Lagrangian Tracking Simulation of Droplet Growth in Turbulence–Turbulence Enhancement of Autoconversion Rate*

RYO ONISHI, KEIGO MATSUDA, AND KEIKO TAKAHASHI

Center for Earth Information Science and Technology, Japan Agency for Marine–Earth Science and Technology, Yokohama, Japan

(Manuscript received 2 October 2014, in final form 7 February 2015)

ABSTRACT

The authors describe the Lagrangian cloud simulator (LCS), which simulates droplet growth in air turbulence. The LCS adopts the Euler–Lagrangian framework and can provide reference data for cloud microphysical models by tracking the growth of particles individually. The collisional growth in a stagnant flow is calculated by the LCS and also by solving the stochastic collision–coalescence equation (SCE). Good agreement is obtained between the LCS and SCE simulations. Comparisons between the results for stagnant and turbulent flows confirm that in-cloud turbulence enhances collisional growth. The enhancement is well predicted by the SCE method if a proper collision model is employed. To quantify the enhancement, the paper defines the time scale of the autoconversion process, in which cloud droplets grow into raindrops through collisions, as the time taken for 10% of the cloud to become rain ($t_{10\%}$). The authors then define the turbulence enhancement factor E_{turb} as $E_{\text{turb}} = \overline{t_{10\%}[\text{NoT}]} / \overline{t_{10\%}[\text{T}]}$, where the overbar denotes the mean value of the LCS runs and the subscripts NoT and T indicate stagnant (nonturbulent) flow and turbulent flow simulations, respectively. It was found that the enhancement factor increases linearly with the energy dissipation rate, while it does not show a consistent dependence on the Reynolds number. The levels of statistical fluctuations in the autoconversion time scales were directly obtained for the first time. It is shown that the relative standard deviation of $t_{10\%}$ simply follows the power law that the binomial distribution theory predicts, independently of the flow conditions.

1. Introduction

Several mechanisms have been proposed in previous works to explain the rapid increase in the size of cloud droplets, which often results in the fast initiation of rain in the early stages of cloud development. Examples of these mechanisms include the turbulence-enhanced collision of cloud droplets (Falkovich and Pumir 2007; Grabowski and Wang 2009), turbulence entrainment (Blyth 1993; Krueger et al. 1997), giant cloud condensate nuclei (Yin et al. 2000; Van Den Heever and Cotton 2007), and turbulent dispersions of cloud droplets (Sidi

et al. 2009). The first of these is the mechanism that has received the most attention and has led to extensive research on particle collisions in turbulence (Sundaram and Collins 1997; Wang et al. 2000; Saw et al. 2008; Onishi et al. 2009; Dallas and Vassilicos 2011 and references therein).

One direction that research in this area has taken is the simulation of collisional growth by solving the stochastic collision–coalescence equation (SCE). Such research relies on accurate collision models, which consist of models for the collision kernel $K_c(r_1, r_2)$ (where r_i is the particle radius), the collision efficiency $E_c(r_1, r_2)$, and the coalescence efficiency $E_{\text{coal}}(r_1, r_2)$. Furthermore, models that take into account the influence of turbulence are required. Several turbulent collision models have been proposed, but most of them are based on databases obtained from direct numerical simulations (DNSs) in which the gravity acting on the particles is neglected. The neglect of gravity is justified only for suitably small particles (Onishi et al. 2009); otherwise, gravity influences the collision statistics in a complex manner (Grabowski and Vaillancourt 1999; Woittiez

* Supplemental information related to this paper is available at the Journals Online website: <http://dx.doi.org/10.1175/JAS-D-14-0292.s1>.

Corresponding author address: Ryo Onishi, Japan Agency for Marine–Earth Science and Technology, 3173-25 Showa-machi, Kanazawa-ku, Yokohama, Kanagawa 236-0001, Japan.
E-mail: onishi.ryo@jamstec.go.jp

et al. 2009). More seriously, hydrodynamic interactions were also often neglected in previous DNSs except for those of Ayala et al. (2007) and Onishi et al. (2013). These interactions can cause particles to avoid collisions, thus often leading to a collision efficiency $E_c < 1$. The interactions are very complex and thus modeling the collision efficiency is very difficult. Strictly speaking, $E_c(r_1, r_2)$ depends on the number concentration of the surrounding particles (Wang et al. 2008). This dependence increases the computational cost of solving the SCE. Furthermore, it also makes modeling E_c more complex. Thus, simplifications of the modeling of E_c and the SCE calculation scheme are highly desired. However, in order to check the validity of any simplifications made, we require a reference simulation that does not rely on the SCE.

This study proposes a direct simulation of droplet growth in turbulent air that is not dependent on the SCE. The DNS is named the Lagrangian cloud simulator (LCS) because it adopts the Euler–Lagrangian framework. It is based on parallel code for interacting inertial particles in isotropic turbulence (PIPIT) developed in Onishi et al. (2013), in conjunction with the collisional growth component developed in Onishi et al. (2011b). One strength of PIPIT, and therefore also of the LCS, is that, using a superposition method, hydrodynamic interactions are taken into account. The current LCS code already includes components for calculating the scalar transportation, humidity as a passive scalar, and temperature as the active scalar that influences the flow through the Boussinesq approximation. The LCS can thus directly simulate the size evolution of droplets caused by condensation as well as collision–coalescence in a turbulent flow box. This study, however, focuses on collisional growth only, leaving condensation growth for future work. The LCS should not, of course, be considered an all-encompassing computation; it is based on point-particle simulation with one-way coupling, in which droplet sizes and the influence of droplet motion on the flow are neglected when determining the kinetic motions of the particles and the flow (note that the particle sizes are considered when detecting collisions). This limits the application of the LCS to the small-particle Reynolds number regime. Despite this limitation, the LCS can provide reliable data regarding the autoconversion term, which expresses the conversion process from the cloud category (small) to the rain category (large) drops caused by collisions between the small cloud droplets. This conversion is the key process in rapid rain initiation. To the best of the authors' knowledge, this study is the first attempt at obtaining autoconversion data from a direct numerical simulation in which hydrodynamic interactions are considered. Reade and Collins (2000) numerically investigated the size evolution of initially monodisperse

particles. However, they did not consider hydrodynamic interactions or obtain autoconversion data.

In addition, the LCS can also provide reference data for investigations into the fluctuations in droplet collisional growth. The kinetic approach based on the SCE simply neglects stochastic correlations of droplet numbers in different sizes. It therefore cannot address the level and nature of stochastic correlations and fluctuations. Kostinski and Shaw (2005) showed that stochastic fluctuations can lead to a factor-of-10 acceleration in the growth of a few lucky drops. Pinsky et al. (2008) stated that lucky droplets, which experience collisions several times faster than other droplets, turn out to be large enough to trigger an intense period of collisions that lasts for a shorter time than that predicted by the SCE, in which only the mean values of kernels are used. In general, since the local rates of collision events vary in space and time, size distribution modeling should consider the stochastic nature of the coagulation process.

Through a Monte Carlo simulation, Uchida and Ohta (1969) found that a small number of large droplets grow more rapidly when distributed heterogeneously than homogeneously. Robertson (1974) found that the statistical effect is significant for the collisional growth time if the initial droplet radii are smaller than $30\text{ }\mu\text{m}$. In their Monte Carlo simulations, the coalescence of collector drops was neglected in order to avoid the need for a direct simulation. However, Gillespie (1975) outlined a rigorous direct Monte Carlo simulation that included collector-drop coalescence; Gillespie (1975) and later Valioulis and List (1984) confirmed that with decreasing total number concentration of droplets, the size distributions predicted by the SCE increasingly differed from the true stochastic average distribution owing to increasing particle stochastic correlation. Pruppacher and Klett (1998) pointed out that the literature supports the notion that the SCE is not suitable for very accurate simulations of rain formation. Wang et al. (2006b) and Alfonso et al. (2008) revisited this problem and confirmed that notion. Direct Monte Carlo simulation is, in practice, feasible only for a system containing a small number of droplets since the total number of states increases very quickly with the number of droplets. In contrast, the LCS can handle a large number of droplets [the maximum number of droplets is $O(10^7)$ in the present study] and is therefore a promising alternative to Monte Carlo simulation.

In the following section, we describe the LCS in detail. Stochastic and bulk parameterizations for the autoconversion in microphysical cloud models are introduced in section 3. The LCS is compared with stochastic and bulk simulations and the fluctuations in the autoconversion statistics are investigated in section 4. The study is concluded in section 5.

TABLE 1. Case configurations and typical turbulent statistics. The Reynolds number $Re = U_0 L_0 / \nu$, where U_0 is the representative velocity and L_0 is the representative length. The RMS of the flow velocity fluctuation is u' , $k_{\max} (= N/2)$ is the maximum wavenumber, l_η is the Kolmogorov scale, L_f is the longitudinal integral scale, and Re_λ is the Taylor-microscale-based Reynolds number.

	N^3	L_0 (m)	Re	u'/U_0	$k_{\max} l_\eta / L_0$	L_f / L_0	Re_λ	ε ($\text{cm}^2 \text{s}^{-3}$)
NoT	32^3	0.0127	0	0	—	—	0	0
T100	96^3	0.0180	97.4	1.00	2.04	1.64	66.1	100
T	96^3	0.0127	97.4	1.00	2.04	1.64	66.1	400
T1000	96^3	0.0101	97.4	1.00	2.04	1.64	66.1	1000
TR127	256^3	0.0338	360	0.98	2.06	1.44	127	400
TR206	512^3	0.0669	908	1.00	2.06	1.43	206	400

2. Lagrangian cloud simulator

a. Overview

The LCS adopts the Euler–Lagrangian hybrid framework, in which the flow motion is computed using an Euler method and the particle motion by a Lagrangian tracking method. To include the phase change of the water droplets, the humidity and temperature fields are calculated using the same Euler method as that for the flow. In this paper, however, focus is centered on collisional growth only. The flow and particle components are introduced individually.

b. Flow phase

1) GOVERNING EQUATIONS AND NUMERICAL METHODS

First we solve the three-dimensional continuity and Navier–Stokes equations for incompressible flows:

$$\nabla \cdot \mathbf{U} = 0 \quad \text{and} \quad (1)$$

$$\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{U} + \mathbf{F}(\mathbf{x}, t), \quad (2)$$

where $\mathbf{U}$ is air velocity, ρ is density, p is pressure, $\mathbf{F}$ is external forcing, and $\mathbf{x}$ is position. The kinematic viscosity ν is set to 1.5×10^{-5} , which is the value for atmospheric air at 1013 hPa and 298 K, throughout this study. The external forcing is needed to achieve a statistically steady state. This study employs reduced-communication forcing (RCF) (Onishi et al. 2011a), which is suitable for massively parallel finite-difference models (FDM), to maintain the kinetic energy with $|\mathbf{k}| < 2.5$, where $\mathbf{k}$ is a wave vector. Spatial derivatives are calculated using fourth-order central differences. The conservative scheme of Morinishi et al. (1998) is employed for the advection term and the second-order Runge–Kutta scheme for time integration. To solve the velocity–pressure coupling we use the highly simplified marker and cell (HSMAC) scheme (Hirt and Cook 1972), iterating until the root-mean square (RMS) of the velocity divergence becomes smaller than δ/Δ , where Δ is the grid spacing and δ is chosen to be 10^{-3} .

2) COMPUTATIONAL PARAMETERS

Table 1 summarizes the computational parameters for the flow calculation as well as the obtained flow statistics. The governing equations were discretized using a cubic domain of length $2\pi L_0$ (where L_0 is the representative length); periodic boundary conditions were applied in all three directions. The flow cube was discretized uniformly into N^3 grid points, resulting in the grid spacing $\Delta = 2\pi L_0 / N$. The bulk Reynolds number Re is defined by $Re = U_0 L_0 / \nu$, where U_0 is the representative velocity. In the present simulations, the forcing level was set so that we could obtain steady flows with $u'/U_0 \sim 1$, where u' is the RMS of the flow velocity fluctuations. In cases T100, T, and T1000, the same number of flow grids with the same Reynolds number were calculated, but for energy dissipation rates of 100, 400, and $1000 \text{ cm}^3 \text{s}^{-2}$, respectively, which are in the typical range observed in turbulent atmospheric clouds (e.g., Jonas 1996; Pruppacher and Klett 1998). Cases T, TR127, and TR206 obtained flows using the same energy dissipation rate ($400 \text{ cm}^3 \text{s}^{-2}$) but with different Taylor-microscale-based Reynolds numbers, Re_λ .

The Reynolds numbers computed in this study are relatively small compared to those used in other cutting-edge studies. However, the computational cost of each simulation was still quite large because each run required long time scale integrations. Typically millions of time steps were required to span the growth time scale, which was in the order of several minutes, making it hundreds of time longer than the flow integral time scale.

c. Particle phase

1) GOVERNING EQUATIONS AND NUMERICAL METHODS

Under the limit of a large ratio of the density of the particle material to that of the fluid ($\rho_p / \rho_f \gg 1$), the governing equation for water droplets is given by

$$\frac{d\mathbf{V}}{dt} = -\frac{f}{\tau_p} \{ \mathbf{V} - [\mathbf{U}(\mathbf{x}, t) + \mathbf{u}(\mathbf{x}, t)] \} + \mathbf{F}_{\text{impulse}} + \mathbf{g}, \quad (3)$$

where $\mathbf{V}$ is the particle velocity, $\mathbf{U}$ is the air velocity at the position of the droplet, $\mathbf{u}$ is the disturbance flow velocity due to the surrounding droplets, and τ_p is the particle relaxation time defined as $\tau_p = (2/9)(\rho_p/\rho_f)(r^2/\nu)$, where r is the particle radius. Here $\mathbf{F}_{\text{impulse}}$ denotes the impulsive force due to collisions and $\mathbf{g}$ is the gravity vector $[-g, 0, 0]$, where g is the gravitational acceleration. The ratio of the density of the particle material to that of the fluid, ρ_p/ρ_f , was set to 8.43×10^2 at 1013 hPa and 298 K. The drag coefficient f is defined as the ratio between the nonlinear drag and the Stokes drag (i.e., the linear drag). The nonlinear drag coefficient was formulated as (Rowe and Henwood 1961)

$$f = (1 + 0.15\text{Re}_p^{0.687}), \quad (4)$$

where Re_p is the particle Reynolds number defined as

$$\text{Re}_p = \frac{2r|\mathbf{V}(\mathbf{x}) - \mathbf{U}(\mathbf{x})|}{\nu}. \quad (5)$$

The second-order Runge–Kutta method was used for the time integration. The flow velocity at the droplet position was linearly interpolated from the adjacent grid values. This simple linear interpolation was justified through comparisons with the cubic Hermitian, cubic Lagrangian, and fifth-order Lagrangian interpolations from Sundaram and Collins (1996). The particle mass and volume fractions were so dilute that flow modulation was ignored and only binary collisions were considered. It was assumed that colliding particles immediately united and

conserved mass and momentum; that is, the coalescence efficiency was unity. This assumption can be justified by the fact that most of the particles were smaller than $100 \mu\text{m}$ and the surface tension was strong enough to inhibit the subsequent breakups. The fact that weak charges exist in natural clouds may also support this assumption. Laboratory studies of small colliding droplets show that the coalescence efficiency is close to unity if the droplets are charged and an electrical field is present (Rogers and Yau 1989).

2) HYDRODYNAMIC INTERACTION CALCULATION

One important physical process, which is often neglected because of the high computational cost associated with simulating it, is the hydrodynamic interaction (HI) between particles. While moving in a flow medium, a particle induces a flow disturbance in its vicinity, which may lead to a collision efficiency $E_c < 1$. Note that E_c is defined by

$$E_c(r_1, r_2) = \frac{K_c(r_1, r_2)[\text{HI}]}{K_c(r_1, r_2)[\text{NoHI}]}, \quad (6)$$

where [HI] indicates that the HI is considered and [NoHI] indicates that it is not. If we assume the disturbance flow to be a Stokes flow, which is valid for $\text{Re}_p \leq 1$ (strictly speaking $\text{Re}_p \leq 0.1$, but is often relaxed to $\text{Re}_p \leq 1$), the disturbance at $\mathbf{x}$ due to a droplet at $\mathbf{y}$ becomes

$$\mathbf{u}_{\text{St}}[\mathbf{x}; r, \mathbf{U}_{\text{rel}}(\mathbf{y})] = \begin{cases} \left[-\frac{3}{4} \frac{r}{d} + \frac{3}{4} \left(\frac{r}{d} \right)^3 \right] [\mathbf{U}_{\text{rel}}(\mathbf{y}) \cdot \mathbf{d}] \frac{\mathbf{d}}{d^2} - \left[\frac{3}{4} \frac{r}{d} + \frac{1}{4} \left(\frac{r}{d} \right)^3 \right] \mathbf{U}_{\text{rel}}(\mathbf{y}) & \text{for } d > r, \\ 0 & \text{for } d \leq r \end{cases}, \quad (7)$$

where r is the particle radius, d ($=|\mathbf{d}| = |\mathbf{x} - \mathbf{y}|$) is the distance from the particle center, and $\mathbf{U}_{\text{rel}}(\mathbf{y}) = \mathbf{U}(\mathbf{y}) - \mathbf{V}(\mathbf{y})$.

The disturbance flow field at point $\mathbf{x}$ in a system containing N_p particles can be written as

$$\mathbf{u}(\mathbf{x}) = \sum_{k=1}^{N_p} \mathbf{u}_{\text{St}}^k(\mathbf{x}), \quad (8)$$

where

$$\mathbf{u}_{\text{St}}^k(\mathbf{x}) \equiv \mathbf{u}_{\text{St}}(\mathbf{x}; r^k, \mathbf{U}_{\text{rel}}^k). \quad (9)$$

Here, a superscript k indicates the disturbance due to the k th particle.

Since the Stokes disturbance flows above are governed by a linear equation, they can be superimposed to

satisfy the same local Stokes equation. The superposition method for calculating the hydrodynamic interaction stands on this basis (Pruppacher and Klett 1998). However, Wang et al. (2005) pointed out that the original superposition method (OSM) does not satisfy the no-slip boundary condition for a system of multiple particles. Ayala et al. (2007) developed an iterative superposition method (ItrSM) that is more reliable but computationally expensive owing to its iteration procedure. Recently, Onishi et al. (2013) proposed an intermediate method, named the binary-based superposition method (BiSM), between OSM and ItrSM in terms of both computational cost and reliability. BiSM assumes that interactions via three or more particles are negligible. This dramatically reduces computational cost but maintains reliability as long as the particle number

TABLE 2. Case configurations.

	Flow	HI	n_{p0} (m^{-3})	N_{p0}
NoT-NoHI	Stagnant	No	1.42×10^8	7.24×10^4
NoT-HI		Yes		7.24×10^4
T-NoHI	Turbulent	No	1.42×10^8	7.24×10^4
T-HI		Yes		7.24×10^4
T100-HI		Yes		2.05×10^5
T1000-HI		Yes		3.64×10^4
TR127-HI		Yes		1.36×10^6
TR206-HI		Yes		1.06×10^7

concentration is small, as observed in atmospheric clouds.

The superposition method is valid only for small particles whose particle Reynolds number is smaller than unity. This requirement corresponds to $r < 40 \mu\text{m}$ (e.g., Fig. 4 in Wang et al. 2006a). Since the superposition method is not accurate for larger droplets, the associated error will grow. It should also be noted that the superposition method, which assumes the Stokes linear flow, becomes incompatible with the nonlinear drag model for such large droplets. However, the streamlines around the particle remain symmetrical with respect to the flow axis, and no standing eddy is present as far as Re_p is smaller than 20, which corresponds to $r < 150 \mu\text{m}$. Therefore, we do not anticipate a substantial error in this stretched range.

3) COMPUTATIONAL PARAMETERS

The initial size distribution was set to the following exponential form (e.g., Soong 1974):

$$n_{p0}(m) = \frac{n_0}{\bar{m}} \exp(-m/\bar{m}). \quad (10)$$

Here, $\bar{m}$ is the mass of a droplet with a radius of $15 \mu\text{m}$ and the initial number density $n_0 = 1.42 \times 10^8 \text{m}^{-3}$ (Table 2). The corresponding initial liquid water content was set to 2.0g m^{-3} . The exponential distribution of size has a problem: it contains very small droplets. The small-particle relaxation times for such small droplets require an extremely small time step. To avoid this, we considered particles with radii smaller than $4 \mu\text{m}$ to be tracer particles. We confirmed that this treatment does not affect the results (not shown).

Periodic boundary conditions were applied in all three directions; for example, if a droplet passes through the bottom boundary, it arrives immediately at the top boundary. Such periodicity in the direction of gravity leads to an extreme situation in that particles under a stagnant flow grow only in fixed vertical columns, which terminates the collisional growth very quickly. To avoid this side effect of periodicity, random perturbations were added in the lateral directions when droplets were

reintroduced from the top boundary. This treatment may have induced excess collisions, but the results obtained (as will be seen in Fig. 1a) justify its use. In the case of turbulent flow, artificial perturbations were not necessary because the turbulent fluctuations already served this purpose.

3. Microphysical cloud model

a. Cloud microphysics simulation

Cloud microphysics simulations consider continuous values for the water content. For example, the spectral-bin scheme prognoses the mass distribution function. We define $\xi(\psi, t)$, where $\psi = \log_{10} r$, as

$$\xi(\psi, t) d\psi = n_p(r, t) m(r) dr, \quad (11)$$

where n_p is the number density function. As $n_p(m, t) = n_p(r, t) dr/dm$, we obtain

$$n_p(m, t) = \frac{\xi(\psi, t)}{3m^2 \ln 10}. \quad (12)$$

The coagulation growth of droplets is modeled by the SCE:

$$\begin{aligned} \frac{\partial n_p(m, t)}{\partial t} &= \int_0^{m/2} K_{\text{coag}}(m - m', m') n_p(m - m', t) n_p(m', t) dm' \\ &\quad - \int_0^\infty K_{\text{coag}}(m, m') n_p(m, t) n_p(m', t) dm'. \end{aligned} \quad (13)$$

The coagulation kernel K_{coag} is equal to the product $E_{\text{coal}} E_c K_c$, where E_{coal} , E_c , and K_c are the coalescence efficiency, collision efficiency, and collision kernel, respectively.

The coagulation component of the spectral bin model in the Multi-Scale Simulator for the Geoenvironment (MSSG-Bin) cloud physics model (Onishi and Takahashi 2012) was used when solving the SCE. The mass coordinate m was discretized as $m_k = 2^{1/s} m_{k-1}$, where s was

set to 16. The representative radius of the first bin was $2.7\ \mu\text{m}$ and 528 classes were calculated, the largest class of which had a representative radius of $5.4\ \text{mm}$.

Strictly speaking, $E_c(r_1, r_2)$ depends on the number concentration n_p of the surrounding particles. This dependence increases the computational cost associated with solving the SCE. Furthermore, it makes the modeling of E_c more complex. This fact is often overlooked by cloud microphysics researchers. It is frequently assumed that the number density is sufficiently low for the n_p dependence to be ignored. However, it was clarified that typical cloud conditions can result in nonnegligible n_p dependence because the length of influence of the disturbance flow is tens of times larger than the particles' radii (Wang et al. 2008).

The gravitational collision kernel describes the collisions due to the settling velocity difference in the form of

$$K_{c,\text{grav}}(r_1, r_2) = \pi R_{12}^2 |V_\infty(r_1) - V_\infty(r_2)|, \quad (14)$$

where $R_{12} (= r_1 + r_2)$ is the collision radius and V_∞ is the gravitational particle settling velocity. As for turbulent

collision kernels, the most well known is that developed by Saffman and Turner (1956). The Saffman and Turner model is for particles with no inertia—that is, for $\text{St} = 0$, where St is the Stokes number defined as $\text{St} = \tau_p/\tau_\eta$ and τ_η is the Kolmogorov time. In reality, cloud droplets and raindrops have finite inertia, which results in the collision rate being on the order of 10 times higher. Many models have been proposed for particles with finite inertia, such as the Ayala model (Ayala et al. 2008a), the Zaichik model (Zaichik et al. 2003), the Derevyanko model (Derevyanko et al. 2008), and the Onishi model (see appendix), but all of these rely on empirical parameters.

There are a number of parameterizations for the collision efficiency as well. Hall (1980) compiled and tabulated the collision efficiency of droplets. Pinsky et al. (2001, hereafter PKS2001) tabulated the collision efficiencies for a wide range of particle Reynolds numbers. Long (1974) approximated the collision efficiency between large and small droplets as

$$E_c(r_L, r_s) = \begin{cases} 4.5 \times 10^8 r_L^2 \times \max(1 - 3 \times 10^{-6}/r_s, 0) & \text{for } r_L < 50 \times 10^{-6} \text{ m,} \\ 1 & \text{otherwise} \end{cases}, \quad (15)$$

where r_L and r_s are the radii of the collector drop and the small droplet, respectively. All of the above three models are for stagnant flows. Wang et al. (2008, hereafter WARG2008) tabulated the turbulence enhancement of the collision efficiency η_E , which is defined as

$$\eta_E(r_1, r_2) = \frac{E_c(r_1, r_2)[\text{T}]}{E_c(r_1, r_2)[\text{NoT}]}, \quad (16)$$

where [T] and [NoT] indicate the turbulent flow case and the stagnant (nonturbulent) flow case, respectively. By multiplying the collision efficiencies for the stagnant flow by η_E , we obtain collision efficiencies for turbulent flows. All the tabulated collision efficiency values in the referenced literature were linearly interpolated into the required tables for the present SCE simulations. For the coalescence efficiency, we assumed $E_{\text{coal}} = 1$ in the stochastic simulations as well as in the LCS simulations.

The spectral-bin microphysics cloud model is computationally very costly and the so-called bulk cloud microphysics model is usually used for cloud simulations. The bulk model prognoses the cloud water content (mass of droplets with radii smaller than $40\ \mu\text{m}$ in a unit volume) L_c and the rainwater content (mass of drops with radii larger than $40\ \mu\text{m}$ in a unit volume) L_r in each numerical grid and calculates their time evolutions. The time evolution of the cloud water content can be written as

$$\frac{\partial L_c}{\partial t} = -\text{ADV}(L_c) + \text{DIFF}(L_c) - P_{\text{auto}} - P_{\text{accr}} + P_{\text{actv}}, \quad (17)$$

where ADV and DIFF are the advection and diffusion terms, respectively. The subscript auto indicates the autoconversion term, accr indicates the accretion term, and actv indicates the activation term. The autoconversion term describes the conversion from cloud to rain due to collisions between cloud droplets, while the accretion term describes the conversion from cloud to rain due to raindrops collecting cloud droplets. The activation term describes the nucleation of tiny cloud droplets due to aerosol activation (dry aerosols to wet aerosols).

b. Autoconversion rate

Autoconversion parameterization describes the rate of conversion of water droplets from the cloud droplet category to raindrops due to collisions with other cloud droplets. It is a significant and sensitive process in bulk microphysical cloud models. Kessler (1969) proposed one of the first common expressions that assumes a linear dependence on the cloud mixing ratio $q_c (= L_c/\rho_{\text{dry}}$, where ρ_{dry} is the dry air mass density):

$$P_{\text{auto}} = \alpha \rho (q_c - q_{c,0}) H(q_c - q_{c,0}). \quad (18)$$

Here, α is a tuning constant, $q_{c,0}$ is the threshold water mixing ratio, and H is the Heaviside step function. In this

study $\alpha = 10^{-3} \text{ s}^{-1}$ and $q_{c,0} = 0.5 \times 10^{-3} \text{ kg kg}^{-1}$ following the work of [Reisner et al. \(1998\)](#). [The values for α and $q_{c,0}$ are rather arbitrary. [Khairoutdinov and Kogan \(2000\)](#) chose $\alpha = 10^{-3} \text{ s}^{-1}$ and $q_{c,0} = 10^{-3} \text{ kg kg}^{-1}$ for deep cumulus clouds. [Thompson et al. \(2004\)](#) chose $\alpha = 10^{-3} \text{ s}^{-1}$ and $q_{c,0} = 0.35 \times 10^{-3} \text{ kg kg}^{-1}$.]

The model developed by [Seifert et al. \(2010, hereafter SNS2010\)](#), and that by [Franklin \(2008\)](#), explicitly consider turbulent collision enhancement. The former considers the turbulence enhancement of the collision efficiency η_E , while the latter ignores it while showing that η_E is not substantial. Since η_E is influential in the present simulations, as will be shown later, our discussion adopts the SNS2010 model. SNS2010 assumed a gamma distribution of cloud droplets written as $n_p(m) = Am^\mu e^{-Bm}$, where m is the particle mass and μ is the shape parameter. The autoconversion parameterization was subsequently proposed as

$$P_{\text{auto}} = \frac{k_{cc}}{20m^*} \frac{(\mu+2)(\mu+4)}{(\mu+1)^2} L_c^2 \bar{m}_c^{-2} \left[1 + \frac{\Phi_{\text{auto}}(\tau)}{(1-\tau)^2} \right], \quad (19)$$

where $m^* (= 2.6 \times 10^{-10} \text{ kg})$ is the separating mass between cloud droplets and raindrops and $\bar{m}_c$ is the mean mass of the cloud droplets ($\bar{m}_c = L_c/N_c \approx L_c/N_p$, where N_c is the number concentration of the cloud droplets and N_p is the combined number concentration of droplets and drops). The rain mass ratio $\tau = L_r/(L_c + L_r)$ describes the evolution of the autoconversion process during a typical rain formation event. The universal function $\Phi_{\text{auto}}(\tau)$ is given by

$$\Phi_{\text{auto}}(\tau) = 600\tau^{0.68}(1 - \tau^{0.68})^3. \quad (20)$$

The kernel parameter for autoconversion was given by $k_{cc} = k_{cc,0} = 9.44 \times 10^9 \text{ s}^{-1} \text{ kg}^{-2}$ on account of the piecewise polynomial approximation of the gravitational collection kernel of [Long \(1974\)](#). The constant $k_{cc,0}$ is for the nonturbulent case. SNS2010, on the other hand, conducted more than 10 000 simulations using the turbulent collision kernel of [Ayala et al. \(2008a\)](#) with the product of the collision efficiencies of [PKS2001](#) and the turbulence enhancement η_E of [WARG2008](#) and suggested the parameterization $k_{cc}(\bar{r}_c, \mu, \varepsilon, \text{Re}_\lambda)$ for k_{cc} , where

$$k_{cc}(\bar{r}_c, \mu, \varepsilon, \text{Re}_\lambda) = k_{cc,0} \left\{ 1 + \varepsilon \text{Re}_\lambda^{1/4} \left[\alpha_{cc}(\mu) \exp \left\{ - \left[\frac{\bar{r}_c - r_{cc}(\mu)}{\sigma_{cc}(\mu)} \right]^2 \right\} + \beta_{cc} \right] \right\}. \quad (21)$$

Here, $\bar{r}_c$ is the mean droplet radius and $\alpha_{cc}(\mu)$, $r_{cc}(\mu)$, and $\sigma_{cc}(\mu)$ are empirical coefficients that depend on the shape parameter μ (see Table I in [SNS2010](#)). Here we define the turbulence enhancement η_{SNS} as

$$\eta_{\text{SNS}} (= k_{cc}/k_{cc,0}) = 1 + C_{\text{SNS}} \varepsilon \text{Re}_\lambda^{1/4}, \quad (22)$$

where C_{SNS} is the term between brackets in Eq. (21) and is a function of $\bar{r}_c$ and μ . This form is examined in the discussion section.

The Kessler model can estimate the autoconversion rate if L_c is given, while the SNS2010 model can be used if L_c , L_r , N_p , μ , ε , and Re_λ are known. LCS can provide all but the shape parameter μ . The initial shape parameter for the LCS simulations was set to zero [see Eq. (10)] in this study. Thus, we simply assumed that the value of zero holds over the whole time integration since the form of the size distribution does not change much in the autoconversion-dominated early stage of growth. (This will be seen below in [Figs. 2 and 4](#).) In order for comparisons to be made with model estimates and the LCS results regarding the autoconversion, the model estimates at each time step were calculated with the aid of the LCS data.

4. Numerical results and discussion

a. Collisional growth in the stagnant flow

[Figure 1](#) shows the time evolutions of the autoconversion and accretion rates obtained from the LCS simulations for a stagnant flow, together with model predictions. [Figure 1a](#) shows the case without HI (NoT-NoHI) and [Fig. 1b](#) shows that with HI (NoT-HI). The shaded areas for the LCS results represent the spread of five runs and the dashed lines inside show the runs' mean value. [Figure 1a](#) shows that the LCS results and the SCE calculation with the gravitational collision kernel are in good agreement throughout the whole time integration, confirming the reliability of LCS. This good agreement also supports the notion that the SCE may be used with confidence when collision growth is predominantly controlled by cloud droplet self-collection (section 15.3.1.3 in [Pruppacher and Klett 1998](#)).

The comparison between [Figs. 1a and 1b](#) confirms that HI suppresses the conversion. The SCE simulations with collision efficiency models can predict the change induced by the inclusion of HI. The SCE result with the Hall model and that with the [PKS2001](#) model are almost identical. This was as expected because the [PKS2001](#)

FIG. 1. Time evolutions of the autoconversion rate, accretion rate, and the rain mass ratio for stagnant flow (a) without considering the hydrodynamic interaction (NoT-NoHI) and (b) considering the hydrodynamic interaction (NoT-HI).

model utilizes recent computer simulation techniques to extend the reliable particle size further than that in the Hall model. The reliabilities for small particles are similar between the two. Both of these SCEs slightly overestimate the autoconversion rate, while that with the Long model underestimates it. Interestingly, the Long model, which is based on a very simple parameterization [Eq. (15)], seems to be most suitable for

modeling the accretion rate. However, the simplified estimate of $E_c = 1$ causes a serious overestimate in turbulent flow cases (not shown). Therefore, this study employed the [PKS2001](#) model from this point onward. [SNS2010](#) provides a better estimate than the Kessler model, but the error becomes larger toward the end of the time integration. The late stage discrepancy would be partly attributed to the simple assumption of $\mu = 0$ (time constant) in the present study, despite the fact that the size distribution shape evolves in time.

[Figure 2](#) shows temporal evolutions of the particle size distribution for a stagnant flow ([Fig. 2a](#)) without HI (NoT-NoHI) and ([Fig. 2b](#)) with HI (NoT-HI). The black solid lines show the initial size distributions. The shaded areas for the LCS results represent the spread of five runs and the lines within show the mean. The results for the SCE with the gravitational collision kernel and the [PKS2001](#) collision efficiency are plotted in [Fig. 2b](#). It is clear that the size broadening occurs to a lesser extent in the HI case than in the NoHI case, corresponding to the lower conversion rate in [Fig. 2b](#) than in [Fig. 2a](#). The SCE predictions agree well with the mean LCS results for both the HI and NoHI cases, confirming the reliability of the HI calculation in LCS. The collision efficiency model used does not consider number concentration dependence, such as was reported in [WARG2008](#). However, good agreements between LCS simulations and SCE simulations with the simple collision efficiency model indicate that the number concentration dependence of the collision efficiency would not be relevant for the system with droplets of various sizes.

b. Turbulence collision enhancement

[Figure 3](#) shows the time evolution of the autoconversion and accretion rates inclusive of HI for a turbulent flow with $\varepsilon = 400 \text{ cm}^2 \text{ s}^{-3}$ (T-HI case). As before, the shaded areas show the spread for five LCS runs and the dashed lines show the mean. In the SCE simulation, the Onishi model (see [appendix](#)) was employed for the turbulent collision kernel. To obtain collision efficiencies for turbulent flows, we multiplied the [PKS2001](#) collision efficiencies by the η_E of [WARG2008](#). The figure clearly shows that the [PKS2001](#) collision efficiency (for a stagnant flow) reduces the collision rates too far but η_E restores the rates to the correct level.

[Figure 4](#) shows the temporal evolution of the particle size distribution in a turbulent flow. In accordance with the agreement obtained between the SCE and LCS for the conversion rates, the figure shows good correspondence between the two sets of results. This further verifies the reliability of the LCS, as well as the SCE simulation with the Onishi collision kernel model, and

FIG. 2. Temporal changes in the size distribution due to collisional growth for a stagnant flow (a) without the hydrodynamic interactions (NoT-NoHI) and (b) with the hydrodynamic interactions (NoT-HI). The [PKS2001](#) collision efficiency was used in the SCE calculation for the HI case.

the [PKS2001](#) collision efficiency multiplied by the η_E of [WARG2008](#).

Figure 5 shows the autoconversion rates in turbulent flows for different energy dissipation rates. Note that [WARG2008](#) provided η_E for $\varepsilon = 100$ and $400 \text{ cm}^2 \text{ s}^{-3}$, but not for $\varepsilon = 1000 \text{ cm}^2 \text{ s}^{-3}$, which was one of the values used in this study. For the $\varepsilon = 1000 \text{ cm}^2 \text{ s}^{-3}$ case, we simply used the value of η_E for $\varepsilon = 400 \text{ cm}^2 \text{ s}^{-3}$. This

choice was justified by the good agreement between the LCS and SCE simulations. It also implies that the ε dependence of the collision efficiency is not critical when droplets of various sizes are included in the system. The maximum values in the figure clearly show that the autoconversion rate increases and reaches its highest value earlier for larger energy dissipation rates. It is also shown that the SCE simulation can predict the ε dependence. The shift in time of the peak autoconversion rate is caused by the decay of the autoconversion rate occurring earlier, which is due to the quicker consumption of cloud droplets for larger ε . The maximum value of the autoconversion rate is determined by the balance between the turbulence collision enhancement and the enhanced consumption of the droplets; therefore, it is not a good measure of the turbulence enhancement alone.

This paper proposes a quantitative measurement method for the turbulence enhancement, focusing on the time scale of the autoconversion process. We first define the time required for a cloud to convert 10% of its cloud mass into rain category drops as $t_{10\%}$. Figures 1 and 3 show that, roughly speaking, the autoconversion rate is larger than the accretion rate for the period of $t < t_{10\%}$. After this stage, the accretion process plays a dominant role in the growth. As such, the duration of this stage can be used as a measure of the autoconversion time scale T_{auto} . Thus, this study considers $T_{\text{auto}} = \overline{t_{10\%}}$, where the overbar indicates the mean value. Note that $t_{10\%}$ in each LCS run fluctuates around the mean value, while it is a unique deterministic value in each SCE simulation. As the autoconversion rate has the unit of inverse time, the effective autoconversion rate is proportional to T_{auto}^{-1} . Consequently, we can define the turbulence enhancement factor as

$$E_{\text{turb}} = \frac{\overline{t_{10\%}}[\text{NoT}]}{\overline{t_{10\%}}[\text{T}]} \quad (23)$$

Table 3 lists the values of $t_{10\%}$ and E_{turb} obtained using the present LCS. [Xue et al. \(2008\)](#) investigated turbulence enhancement in collision growth using a SCE simulation based on the Ayala model. They defined a characteristic time for the growth of cloud droplets as the time that the radar reflectivity reaches 20 dBZ and reported 20%–29% enhancement for $\varepsilon = 100 \text{ cm}^2 \text{ s}^{-3}$ and 31%–41% enhancement for $\varepsilon = 400 \text{ cm}^2 \text{ s}^{-3}$. Our 8% enhancement for the former case is slightly smaller, while our 42% enhancement for the latter is in good agreement. [Pinsky and Khain \(2002\)](#) listed turbulence enhancement in terms of collision kernels due to an increase in both the swept volume and collision efficiency. The listed factors ranged from 1.2 to 2 for droplet radii in

FIG. 3. Temporal evolutions of the autoconversion rate, accretion rate, and the rain mass ratio for the T-HI case. The product of the **PKS2001** collision efficiency and η_E of **WARG2008** was used as the collision efficiency for the turbulent flow.

droplet pairs ranging from 10 to 40 μm in a turbulent medium with $\varepsilon = 100\text{--}200\text{ cm}^2\text{ s}^{-3}$. The present enhancement factor for $\varepsilon = 100\text{ cm}^2\text{ s}^{-3}$ is 1.09, which is slightly smaller than the range in **Pinsky and Khain (2002)** but of a similar magnitude.

Figure 6a contains the plots of E_{turb} , listed in Table 3, showing that the enhancement factor can be well approximated by a linear function of the energy dissipation rate:

$$E_{\text{turb}} \sim 1 + C_e \varepsilon. \quad (24)$$

Here, $C_e = 11.0\text{ s}^3\text{ m}^{-2}$. This linear function with respect to ε agrees with the enhancement parameterization in **SNS2010** [see Eq. (22)]. When comparing our E_{turb} to η_{SNS} , we need to estimate C_{SNS} , which is a function of $\bar{r}_c$ and μ . In the present HI simulations, $\bar{r}_c$ grew from 15.0 μm to 19.0 $\sim$ 20.1 μm (depends on the flow condition) for the autoconversion growth time $\bar{t}_{10\%}$. Integration of C_{SNS} , assuming $\mu = 0$, and with respect to $\bar{r}_c$ from 15.0 to 20.0 μm yields $C_{\text{SNS}} = 5.22$. That is, we obtain $\eta_{\text{SNS}} = 1 + 5.22\varepsilon\text{Re}_\lambda^{1/4}$. Substitution of $\text{Re}_\lambda = 66.0$

into this yields $\eta_{\text{SNS}} = 1 + 14.9\varepsilon$, which is in good agreement with Eq. (24).

Equation (22) further predicts a Reynolds number dependence of the turbulence enhancement. Figure 6b shows the turbulence enhancement for $\varepsilon = 400\text{ cm}^2\text{ s}^{-3}$, where η_{SNS} and E_{turb} show a significant discrepancy. Turbulence enhancement η_{SNS} increases as Re_λ increases, while E_{turb} does not show a consistent Reynolds number dependence. A careful comparison between the two small Re_λ cases (i.e., T-HI and TR127-HI) shows a small increase in E_{turb} due to an increase in Re_λ . **SNS2010**, which relied on the DNS results of **Ayala et al. (2008b)** for Re_λ ranging from 23 to 72, may have captured this Reynolds number dependence. Recent high-performance DNS data have clarified that the collision kernels for particles with St smaller than unity would slightly decrease with increasing Re_λ for high Reynolds numbers (**Onishi and Vassilicos 2014; Onishi et al. 2013; Rosa et al. 2013**). The Reynolds number dependence in **SNS2010** would not be valid for atmospheric turbulent clouds in which observations have shown $\text{Re}_\lambda \sim 10^3\text{--}10^4$.

FIG. 4. Temporal change in the size distribution due to collisional growth for the T-HI case. The Onishi collision kernel with the product of the [PKS2001](#) collision efficiency and the η_E of [WARG2008](#) was used for the SCE simulation.

c. Fluctuation in autoconversion time scales

To investigate the statistical fluctuations, we calculate the standard deviations of bulk cloud microphysics properties such as $t_{10\%}$ and $\tau_{10\%}$ (rain mass ratio τ at $t = t_{10\%}$). We define the probability of a cloud droplet with an initial radius smaller than $40 \mu\text{m}$ (all the droplets are initially smaller than $40 \mu\text{m}$ in the present setting) becoming larger than $40 \mu\text{m}$ within $t_{10\%}$ as p (constant). Assuming that independent yes–no trials are conducted for N_{p0} times (although independent trials are not particularly accurate for coalescence events), each of which yields success with p , the binomial distribution theory shows that the expected number of drops larger than $40 \mu\text{m}$ at $t_{10\%}$ is $N_{p0}p$ and that the standard deviation of the number is $\sqrt{N_{p0}p(1-p)}$. This leads to the relative standard deviation γ of $\tau_{10\%}$:

$$\gamma(\tau_{10\%}) = \sqrt{\frac{1-p}{N_{p0}p}}. \quad (25)$$

We investigate the confidence level for the calculated deviations. In general, a large number of samples are required to accurately estimate the population's statistical deviation. However, it is difficult to obtain such large numbers, so instead, we need to know the level of confidence for each calculated variance for a limited number of samples. We define σ^{*2} as the population variance and σ_n^2 as the variance of n samples. If the probability of variable X is distributed normally, $\sum_{i=1}^n (X_i - \bar{X})^2 / \sigma^{*2}$ follows the

FIG. 5. Autoconversion rates for different energy dissipation rates. The Onishi collision kernel with the product of the [PKS2001](#) collision efficiency and η_E of [WARG2008](#) was used for the SCE simulations.

chi distribution with $n - 1$ degrees of freedom, whose variance is $2(n - 1)$. As $\sigma_n^2 = (n - 1)^{-1} \sum_{i=1}^n (X_i - \bar{X})^2$, the standard deviation of σ_n^2 / σ^{*2} becomes

$$\text{stdev}\left(\frac{\sigma_n^2}{\sigma^{*2}}\right) = \sqrt{\frac{2}{n-1}}. \quad (26)$$

This means that the estimated error level of the standard deviation, for say 100 samples, is 14%.

[Table 4](#) shows the relative standard deviations (standard deviation divided by the mean) of $t_{10\%}$ and $\tau_{10\%}$ —that is, $\gamma(t_{10\%})$ and $\gamma(\tau_{10\%})$ respectively. The substitution of $p = 0.1$ and $N_{p0} = 72\,400$ into Eq. (25) gives $\gamma = 0.0111$. The calculated value for the corresponding case was 0.0616 for the T-HI case, which is larger than the prediction. Interestingly, [Table 4](#) shows that $\gamma(\tau_{10\%})$ for T-HI is smaller than that for NoT-HI. This means that the turbulence reduces the level of statistical fluctuations, which contradicts the intuitive prediction that turbulence induces extra statistical fluctuations. This somewhat surprising result can be explained by the fact

TABLE 3. Time scales of the autoconversion rate for different energy dissipation rates and Reynolds numbers. E_{turb} is the turbulence enhancement factor defined by Eq. (23).

	No. of runs	Re_λ	ε ($\text{cm}^2 \text{s}^{-3}$)	$\overline{t_{10\%}}$ (s)	E_{turb}
NoT-HI	100	0	0	270.2	1.00 (reference)
T100-HI	30	66	100	250.3	1.08
T-HI	100	66	400	189.8	1.42
T1000-HI	30	66	1000	129.1	2.09
TR127-HI	30	127	400	186.2	1.45
TR206-HI	5	206	400	190.7	1.42

FIG. 6. Turbulence enhancement factors as a function of (left) the energy dissipation rate ϵ and (right) the Taylor-microscale-based Reynolds number Re_λ . $Re_\lambda = 66$ for (left) and $\epsilon = 400 \text{ cm}^2 \text{ s}^{-3}$ for (right).

that the turbulence lessens the high size dependence of the gravity-induced collision rate. For example, $K_{\text{coal}}(43 \mu\text{m}, 30 \mu\text{m})/K_{\text{coal}}(38 \mu\text{m}, 30 \mu\text{m})$ (note that $38 = 2^{1/3} \times 30$ and $43 = 3^{1/3} \times 30$) was 1.4 for a turbulent flow (estimated using the Onishi kernel model with the PKS2001 collision efficiency multiplied by η_E of WARG2008), which is much smaller than 2.9 for a stagnant flow (estimated from the gravitational kernel model with the PKS2001 collision efficiency). This implies that p is less dependent on r in turbulent flows than in stagnant flows.

Figure 7 plots $\gamma(\tau_{10\%})$ against the initial number of droplets. The error bars were estimated by substituting the number of runs into Eq. (26). It is clear that all the plots are larger than the values predicted by Eq. (25) and approximately follow $\gamma(\tau_{10\%}) = 5\sqrt{(1-p)/(N_{p0}p)}$ irrespective of Re_λ . The power law of this relationship is as predicted by Eq. (25). The source of the factor 5 is not very clear. While some fraction of it can be attributed to the fact that p is not a constant and depends on the initial size of the droplet, other parts can be due to the fact that the independence assumption for the trials used for the binomial distribution theory is not valid for coalescence events. One conjecture that can be gleaned from the fact that a number of plots from various flow conditions

more or less collapse into the power law function of the number of droplets is that the flow state does not have substantial influence on autoconversion fluctuations.

Note that $\gamma(\tau_{10\%})/\gamma(t_{10\%})$ and in Table 4 is around three. The ratio $\gamma(\tau_{10\%})/\gamma(t_{10\%})$ depends on the function relating τ and t . We can assume this function to be exponential for $t < t_{10\%}$, based on the time evolutions of τ in Figs. 1 and 3:

$$\tau = C_\tau t^\alpha. \quad (27)$$

Here, C_τ is a coefficient. For $t = \overline{t_{10\%}}$, we obtain $\tau_{10\%} = C_\tau \overline{t_{10\%}}^\alpha$ and $(\delta\tau/\delta t)_{t=t_{10\%}} = C_\tau \alpha \overline{t_{10\%}}^{\alpha-1}$, leading to $(\delta\tau/\tau_{10\%})_{t=t_{10\%}} = \alpha(\delta t/\overline{t_{10\%}})_{t=t_{10\%}}$. That is, $\gamma(\tau_{10\%})/\gamma(t_{10\%})$ corresponds to α , irrespective of C_τ . When $\alpha \approx 3$ this indicates that the total conversion rate ($P_{\text{auto}} + P_{\text{accr}}$) is proportional to the square of time since τ is obtained by integrating the total conversion rate with respect to time. Also, $\alpha \approx 3$, together with $\gamma(\tau_{10\%}) = 5\sqrt{(1-p)/(N_{p0}p)}$, suggests that $\gamma(t_{10\%})$ roughly follows $1.7\sqrt{(1-p)/(N_{p0}p)}$.

The relative standard deviation of $t_{10\%}$ (defined as the autoconversion time scale) shows the level of statistical fluctuations in P_{auto} . When Re_λ is as high as 10^3 and 10^4 , as is the case in atmospheric turbulent clouds, with

TABLE 4. Relative standard deviations of $t_{10\%}$ and $\tau_{10\%}$.

	No. of runs	N_{p0}	$\gamma(t_{10\%})$	$\gamma(\tau_{10\%})$	$\gamma(\tau_{10\%})/\gamma(t_{10\%})$
NoT-HI	100	7.24×10^4	0.0260	0.0867	3.33
T100-HI	30	2.05×10^5	0.0141	0.0438	3.11
T-HI	100	7.24×10^4	0.0213	0.0616	2.89
T1000-HI	30	3.64×10^4	0.0236	0.0698	2.96
TR127-HI	30	1.36×10^6	0.004 70	0.0152	3.23
TR206-HI	5	1.06×10^7	0.001 08	0.003 33	3.08

FIG. 7. Calculated relative standard deviations against the initial number of droplets. Data from Table 4. The solid line illustrates Eq. (25) and the dashed line shows $\gamma = 5\sqrt{(1-p)/(N_{p0}p)}$.

$n_{p0} = 1.42 \times 10^8 \text{ m}^{-3}$ and $\varepsilon = 400 \text{ cm}^2 \text{ s}^{-3}$ (the same conditions as the T-HI case), we obtain $N_{p0} = 1.30 \times 10^{10}$ and 4.09×10^{14} , respectively. Substitutions of these values into $\gamma(t_{10\%}) = 1.7\sqrt{(1-p)/(N_{p0}p)}$ give $\gamma(t_{10\%})$ values of 4.5×10^{-5} and 2.5×10^{-7} , respectively. These small but finite statistical fluctuations due to the stochastic nature of collisions may make a difference in weather simulations.

5. Concluding remarks

We developed the Lagrangian cloud simulator (LCS) based on the parallel code for interacting inertial particles in homogeneous isotropic turbulence (PIPIT in Onishi et al. 2013). The collisional growth component developed in Onishi et al. (2011b) and a condensational growth component were implemented in PIPIT to complete the LCS. The LCS calculates the flow using an Euler method and tracks each droplet with a Lagrangian model; that is, it adopts the so-called Euler–Lagrangian approach. The ultimate objective of the LCS is to numerically simulate droplet growth induced by both collisions and condensation while taking into account the general flow. This study focused on the collisional growth and reported for the first time direct simulation results of the auto-conversion rate obtained by tracking particle motions in turbulent flows.

The LCS started with small droplets exponentially distributed in size. Each droplet was tracked by computing its hydrodynamic interactions and collision events—that is, when it overlapped with another drop or

droplet. Pairs of overlapped droplets united immediately conserving mass and momentum. The droplets were classified into cloud and rain categories depending on their size: droplets with radii smaller than $40 \mu\text{m}$ were considered cloud droplets, while drops with $r > 40 \mu\text{m}$ were raindrops. Next we obtained the time evolutions of the cloud and rainwater contents and calculated the rate of conversion from cloud to rain, which was a combination of the autoconversion and accretion rates. The calculated size evolutions of the droplets and the auto-conversion rates were compared with those obtained from the stochastic collision–coalescence equation (SCE) and with those obtained for bulk cloud microphysical parameterizations. Collision kernel and collision efficiency models were required for the SCE simulation. The gravitational model for stagnant flows and the Onishi model (see the appendix) for turbulent flows were employed for the collision kernel, and the Long model (Long 1974), Hall model (Hall 1980), and PKS2001 model (PKS2001) were employed for the collision efficiency for stagnant flows. The collision efficiencies for turbulent flows were obtained by multiplying the PKS2001 efficiencies by the turbulence enhancement of the efficiency η_E tabulated in WARG2008. The collision efficiency model used does not consider the number concentration dependence that was reported in WARG2008. The good agreements between the SCE simulations with the simple collision efficiency model and the LCS simulations indicate that the collision efficiency number concentration dependence would not be relevant to typical cloud systems containing droplets with various sizes.

First, we investigated collisional growth in a stagnant flow without hydrodynamic interactions (i.e., the droplets themselves did not create flow disturbances). Perfect agreement between the results of the LCS and the SCE with a gravitational kernel model were observed for both the size evolutions and conversion rates. This confirms the reliabilities of both of these simulations. This good agreement also supports the notion that the SCE may be used with confidence when the collision growth is predominantly controlled by the self-collection of cloud droplets (section 15.3.1.3 in Pruppacher and Klett 1998). Comparisons of stagnant and turbulent flow results have confirmed that droplet growth is enhanced by turbulence and that this enhancement can be predicted accurately if the turbulent collision kernel is used. While the SCE simulation results are dependent on the collision efficiency model, this study has also shown the reliability of the PKS2001 and Hall collision efficiency models for stagnant flows and the product of the PKS2001 efficiency and η_E of WARG2008 for turbulent flows.

In this work, two bulk parameterizations were examined: the Kessler model and the model of SNS2010 for the autoconversion rate. The main difference between the two is that the former does not explicitly consider turbulence enhancement but the latter does. The Kessler model significantly overestimates the autoconversion rate with a monotonically decreasing trend for a stagnant flow, but its prediction for a turbulent flow is in a similar range to that of the LCS for an energy dissipation rate of $\varepsilon = 400 \text{ cm}^2 \text{ s}^{-3}$. This indicates that the empirical parameters in the Kessler model were tuned for the microphysical process in typical cloud turbulence when ε was $O(100 \text{ cm}^2 \text{ s}^{-3})$. In contrast, the SNS2010 model works well overall for both stagnant and turbulent flows qualitatively as well as quantitatively. Another focus of this study was the time period in which 10% of a cloud's water content is converted to rain category drops, defined as $t_{10\%}$. In general, the autoconversion rates were larger than the accretion rates for $t < t_{10\%}$. We therefore consider $t_{10\%}$ to be the autoconversion time scale and define the turbulence enhancement factor E_{turb} as $E_{\text{turb}} = \overline{t_{10\%}}[\text{NoT}] / \overline{t_{10\%}}[\text{T}]$, where the overbar denotes the mean value over all LCS runs and [NoT] and [T] indicate stagnant (nonturbulent) and turbulent flow simulations, respectively. For a fixed Reynolds number, it was found that the enhancement factor E_{turb} increases linearly with ε according to $E_{\text{turb}} = 1 + C_\varepsilon \varepsilon$ and that the enhancement agrees with the turbulence enhancement in the SNS2010 model. On the other hand, unlike SNS2010 model predictions, the enhancement factors do not show a consistent Reynolds number dependence.

We also investigated the fluctuations in $t_{10\%}$. If we assume that the probability of a cloud droplet becoming a raindrop within $\overline{t_{10\%}}$ is a constant p for any droplet at $t = 0$, binomial distribution theory predicts that the relative standard deviation of $\tau_{10\%}$, $\gamma(\tau_{10\%})$ is inversely proportional to the square root of the initial number of droplets. This relationship was apparent in the LCS data, and the plots from various flow conditions seem to collapse into a line that follows the relationship. This suggests that the flow state does not have substantial influence on autoconversion fluctuations.

Strictly speaking, the LCS results for $\gamma(\tau_{10\%})$ do not follow binomial distribution theory perfectly. The coefficient obtained is larger than the prediction by approximately a factor of 5. This is partly because the probability p is actually not constant but depends on the particle size and flow conditions. This variability adds extra statistical fluctuations. Interestingly, it was shown that turbulence slightly reduces the extra fluctuations because it lessens the size dependence of p . The fact that binomial distribution theory cannot accurately predict coalescence events when trial independence is assumed

also contributes. We also estimated the statistical fluctuations in the autoconversion time scale for atmospheric cloud turbulence with $\text{Re}_\lambda = 10^3$ – 10^4 and $\varepsilon = 400 \text{ cm}^2 \text{ s}^{-3}$. The estimated relative fluctuations are $O(10^{-7}$ – $10^{-5})$. This may appear very small, but they can have an effect in weather simulations as a result of the nonlinear nature of cloud physics.

Acknowledgments. This research was supported by a Grant-in-Aid from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan for Young Scientists (B) 24760152. The numerical simulations presented were carried out on the supercomputers, including the Earth Simulator, of the Japan Agency for Marine-Earth Science and Technology. The authors are grateful to Dr. Orlando Ayala for providing the collision kernel values obtained from the Ayala model.

APPENDIX

Modified Onishi Model

Wang et al. (1998) expressed the collisions between bidispersed droplets based on the spherical formulation as

$$K_c(r_1, r_2) = 2\pi R_{12}^2 \langle |w_r| \rangle g_{12}(R_{12}), \quad (\text{A1})$$

where angle brackets denote an ensemble average, $R_{12} = r_1 + r_2$ is the collision radius, $|w_r|$ is the radial relative velocity at contact, and $g_{12}(R_{12})$ is the radial distribution function at contact. The term $g_{12}(R_{12})$ represents the clustering effect, which is equal to unity when the particles are uniformly distributed. The value of $g_{12}(R)$ becomes $O(10)$ when the particle Stokes number ($\text{St} = \tau_p / \tau_\eta$, where τ_η is the Kolmogorov time) ranges from $O(0.1)$ to $O(1)$.

Onishi (2005) employs the model of Wang et al. (2000) for $\langle |w_r| \rangle$ but proposes an original model for the clustering effect in monodisperse systems that has better physical support than previous models and includes the Reynolds number dependence of the clustering effect. The key in the Onishi model for $g_{11}(R)$ [note that we use $R(=2r)$ instead of $R_{11}(=2r)$] are the following empirical scalings:

$$g_{11}(R) - 1 \propto \begin{cases} \text{St}^2 & \text{for } \text{St} \leq 0.4 \\ \text{St}^{-2} \text{Re}_\lambda & \text{for } 2 < \text{St} \end{cases} \quad (\text{A2})$$

The first scaling is consistent with the analytical solution of $g_{11}(R) - 1 \propto \text{St}^2$ for $\text{St} \ll 1$ particles (Wang et al. 2000). The substitution of $\text{Re}_\lambda \propto (l_\lambda / l_\eta)^2$ and $\text{St} \propto (r / l_\eta)^2$ into the above scalings yields

FIG. A1. The radial distribution function at the contact of monodisperse particles, $g_{11}(R)$, for $St = 0.1, 0.2, 0.4, 0.6$, and 1 . The plotted points are the DNS results, whose values are listed in Table A1, and the lines are the model prediction.

$$g_{11}(R) - 1 \propto \begin{cases} (r/l_\eta)^4 & \text{for } St \leq 0.4 \\ (r/l_\eta)^{-2} (r/l_\lambda)^{-2} & \text{for } 2 < St \end{cases}. \quad (\text{A3})$$

This reveals that the clustering of particles with small inertia is dominated by the Kolmogorov scale and that of particles with larger inertia by a larger length scale—that is, the Taylor microscale. This is physically reasonable. The clustering of particles with even higher inertia would be dominated by a larger scale still (i.e., the integral scale), but $g_{11}(R) - 1$ itself would be negligibly small so such a range is not worth discussing for cloud droplets.

Equation (A2) can be rewritten as

$$g_{11}(R) - 1 = \begin{cases} A_1 St^2 & (\equiv y_1) \text{ for } St < St_a \\ A_2 Re_\lambda St^{-2} & (\equiv y_2) \text{ for } St_a \leq St \end{cases}, \quad (\text{A4})$$

where A_1 and A_2 were empirically determined to be 110 and 0.38 (95 and 0.32 in the original model), respectively. The domain boundary St_a is $(A_2/A_1)^{1/4} Re_\lambda^{1/4}$. To smoothly connect the two formulations in Eq. (A4), a tanh smoothing function z_a was employed as

$$g_{11}(R) - 1 = y_1 z_a^\alpha + y_2 (1 - z_a)^\alpha. \quad (\text{A5})$$

Here,

$$z_a(St) = \frac{1}{2} \left(1 - \tanh \frac{\log_{10} St - \log_{10} St_a}{C_a} \right), \quad (\text{A6})$$

where C_a is parameterized as

TABLE A1. Values of $g_{11}(R)$ obtained from the DNS in Onishi et al. (2013) and Onishi and Vassilicos (2014). The $\pm$ value shows the standard deviation obtained from more than three runs.

Re_λ	$St = 0.1$	$St = 0.4$	$St = 0.6$	$St = 1$
49.0	2.07 ± 0.82	16.3 ± 3.4	24.9 ± 2.1	17.3 ± 2.4
82.7	2.10 ± 0.79	16.8 ± 1.2	26.2 ± 1.1	23.2 ± 0.6
132	1.98 ± 0.38	16.4 ± 0.5	26.2 ± 0.7	25.2 ± 0.8
209	2.05 ± 0.14	16.0 ± 0.4	25.8 ± 0.1	26.5 ± 0.2
340	2.06 ± 0.05	15.4 ± 0.2	24.5 ± 0.1	25.9 ± 0.1
527	2.03	14.7 ± 0.4	23.3	27.2

$$C_a = a_c Re_\lambda^{b_c}. \quad (\text{A7})$$

The values of a_c and b_c were set to 0.06 and 0.30, respectively (0.035 and 0.50 in the original model). The superscript α in Eq. (A5) is parameterized as

$$\alpha = \log_2 a_\alpha Re_\lambda^{b_\alpha}, \quad (\text{A8})$$

where a_α and b_α were set to 0.26 and 0.50, respectively (0.092 and 0.83 in the original model). The empirical coefficients (i.e., $A_1, A_2, a_c, b_c, a_\alpha$, and b_α) were updated according to the recent dataset in Onishi et al. (2013) and Onishi and Vassilicos (2014).

Figure A1 shows a comparison between direct numerical simulation results (Onishi et al. 2013; Onishi and Vassilicos 2014), whose values are compiled in Table A1, and the present model for $g_{11}(R)$ for $St = 0.1, 0.2, 0.4, 0.6$, and 1 .

This comparison shows that the present $g_{11}(R)$ model is reliable at least for the tested Stokes numbers and the Reynolds numbers of $Re_\lambda < 530$. The present model employs several empirical functions and parameters and, therefore, its reliability is uncertain for higher Reynolds numbers, such as $Re_\lambda = 10^3$ – 10^4 , which is the case in atmospheric clouds. However, the excellent agreement for $Re_\lambda < 530$ suggests that the possible errors for higher Re_λ would not be large.

The present Onishi collision kernel model is compared with the Ayala model for a monodisperse system with $r = 20 \mu\text{m}$ droplets in Fig. A2. The two models agree for low Re_λ but differ significantly in high Re_λ . This suggests that the two models would lead to significantly different results in droplet growth in turbulent clouds, whose Re_λ ranges over 10^3 – 10^4 , thus implying the significance of the Reynolds number dependence of collision statistics.

To determine the clustering effect for bidisperse systems, an empirical formulation of the form $g_{12} = f(g_{11}, g_{22})$ by Zhou et al. (2001) was employed. The turbulent collision kernel formulated above does not include the collision contribution of gravitational settlings. To introduce the gravity contribution, the following simple

FIG. A2. Turbulent collision kernels among monodisperse droplets with $20\ \mu\text{m}$ in radius against the energy dissipation rate for different Reynolds numbers obtained from the Ayala model (thin lines) and the present Onishi model (thick lines).

formulation was employed to obtain the total collision kernel:

$$K_{c,\text{total}}(r_1, r_2) = [K_{c,\text{turb,nog}}^2(r_1, r_2) + K_{c,\text{grav}}^2(r_1, r_2)]^{1/2}. \quad (\text{A9})$$

Here, $K_{c,\text{turb,nog}}$ denotes the turbulent collision kernel for the no-gravity condition.

The Fortran 90 code used to calculate the above Onishi model is provided as a supplemental material.

REFERENCES

- Alfonso, L., G. B. Raga, and D. Baumgardner, 2008: The validity of the kinetic collection equation revisited. *Atmos. Chem. Phys.*, **8**, 969–982, doi:10.5194/acp-8-969-2008.
- Ayala, O., W. W. Grabowski, and L.-P. Wang, 2007: A hybrid approach for simulating turbulent collisions of hydrodynamically-interacting particles. *J. Comput. Phys.*, **225**, 51–73, doi:10.1016/j.jcp.2006.11.016.
- , B. Rosa, and L.-P. Wang, 2008a: Effects of turbulence on the geometric collision rate of sedimenting droplets. Part 2. Theory and parameterization. *New J. Phys.*, **10**, 075016, doi:10.1088/1367-2630/10/7/075016.
- , —, —, and W. W. Grabowski, 2008b: Effects of turbulence on the geometric collision rate of sedimenting droplets. Part 1. Results from direct numerical simulation. *New J. Phys.*, **10**, 075015, doi:10.1088/1367-2630/10/7/075015.
- Blyth, A. M., 1993: Entrainment in cumulus clouds. *J. Appl. Meteor.*, **32**, 626–641, doi:10.1175/1520-0450(1993)032<0626:EICC>2.0.CO;2.
- Dallas, V., and J. C. Vassilicos, 2011: Rapid growth of cloud droplets by turbulence. *Phys. Rev.*, **84E**, 046315, doi:10.1103/PhysRevE.84.046315.
- Derevyanko, S., G. Falkovich, and S. Turitsyn, 2008: Evolution of non-uniformly seeded warm clouds in idealized turbulent conditions. *New J. Phys.*, **10**, 075019, doi:10.1088/1367-2630/10/7/075019.
- Falkovich, G., and A. Pumir, 2007: Sling effect in collisions of water droplets in turbulent clouds. *J. Atmos. Sci.*, **64**, 4497–4505, doi:10.1175/2007JAS2371.1.
- Franklin, C. N., 2008: A warm rain microphysics parameterization that includes the effect of turbulence. *J. Atmos. Sci.*, **65**, 1795–1816, doi:10.1175/2007JAS2556.1.
- Gillespie, D. T., 1975: An exact method for numerically simulating the stochastic coalescence process in a cloud. *J. Atmos. Sci.*, **32**, 1977–1989, doi:10.1175/1520-0469(1975)032<1977:AEMFNS>2.0.CO;2.
- Grabowski, W. W., and P. A. Vaillancourt, 1999: Comments on “Preferential concentration of cloud droplets by turbulence: Effects on the early evolution of cumulus cloud droplet spectra.” *J. Atmos. Sci.*, **56**, 1433–1436, doi:10.1175/1520-0469(1999)056<1433:COPCOC>2.0.CO;2.
- , and L.-P. Wang, 2009: Diffusional and accretional growth of water drops in a rising adiabatic parcel: Effects of the turbulent collision kernel. *Atmos. Chem. Phys.*, **9**, 2335–2353, doi:10.5194/acp-9-2335-2009.
- Hall, W. D., 1980: A detailed microphysical model within a two-dimensional dynamic framework: Model description and preliminary results. *J. Atmos. Sci.*, **37**, 2486–2507, doi:10.1175/1520-0469(1980)037<2486:ADMMWA>2.0.CO;2.
- Hirt, C. W., and J. L. Cook, 1972: Calculating three-dimensional flows around structures and over rough terrain. *J. Comput. Phys.*, **10**, 324–340, doi:10.1016/0021-9991(72)90070-8.
- Jonas, P. R., 1996: Turbulence and cloud microphysics. *Atmos. Res.*, **40**, 283–306, doi:10.1016/0169-8095(95)00035-6.
- Kessler, E., 1969: *On the Distribution and Continuity of Water Substance in Atmospheric Circulations*. Meteor. Monogr., No. 32, Amer. Meteor. Soc., 84 pp.
- Khairoutdinov, M., and Y. Kogan, 2000: A new cloud physics parameterization in a large-eddy simulation model of marine stratocumulus. *Mon. Wea. Rev.*, **128**, 229–243, doi:10.1175/1520-0493(2000)128<0229:ANCPPI>2.0.CO;2.
- Kostinski, A., and R. Shaw, 2005: Fluctuations and luck in droplet growth by coalescence. *Bull. Amer. Meteor. Soc.*, **86**, 235–244, doi:10.1175/BAMS-86-2-235.
- Krueger, S. K., C.-W. Su, and P. A. McMurtry, 1997: Modeling entrainment and finescale mixing in cumulus clouds. *J. Atmos. Sci.*, **54**, 2697–2712, doi:10.1175/1520-0469(1997)054<2697:MEAFMI>2.0.CO;2.
- Long, A. B., 1974: Solutions to the droplet collection equation for polynomial kernels. *J. Atmos. Sci.*, **31**, 1040–1052, doi:10.1175/1520-0469(1974)031<1040:STTDCE>2.0.CO;2.
- Morinishi, Y., T. S. Lund, O. V. Vasilyev, and P. Moin, 1998: Fully conservative higher order finite difference schemes for incompressible flow. *J. Comput. Phys.*, **143**, 90–124, doi:10.1006/jcph.1998.5962.
- Onishi, R., 2005: Numerical simulations of chemical reaction and droplet growth in environmental turbulent flows. Ph.D. dissertation, Kyoto University, 144 pp.
- , and K. Takahashi, 2012: A warm-bin-cold-bulk cloud microphysical model. *J. Atmos. Sci.*, **69**, 1474–1497, doi:10.1175/JAS-D-11-0166.1.
- , and J. C. Vassilicos, 2014: Collision statistics of inertial particles in two-dimensional homogeneous isotropic turbulence with an inverse cascade. *J. Fluid Mech.*, **745**, 279–299, doi:10.1017/jfm.2014.97.
- , K. Takahashi, and S. Komori, 2009: Influence of gravity on collisions of monodispersed droplets in homogeneous isotropic turbulence. *Phys. Fluids*, **21**, 125108, doi:10.1063/1.3276906.

- , Y. Baba, and K. Takahashi, 2011a: Large-scale forcing with less communication in finite-difference simulations of stationary isotropic turbulence. *J. Comput. Phys.*, **230**, 4088–4099, doi:[10.1016/j.jcp.2011.02.034](https://doi.org/10.1016/j.jcp.2011.02.034).
- , K. Matsuda, K. Takahashi, R. Kurose, and S. Komori, 2011b: Linear and nonlinear inversion schemes to retrieve collision kernel values from droplet size distribution change. *Int. J. Multiphase Flow*, **37**, 125–135, doi:[10.1016/j.ijmultiphaseflow.2010.10.004](https://doi.org/10.1016/j.ijmultiphaseflow.2010.10.004).
- , K. Takahashi, and J. C. Vassilicos, 2013: An efficient parallel simulation of interacting inertial particles in homogeneous isotropic turbulence. *J. Comput. Phys.*, **242**, 809–827, doi:[10.1016/j.jcp.2013.02.027](https://doi.org/10.1016/j.jcp.2013.02.027).
- Pinsky, M. B., and A. P. Khain, 2002: Effects of in-cloud nucleation and turbulence on droplet spectrum formation in cumulus clouds. *Quart. J. Roy. Meteor. Soc.*, **128**, 501, doi:[10.1256/003590002321042072](https://doi.org/10.1256/003590002321042072).
- , —, and M. Shapiro, 2001: Collision efficiency of drops in a wide range of Reynolds numbers: Effects of pressure on spectrum evolution. *J. Atmos. Sci.*, **58**, 742–764, doi:[10.1175/1520-0469\(2001\)058<0742:CEODIA>2.0.CO;2](https://doi.org/10.1175/1520-0469(2001)058<0742:CEODIA>2.0.CO;2).
- , —, and H. Krugliak, 2008: Collisions of cloud droplets in a turbulent flow. Part V: Application of detailed tables of turbulent collision rate enhancement to simulation of droplet spectra evolution. *J. Atmos. Sci.*, **65**, 357–374, doi:[10.1175/2007JAS2358.1](https://doi.org/10.1175/2007JAS2358.1).
- Pruppacher, H. R., and J. D. Klett, 1998: *Microphysics of Clouds and Precipitation*. Kluwer Academic Publisher, 954 pp.
- Reade, W. C., and L. Collins, 2000: A numerical study of the particle size distribution of an aerosol undergoing turbulent coagulation. *J. Fluid Mech.*, **415**, 45–64, doi:[10.1017/S0022112000008521](https://doi.org/10.1017/S0022112000008521).
- Reisner, J., R. M. Rasmussen, and R. T. Bruintjes, 1998: Explicit forecasting of supercooled liquid water in winter storms using the MM5 mesoscale model. *Quart. J. Roy. Meteor. Soc.*, **124**, 1071–1107, doi:[10.1002/qj.49712454804](https://doi.org/10.1002/qj.49712454804).
- Robertson, D., 1974: Monte Carlo simulations of drop growth by accretion. *J. Atmos. Sci.*, **31**, 1344–1350, doi:[10.1175/1520-0469\(1974\)031<1344:MCSODG>2.0.CO;2](https://doi.org/10.1175/1520-0469(1974)031<1344:MCSODG>2.0.CO;2).
- Rogers, R. R., and M. K. Yau, 1989: *A Short Course in Cloud Physics*. 3rd ed. Butterworth-Heinemann, 304 pp.
- Rosa, B., H. Parishani, O. Ayala, W. W. Grabowski, and L.-P. Wang, 2013: Kinematic and dynamic collision statistics of cloud droplets from high-resolution simulations. *New J. Phys.*, **15**, 045032, doi:[10.1088/1367-2630/15/4/045032](https://doi.org/10.1088/1367-2630/15/4/045032).
- Rowe, P. N., and G. A. Henwood, 1961: Drag forces in a hydraulic model of a fluidized bed. I. *Trans. Inst. Chem. Eng.*, **39**, 43.
- Saffman, P. G., and J. S. Turner, 1956: On the collision of drops in turbulent clouds. *J. Fluid Mech.*, **1**, 16–30, doi:[10.1017/S0022112056000020](https://doi.org/10.1017/S0022112056000020).
- Saw, E. W., R. A. Shaw, S. Ayyalasomayajula, P. Y. Chuang, and A. Gylfason, 2008: Inertial clustering of particles in high-Reynolds-number turbulence. *Phys. Rev. Lett.*, **100**, 214501, doi:[10.1103/PhysRevLett.100.214501](https://doi.org/10.1103/PhysRevLett.100.214501).
- Seifert, A., L. Nuijens, and B. Stevens, 2010: Turbulence effects on warm-rain autoconversion in precipitating shallow convection. *Quart. J. Roy. Meteor. Soc.*, **136**, 1753–1762, doi:[10.1002/qj.684](https://doi.org/10.1002/qj.684).
- Sidin, R. S. R., R. H. A. Ijzermans, and M. W. Reeks, 2009: A Lagrangian approach to droplet condensation in atmospheric clouds. *Phys. Fluids*, **21**, 106603, doi:[10.1063/1.3244646](https://doi.org/10.1063/1.3244646).
- Soong, S. T., 1974: Numerical simulation of warm rain development in an axisymmetric cloud model. *J. Atmos. Sci.*, **31**, 1262–1285, doi:[10.1175/1520-0469\(1974\)031<1262:NSOWRD>2.0.CO;2](https://doi.org/10.1175/1520-0469(1974)031<1262:NSOWRD>2.0.CO;2).
- Sundaram, S., and L. Collins, 1996: Numerical considerations in simulating a turbulent suspension of finite-volume particles. *J. Comput. Phys.*, **124**, 337–350, doi:[10.1006/jcph.1996.0064](https://doi.org/10.1006/jcph.1996.0064).
- , and L. R. Collins, 1997: Collision statistics in an isotropic particle-laden turbulent suspension. Part 1. Direct numerical simulations. *J. Fluid Mech.*, **335**, 75–109, doi:[10.1017/S0022112096004454](https://doi.org/10.1017/S0022112096004454).
- Thompson, G., R. M. Rasmussen, and K. Manning, 2004: Explicit forecasts of winter precipitation using an improved bulk microphysics scheme. Part I: Description and sensitivity analysis. *Mon. Wea. Rev.*, **132**, 519–542, doi:[10.1175/1520-0493\(2004\)132<0519:EFOWPU>2.0.CO;2](https://doi.org/10.1175/1520-0493(2004)132<0519:EFOWPU>2.0.CO;2).
- Uchida, E., and S. Ohta, 1969: A simulation approach to the formation of precipitation particles using the Monte Carlo method. *J. Meteor. Soc. Japan*, **47**, 279–291.
- Valioulis, I., and E. J. List, 1984: A numerical evaluation of the stochastic completeness of the kinetic coagulation equation. *J. Atmos. Sci.*, **41**, 2516–2529, doi:[10.1175/1520-0469\(1984\)041<2516:ANEOTS>2.0.CO;2](https://doi.org/10.1175/1520-0469(1984)041<2516:ANEOTS>2.0.CO;2).
- Van Den Heever, S. C., and W. R. Cotton, 2007: Urban aerosol impacts on downwind convective storms. *J. Appl. Meteor. Climatol.*, **46**, 828–850, doi:[10.1175/JAM2492.1](https://doi.org/10.1175/JAM2492.1).
- Wang, L.-P., A. S. Wexler, and Y. Zhou, 1998: Statistical mechanical description and modelling of turbulent coagulations. *Phys. Fluids*, **10**, 2647–2651, doi:[10.1063/1.869777](https://doi.org/10.1063/1.869777).
- , —, and —, 2000: Statistical mechanical description and modelling of turbulent collision of inertial particles. *J. Fluid Mech.*, **415**, 117–153, doi:[10.1017/S0022112000008661](https://doi.org/10.1017/S0022112000008661).
- , O. Ayala, and W. W. Grabowski, 2005: Improved formulations of the superposition method. *J. Atmos. Sci.*, **62**, 1255–1266, doi:[10.1175/JAS3397.1](https://doi.org/10.1175/JAS3397.1).
- , —, Y. Xue, and W. W. Grabowski, 2006a: Comments on “Droplets to drops by turbulent coagulation.” *J. Atmos. Sci.*, **63**, 2397–2401, doi:[10.1175/JAS3750.1](https://doi.org/10.1175/JAS3750.1).
- , Y. Xue, O. Ayala, and W. Grabowski, 2006b: Effects of stochastic coalescence and air turbulence on the size distribution of cloud droplets. *Atmos. Res.*, **82**, 416–432, doi:[10.1016/j.atmosres.2005.12.011](https://doi.org/10.1016/j.atmosres.2005.12.011).
- , O. Ayala, B. Rosa, and W. W. Grabowski, 2008: Turbulent collision efficiency of heavy particles relevant to cloud droplets. *New J. Phys.*, **10**, 075013, doi:[10.1088/1367-2630/10/7/075013](https://doi.org/10.1088/1367-2630/10/7/075013).
- Woittiez, E. J. P., H. J. J. Jonker, and L. M. Portela, 2009: On the combined effects of turbulence and gravity on droplet collisions in clouds: A numerical study. *J. Atmos. Sci.*, **66**, 1926–1943, doi:[10.1175/2005JAS2669.1](https://doi.org/10.1175/2005JAS2669.1).
- Xue, Y., L.-P. Wang, and W. Grabowski, 2008: Growth of cloud droplets by turbulent collision-coalescence. *J. Atmos. Sci.*, **65**, 331–356, doi:[10.1175/2007JAS2406.1](https://doi.org/10.1175/2007JAS2406.1).
- Yin, Y., Z. Levin, T. G. Reisner, and S. Tzivion, 2000: The effects of giant cloud condensation nuclei on the development of precipitation in convective clouds—A numerical study. *Atmos. Res.*, **53**, 91–116, doi:[10.1016/S0169-8095\(99\)00046-0](https://doi.org/10.1016/S0169-8095(99)00046-0).
- Zaichik, L., O. Simonin, and V. Alipchenkov, 2003: Two statistical models for predicting collision rates of inertial particles in homogeneous isotropic turbulence. *Phys. Fluids*, **15**, 2995–3005, doi:[10.1063/1.1608014](https://doi.org/10.1063/1.1608014).
- Zhou, Y., A. S. Wexler, and L.-P. Wang, 2001: Modelling turbulent collision of bidisperse inertial particles. *J. Fluid Mech.*, **433**, 77–104, doi:[10.1017/S0022112000003372](https://doi.org/10.1017/S0022112000003372).