CORRESPONDENCE

Comments on "A Unified Representation of Deep Moist Convection in Numerical Modeling of the Atmosphere. Part I"

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Arakawa and Wu (2013, hereafter AW13) recently developed a formal approach to a unified parameterization of atmospheric convection for high-resolution numerical models. The work was based on ideas formulated by Arakawa et al. (2011) and was designed to parameterize the eddy transport of moist static energy by convection. AW13 contained two main points: 1) a new formulation for convective transport valid for both small and large convective cloud fractions and 2) a closure in terms of convective cloud fraction suitable for such situations. Thus, the key parameter in this approach is convective cloud fraction σ. Since only updrafts were considered in AW13, convective cloud fraction was synonymous to updraft fraction. To avoid any confusion, we will use "updraft fraction" exclusively in this note to mean convective cloud fraction in AW13, unless explicitly stated otherwise.

In conventional parameterization, it is assumed that $\sigma \ll 1$. This assumption is no longer valid when the horizontal resolution of numerical models approaches a few to a few tens of kilometers, since in such situations updraft fraction can be comparable to unity. Therefore, AW13 argue that the conventional approach to formulating an expression for convective transport must include a factor $1-\sigma$ in order to unify the parameterization for the full range of model resolutions so that it is scale aware and valid for large updraft fractions. AW13 raised an important issue in future convective parameterization development. In this note we intend to show that the assumption of $\sigma \ll 1$ was unnecessary in the conventional approach and the scale-awareness factor $1-\sigma$ was already built in implicitly, although not recognized for the last 40 years. Therefore, the formulation for convective transport in conventional parameterizations should work well even in the gray zone where large updraft fractions exist.

To begin, we use the same notation as in AW13. To parameterize the vertical transport of moist static energy h by convective updrafts $(\overline{w'h'})$, we note that any variable ψ can be written as

$$\psi = \overline{\psi} + \psi' \quad \text{and} \quad \overline{\psi} = \sigma\psi_c + (1-\sigma)\tilde{\psi}, \tag{1}$$

where subscript c denotes mean value in convective updrafts, a tilde denotes environmental value, and an overbar denotes average over a numerical model grid box. After simple manipulation, we express the vertical eddy transport of h by convection as

$$\overline{w'h'} = \frac{\sigma}{1-\sigma}(w_c - \overline{w})(h_c - \overline{h}). \tag{2}$$

It can also be expressed in terms of the difference between updraft and environmental values after substituting Eq. (1) into Eq. (2) for $\overline{w}$ and $\overline{h}$:

$$\overline{w'h'} = \sigma(1-\sigma)(w_c - \tilde{w})(h_c - \tilde{h}). \tag{3}$$

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This is identical to Eq. (9) in AW13 for vertical transport of moist static energy by convection in their unified parameterization approach.

It is noted that in AW13 it was assumed that convection consists of updrafts only. Thus, the environment represents the true convection-free and downdraft parts of a model grid box. In reality, convection contains multiple updrafts and/or downdrafts. Equation (3) can be modified as (Liu et al. 2015)

$$\overline{w'h'} = \sum_i \sigma_i(1-\sigma_i)(w_i - \tilde{w})(h_i - \tilde{h}) - \sum_i \sigma_i(w_i - \tilde{w})\sum_{j\neq i}\sigma_j(h_j - \tilde{h}). \qquad (3a)$$

Here i and j represent an updraft and/or a downdraft, and the summation is over all convective updrafts and downdrafts. The convective cloud fraction includes all updraft and downdraft areas $\sigma = \sum_i \sigma_i$, and the environment now represents the true convection-free region of a model grid box. With this in mind and without losing generality, we will continue the discussion with the ''updraft only'' scenario as in AW13.

Before proceeding further, it is appropriate to summarize the gist of AW13. First, AW13 demonstrates that the moist static energy source from convective parameterization in numerical models does not converge to convection-resolving results as the model resolution increases. It argues that this problem is related to σ, which becomes nonnegligible when the model resolution increases. As shown in Eq. (3), vertical eddy transport by convection involves both σ and $1-\sigma$. Then AW13 shows that the conventional approach does not consider the $1-\sigma$ factor. Finally, adopting the unified parameterization approach, AW13 proposes a way to determine σ:

$$\sigma = \frac{(\overline{w'h'})_E}{\Delta w \Delta h + (\overline{w'h'})_E}, \qquad (4)$$

where $\Delta w = (w_c - \tilde{w})$ and $\Delta h = (h_c - \tilde{h})$, and $(\overline{w'h'})_E$ is the equilibrium vertical eddy transport of moist static energy required for the full adjustment responding to grid-scale destabilization and is assumed to be known.

Now we consider the conventional approach. In conventional convective parameterizations, it is assumed that (i) $\sigma \ll 1$ and (ii) $\overline{w} \ll w_c$. Therefore, Eq. (2) becomes

$$\overline{w'h'} \approx \sigma w_c(h_c - \overline{h}), \qquad (5)$$

where σw_c is typically treated as the updraft mass flux after being multiplied by air density. Equation (5) is the formulation used for convective transport in current mass flux–based parameterization schemes, and we call this the conventional approach. It is the same as Eq. (7) combined with Eq. (6) in AW13:

$$\overline{h} = \tilde{h}$$

[Eq. (6) in AW13] and

$$\overline{w'h'} \approx \sigma w_c(h_c - \tilde{h})$$

[Eq. (7) in AW13]. However, we note that AW13 refer to their Eq. (7) alone as the conventional approach in the rest of their paper. Since $\tilde{h}$ is not a model predicted variable, AW13's Eqs. (6) and (7) are always used together in current climate and numerical weather prediction models. Therefore, it is more accurate to call their Eq. (7) combined with their Eq. (6), or equivalently Eq. (5) above, the conventional approach.

Below we will show that (i) the assumption of $\sigma \ll 1$ is not needed to arrive at Eq. (5) and (ii) Eqs. (5) and (3) are practically the same.

In the past, it has always been believed that one must assume $\sigma \ll 1$ in order to obtain Eq. (5) from Eq. (2). Surprisingly, this turns out to be unnecessary. To show this, we note that

$$(w_c - \overline{w}) = w_c - [\sigma w_c + (1-\sigma)\tilde{w}] = (1-\sigma)(w_c - \tilde{w}). \qquad (6)$$

Substituting Eq. (6) into Eq. (2) gives

$$\overline{w'h'} = \sigma(w_c - \tilde{w})(h_c - \overline{h}). \qquad (7)$$

In general, w_c is much larger than $|\tilde{w}|$ regardless of updraft fraction (e.g., even when a subdomain is largely filled with convection) or model resolution. We examined the cloud-resolving model data from Liu et al. (2015) and found that w_c is larger than $|\tilde{w}|$ by an order of magnitude for subdomains as small as 8 km (figure not shown). Thus, we can safely neglect $\tilde{w}$ and approximate Eq. (7) by

$$\overline{w'h'} \approx \sigma w_c(h_c - \overline{h}). \qquad (8)$$

Note that Eq. (8) is derived from the unified parameterization approach without making any assumption about updraft fraction. But it turns out to be identical to Eq. (5)—the expression derived from the conventional approach. This is because in Eq. (2) the factor

$$\frac{1}{1-\sigma}(w_c - \overline{w}) = (w_c - \tilde{w}) \approx w_c \qquad (9)$$

is independent of updraft fraction (e.g., even when it approaches 1). Therefore, the conventional parameterization for calculating vertical transport by convection is not restricted to $\sigma \ll 1$.

Next we will show that the conventional approach is essentially the same as the unified parameterization approach in AW13. We start with Eq. (5). Note that the $1-\sigma$ factor is implicitly contained in the difference between the updraft mean and the gridbox mean:

$$h_c - \overline{h} = (1-\sigma)(h_c - \tilde{h}). \quad (10)$$

When σ approaches 1, $h_c - \overline{h}$ approaches zero. Substituting Eq. (10) into Eq. (5) gives an alternative expression for vertical transport of moist static energy in the conventional approach:

$$\overline{w'h'} = \sigma(1-\sigma)w_c(h_c - \tilde{h}). \quad (11)$$

Comparing this with Eq. (3), which is the unified parameterization approach in AW13, we see that $\sigma(1-\sigma)$ appears in both equations, and the only difference between them is that the conventional approach uses w_c while the unified approach uses $w_c - \tilde{w}$. Since $|\tilde{w}| \ll w_c$ even when the updraft fraction is large, the vertical transport calculated by Eq. (3) and by Eq. (11) is practically the same. In other words, the conventional approach is already scale aware: the eddy transport approaches zero when σ approaches 1.

To verify this conclusion, we show in Fig. 1 the vertical transport of moisture by convective updrafts in a simulated mesoscale convective complex over the U.S. southern Great Plains using the Weather Research and Forecasting (WRF) Model at 1-km resolution (Liu et al. 2015). Note that this direct calculation bypasses the convective closure issue for determining the individual terms on the right-hand sides of Eqs. (3) and (5) in the vertical transport formulations by assuming that they are known. We divide the model domain into subdomains of different sizes to mimic the grid spacing of large-scale and mesoscale models. As in AW13, a single top-hat-type updraft is used when calculating the parameterized vertical transport. Indeed, for all resolutions the vertical transport using the conventional parameterization [Eq. (5)] and the unified parameterization [Eq. (3)] is very close, supporting our conclusion based on the formulations. The single-updraft parameterization, however, underestimates the explicitly calculated vertical eddy transport, as also noted in AW13. The underestimation is as much as 50% and is across all scales owing to the failure of a single top-hat updraft to capture the internal variability of updrafts as shown in Liu et al. (2015).

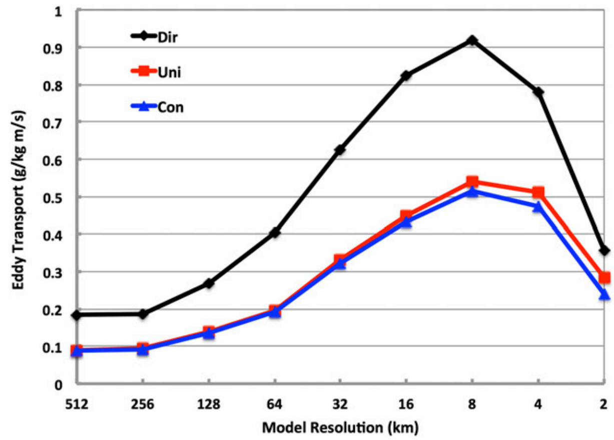

FIG. 1. Vertical transport of water vapor (g kg^{-1} m s^{-1}) at 3.5 km by convective updrafts averaged over different subdomain sizes from 2 to 512 km, estimated from direct calculation (black) and one-updraft approximation, with the conventional approach (blue) and with the unified approach (red) for a mesoscale convective complex. The differences at high resolutions are caused by the neglect of environmental vertical velocity in the conventional approach.

To summarize, AW13 presented a unified approach to parameterizing the vertical transport by convective updrafts for high-resolution numerical models. In their approach, updraft fraction σ is a key scale-aware parameter. They argue that when updraft fraction is no longer negligible, as is the case in high-resolution models, the conventional parameterization must be modified by a factor $1-\sigma$ to make it scale aware. However, we note that AW13's definition of the conventional approach [their Eq. (7)] would be more consistent with what is currently used in numerical models if their Eq. (7) were used together with their Eq. (6). We show that (i) the assumption of $\sigma \ll 1$ is unnecessary and (ii) the conventional approach as defined in Eq. (5) and used in current numerical models already contains a scale-aware factor $1-\sigma$ implicitly and can be applied to gray zone where large convective cloud fractions exist. Direct calculation of convective transport of moisture from convection-resolving simulation, in which the convective closure issue is bypassed, supports our conclusion.

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