an f plane with hexagonal periodicity (see the appendix) under uniform rotation without spherical metrics. NICAM solves fully compressible nonhydrostatic governing equations originally formulated with an icosahedral grid on a sphere and the terrain-following coordinate in the vertical. The prognostic equations are for momentum in three dimensions including perturbation density, total energy, and densities of water components. They were formulated in conservation form to guarantee the conservation of prognostic variables. Previous studies (e.g., Fudeyasu et al. 2010) have shown that NICAM can successfully reproduce the entire life cycle of a TC.

In the present numerical simulation, we used the explicit cloud microphysics scheme of the NICAM single-moment water 6 model (NSW6; Tomita 2008), which solves for six categories of hydrometeors including water vapor, cloud water, cloud ice, rain, snow, and graupel. Turbulent closure was calculated using the level-3 Mellor–Yamada–Nakanishi–Niino scheme (MYNN3; Nakanishi and Niino 2004; Nakanishi and Niino 2006), which parameterizes the vertical mixing and contributes to both the planetary boundary and the free atmosphere (Noda et al. 2010). The radiation scheme used was mstrnX (Sekiguchi and Nakajima 2008), and the bulk surface flux over the ocean was calculated following Louis (1979) and Moon et al. (2007). We used fourth-order horizontal numerical diffusion.

A grid size of 2.3 km was used for the entire domain, which covered a periodic regular hexagonal area, with the apothem length of 1250 km on the f plane. We used 92 vertical levels up to 45 km. The grid spacing increased gradually from 160 to 400 m within the lower troposphere and lower stratosphere to resolve the dynamical processes occurring near the tropopause. Above that level, the grid spacing increased gradually again.

The model was initialized with an axisymmetric cyclonic vortex formulated by Rotunno and Emanuel (1987). The vortex had a maximum azimuthal wind speed of 12 m s\(^{-1}\) with a radius of 80 km at the surface that linearly decreased to zero at 16 km. The wind fields, mass, and thermodynamic fields were obtained on the basis of a balance equation to satisfy the thermal wind balance. The initial water vapor mixing ratio and the environmental sounding were assumed to be horizontally homogeneous. The environmental vertical profile was calculated by conducting a radiative–convective equilibrium simulation without a vortex over a fixed sea surface temperature (SST) of 31°C until statistical equilibrium was reached. For the TC simulation, the SST was fixed at the same value as that used for the calculation of the environmental profile. The environment was assumed to be quiescent, and there was no background flow. Uniform rotation at 18°N was given as a constant Coriolis parameter.

3. Results

The results of the numerical simulation are shown in this section. In section 3a, we show the structure and time evolution of the perturbation temperature and the minimum sea surface pressure of a simulated TC. In section 3b, the processes occurring at upper levels in the eye are investigated via potential temperature budget analysis, and we show that upper-level warming is dominated via advective warming of the azimuthally averaged secondary circulation. In section 3c, the causes of azimuthally averaged secondary circulation are investigated using Sawyer–Eliassen diagnosis.
a. Time evolution and structure of simulated tropical cyclone

Figure 1 shows the time–height Hovmöller plot of the perturbation temperature at the storm center and the time evolution of the minimum sea surface pressure (MSLP). The center of a TC ($r_c$) is determined by the mass center in the model defined as the column mass centroid, given by

$$r_c = \frac{\int \int p_s \, dS}{\int \int p_s \, dS},$$

where $p_s$ is surface pressure (Wang 2007). The integration domain of the above equation is a circular area within a radius of 70 km from the surface vortex center, which is approximately 3 times that of the maximum wind speed at the quasi-steady stage. The perturbation temperature was calculated relative to the environmental profile, which is the time-evolving azimuthal-mean temperature at a radius of 500 km. It may appear that the choice of a 500-km radius is too small for the reference for the environment. We have examined the case where the perturbation from the azimuthal-mean temperature is defined with respect to a reference at radius of 1000 km. Although the magnitudes of the temperature perturbation showed differences, the behavior of the warm core was qualitatively similar. Therefore, we used a radius of 500 km to define the environmental structure throughout this study. After an initial slight increase, the MSLP continued to drop until 170 h elapsed. Then, a quasi-steady stage was achieved. The cyclone reached a minimum central surface pressure of 897 hPa at approximately 188 h.

The maximum perturbation temperature initially formed near 9-km height, which is somewhat lower than that shown in the observational studies of Hawkins and Rubsam (1968) and Hawkins and Imbembo (1976) at approximately 90 h. At approximately 80 h, an additional core appeared near 16-km height, which corresponds to that of the tropopause in the environmental profiles. The lower core among the two began to ascend and decay at 110 h, after which time the two maxima merged. A similar ascent of warm core is reported in the study of Hurricane Daisy by Simpson et al. (1998). The upper-level warm core in the quasi-steady state is higher than that shown in some previous observational studies (Hawkins and Rubsam 1968; Hawkins and Imbembo 1976); however, similar high warm-core structures were reported in previous studies of idealized and realistic numerical simulations (Persing and Montgomery 2003; Chen and Zhang 2013; Wang and Wang 2014). Notably, instead of an upper warm core, cases were reported with the warm core being maximized at the middle level of the troposphere or the existence of dual maxima accompanied by a dominant midlevel maximum (e.g., Hawkins and Rubsam 1968; Halverson et al. 2006; Stern and Nolan 2012).

Figure 2 shows radial–vertical cross sections of the azimuthally averaged TC structure between 160 and
210 h of the simulation. The maximum tangential wind reached 80 m s\(^{-1}\) near the surface (500-m height) at a radius of approximately 20 km, and tilted slightly outward with height (Fig. 2a). The tangential wind decreased with height, and the maximum tangential wind near the tropopause (i.e., \(z = 16\) km) still exceeded 25 m s\(^{-1}\). Strong inflow occurred in the boundary layer below approximately 1 km with a maximum value of approximately 25 m s\(^{-1}\) at a 20-km radius just outside the radius of the maximum tangential wind (Fig. 2b). An outflow layer was located in the upper troposphere outside the eyewall with its roots at lower levels in the eyewall having a radius of approximately 20-km radius and a height of 6 km. Above this outflow layer, a weak radial inflow layer of approximately 1 m s\(^{-1}\) occurred near 17-km height. A similar radial inflow layer above the upper-level outflow layer together with an upper-level warm core have been reported in previous idealized and realistic studies on the basis of numerical simulations (e.g., Rotunno and Emanuel 1987; Chen and Zhang 2013; Wang and Wang 2014). In this mature stage, a warm core was formed with a maximum temperature anomaly of 20°C at 15-km height (Fig. 2c). The vertical velocity field shown in Fig. 2d indicates a weak descending motion in the eye and a strong updraft in the eyewall.

b. Contributions of the upper-level dynamical process

To investigate the processes of upper-level warming, the budget of potential temperature \(\theta\) in the inner core was examined. The budget of azimuthal-mean \(\theta\) is given by the following equation:

\[
\frac{\partial \theta}{\partial t} = \text{TADV} + \text{RAD} + \text{MP} + \text{DIFF},
\]

where TADV is the tendency due to advection (total advection including both azimuthal-mean and eddy
components); RAD and MP are radiation and cloud microphysics terms, respectively; and DIFF is the tendency due to turbulence (i.e., vertical mixing), dissipative heating, and horizontal fourth-order numerical diffusion.

Figures 3a and 3b show radial–vertical cross sections of the actual change of $u$ and the sum of the budget terms in the eye. Because the magnitude of tendencies of the budget terms in the eyewall is significantly larger than that in the eye, the eyewall region is masked in Fig. 3 in order to focus on the tendencies in the eye. The tendencies were calculated for the integration period between 150 and 160 h by using output data with a 6-min time interval. Such high-frequency data are necessary for calculating tendencies because coarser sampling data would lead to large errors due to aliasing of vertical velocities. Figure 3c shows that the sum of all tendencies agreed qualitatively well with the actual change of $u$ above 6-km height in the eye, although the sum of the tendencies overestimated the actual warming, particularly just above the eyewall (at 14–16 km) and in the lower troposphere below 6-km height in the eye. However, because the aim of the present study is relevant to the upper-level in the eye, the present analysis with the 6-min time interval is sufficient for discussion of upper-level warming within the eye.

Figure 4 shows radius–height cross sections of the azimuthal-mean fields of all budget terms in the inner-core regions of the simulated TC at the end of the development stage averaged between 150- and 160-h integration time by using 6-min data. It is apparent in the figure that the tendency of $u$ near 14-km height in the eye is dominated by warming due to total advection. This advective warming at upper levels in the eye is caused by the flow from the lower stratosphere into the eye. Zhang and Chen (2012) argued that such flow is associated with compensating subsidence of CBs at early stages of development. In the present case, however, the flow existed
at the end of the development stage and even at the mature stage, as shown in Fig. 2. Although the upper-level warming occurred through the intensification process of the TC, the convective updrafts become weaker because of the stabilizing effect of the warm-core development (not shown). Large contributions of MP were noted at the interface of the eye and the eyewall regions, indicating evaporation and sublimation cooling. This effect has been reported in previous studies of Zhang et al. (2002) and Stern and Zhang (2013). The contributions of RAD and MP to upper-level warming were small.

Total advection (TADV) was decomposed into the azimuthal-mean component and the asymmetric eddy component in the same manner as that reported by Stern and Zhang (2013). Using $\bar{u}$, radial wind $u$, and vertical wind $w$, we calculated the azimuthal means of the advection term and covariances between $u'$ and $\theta'$ and $w'$ and $\theta'$. Then, the azimuthal-mean advection $\text{ADVM} = -\frac{\partial}{\partial r}(\bar{u} \bar{w}) - w \frac{\partial \bar{u}}{\partial r}$ and eddy advection $\text{ADVE} = -(1/r)\frac{\partial}{\partial r}(ru'w') - (\partial w'/\partial w')w'$ were calculated. In the above definitions, the overbars and primes represent the azimuthal mean and the deviation from the azimuthal mean, respectively. These calculation procedures can introduce nonnegligible errors, defined as $\text{ERR} = (\text{ADVM} + \text{ADVE}) - \text{TADV}$. Actually, relatively large errors can be seen outside the eye, particularly at the eyewall and the outflow region (not shown).

Figures 5a and 5b show ADVM and ADVE, respectively, within the eye. The positive contribution of ADVM overwhelmed the negative contribution of ADVE near 15-km height. At the interface of the eye and eyewall, ADVM had a large positive value, and ADVE was positive from 7- to 14-km heights, which together formed the strong warming tendency at the

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**Fig. 4.** Radial–vertical cross sections of the tendencies on azimuthal-mean $\theta$ from (a) total advection, (b) radiative heating, (c) cloud microphysics, and (d) diffusion averaged between 150- and 160-h integration times from 6-min data. (e) The sum of all terms. The contour interval is 0.1 K h$^{-1}$, except in (b), where it is 0.01 K h$^{-1}$. The eyewall region is masked.

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interface observed in TADV. On the contrary, ADVM was negative slightly inside the interface from 10 to 14 km. As shown in Fig. 5c, the overall ERR was small within the eye, although somewhat large errors appeared near the top of the eyewall. In addition, errors were relatively large at the outer region (not shown), although they did not affect our analysis within the eye. Thus, the decomposition is sufficiently accurate for the above discussion on the relative roles of azimuthal-mean and eddy advection within the eye.

c. Sawyer–Eliassen diagnosis

From the above budget analysis, it is evident that the upper-level warming is primarily due to the azimuthal-mean flow advection. To determine the driving mechanism for this azimuthal-mean flow (i.e., the flow from the lower stratosphere into the eye), secondary circulation was investigated using a balanced dynamics approach. The azimuthal-mean secondary circulation can be diagnosed by solving the Sawyer–Eliassen equation (Eliassen 1951). Because it is a linear partial differential equation, the solutions to each forcing are additive (e.g., Fudeyasu et al. 2010; Fudeyasu and Wang 2011). This feature allows us to evaluate the contributions of forcings to the total tendency due to the advection. The Sawyer–Eliassen equation in height coordinates can be written as (Pendergrass and Willoughby 2009)

$$\frac{\partial}{\partial r} \left( \frac{N^2}{R} \frac{\partial \psi}{\partial r} \right) + \frac{\partial}{\partial z} \left( \frac{N^2}{R} \frac{\partial \psi}{\partial z} - B \frac{\partial \psi}{\partial r} \right) = \frac{\partial}{\partial r} \left( \frac{Q}{\theta} \right) + \frac{\partial}{\partial z} \left( \frac{\gamma Q - \xi M}{\theta} \right),$$

where $r$ is the radius; $z$ is the height; $\psi$ is the azimuthal-mean transverse streamfunction; $\theta$ is the potential temperature; $N^2$ and $B$ are the vertical and radial gradients of buoyancy $b = g \ln(\theta/\theta_0)$, where $g$ is the gravitational
acceleration and \( \theta_0 \) is a constant; \( \gamma = (\nu^2/r + fu)/g \), \( R = r\theta \), \( \Gamma^2 = \Gamma^2 - \gamma B \), \( \xi = 2\nu/r + f \), where \( \rho \) is the density, \( \nu \) is the tangential velocity, \( f \) is the Coriolis parameter, and \( \Gamma^2 = \xi(\partial u/\partial r + u/r + f) \) is the inertial stability; \( \mathbf{M} \) is the force in the tangential direction; and \( \mathbf{Q} \) is the buoyancy source. The detailed derivation was reported by Pendergrass and Willoughby (2009); however, the terms associated with the tendency of the pressure gradient \( \gamma \) neglected in the study of Pendergrass and Willoughby (2009) are included in this study for mathematical completeness and simplicity. In the following analysis, \( \mathbf{M} \) is decomposed into the axisymmetric diffusion and the eddy advection of azimuthal wind. The buoyancy source is also decomposed into the axisymmetric diabatic heating, heating due to turbulent vertical and numerical horizontal diffusions, and the eddy advection of potential temperature.

Because the Sawyer–Eliassen equation is based on the assumption of gradient wind balance, it is necessary to verify the extent to which the simulated TC satisfies the assumption. We investigated the radial–height distributions of the difference between the pressure gradient term \([-1/\rho(\partial p/\partial r)]\) and the rotational term \[(f + \nu/r)u\] of the radial wind balance normalized by the rotational term, which represents the degree of accuracy of the assumption of the gradient wind balance averaged between 150 and 160 h of simulation (Fig. 6). Relatively large imbalances of approximately 10% were observed in the boundary layer under the eyewall because of surface friction, which is consistent with previous studies (e.g., Kepert and Wang 2001; Smith and Montgomery 2008). Large imbalances existed in the outer upper region. At this region, the radial wind speed had a somewhat large value relative to the tangential wind speed; thus, the contribution of the advection of the radial wind is significant. However, forcings in the outer upper region hardly caused the flow penetrating the eye region because the strong inertial stability region extended up to 18-km height in the simulated TC, which is shown in Fig. 2a. Therefore, it can be safely considered that the Sawyer–Eliassen equation is applicable for investigating the upper-level warming.

Figure 7f shows the tendency of azimuthal-mean \( \theta \) from the total advection obtained from the Sawyer–Eliassen equation with all forcing terms. The corresponding distribution of the diagnosed streamfunction
is shown in Fig. 8f. Figure 7f can be compared to Fig. 5a, which shows that the simulated secondary circulation can be well explained by the balanced dynamics. In particular, the solution to the Sawyer–Eliassen equation captured the characteristic distribution patterns of the upper-level warming near 14-km height and the warming at the interface of the eyewall at the middle level. More precisely, the absolute value of the tendency of the balanced dynamics in the middle of the troposphere tended to be larger than that of the simulated total tendency shown in Fig. 5a. The possible reasons for differences are the lack of the imbalance effects, sampling frequency, and interpolation errors; however, the factor most responsible for the differences is unclear. Then, we investigated the contribution of each forcing to the secondary circulation in the simulated TC, such as contributions of the diabatic heating, heating due to diffusion and the eddy advection of $\theta$, axisymmetric diffusion, and eddy advection of the azimuthal wind. The distributions of tendencies from induced flow advection are shown in Figs. 7a–e. Among them, the advection due to diabatic heating from cloud microphysics (Fig. 7a) yielded the largest positive tendency in the eye, with maxima near 7-, 13-, and 15-km heights. On the contrary, the advection by diffusion of the azimuthal wind (Fig. 7d) caused a large negative tendency in the eye with minima at the corresponding heights to the maxima of those by diabatic heating (Fig. 5a). Interestingly, these two tendencies showed a notable strong negative correlation, although there was no obvious necessity for such results. A possible reason is the increase of Rossby depth, as is discussed in section 4. Although the contributions of the eddy advection of $\theta$ (Fig. 7c) to the upper-level warming were positive, they were significantly smaller than those by the diabatic heating.
4. Diagnosis by a forced problem

a. Hypothesis

Although the height of maxima of the actual advective warming tendency is determined by the sum of forced circulations induced by all forcings, in the previous section, it was shown that upper-level warming is primarily caused by secondary circulation in response to the diabatic heating by cloud microphysics. However, the driving mechanism for the downward flow from the stratosphere to the eye has not been reported thus far. We believe that it depends on a specific structure of the TC vortex because the upper-level warm-core maximum did not always emerge in observations or simulations. Therefore, we focused on two possible effects: 1) the distribution of diabatic heating in the upper level and 2) relative height between the top of the vortex and the tropopause.

When diabatic heating due to cloud microphysics exists in the upper level where the static stability is strong, upper-level circulations are induced. However, we determined that the distributions of the diabatic heating in the simulated TC were not peculiar in this case; rather, they were compatible with heating distributions inferred from vertical wind distributions for cases in which upper-level warm-core maxima do not exist (e.g., Frank 1977; Liu et al. 1997, 1999). In addition, it is believed that if cloud microphysics schemes are changed or modified, the distributions of the diabatic heating also change. However, the previous studies did not report significant changes in the height and structure of a warm core when examining the impacts of cloud microphysical processes.

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**FIG. 8.** Radial–vertical cross sections of streamfunction of forced circulation induced by (a) diabatic heating, (b) heating due to diffusion, (c) eddy advection of $\theta$, (d) axisymmetric diffusion, (e) eddy advection of azimuthal wind, and (f) all heatings and forcings. The contour interval is $2 \times 10^3$ kg s$^{-1}$. Dashed lines indicate negative values.
on TCs (Pattnaik and Krishnamurti 2007; Stern and Nolan 2012). Therefore, it is unlikely that the existence of strong heating at the upper level alone is the primary cause of the flow from the lower stratosphere, at least in realistic situations. When the top of the vortex is higher than the tropopause, a forced circulation extends into upper levels because of the vertically oriented aspect ratio of streamfunctions associated with the strong inertial stability in the lower stratosphere. The scales of the induced circulation are well known to be determined by the relative magnitude of inertial and static stabilities (Shapiro and Willoughby 1982; Holland and Merrill 1984). As inertial stability strengthens, the horizontal scale of circulation becomes shorter. Distributions of stabilities may lead to a remote effect at some distance from the forcing region.

Figure 9a shows the distributions of potential temperature and angular momentum averaged between 40 and 60 h of simulation at the early stage of development before upper-level warming. At this stage, the tangential wind distribution is confined to the relatively low static stability region (i.e., the troposphere) below 16-km height (Fig. 9c). Therefore, a high inertial stability region is also bounded in this low static stability region, as shown in Fig. 9b. The corresponding distribution of local deformation radius $R_d^2 = I^2 N_0^2 f^2 N^2$, which is the measure of the aspect ratio of response, is shown Fig. 9d, where $N_0 = 1 \times 10^{-2} \text{s}^{-1}$. This indicates that induced flow hardly occurs in the high static stability region, where there will be more increase in $\theta$ for a given vertical displacement (i.e., the stratosphere) above 16 km. Figure 10a shows the distributions of potential temperature and angular momentum of the simulation averaged.
between 160 and 210 h. The contours of angular momentum are shown to be crowded in the eye region, which extends above 14-km height. This result corresponds to the vertical extension of the high inertial stability distribution, as shown in Fig. 10b, which can be compared to that in Fig. 9b. The corresponding distribution of static stability is shown in Fig. 10c. The static stability is changed at the warm-core region, although the change is significantly smaller than that of inertial stability. The change in distribution of the local deformation radius from that in Figs. 9d and 10d is determined to be dominated by the effect of change in inertial stability.

The effect of the change in deformation radius to the vertical extent of forced circulation can be seen from the distribution of Rossby depth $g = \frac{f^2}{N^2}$, where $k$ is the horizontal wavenumber. Figures 9f and 10f show the distributions of Rossby depth calculated using $k = 30$ km. It is evident that the depth of circulation increased and the effect extended to the height of the lower stratosphere as the TC developed. The effect of the increase in Rossby depth was discussed by Schubert and McNoldy (2010). They determined that the large variations of Rossby depth with vortex strength also have large impacts on the vertical penetration of Ekman pumping; strong vortices have sufficiently large Rossby depths to allow Ekman pumping to deeply penetrate the troposphere. Although their study is relevant to the Ekman pumping, it can be inferred that the same vertical extension will occur in forced circulation due to diabatic heating.

Therefore, we hypothesize that the occurrence of the downward flow from the high static stability region is closely related to the increase in the Rossby depth because of the increase of the strong-inertial-stability region in the high static stability region, which corresponds to the relatively high top of the vortex that penetrates the lower stratosphere.

b. Analysis of forced circulations of idealized vortices

Because it is difficult to control the structures of TCs and the distributions of diabatic heating in numerical
simulations, we examined the sensitivity of the induced circulations in various balanced vortex profiles with the same heating distribution. Our methods are similar to those reported by Pendergrass and Willoughby (2009). The thermodynamic structure of the unperturbed atmosphere is shown in Fig. 11a: the bottom temperature was 304 K, and the lapse rate was 6.67 K km\(^{-1}\) below 16-km height and isothermal above that height. The balanced primary vortex was based on an empirically derived sectionally continuous algebraic profile (Willoughby et al. 2006). The tangential wind profile \(v(r, z)\) is defined by

\[
v(r, z) = \begin{cases} 
V_i = V_{\text{max}}(z) \left[ \frac{r}{R_{\text{max}}(z)} \right]^n, & 0 \leq r \leq R_1(z), \\
V'[1 - A(x)] + V_o A(x), & R_1(z) \leq r \leq R_2(z), \\
V_o = V_{\text{max}}(z) \exp \left[ -\frac{r - R_{\text{max}}(z)}{X_1} \right], & R_2(z) \leq r,
\end{cases}
\]

where \(V_i\) and \(V_o\) are the tangential wind profiles inside the eye and beyond the transition zone \([R_1(z) \leq r \leq R_2(z)]\), respectively; \(V_{\text{max}}(z)\) and \(R_{\text{max}}(z)\) are the maximum wind and the radius of maximum wind speed, respectively; \(X_1\) is an e-folding distance; and \(A(x)\) is a polynomial bellramp function that varies smoothly from 0 to 1 in the transition zone as a function of its argument \(x = (r - R_1)/(R_2 - R_1)\). The mass and thermodynamic fields associated with the vortex were obtained by solving the nonlinear balance equations as described by Pendergrass and Willoughby (2009). In both vortices, \(R_{\text{max}}(z)\) is 30 km at the surface and slopes outward by 16 km to the tops of the vortices. The maximum wind decreases with height according to a polynomial function \(w_i = \xi^2(3 - 2\xi)\), which varies smoothly from 0 to 1 between the surface and height of the vortex \(z_{\text{top}}\) as a function of its argument \(\xi = (z_{\text{top}} - z)/z_{\text{top}}\) (Willoughby et al. 2006).
To investigate the effects of the relatively strong inertial stability in the strong-static-stability region, two vortex structures were examined. The first was a high vortex in which the top penetrates the relatively-strong-static-stability region, and the second was a low vortex confined to the low static stability region. The values of \( z_{\text{top}} \) and \( V_{\text{max}}(0) \) are 22 km and 80 m s\(^{-1}\) in the former case and 17 km and 55 m s\(^{-1}\) in the latter case. The radial–vertical cross sections of the tangential wind profile are shown in Figs. 12b and 13b. To focus on the effect of increase in inertial stability in the high static stability region, these structures were chosen, so that the effect of temperature perturbation associated with the balanced primary vortex was minimized in the high static stability region. In both cases, the peak of the temperature perturbation was located at approximately 6-km height.

The heat source was specified by the following function:

\[
Q(r, z) = Q_0[x_q(1 - x_q)]^3w_4(\xi_q),
\]

\[
\frac{|r - R_{q_{\text{max}}}(z)|}{L_h} \leq 1, \quad 0 \leq \xi_q \leq 1,
\]

where

\[
x_q = \frac{|r - R_{q_{\text{max}}}(z)|}{L_h} + \frac{1}{2},
\]

\[
\xi_q = \frac{z_{\text{q,top}} - z}{z_{\text{q,top}}},
\]

\[
w_4(\xi_q) = \xi_q^5(126 - \xi_q)(420 - \xi_q)[540 - \xi_q(315 - 70\xi_q)]\]

Here \( R_{q_{\text{max}}(0)} \) is 28 km, 2 km inside the radius of maximum wind (RMW), and \( R_{q_{\text{max}}}(z) \) slopes outward 16 km linearly as \( R_{\text{max}} \). The variable \( Q_0 \) is scaled to produce a 1 m s\(^{-1}\) updraft at the bottom. The heating profile is shown in Fig. 11b.

Figure 12a shows a radial–vertical cross section of the mass streamfunction of the secondary circulation induced by diabatic heating with a high vortex up to
200-km radii. The green line at 16-km height indicates the height at which the static stability of the background profile changed (e.g., the background tropopause). The dipole of streamfunction existed at the upper area of the low static stability region. The maximum was just outside RMW at 10-km height, and the minimum appeared just inside the RMW at 12-km height. A steep positive horizontal gradient of streamfunction appeared, and upward flow was excited at the location of heating. Negative vertical gradients of streamfunction appeared between heights of 14 and 16 km, which indicates the location at which outflow was induced. In the inner region, the sign of the horizontal gradient was negative, and downward flow occurred. The peak was located at 12-km height. The vertical gradient of the streamfunction was weakly negative at approximately 17-km height and positive above that height. The distribution of the streamfunction had minima along the vertical coordinate at that level, which indicates the existence of the inflow region above. At that height, the horizontal gradient was weakly negative, and downflow in the strong-static-stability region was induced. These results occurred because the isolines of potential temperature and angular momentum were nearly parallel near the top of the heating region. The streamfunction was elongated along the isolines in that area, which resulted in occurrence of inflow at approximately 17-km height.

Figure 12b shows a radial–vertical cross section up to 100-km radius of the tendency on \( \theta \) from the advection due to secondary circulation induced by the heating source. The black thick lines in the figure show the distribution of the azimuthal wind speed. A strong negative tendency associated with the updraft was observed at the area in which the heating source was placed. The tendency was positive inside the RMW because of the downdraft. Although the downward peak appeared at 12-km height, the tendency associated with the downdraft showed a minimum at the same height. A similar inconsistency was reported by Stern and Zhang (2013). They determined that the mean descent tended to be maximized in the upper troposphere, whereas the vertical component of the azimuthally averaged advective warming was maximized in the midtroposphere in their numerical simulation using the moist-tropical sounding of Dunion (2011) as their environmental sounding. They attributed the result to the small amount of static stability around the height of the peak of mean descent. They also suggested that the presence of weak descent above the height of the base of the tropical tropopause layer led to secondary upper-level maximum warming in their case. Similarly, the inconsistency in this study was
caused by the temperature anomaly associated with the balanced vortex that weakened the static stability above 12 km. Inside the RMW, a relatively strong positive tendency was observed between heights of 16 and 18 km, in which the background static stability was strong.

Figure 13a shows a radial–vertical cross section of the mass streamfunction of secondary circulation for the low vortex. As in the case of the high vortex, the dipole of the streamfunction was located in the upper area of the weak-static-stability region, although the height was lower. The altitudes at which the maximum and minimum existed were approximately 9 and 10 km, respectively. The height at which the outflow occurred was also lower, at approximately 12 km. Unlike that in the case of the high vortex, downward flow was not induced in and around the strong-static-stability region.

Figure 13b shows a radial–vertical cross section of the tendency on $\theta$ from the advection due to the secondary circulation with the low vortex. The tendency of $\theta$ was negative associated with the upward wind around the RMW, as with the case of the high vortex. However, the relatively strong positive tendency in the strong-static-stability region that was observed in the case of the high vortex did not appear in this case.

These results indicate that if inertial stability increases in a strong-static-stability region and the static stability decreases relative to the inertial stability, vertical wind can be induced in the strong-static-stability region. In this study, artificial balanced vortex structures were selected, so that the effect of temperature anomalies associated with the tangential wind distributions to the static stability at the strong-static-stability region were minimized, in order to focus on the effect of the increase in inertial stability in that region. In TC-like vortices, however, the temperature anomaly at the center, which is in quasi-balance with the wind speed distribution, is positive, and the temperature anomaly can decrease static stability above its maxima. This reduction of static stability can intensify vertical elongation of forced circulation above temperature anomaly maxima due to the increase in inertial stability. Therefore, the distinct distributions of static and inertial stabilities are robust; thus, the results with the specified stabilities can be applicable to realistic vortex structures. If angular momentum is transported to the lower stratosphere due to processes such as CBs and heights of the vortices become sufficiently tall, downward flow from the lower stratosphere can be induced by diabatic heating, and warm cores can be developed near the tropopause.

5. Sensitivity tests

To examine the proposed mechanism described in section 4, the relation between upper-level warm cores and inertial stability were examined from numerical results. To save computational resources, the simulations shown in this section were conducted with 4.8-km grid spacing and 45 vertical levels. All other settings were identical to those described in section 2.

The time evolution of the MSLP and the time–height Hovmöller plot of the perturbation temperature at the storm center with 4.8-km grid spacing over 168-h integration is shown in Fig. 14a. In this simulation, the TC reached a mature stage at approximately 130 h. The overall evolution of the warm core was similar to that with 2.4-km grid spacing, although the intensification rate and the maximum intensity were somewhat greater. A maximum of perturbation temperature developed in the middle of the troposphere, and an additional maximum appeared near the tropopause.

We used the results of numerical simulation by varying the vertical mixing length of the turbulence scheme to examine the warm-core evolution. This process was motivated by the results of previous studies (e.g., Kanada et al. 2012), in which turbulence schemes were shown to have significant effects on the behaviors of simulated TCs. In the MYNN scheme, which is the turbulence scheme used in this study, the vertical mixing length $L$ is diagnostically determined. Here we set a constant value to the vertical mixing length in the framework of the MYNN to obtain a variety of behaviors in the TC simulations. As is subsequently discussed, the behaviors of TCs do not systematically change with the specified value of the vertical mixing length; rather, the behaviors of formed TCs diverge. However, for the present purpose, it was desirable to obtain diverse samples of various behaviors of TCs because such a method is useful for examining the relation between structures of formed vortices and secondary circulations. A complete explanation of the manner in which the vertical mixing length affects the TC structure is beyond the scope of the present study.

The time evolutions of the MSLP and the time–height Hovmöller plot of the perturbation temperature of simulations at which the vertical mixing length is explicitly given as $L = 12.5, 25, 50, 100,$ and 200 m are shown in Fig. 14. In the simulations with $L = 12.5, 25,$ and 50 m, MSLPs dropped much faster than that in the control case, and they achieved maximum intensities at approximately 80 h. In the case of $L = 100$ m, the pressure drop rate was moderate and weak relative to that in simulations with smaller values of vertical mixing length. In the case of $L = 200$ m, the pressure drop rate was slower than that in the control case, although the strongest intensity was achieved among the simulations in which the vertical mixing length was explicitly given. In all of the sensitivity experiments, warm cores developed
at approximately 9-km height; however, in the simulations with $L = 12.5$ and 25 m, upper-level perturbation temperature maxima near the tropopause did not develop. In the simulations with $L = 50$, 100, and 200 m, upper-level perturbation temperature maxima appeared, although not continuously. The timings of appearance of the upper-level maxima correspond to those in which TCs reached maximum intensities at approximately 90 h in the simulations with $L = 50$ and 100 m and 130 h in the simulation with $L = 200$ m. These results seem to demonstrate the close relation between intensity and upper-level warming in the life cycle, although the relation between occurrence of upper-level warming and maximum intensity is not clear.

Figure 15a shows the vertical profile of vertical wind speed averaged within a radius of 20 km, averaged between 40 h just before the time when TCs reached their maximum intensities (i.e., the development stages). The vertical wind speed is normalized by its magnitude at 14-km height in order to compare the depths of secondary circulations. The downward-flow regions are deeper in the control simulation and the simulations with $L = 50$, 100, and 200 m where upper-level warm-core maxima formed than those in the simulations with $L = 12.5$ and 25 m where upper-level warm-core maxima did not appear. Figure 15b shows the vertical profile of inertial stability averaged between the corresponding times. Inertial stabilities at about 16-km height are more than several times stronger in the control simulation and the simulations with $L = 50$, 100, and 200 m than those in the simulations with $L = 12.5$ and 25 m. These results are consistent with the mechanism discussed in section 4.

6. Summary

To examine the upper-level warming process in a TC, a numerical experiment was conducted using NICAM. During the early stage of development, a maximum of perturbation temperature at the center evolves near 9-km height, which is somewhat lower than that shown
in some observational studies (Hawkins and Rubsam 1968; Hawkins and Imbembo 1976). In the middle of the development stage, an additional maximum of perturbation temperature appears near 16 km, which corresponds to the height of the tropopause of the environmental profiles. The maximum at this height is greatest in the quasi-steady state. This vortex structure, which has a warm core near the height of the tropopause is sustained for more than 30 h and lasts until the end of the simulation time.

To investigate the contributions of the processes to the upper-level warming, the inner-core budgets of potential temperature were analyzed at the end of the development stage. The total tendency of potential temperature near 16-km height is dominated by heating due to the total advection associated with subsidence from the lower stratosphere into the eye. The decomposition of the tendency due to the total advection into azimuthal-mean and eddy components reveals that the large positive contribution of the azimuthal-mean component overwhelms the small negative contribution of the eddy component.

The results of decomposition of the potential temperature advection motivated us to conduct the analysis of the secondary circulation via the balanced dynamics approach to investigate the driving mechanism of the flow from the lower stratosphere into the eye. The Sawyer–Eliassen equation used to diagnose the azimuthal-mean secondary circulation is a linear partial differential equation that evaluates the contribution of heatings and forcings to the tendency due to advection. Although the contributions of the eddy terms are nonnegligible, those of diabatic heating and diffusion of tangential wind are dominant in the eye. The contributions of diabatic heating by cloud microphysics and diffusion of tangential wind are positive and negative, respectively. A strong negative correlation exists between the tendencies due to the two forcings, suggesting that diffusion of tangential wind tend to damp the diabatic heating of cloud microphysics. In any case, the diabatic heating due to cloud microphysics is identified as the main cause of upper warming, although in reality, the height of the maximum temperature anomaly is determined by a balance among forced secondary circulations.

To determine the reason for the downward flow from the lower stratosphere, a forced problem was solved to investigate the effects of relative heights between the
top of the eyewall and the tropopause. The responses with two vortex structures to the heating distribution were examined. The results show that the change in the local Rossby deformation radius due to the change in inertial stability causes an extension of the response to heating and upper-level warming even if the effect of temperature anomalies to the distribution of static stability is small. The results of the sensitivity experiments are consistent with this mechanism. Although previous studies suggested that the upper-level warming is related to CBs generally observed during the early stages of TCs, the present study describes a mechanism that explains the existence of upper-level warming in the mature stages of TCs. The upper-level warming related to CBs and that discussed in the present study are not mutually exclusive.

To the best of the authors’ knowledge, no observational evidence has been reported thus far that supports the mechanism discussed in the present study. A possible method for obtaining useful information to validate the mechanism is to measure the ozone concentration in the eye. Because stratospheric air has a higher ozone concentration, the proportion of stratospheric air that has descended into the eye from above can be inferred through ozone measurements in the eye. Carsey and Willoughby (2005) measured ozone in the lower troposphere in Hurricanes Floyd and Georges in the Atlantic. An estimation of the amount of descent was conducted on the basis of the observed temperature increase. The estimated amount was approximately 2 km, although the accuracies of the assumptions were unclear. Using ozone as complement to the estimate gave a 1-km-or-less distance of subsidence, or a descent rate of 1 cm s⁻¹. They found only 10%–20% of the air at the aircraft could be attributed to the mass flux transport across the tropopause. It is unclear whether their study included a case where the mechanism discussed in the present study was operating. However, if additional observations are conducted for the upper troposphere in tropical cyclones, the case of higher contribution of lower stratospheric air is expected to be observed.

The present analysis suggests that TCs can be even stronger than those expected by theories in which the TC vortex is confined to the troposphere (e.g., Emanuel 1986). To provide a better understanding of TC intensities, TC theories must be revised to consider the effects of the stratosphere and the dynamics occurring in the eye.

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APPENDIX

Modification of NICAM

Here, we describe a simplification of NICAM with hexagonal periodicity, which is used in this study.

In NICAM, the global spherical calculation domain is covered by 10 diamond-shaped domains. In the model used in this study, we selected one diamond-shaped domain to perform an f-plane calculation. All spherical metrics were removed, and the diamond was set on a plane. We imposed a doubly periodic boundary condition to the diamond. Although such periodicity is generally believed to cause distortion, we determined that this geometry has a desirable structure of the hexagonal periodicity for theoretical studies, as shown in Fig. A1. The black thick lines in the figure indicate the calculation domain in this model. Outward fluxes across side A–B are equivalent to the inward flux across side D–C. The dark gray, light green, light gray, and dark green areas inside the region surrounded by the black thick lines correspond to the hatched areas with corresponding colors. Therefore, if a double periodic boundary condition is imposed onto a diamond-shaped domain with side length L, then a unit cell is a regular hexagonal domain with side length \( l = \frac{L}{\sqrt{3}} \), which is represented as the hatched area in the figure. A cell with regular hexagonal geometry is closer to a circle than...
a cell with square or regular triangular geometry and is, therefore, superior from the perspective of isotropy.

REFERENCES


