Entropy Production and Climate Efficiency

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ABSTRACT

Earth's climate system is a heat engine, absorbing solar radiation at a mean input temperature T_{in} and emitting terrestrial radiation at a lower, mean output temperature $T_{out} < T_{in}$. These mean temperatures, defined as the ratio of the energy to entropy input or output, determine the Carnot efficiency of the system. The climate system, however, does no external work, and hence its work efficiency is zero. The system does produce entropy and exports it to space. The efficiency associated with this entropy production is defined for two distinct representations of the climate system. The first defines the system as the sum of the various material subsystems, with the solar and terrestrial radiation fields constituting the surroundings. The second defines the system as a control volume that includes the material and radiation systems below the top of the atmosphere. These two complementary representations are contrasted using a radiative–convective equilibrium model of the climate system. The efficiency of Earth's climate system based on its material entropy production is estimated using the two representations.

1. Introduction

The motions of Earth's atmosphere and ocean have long been viewed (e.g., Shaw 1930) as manifestations of a global heat engine driven by the absorption of energy in the form of solar radiation and its emission back to space in the form of terrestrial radiation. The determination of the efficiency of this heat engine is a fundamental problem of climate science (e.g., Lorenz 1967). The physics of a heat engine like the climate system is governed by the laws of thermodynamics (Fig. 1). The first law is a statement of energy conservation between a system and its surroundings. The rate of change of the total energy E of the system is the sum of the rate of heating $\dot{Q}$ of the system less the rate of working $\dot{W}$ by the system on its surroundings:

$$\frac{dE}{dt} = \dot{Q}_{in} - \dot{Q}_{out} - \dot{W}, \quad (1.1)$$

where the heating is partitioned into a heating input ($\dot{Q}_{in} > 0$) and a heating output ($\dot{Q}_{out} > 0$). [Here the overdot denotes the rate of a process (e.g., per unit time).] The second law of thermodynamics is a statement of the entropy increase of a system as a result of its thermal interaction with its surroundings. The rate of change of the entropy S of the system is the sum of the entropy change rates due to the heating and its internal, irreversible entropy production $\dot{\Sigma}_{irr} > 0$ (e.g., Kondepudi and Prigogine 1998):

$$\frac{dS}{dt} = \dot{S}_{in} - \dot{S}_{out} + \dot{\Sigma}_{irr}. \quad (1.2)$$

(The Greek letter sigma, either upper or lower case, with an overdot is used exclusively here to refer to positive-definite, irreversible entropy production rates.) Only the heating (and not the working) leads to a change in the entropy of the system. The temperatures of heating input and output (T_{in} and T_{out}, respectively) are defined as the ratio of the heating to its associated entropy change rate:

$$T_{in} \equiv \dot{Q}_{in}/\dot{S}_{in} \quad \text{and} \quad T_{out} \equiv \dot{Q}_{out}/\dot{S}_{out}. \quad (1.3)$$

Assuming a statistical steady state, the energy and entropy of the system remains constant. Then the two laws in (1.1) and (1.2) can be combined using (1.3) to yield the heat-engine relation:

$$\eta\dot{Q}_{in} = \dot{W} + T_{out}\dot{\Sigma}_{irr}, \quad (1.4)$$

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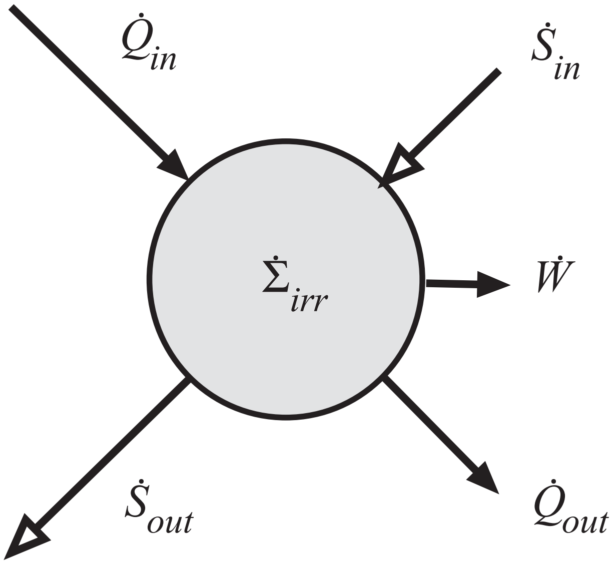

FIG. 1. The generic heat engine (shaded circle) receives heat at the rate $\dot{Q}_{\rm in}$ at the temperature $T_{\rm in}$ and outputs heat at the rate $\dot{Q}_{\rm out}$ at the temperature $T_{\rm out} < T_{\rm in}$. Work on its surroundings is done at the rate $\dot{W}$. Entropically, the engine receives entropy at the rate $\dot{S}_{\rm in}$, produces entropy internally and irreversibly at the rate $\dot{\Sigma}_{\rm irr}$, and exports entropy at the rate $\dot{S}_{\rm out}$. The input and output temperatures are defined by $T_{\rm in} = \dot{Q}_{\rm in}/\dot{S}_{\rm in}$ and $T_{\rm out} = \dot{Q}_{\rm out}/\dot{S}_{\rm out}$, respectively.

where $\eta = (T_{\rm in} - T_{\rm out})/T_{\rm in}$ is the Carnot efficiency. This relation indicates that only a fraction η of $\dot{Q}_{\rm in}$ is utilized by the system in doing work on its surroundings and in producing entropy internally. This internal entropy production is associated with frictional dissipation of kinetic energy and internal heat exchange within the system by radiation, conduction, and diffusion, as well as any irreversible chemical reactions and phase changes. Motivated by the relation (1.4), engineers seek to maximize $\dot{W}$ of their engines by minimizing the entropy production $T_{\rm out}\dot{\Sigma}_{\rm irr}$ that is viewed as "lost work" (e.g., Bejan 2006). In contrast, the climate system produces no external work (i.e., $\dot{W} = 0$; e.g., Goody 1995, section 2.4.3), but the internal activity of the climate machine constantly produces entropy. Globally, the irreversible entropy production of the system measures this internal activity, which includes the redistribution of heat, moisture, and salt down the temperature, moisture, and salinity gradients in the atmosphere and ocean (both vertically and horizontally) by diffusion. Then the entropy production may be viewed as "spent work" or "consumed work."

The idea that irreversible entropy production is a valid tool to estimate the climate efficiency is not without precedent. Brunt (1939) compares the eddy dissipation of kinetic energy with the incoming solar radiation to obtain an estimate of 2% for the conversion of the radiation to kinetic energy. Wulf and Davis (1952) formally incorporate Brunt's dissipation in an entropy approach of the efficiency. Lorenz (1967) makes estimates similar to Brunt's for the general circulation. In addition to frictional dissipation, Pauluis and Held (2002a,b) emphasize the entropy production due to the diffusion of heat, the diffusion of water vapor, the irreversibility of phase changes, and the frictional dissipation resulting from falling precipitation.

All these dissipative processes produce entropy, and that entropy production, always positive, is a measure of the internal activity of the climate system. The heat-engine relation in (1.4) provides the structure to formally define an efficiency associated with these entropy-producing processes. In particular, this structure requires determination of the heating input and the output temperature. The latter differs from the temperature, say, of where the frictional dissipation occurs.

In addition to this material entropy production, it may also be necessary to include in the analysis the entropy production associated with the radiation field. Entropy is produced by the conversion of the high-energy, low-entropy solar photons into the low-energy, high-entropy photons of the outgoing longwave radiation (e.g., Wu and Liu 2010a). In general, the irreversible entropy production is composed of material and radiation components: $\dot{\Sigma}_{\rm irr} = \dot{\Sigma}_{\rm mat} + \dot{\Sigma}_{\rm rad}$. Then the climate efficiency associated with the material entropy production may be defined as

$$\eta_{\rm mat} \equiv \frac{T_{\rm out}\dot{\Sigma}_{\rm mat}}{\dot{Q}_{\rm in}}. \tag{1.5}$$

The purpose of this study is to quantify the material entropy production efficiency (1.5) of Earth's climate system. It is shown that the results are dependent on the definition of the climate system. Two different representations of the system appear relevant. The first representation defines the climate system as the sum of its material components, including the atmosphere, ocean, land, and cryosphere. This material system constantly exchanges energy with the surrounding radiation field. The second representation uses the engineering concept of a control volume (e.g., Bejan 2006) and defines the climate system as the sum of the material system and the radiation contained within its volume. The control volume exchanges energy and entropy only through the top-of-the-atmosphere (TOA) radiation fluxes. These two different representations provide complementary insights into the extent of the entropy production and, hence, into the efficiency of the climate system. Goody (2000, section 2) has discussed the difference in the entropy balance equation for the two representations and suggests that the material system is preferable. Nonetheless, neither representation is more fundamental, and both are examined here.

These two representations of the climate system clearly differentiate between the system and its surroundings. In contrast, the climate efficiency analyses of Johnson (1989, 2000) and Lucarini (2009) do not clearly distinguish between the climate system and its surroundings. These analyses include internal heating by sensible and latent heat fluxes with the radiative heating as part of the external heating of the climate system by its surroundings. They further subdivide the material climate system into regions of mean warming and cooling by all diabatic processes. As a consequence, the interaction between the system and its surroundings is improperly handled in their formulations of climate efficiency. [It is noted that this shortcoming does not invalidate the necessity of climate models to simulate the system's entropics accurately (Johnson 1997).] The present approach treats the interaction properly.

Section 2 presents the energetics and entropics of the material representation of the climate system for which the radiation field constitutes the surroundings. The heat-engine relation for the material system is derived, and it is shown that the system does no external work. Thus, the productivity of the system is measured solely by its irreversible entropy production. The analysis accurately includes nonequilibrium thermodynamics (de Groot and Mazur 1984). Section 3 presents the energetics and entropics of the time-dependent radiation subsystem following Essex (1984) and Callies and Herbert (1988). Section 4 derives the heat-engine relation for the control-volume representation of the climate system, combining both the material and radiation subsystems below the top of the atmosphere. Section 5 applies both the material-system and control-volume representations to an idealized model climate system in radiative–convective equilibrium. Such radiative–convective models are useful tools in climate studies (e.g., Ramanathan and Coakley 1978), but their entropy aspects are rarely examined. The single-slab atmosphere model used here illustrates the various abstract aspects of the entropy production, particularly those associated with the irreversible radiation entropy production. The bulk atmospheric budgets found here are broadly consistent with the multilevel radiative–convective models of Li et al. (1994) and Wu and Liu (2010b) that do examine radiation entropics. Section 6 presents the conclusions and estimates the efficiency of Earth's climate system using the two representations.

2. The material climate system

The material climate system is compactly described using the fluid mechanical equations for a multicomponent fluid. The specific energy e is the sum of the specific kinetic, internal, and gravitational energies: $e = k + u + \Phi$. The statement of its conservation is

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho e \mathbf{v}) = \dot{q} - \nabla \cdot (p\mathbf{v} + \mathbf{v} \cdot \underset{\approx}{\tau}) - \nabla \cdot \mathbf{J}_e, \qquad (2.1)$$

where ρ is the density, p is the pressure, $\mathbf{v}$ is the mass-weighted, barycentric velocity, and $\underset{\approx}{\tau}$ is the stress tensor. Here, the radiative heating rate per unit volume is $\dot{q}$ (defined in section 3) and $\mathbf{J}_e$ is the material energy flux due to conduction and diffusion (e.g., de Groot and Mazur 1984). We neglect the small geothermal heating of the climate system. Integration over the volume V of the climate system yields the statement of the first law of thermodynamics:

$$\frac{dE_{\mathrm{mat}}}{dt} = \dot{Q}_{\mathrm{in}} - \dot{Q}_{\mathrm{out}} - \dot{W}, \qquad (2.2)$$

where $E_{\mathrm{mat}} = \int \rho e\, dV$ is the total energy of the material system. The derivation of (2.2) assumes that the system is closed and that the mass and material energy fluxes $\rho\mathbf{v}$ and $\mathbf{J}_e$ vanish at the boundaries of the system. The radiation heating is partitioned into contributions due to the heating input $\dot{Q}_{\mathrm{in}}$ and the heating output $\dot{Q}_{\mathrm{out}}$. The total rate of working by the climate system on its surroundings is as follows:

$$\dot{W} = \int_V \nabla \cdot (p\mathbf{v} + \mathbf{v} \cdot \underset{\approx}{\tau})\, dV = \int_A (p\mathbf{v} + \mathbf{v} \cdot \underset{\approx}{\tau}) \cdot \mathbf{n}\, dA, \qquad (2.3)$$

and it vanishes at the top of the climate system, where the pressure and surface stress tends to zero. Here and elsewhere, the area integral is taken over the area of the TOA, and $\mathbf{n}$ is the outward unit vector normal to the surface.

The specific material entropy s satisfies

$$\frac{\partial \rho s}{\partial t} + \nabla \cdot (\rho s \mathbf{v}) = \frac{\dot{q}}{T} - \nabla \cdot \mathbf{J}_s + \dot{\sigma}_{\mathrm{mat}}, \qquad (2.4)$$

where the entropy flux is $\mathbf{J}_s = (\mathbf{J}_e - \mu_k \mathbf{J}_k)/T$ with μ_k being the specific chemical potential of the kth component of the multicomponent system. The summation convention is assumed for repeated indices. The rate of irreversible material entropy production per unit volume $\dot{\sigma}_{\mathrm{mat}}$ is, following de Groot and Mazur (1984),

$$\dot{\sigma}_{\mathrm{mat}} = \frac{\dot{\phi}}{T} - \frac{1}{T^2}\mathbf{J}_e \cdot \nabla T - \frac{1}{T}\mathbf{J}_k \cdot \nabla\left(\frac{\mu_k}{T}\right) - \frac{\mu_k}{T}\dot{r}_k, \qquad (2.5)$$

where $\dot{\phi}$ is the viscous dissipation rate of kinetic energy, $\mathbf{J}_k$ is the diffusive mass flux of the kth component, and $\dot{r}_k$ is the chemical reaction/phase-change rate of the kth component. Integration of (2.4) over the volume of the system yields, with $\dot{\Sigma}_{mat} = \int \dot{\sigma}_{mat}\, dV$ being the total rate of material entropy production,

$$\frac{dS_{mat}}{dt} = \int \left(\frac{\dot{q}_{in}}{T} - \frac{\dot{q}_{out}}{T} \right) dV + \dot{\Sigma}_{mat} \tag{2.6}$$

or

$$\frac{dS_{mat}}{dt} = \frac{\dot{Q}_{in}}{T_{in}} - \frac{\dot{Q}_{out}}{T_{out}} + \dot{\Sigma}_{mat}. \tag{2.7}$$

Here T_{in} and T_{out} are the entropic input and output temperatures defined by

$$\frac{1}{T_{in}} \equiv \int \left(\frac{\dot{q}_{in}}{T} \right) dV/\dot{Q}_{in}, \quad \frac{1}{T_{out}} \equiv \int \left(\frac{\dot{q}_{out}}{T} \right) dV/\dot{Q}_{out}. \tag{2.8}$$

These Carnot temperatures are the heating-averaged temperatures of the material system, where the radiative heating is increasing or decreasing the system's entropy.

For a cyclic process or for a system in a steady state, the energy equation (2.2) and entropy equation (2.6) combine to yield the heat-engine relation in the form

$$\eta \dot{Q}_{in} = \dot{W} + T_{out}\dot{\Sigma}_{mat}, \tag{2.9}$$

where the Carnot efficiency is $\eta = (T_{in} - T_{out})/T_{in}$ Equation (2.9) agrees with the general discussion in the introduction [see (1.4)]. Then, because the working vanishes ($\dot{W} = 0$), the material entropy production efficiency, defined as $\eta_{mat} = T_{out}\dot{\Sigma}_{mat}/\dot{Q}_{in}$, equals the Carnot efficiency.

The criterion for the partitioning of the heating into input and output components requires careful consideration for the complex climate system. The partitioning is not unique, and three different partitionings appear relevant. The first applies a strict interpretation of the distinction between the material system and its radiation surroundings. This case requires that the energy/entropy input to the system is by the absorption of both solar and terrestrial radiation and that the output is solely by emission of terrestrial radiation. As a consequence, the net heat input is greater than that due to the absorption of solar radiation alone. This first case for the material climate system is denoted as MS1. Alternatively, a second case adopts a perspective from the top of the atmosphere and defines the input to the system as that due solely to the absorption of solar radiation. Then the output is that associated with the net emission of terrestrial radiation. This second case (referred to as MS2) takes $\dot{Q}_{in} = \dot{Q}_{solar}$ and $\dot{Q}_{out} = \dot{Q}_{terr}$. Last, a third case employs the expression of the steady-state, radiation heating as the convergence of a radiation energy flux $\mathbf{F}$ (defined in section 3). Then the entropy production by the radiation output in (2.6) becomes

$$\int \frac{\dot{q}_{out}}{T} dV = \int \frac{\nabla \cdot \mathbf{F}_{terr}}{T} dV = \int_A \left(\frac{\mathbf{F}_{terr}}{T} \right) \cdot \mathbf{n} dA + \dot{\Sigma}_{terr}, \tag{2.10}$$

with

$$\dot{\Sigma}_{terr} = \int \mathbf{F}_{terr} \cdot \nabla \left(\frac{1}{T} \right) dV. \tag{2.11}$$

There is a flux of entropy out of the system at the top of the atmosphere and an irreversible entropy production associated with the internal emissions and absorptions of terrestrial radiation by the material system. Then (2.7)–(2.9) are modified to include this production. In particular, the heat-engine relation (2.9) becomes

$$\eta = \eta_{mat} + \eta_{terr}, \tag{2.12}$$

with $\eta_{terr} = T_{out}\dot{\Sigma}_{terr}/\dot{Q}_{in}$. The case using (2.10)–(2.12) is referred to as MS3. All three cases are illustrated using a radiative–convective equilibrium model in section 5.

3. The radiation system

The time-dependent radiative transfer equations (e.g., Essex 1984; Callies and Herbert 1988), assuming local thermodynamic equilibrium and conservative scattering, provide an accurate description of the interaction between the radiation field and the material system. The spectral energy radiance $I_\nu(\mathbf{r}, \mathbf{\Omega}, t)$ represents the radiation energy as a function of photon frequency ν, position $\mathbf{r}$, direction $\mathbf{\Omega}$, and time t. Integration of the spectral radiance over all frequencies and over all solid angles yields the radiation energy density $u_{rad} = 1/c \iint_{4\pi} I_\nu \, d\omega \, d\nu$, defined as the energy per unit volume. Here, c is the speed of light. The unit vector $\mathbf{\Omega}$ is defined by the zenith angle ϑ and azimuthal angle φ with elemental solid angle $d\omega = \sin\vartheta d\vartheta d\varphi$. The rate of change of the radiation energy density is governed by $\mathbf{F}$ and by $\dot{q}$ associated with the emission and absorption of radiation:

$$\frac{\partial u_{rad}}{\partial t} + \nabla \cdot \mathbf{F} = -\dot{q}. \tag{3.1}$$

The energy flux density vector is

$$\mathbf{F} = \int_0^\infty \int_{4\pi} I_\nu \mathbf{m} \, d\omega \, d\nu, \tag{3.2}$$

where $\mathbf{m}$ is the unit vector normal to the surface element through which the radiation passes. Because energy is conserved between the material and radiation systems during emission and absorption, the material heating rate per unit volume $\dot{q} = \int \dot{q}_\nu \, d\nu$ is the integral over frequency of the spectral heating rate:

$$\dot{q}_\nu = \int_{4\pi} \varepsilon_\nu (I_\nu - B_\nu) \, d\omega, \tag{3.3}$$

where ε_ν is the emissivity and $B_\nu(T)$ is the Planck function at the temperature T of the matter. The conservative scattering makes no direct contribution in (3.1) because the scattering only changes the direction of the photons. The integral of the radiation energy density equation (3.1) over the volume of the climate system yields an equation for the radiation energy $E_{\rm rad}$:

$$\frac{dE_{\rm rad}}{dt} + \int_A \mathbf{F} \cdot \mathbf{n} \, dA = -\dot{Q}, \tag{3.4}$$

where $E_{\rm rad} = \int u_{\rm rad} \, dV$ and the heating rate $\dot{Q} = \int_V \dot{q} \, dV$ of the matter by the radiation field represents a loss of radiation energy in (3.4).

In addition to energy, a radiation field also contains and transports entropy. The spectral entropy radiance $L_\nu = L_\nu(\mathbf{r}, \mathbf{\Omega}, t)$ is a known function of I_ν (e.g., Rosen 1954). The radiation entropy density is $s_{\rm rad} = 1/c \iint_{4\pi} L_\nu \, d\omega \, d\nu$. The rate of change of this entropy is described by

$$\frac{\partial s_{\rm rad}}{\partial t} + \nabla \cdot \mathbf{J} = \dot{s}_{\rm terr} + \dot{s}_{\rm asr} + \dot{\sigma}_{\rm ssr}, \tag{3.5}$$

where the radiation entropy flux density vector $\mathbf{J}$ is defined analogously to (3.2), with L_ν replacing I_ν. The last term in (3.5) describes the rate of irreversible entropy production due to the scattering of solar radiation and is positive definite (e.g., Callies and Herbert 1988).

The radiation entropy production due to the absorption and emission in (3.5) is separated spectrally into solar and terrestrial contributions. We have

$$\dot{s}_{\rm asr} = -\int_{\rm solar} \int_{4\pi} \frac{\varepsilon_\nu I_\nu}{T_\nu} \, d\omega \, d\nu,$$
$$\dot{s}_{\rm terr} = -\int_{\rm terr} \int_{4\pi} \varepsilon_\nu \frac{[I_\nu - B_\nu(T)]}{T_\nu} \, d\omega \, d\nu, \tag{3.6}$$

where the radiation/brightness temperature $T_\nu(\mathbf{r}, \mathbf{\Omega}, t)$ satisfies the relation $I_\nu(\mathbf{r}, \mathbf{\Omega}, t) = B_\nu(T_\nu)$. These sources may be positive or negative, depending upon whether the emission or absorption of radiation dominates. For example, if the material temperature is greater than the radiation temperature, $T > T_\nu$, then emission dominates the absorption, and the entropy of the terrestrial radiation field would increase, $\dot{s}_{\rm terr} > 0$. In contrast, emission may be neglected over the solar part of the spectrum, and the absorption of solar radiation is a sink of radiation entropy, $\dot{s}_{\rm asr} < 0$.

The integral of the radiation entropy equation (3.5) over the volume of the climate system can be separated spectrally into relations for the solar and terrestrial entropy components. We find

$$\frac{dS_{\rm solar}}{dt} + \int_A \mathbf{J}_{\rm solar} \cdot \mathbf{n} \, dA = \int_V (\dot{s}_{\rm asr} + \dot{\sigma}_{\rm ssr}) \, dV \quad \text{and} \tag{3.7}$$

$$\frac{dS_{\rm terr}}{dt} + \int_A \mathbf{J}_{\rm terr} \cdot \mathbf{n} \, dA = \int_V \dot{s}_{\rm terr} \, dV, \tag{3.8}$$

respectively. Here, the total entropy of the system is $S_{\rm rad} = \int s_{\rm rad} \, dV = S_{\rm solar} + S_{\rm terr}$.

4. The control-volume climate system

The climate system is represented here as the sum of the material and radiation systems contained in a control volume lying below the top of the atmosphere. Then the energy and entropy budgets of the combined system are the sums of the respective material and radiation budgets. One finds, for the total energy $E = E_{\rm mat} + E_{\rm rad}$ and the total entropy $S = S_{\rm mat} + S_{\rm rad}$,

$$\frac{dE}{dt} + \int_A \mathbf{F} \cdot \mathbf{n} \, dA = -\dot{W} \quad \text{and} \tag{4.1}$$

$$\frac{dS}{dt} + \int_A \mathbf{J} \cdot \mathbf{n} \, dA = \int_V (\dot{\sigma}_{\rm mat} + \dot{\sigma}_{\rm terr} + \dot{\sigma}_{\rm asr} + \dot{\sigma}_{\rm ssr}) \, dV, \tag{4.2}$$

where the rate of irreversible entropy production due to the absorption of solar radiation is

$$\dot{\sigma}_{\rm asr} \equiv \frac{\dot{q}_{\rm solar}}{T} + \dot{s}_{\rm asr} = \int_{\rm solar} \int_{4\pi} \varepsilon_\nu I_\nu \left(\frac{1}{T} - \frac{1}{T_\nu} \right) d\omega \, d\nu \ge 0, \tag{4.3}$$

and that due to the absorption and emission of terrestrial radiation for the total system is

$$\dot{\sigma}_{\rm terr} \equiv \frac{\dot{q}_{\rm terr}}{T} + \dot{s}_{\rm terr} = \int_{\rm terr} \int_{4\pi} \varepsilon_\nu [I_\nu - B_\nu(T)] \left(\frac{1}{T} - \frac{1}{T_\nu} \right) d\omega \, d\nu \ge 0. \tag{4.4}$$

With the climate system doing no external work, $\dot{W} = 0$, the total energy of the system is governed by the net flux of radiation energy (4.1). In contrast, the total entropy of the system is governed by the net flux of entropy and the irreversible entropy production due to the material processes (2.5), the absorption and emission of terrestrial radiation (4.4), the absorption of solar radiation (4.3), and the scattering of solar radiation (3.5).

The result contained in (4.4) warrants some discussion. It is identical to the ''irreversible source of radiation entropy'' (15) of Goody and Abdou (1996). Mathematically, the integrand is nonnegative, because the radiation temperature is proportional to the radiance. It is zero for the case of thermodynamic equilibrium when the radiation temperature equals the temperature of the material system. It is emphasized that the individual source terms $\dot{q}_{\rm terr}/T$ and $\dot{s}_{\rm terr}$ for the entropy of the matter and of the radiation vanish separately in the case of thermodynamic equilibrium, for which the radiance field is the isotropic Planck function at the temperature T of the body. In nonequilibrium conditions, the net effect of the interaction of the matter and the radiation is irreversible, and the entropy of the combined system increases. This irreversibility is analogous to that between any two systems that exchange heat (e.g., Kondepudi and Prigogine 1998, section 3.5). The explanation of this irreversibility is straightforward. Consider two systems (systems 1 and 2) with temperatures $T_2 > T_1$. When exchanging heat $dQ > 0$ from system 2 to 1, the entropy of system 1 increases by dQ/T_1, while that of system 2 decreases by $-dQ/T_2$. The total change in entropy is the sum $dQ/T_1 - dQ/T_2 = dQ(1/T_1 - 1/T_2) > 0$ and is greater than zero; hence, the exchange is irreversible. To emphasize this point, we define the spectral heating rate of the matter by the radiation as $\dot{q}_\nu \equiv \varepsilon_\nu[I_\nu - B_\nu(T)]$. Then (4.4) takes the form

$$\dot{\sigma}_{\rm terr} = \int_0^\infty \int_{4\pi} \left(\frac{\dot{q}_\nu}{T} - \frac{\dot{q}_\nu}{T_\nu}\right) d\omega\, d\nu \geq 0, \qquad (4.5)$$

corresponding to the rate of entropy increase by the heat exchange between two systems at the temperatures T_ν and T. Thus, it is physically incorrect to describe (4.4) as an irreversible change solely to the radiation entropy, as done in Goody and Abdou (1996). It results from the entropics of both the radiation and the matter. This argument is further illustrated for the model climate system discussed in section 5.

A heat-engine relation for the combined system follows from (4.1) and (4.2) in the steady state. Then the energy relation in (4.1) yields a balance between the incoming flux of solar radiation and the outgoing fluxes of scattered solar radiation and outgoing longwave radiation:

$$F \equiv F_{\rm ISR} - F_{\rm SSR} - F_{\rm OLR} = 0. \qquad (4.6)$$

In contrast, the entropy balance in (4.2) indicates that the net radiation entropy flux out of the climate system transports the irreversible entropy production

$$J_{\rm OLR} + J_{\rm SSR} - J_{\rm ISR} = \dot{\Sigma}_{\rm mat} + \dot{\Sigma}_{\rm terr} + \dot{\Sigma}_{\rm asr} + \dot{\Sigma}_{\rm ssr} \equiv \dot{\Sigma} > 0, \qquad (4.7)$$

where the entropy production terms are redefined as productions per unit area. The associated input and output Carnot temperatures are then, in contrast to (2.8), defined in terms of the radiation entropics:

$$T_{\rm in} = F_{\rm ISR}/J_{\rm ISR} \quad \text{and} \qquad (4.8a)$$

$$T_{\rm out} = (F_{\rm SSR} + F_{\rm OLR})/(J_{\rm SSR} + J_{\rm OLR}). \qquad (4.8b)$$

The relations (4.6)–(4.8) combine to yield the heat-engine relation

$$\eta F_{\rm ISR} = T_{\rm out}\dot{\Sigma} \qquad (4.9)$$

or

$$\eta = \eta_{\rm mat} + \eta_{\rm terr} + \eta_{\rm asr} + \eta_{\rm ssr}, \qquad (4.10)$$

where, for example, the material entropy production efficiency is $\eta_{\rm mat} = T_{\rm out}\dot{\Sigma}_{\rm mat}/F_{\rm ISR}$.

The general result (4.10) is referred to as control-volume case 1 (CV1). Two additional cases of the control-volume representation are possible. Both cases remove the scattering processes from consideration by taking the incoming radiation to be the absorbed solar radiation. Then the fluxes transform as follows: $F_{\rm ISR} \rightarrow F_{\rm ISR} - F_{\rm SSR} \equiv F_{\rm ASR}$ and $J_{\rm ISR} \rightarrow J_{\rm ISR} - J_{\rm SSR} \equiv J_{\rm ASR}$; and the entropy production caused by scattering is removed from the entropy budget (4.7) and the heat-engine relation (4.10). This case is referred to as CV2. A further refinement, CV3, excludes the solar radiation from the control volume that is then restricted to include only the material subsystem and the terrestrial radiation. In this third case, the system heat input is the absorbed solar radiation with input temperature defined using (2.8a), with the heating input defined by (3.6a).

An exergy analysis (e.g., Bejan 2006) follows by multiplying the entropy equation (4.7) by a constant reference temperature T_0 and adding it to the energy equation (4.6). One finds $F_{\rm Ex}^{\rm ISR} = F_{\rm Ex}^{\rm SSR} + F_{\rm Ex}^{\rm OLR} + T_0\dot{\Sigma}$, where the exergy flux is $F_{\rm Ex} \equiv F - T_0 J$. For the choice $T_0 = T_{\rm out}$, this exergy relation reduces to (4.9). The exergetics is not pursued further.

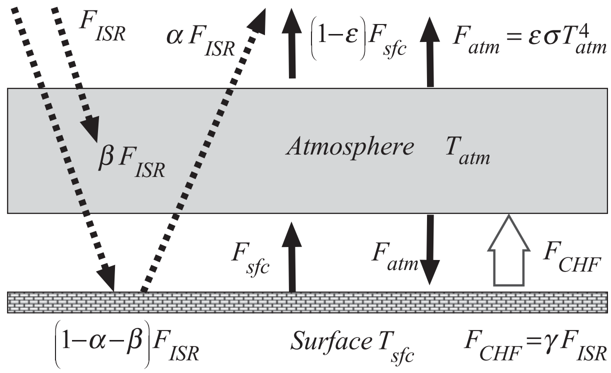

FIG. 2. A radiative–convective equilibrium model of the climate system consists of an isothermal atmosphere at temperature T_{atm} above an isothermal surface at temperature T_{sfc}. The surface reflects a fraction α of the flux of incoming solar radiation F_{ISR} and radiates as a blackbody. The atmosphere absorbs a fraction β of the incoming solar radiation and acts as a graybody with emissivity ε for terrestrial radiation. The solar and terrestrial radiative fluxes are denoted with dotted and solid arrows, respectively. Convection–conduction transports a heat flux $F_{CHF} = \gamma F_{ISR}$ (open arrow) from the surface to the atmosphere.

5. A radiative–convective equilibrium model of the climate system

An idealized model of the climate system in radiative–convective equilibrium illustrates the effect of the representation of the climate system on the Carnot efficiency and on the material entropy production efficiency η_{mat}. The horizontally homogeneous model (Fig. 2) consists of an isothermal atmosphere at the temperature T_{atm} above a land surface at the temperature T_{sfc}. The system reflects a fraction α of the incoming solar radiation. For simplicity, the atmosphere is assumed to be nonreflecting. The atmosphere absorbs a fraction β of the incoming solar radiation and has constant longwave emissivity/absorptivity ε. The surface acts like a blackbody to longwave radiation.

The incoming solar energy and entropy fluxes for the model are $F_{ISR} = \sigma T^4_{sun}\cos\theta_{sun}\Omega_{sun}/\pi$ and $J_{ISR} = (4/3)\sigma T^3_{sun}\cos\theta_{sun}\Omega_{sun}/\pi$ (Wu and Liu 2010a). Here, the temperature, solid angle, and mean zenith angle of the sun are T_{sun}, Ω_{sun}, and θ_{sun}, respectively, and σ is the Stefan–Boltzmann constant. The energy flux of scattered solar radiation flux in the model is $F_{SSR} = \alpha F_{ISR}$. The model entropy flux of this outgoing scattered solar radiation follows Stephens and O'Brien (1993). It is $J_{SSR} = (4/3)\sigma T^3_{sun}\chi(u_L)$, where the dilution function χ is taken to be that for Lambertian scattering with the factor $u_L = \alpha\cos\theta_{sun}\Omega_{sun}/\pi$. The parameter settings are $T_{sun} = 5779$ K, $\cos\theta_{sun} = 1/4$, and $\Omega_{sun} = 6.77\times10^{-5}$ sr (Wu and Liu 2010a), with $\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}. For these settings, $u_L = 1.62\times10^{-6}$, and $\chi(u_L) = 5.54\times10^{-6}$.

The terrestrial energy and entropy fluxes are those for a gray-/blackbody. The blackbody surface emissions are $F_{sfc} = \sigma T^4_{sfc}$ and $J_{sfc} = (4/3)\sigma T^3_{sfc}$. The graybody, upward and downward atmospheric emissions are $F_{atm} = \varepsilon\sigma T^4_{atm}$ and $J_{atm} = (4/3)\varepsilon\sigma T^3_{atm}$.

In addition to radiative exchange, the model surface and atmosphere exchange energy through a convective–conductive heat flux F_{CHF}. By convention, F_{CHF} is positive if energy is transported from the warm surface to the cooler atmosphere. The convective heat flux is assumed to be a fraction γ of the incoming solar radiation: $F_{CHF} = \gamma F_{ISR}$. The model is closed with the specification of the parameters α, β, γ, and ε. The parameter settings are $\alpha = 0.30$, $\beta = 0.10$, $\gamma = 0.25$, and $\varepsilon = 0.95$. These choices are broadly consistent with estimates of Earth's energy budget (e.g., Trenberth et al. 2009). The model energy fluxes are depicted schematically in Fig. 2.

a. Model energetics

For a steady state, the energy fluxes must balance. At the top of the atmosphere, the net incoming flux is the sum of the incoming solar radiation less the solar radiation scattered back to space and the terrestrial radiation escaping the atmosphere:

$$F^{net}_{TOA} = F_{ISR} - \alpha F_{ISR} - (1-\varepsilon)F_{sfc} - F_{atm} = 0. \quad (5.1)$$

The net flux into the atmosphere is the sum of the absorbed solar and terrestrial radiation and the convective heat flux less the terrestrial radiation emitted by the atmosphere:

$$F^{net}_{atm} = \beta F_{ISR} + \varepsilon F_{sfc} + F_{CHF} - 2F_{atm} = 0. \quad (5.2)$$

The net flux into the surface is the sum of the absorbed solar and terrestrial radiation less the sum of the terrestrial radiation emitted and the convective heat flux:

$$F^{net}_{sfc} = (1-\beta)F_{ISR} + F_{atm} - \alpha F_{ISR} - F_{sfc} - F_{CHF} = 0. \quad (5.3)$$

Note that the sum of the atmosphere and surface budgets equals that for the total system (5.1). Together (5.2) and (5.3) can be solved for the temperatures of the atmosphere and the surface. One finds that the surface temperature is given by $(2-\varepsilon)\sigma T^4_{sfc} = 2\sigma T^4_p - \beta F_{ISR} - F_{CHF}$, where T_p is the planetary temperature: $\sigma T^4_p = (1-\alpha)F_{ISR}$. Here, $T_p = 255$ K. Results for the temperatures and fluxes, summarized in Table 1, indicate representative surface and atmosphere temperatures of 278 and 253 K, respectively. Figure 3 summarizes the energetics of the model.

TABLE 1. Temperature T, flux of energy F, and flux of entropy J for the radiative–convective equilibrium model of the climate system (Fig. 2). The radiation fields include the incoming, scattered, and absorbed solar radiation (ISR, SSR, and ASR, respectively), as well as the outgoing longwave radiation (OLR). The material fields include the surface, atmosphere, and convective heat flux (SFC, ATM, and CHF, respectively).

Quantity	ISR	SSR	ASR	OLR	SFC	ATM	CHF
T (K)	5779	5779	5779	255	278	253	2798
F (W m^{-2})	341	102	239	239	341	221	85
J (mW m^{-2} K^{-1})	79	110	55	1248	1632	1166	30

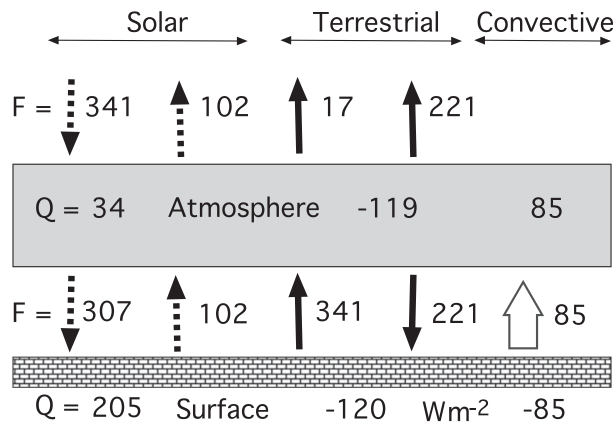

FIG. 3. Energy budget (W m^{-2}) of the radiative–convective equilibrium model. Dashed arrows represent energy fluxes of solar radiation, solid arrows represent fluxes of terrestrial radiation, and the open arrow represents the convective heat flux. Numbers in the atmosphere and below the surface represent the net gain or loss of energy due to heating by the solar, terrestrial, and convective fluxes. The net heating in the atmosphere and surface is zero. The sum of the TOA fluxes is also zero.

b. Model entropics

In contrast to the energy balance, the entropy budgets indicate a net production. At the top of the atmosphere, the net outgoing entropy flux is the sum of the flux of the scattered solar radiation and the terrestrial radiation escaping to space less the incoming solar radiation flux:

$$J_{\mathrm{TOA}}^{\mathrm{net}} = J_{\mathrm{SSR}} + J_{\mathrm{atm}} + (1-\varepsilon)J_{\mathrm{sfc}} - J_{\mathrm{ISR}}. \quad (5.4)$$

The global entropy production $J_{\mathrm{TOA}}^{\mathrm{net}}$ of 1279 mW m^{-2} K^{-1} is consistent with Wu and Liu (2010a). The entropy production of the atmosphere and the surface can be found in a manner similar to that in Planck (1959, section 102). The total entropy production associated with the interaction of the matter with the radiation fields is the sum of the production of entropy in the matter due to heating plus the entropy produced by the emission of radiation less the entropy extracted from the radiation field by absorption. For the atmosphere, we have

$$\dot{\Sigma}_{\mathrm{atm}} = \frac{F_{\mathrm{atm}}^{\mathrm{net}}}{T_{\mathrm{atm}}} + 2J_{\mathrm{atm}} - \varepsilon J_{\mathrm{sfc}} - \beta J_{\mathrm{ISR}}. \quad (5.5)$$

Similarly, for the surface, we have

$$\dot{\Sigma}_{\mathrm{sfc}} = \frac{F_{\mathrm{sfc}}^{\mathrm{net}}}{T_{\mathrm{sfc}}} + J_{\mathrm{SSR}} + J_{\mathrm{sfc}} - J_{\mathrm{atm}} - (1-\beta)J_{\mathrm{ISR}}. \quad (5.6)$$

The net energy flux terms in (5.5) and (5.6) vanish for the steady state where (5.2) and (5.3) hold. Thus, the net productions are associated with the radiation processes. The atmospheric entropy production of 774 mW m^{-2} K^{-1} is dominated by the increase in radiation entropy due to terrestrial emission, both upward and downward, less the absorption of solar radiation and terrestrial radiation emitted by the surface. Similarly, the surface entropy production of 505 mW m^{-2} K^{-1} is dominated by the increase in radiation by terrestrial emission and solar scattering less the absorption of downwelling terrestrial radiation from the atmosphere and of solar radiation.

The sum of (5.5) and (5.6) yields the net entropy production by the climate system. This net production agrees with the net outgoing flux of entropy in (5.4). We find $J_{\mathrm{TOA}}^{\mathrm{net}} = \dot{\Sigma}_{\mathrm{atm}} + \dot{\Sigma}_{\mathrm{sfc}}$. Consistent with the model assumption that only the surface scatters the solar radiation, the production due to the solar scattering (J_{SSR} = 110 mW m^{-2} K^{-1}) is assigned to the surface entropy budget. We note that dividing the scattering processes between the surface and atmosphere would redistribute the scattering production between the two. Such redistribution would affect the local, but not the global, entropics of the model. Figure 4 summarizes the Planckian entropics of the model.

An alternative to the Planck approach is to analyze the entropy production according to the individual processes rather than to their particular location. The irreversible entropy production due to the convection is

$$\dot{\Sigma}_{\mathrm{mat}} = F_{\mathrm{CHF}}(T_{\mathrm{atm}}^{-1} - T_{\mathrm{sfc}}^{-1}) \equiv F_{\mathrm{CHF}}/T_{\mathrm{CHF}}. \quad (5.7)$$

It is positive because the flux is down the temperature gradient from the surface to the atmosphere. The atmosphere gains entropy at the rate of 336 mW m^{-2} K^{-1} while the surface loses entropy at the rate of 306 mW m^{-2} K^{-1} for a total production of 30 mW m^{-2} K^{-1}. This material entropy production is present in both the material and control-volume representations. Figure 5 illustrates the irreversible material entropy production.

The control-volume representation of section 4 documents additional irreversible entropy production associated

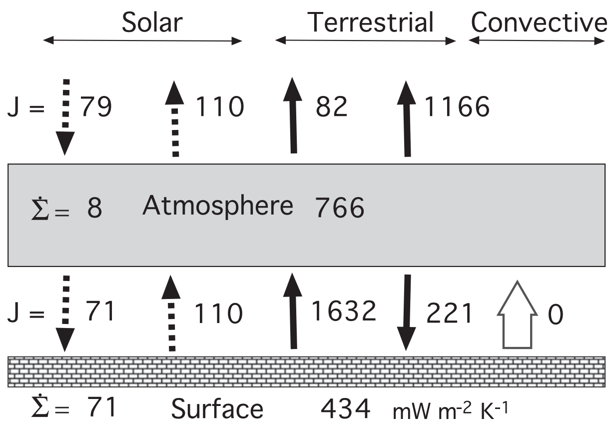

FIG. 4. Entropy budget ($\mathrm{mW\,m^{-2}\,K^{-1}}$) of the radiative–convective equilibrium model. Dashed arrows represent entropy fluxes of solar radiation, and solid arrows represent fluxes of terrestrial radiation. Numbers in the atmosphere and below the surface represent the net production of entropy due to the solar and terrestrial fluxes. The convective heat flux transports no entropy. The net fluxes in the atmosphere and surface sum to 774 and $505\,\mathrm{mW\,m^{-2}\,K^{-1}}$, respectively. Their total of $1279\,\mathrm{mW\,m^{-2}\,K^{-1}}$ equals the sum of the TOA entropy fluxes to space.

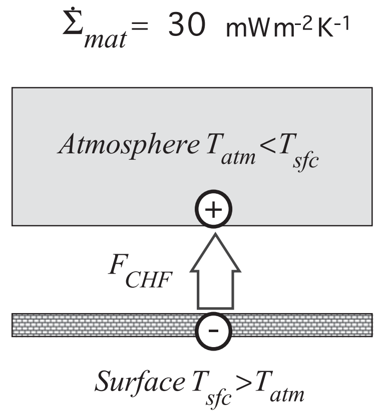

FIG. 5. Schematic diagram of the material entropy production ($\mathrm{mW\,m^{-2}\,K^{-1}}$) due to heating. Convection and conduction transport energy from the warm surface to the cooler atmosphere. The entropy gain (indicated by the plus sign) by the atmosphere is greater than the entropy loss (indicated by the minus sign) by the surface because of its lower temperature. As a consequence, the production is positive: $\dot{\Sigma}_{\mathrm{mat}} = F_{\mathrm{CHF}}(1/T_{\mathrm{atm}} - 1/T_{\mathrm{sfc}}) > 0$.

with the radiation processes. The production due to the scattering of solar radiation,

$$\dot{\Sigma}_{\mathrm{ssr}} = J_{\mathrm{SSR}} - \alpha J_{\mathrm{ISR}}, \tag{5.8}$$

is positive despite energy conservation of the scattered photons because the photons are dispersed over a larger solid angle. The entropy flux in the scattered radiation is $110\,\mathrm{mW\,m^{-2}\,K^{-1}}$ compared to $24\,\mathrm{mW\,m^{-2}\,K^{-1}}$ for the flux of the incident photons scattered. The difference is a net production of $86\,\mathrm{mW\,m^{-2}\,K^{-1}}$. Figure 6 illustrates the irreversible entropy production by scattering.

The absorption of solar radiation entails an increase in material entropy associated with the heating of the matter with a concomitant loss of radiation entropy. We find that

$$\dot{\Sigma}_{\mathrm{asr}} = (1 - \alpha - \beta)(F_{\mathrm{ISR}}/T_{\mathrm{sfc}} - J_{\mathrm{ISR}}) + \beta(F_{\mathrm{ISR}}/T_{\mathrm{atm}} - J_{\mathrm{ISR}}) \tag{5.9}$$

is positive, as the energy is dispersed over the cooler matter. The surface and atmosphere gain entropy at the rates of 734 and $135\,\mathrm{mW\,m^{-2}\,K^{-1}}$, respectively, while the solar beam loses entropy at the rate of $55\,\mathrm{mW\,m^{-2}\,K^{-1}}$ for a net production of $814\,\mathrm{mW\,m^{-2}\,K^{-1}}$. Figure 6 illustrates the irreversible entropy production.

The entropy production due to the absorption and emission of terrestrial radiation is similar in form to that of the absorption of solar radiation but is further augmented by production associated with the emission of radiation. That emission consists of a loss of material entropy with a concomitant gain in radiation entropy. For example, for a body at temperature T that emits a radiation flux of energy F and entropy J, the entropy production is $J - F/T$. Because emission is a spontaneous process, this production must be positive. Hence, $J > F/T$. Each emission and absorption consists of an exchange of entropy between the two systems. One finds that the total production is

$$\dot{\Sigma}_{\mathrm{terr}} = 2(J_{\mathrm{atm}} - F_{\mathrm{atm}}/T_{\mathrm{atm}}) - (J_{\mathrm{atm}} - F_{\mathrm{atm}}/T_{\mathrm{sfc}}) + (J_{\mathrm{sfc}} - F_{\mathrm{sfc}}/T_{\mathrm{sfc}}) - \varepsilon(J_{\mathrm{sfc}} - F_{\mathrm{sfc}}/T_{\mathrm{atm}}). \tag{5.10}$$

Each term in parentheses is positive. The four groupings correspond to the emission and absorption by the atmosphere and the emission and absorption by the surface. The emission terms are positive and larger in amplitude than the absorption terms, which are negative. Table 2 summarizes the terrestrial radiation production budget of (5.10). The net emission by the atmosphere dominates the production. Physically, the downward emission from the atmosphere is absorbed by the surface, and a fraction ε of the surface emission is absorbed by the atmosphere. Mathematically, these processes cancel in the expression (5.10) that can be rewritten as

$$\dot{\Sigma}_{\mathrm{terr}} = (J_{\mathrm{atm}} - F_{\mathrm{atm}}/T_{\mathrm{atm}}) + (1 - \varepsilon)(J_{\mathrm{sfc}} - F_{\mathrm{sfc}}/T_{\mathrm{sfc}}) + (\varepsilon F_{\mathrm{sfc}} - F_{\mathrm{atm}})(1/T_{\mathrm{atm}} - 1/T_{\mathrm{sfc}}). \tag{5.11}$$

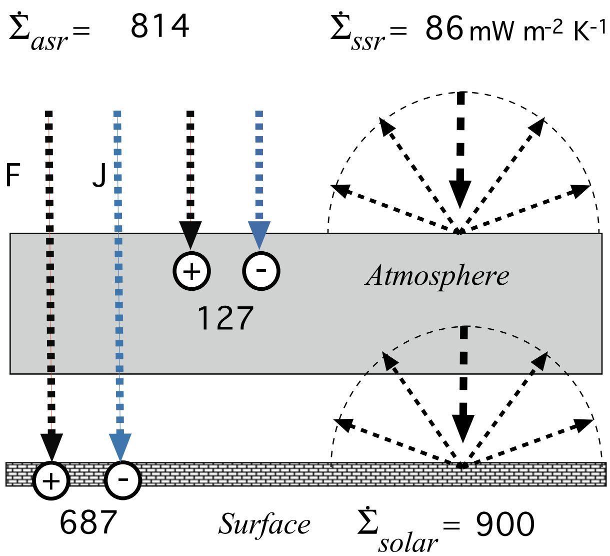

FIG. 6. Schematic diagram of the radiation entropy production ($\mathrm{mW\,m^{-2}\,K^{-1}}$) due to the (left) absorption and (right) scattering of solar radiation. The entropy produced (plus sign) in the matter at temperature T by the absorption of the energy flux (black dashed arrow) of radiation is greater than the entropy flux (blue dashed arrow) of radiation lost (minus sign) from the radiation field: $F_{abs}/T - J_{abs} > 0$. The energy flux scattered upward equals the energy flux incident downward. Because the energy of the scattered radiation is distributed over a larger solid angle (i.e., a larger number of directions), its entropy flux is greater than that of the incident radiation. For generality, the schematic includes scattering by both the atmosphere and the surface.

Each grouping in (5.11) is positive definite. The first two are the contributions to the radiation entropy as a result of the upward emission to space by the atmosphere and the fraction $(1-\varepsilon)$ of the surface emission that escapes to space. The last term in (5.11) expresses the net exchange of radiation energy between the surface and the atmosphere. The net flux is upward from the surface: $\varepsilon F_{sfc} - F_{atm} = 102\,\mathrm{W\,m^{-2}}$ and down the temperature gradient. We note that the associated radiation entropy flux makes no contribution, because the absorption of the radiation cancels with the emission. Similar to the upward convective flux from the surface to the atmosphere, there is an irreversible entropy production, because the atmospheric temperature is less than that of the surface. Figure 7 illustrates the irreversible entropy production by the absorption and emission of terrestrial radiation, as expressed by (5.11).

Consistent with (4.7), the net entropy production $\dot{\Sigma} \equiv \dot{\Sigma}_{mat} + \dot{\Sigma}_{terr} + \dot{\Sigma}_{asr} + \dot{\Sigma}_{ssr} > 0$ is positive and corresponds to the net entropy loss to space: $\dot{\Sigma} = J_{OLR} + J_{SSR} - J_{ISR}$. In summary, the contributions to the irreversible entropy production for the control-volume case are $\dot{\Sigma}_{mat} = 30\,\mathrm{mW\,m^{-2}}$, $\dot{\Sigma}_{ssr} = 87\,\mathrm{mW\,m^{-2}}$, $\dot{\Sigma}_{asr} = 814\,\mathrm{mW\,m^{-2}}$, and $\dot{\Sigma}_{terr} =$ $349\,\mathrm{mW\,m^{-2}}$. We also find that these contributions are consistent with the Planck approach: $\dot{\Sigma} = \dot{\Sigma}_{atm} + \dot{\Sigma}_{sfc}$.

TABLE 2. Entropy production by the emission and absorption of terrestrial radiation summarized by (5.10).

Process	$\dot{\Sigma}_{terr}$ ($\mathrm{mW\,m^{-2}\,K^{-1}}$)
Atmospheric emission	583
Atmospheric absorption	−272
Surface emission	408
Surface absorption	−371
Total	349

c. *Efficiency analysis*

The efficiency of the model climate is analyzed using both the material-system and control-volume representations. Table 3 summarizes the results for six different cases. The three CV cases indicate large Carnot efficiencies but small efficiencies for the material entropy production. The three cases differ in their treatment of the scattering of the solar radiation, as indicated in the energy input column. In the first case, CV1, the energy input is $F_{ISR} = 341\,\mathrm{W\,m^{-2}}$. Then the input temperature is the entropic temperature (4.8a) of the solar radiation $T_{in} = T_{sun}/(4/3)$. The output temperature is a mixture (4.8b) of contributions from the scattered solar radiation and the outgoing longwave radiation, with the effect of the OLR dominating (Table 1). The heat-engine relation in (4.10) holds with the bulk of the entropy production associated with the absorption of solar radiation η_{asr}. The efficiency of the material entropy production is the least, $\eta_{mat} = 2.2\%$. The second case, CV2, ignores the scattering of the solar energy and takes the energy input to be $F_{ASR} = 239\,\mathrm{W\,m^{-2}}$. The input temperature is unchanged from CV1, but the output temperature is reduced to 191 K, the entropic temperature of the OLR, because the scattered radiation is eliminated from the output temperature definition in (4.8b). The material entropy efficiency is slightly increased. The third case, CV3, excludes the solar radiation from the control volume. The output temperature is again that of CV2 for the OLR, but the input temperature becomes the weighted average of the temperatures where the solar radiation is absorbed:

$$T_{in} = (1-\alpha)[(1-\alpha-\beta)/T_{sfc} + \beta/T_{atm}]^{-1}. \qquad (5.12)$$

We find $T_{in} = 275\,\mathrm{K}$. This reduction in input temperature decreases the Carnot efficiency to about a third from the two previous cases. The material entropy production efficiency η_{mat} is very similar for these three control-volume cases.

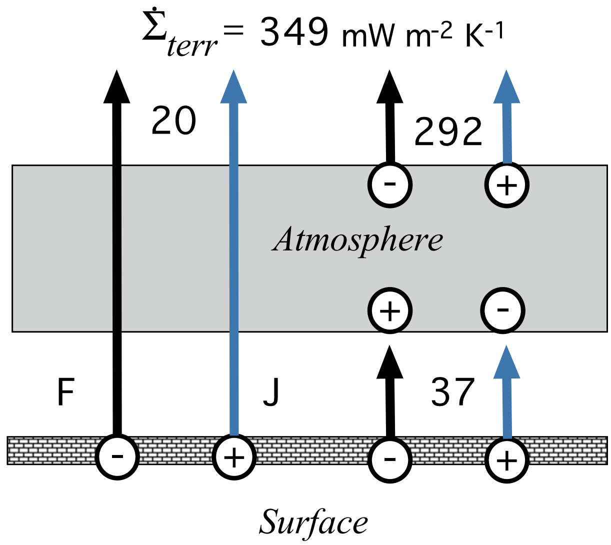

FIG. 7. Schematic diagram of the radiation entropy production by the absorption and emission of terrestrial radiation ($\mathrm{mW\,m^{-2}\,K^{-1}}$). Black arrows represent energy fluxes; blue arrows represent entropy fluxes. Emission of energy decreases the entropy of the emitting matter (minus sign) but increases the entropy of the radiation field (plus sign). Absorption has the opposite effect. The emissions from the surface and the atmosphere to space are irreversible, as is the net exchange of radiation from the surface to the atmosphere. The net entropy emission is defined by the increase in radiation entropy relative to the loss of material entropy $J - F/T$. Analogous to the material entropy production, the net energy exchange produces entropy at the rate $(\varepsilon F_{\mathrm{sfc}} - F_{\mathrm{atm}})(1/T_{\mathrm{atm}} - 1/T_{\mathrm{sfc}}) > 0$.

The three material-system cases (MS1–MS3) of Table 3 correspond to the analysis of section 2, for which the radiation system constitutes the thermodynamic surroundings of the combined surface–atmosphere system. The cases differ in their partitioning of the heat input and output. The case MS1 strictly distinguishes between absorption and emission and takes the input and output heating to be

$$\dot{Q}_{\mathrm{in}} = \int_V \int_0^\infty \int_{4\pi} \varepsilon_\nu I_\nu \, d\omega \, d\nu \, dV \quad \text{and}$$
$$\dot{Q}_{\mathrm{out}} = \int_V \int_0^\infty \int_{4\pi} \varepsilon_\nu B_\nu \, d\omega \, d\nu \, dV, \tag{5.13}$$

respectively. The limits of integration in (5.13) are over all frequencies. Then the terrestrial radiation contributes to both the input and the output. The solar radiation contributes only to the input because of the absence of shortwave emission by the atmosphere. This partitioning results in a large energy flux into the system of $784\,\mathrm{W\,m^{-2}}$. In this situation, the input and output temperatures in (2.8) are very similar, and a small Carnot efficiency results. As required by the heat-engine relation (2.9) with no external working, the efficiency factor for the material entropy production equals the Carnot efficiency of 1.0%. Rigorous application of this approach (MS1) is impractical using accurate, nongray calculations where the photon mean-free paths in some spectral lines are less than 1 m.

Material cases MS2 and MS3 use a heat partitioning that distinguishes between incoming solar and outgoing terrestrial radiation. The partitioning is

$$\dot{Q}_{\mathrm{in}} = \int_V \int_{\mathrm{solar}} \int_{4\pi} \varepsilon_\nu I_\nu \, d\omega \, d\nu \, dV,$$
$$\dot{Q}_{\mathrm{out}} = \int_V \int_{\mathrm{terr}} \int_{4\pi} \varepsilon_\nu (B_\nu - I_\nu) \, d\omega \, d\nu \, dV, \tag{5.14}$$

where the frequency limits of integration differ for the two integrals. The heat input is due solely to the absorption of solar radiation. The heat output is due to the net emission by the terrestrial radiation. Then the input temperature is given by (5.12). The output temperature differs according to the choice of terrestrial heating. Case MS2 uses the formulation (2.8b); then the output temperature is a mean of the surface and atmospheric temperatures. The material efficiency remains low and equals the Carnot efficiency (Table 3). Case MS3 uses the divergence form (2.10) of the terrestrial heating. Then an irreversible radiation entropy production occurs in the budget (Table 3) given by (2.11), $\dot{\Sigma}_{\mathrm{terr}} = (F_{\mathrm{SFC}} - F_{\mathrm{ATM}})(1/T_{\mathrm{atm}} - 1/T_{\mathrm{sfc}}) = 68\,\mathrm{mW\,m^{-2}}$, and the output temperature is the atmospheric temperature of 253 K.

It is worth emphasizing that none of the output temperatures equal the planetary temperature of 255 K that is based on an energy balance. We note that dropping the entropy factor of 4/3 for cases CV2 and CV3 would raise the output temperature to the planetary temperature. However the following simple argument favors a factor greater than unity. The emission of radiation consists of a loss of material entropy with a concomitant gain in radiation entropy. For example, for a body at temperature T that emits a radiation flux of energy F and entropy J, the entropy production is $J - F/T$. Because emission is a spontaneous process, this production must be positive. Hence, the entropy flux must exceed the energy flux divided by the temperature T, $J > F/T$. An entropy factor of unity incorrectly implies that emission is a reversible process.

We note that the efficiency approach of Johnson (2000) and Lucarini (2009) would include F_{CHF} in the calculation of the energy and entropy fluxes in each part of the system. Then both the surface and atmosphere have zero net heating, and the Carnot efficiency is indeterminate.

6. Conclusions

The present analysis successfully applies the theory of irreversible thermodynamics to Earth's climate system.

TABLE 3. Efficiency analysis of the radiative–convective equilibrium model of the climate system (Fig. 2). The climate system is treated as either a control volume (CV) containing matter and radiation or a material system (MS) exclusively. The numerical cases correspond to different assumptions regarding the energy input F_{in}. The Carnot efficiency $\eta = (T_{in} - T_{out})/T_{in}$ is the sum of the efficiencies due to material entropy production η_{mat}, the radiative entropy productions due to the absorption and emission of terrestrial radiation η_{terr}, and the absorption η_{asr} and scattering η_{ssr} of solar radiation.

Climate system	$F_{in} = F_{out}$ (W m^{-2})	T_{in} (K)	T_{out} (K)	η (%)	η_{mat} (%)	η_{terr} (%)	η_{asr} (%)	η_{ssr} (%)
CV1	341	4334	251	94.2	2.2	25.7	59.9	6.4
CV2	239	4334	191	95.6	2.4	27.9	65.2	—
CV3	239	275	191	30.4	2.4	27.9	—	—
MS1	784	266	264	1.0	1.0	—	—	—
MS2	239	275	265	3.4	3.4	—	—	—
MS3	239	275	253	7.8	3.2	4.5	—	—

In the steady state, a heat-engine relation (1.4) can be found treating the system as a single material system (section 2) or as a control-volume system (section 4) that combines the material and radiation subsystems below the top of the atmosphere. Because the climate system does no external work, the net interaction between the system and its surroundings results solely in the production of entropy. It is argued that this material entropy production $\dot{\Sigma}_{mat}$ is the signature of the internal activity of the atmosphere, ocean, and land in absorbing solar radiation, redistributing that energy both vertically and horizontally, and then emitting that energy back out to space as terrestrial radiation.

The efficiency of that interaction $\eta_{mat} = T_{out}\dot{\Sigma}_{mat}/F_{ISR}$ is the product of the entropy production rate times the output temperature divided by the flux of incoming solar radiation. The validity of this formulation has been demonstrated using the radiative–convective equilibrium model of section 5. Here, we apply it to Earth's climate system for either the control-volume or material-system representations of the climate system. The applications are equivalent in the sense that both use the heat-engine relation, but complementary because the representation of the system differs. For the control-volume representation (CV1), the planetary input and output temperatures (4.8), using the observed TOA radiation data from Fig. 1 of Wu and Liu (2010a) are $T_{in} = 4329$ K and $T_{out} = 251$ K. Then the Carnot efficiency $\eta = (T_{in} - T_{out})/T_{in}$ is 94%. Recent estimates using several climate model simulations (Lucarini et al. 2011) suggest Earth's material entropy production rate is about 55 mW m^{-2} K^{-1}. For the incoming solar flux of $F_{ISR} = 342$ W m^{-2}, the efficiency of Earth's material entropy production η_{mat} is therefore about 4%. Alternatively, application of the efficiency relation to the material-system representation (MS2) of Earth's climate system requires both energy and entropy flux estimates from climate models. For example, the energy and entropy radiation data of Table 5 of Goody (2000) for a 6-yr, unsteady, numerical model simulation including clouds implies input and output temperatures $T_{in} = 274$ K and $T_{out} = 251$ K. Using his estimated entropy production of 70 mW m^{-2} K^{-1}, $\eta_{mat} = 7.6\%$ is in close agreement with his Carnot efficiency η of 8.4%.

In addition to estimating the climate efficiency, the heat-engine relation (1.4) has the potential to be a useful metric for steady-state simulations of climate systems, because it incorporates both the first and second laws of thermodynamics. For example, Johnson (1997) hypothesizes that numerical procedures and model parameterizations will lead to unrealistic entropy production rates. The relation (1.4) with no external working leads to

$$\frac{\dot{\Sigma}_{irr}}{\dot{Q}_{in}} = \frac{1}{T_{out}} - \frac{1}{T_{in}}. \tag{6.1}$$

Then increased entropy production at fixed heat input would require a decrease in the output temperature or an increase in the input temperature or a combination of the two. Then the observed "general coldness" of climate models is qualitatively consistent with (6.1). Quantification of the enhanced entropy production by computational methods is required to substantiate the hypothesis.

The current analysis provides a foundation for further research. Improved estimates of Earth's radiation energy and entropy budgets should be made, along with those for the material entropy budget of the climate system. The material entropy production consists of positive-definite components that allow for intercomparison. For example, Lucarini et al. (2011, their Table 2) estimate that the vertical/convective (as opposed to horizontal/baroclinic) entropy production contributes about 90% of the total production. Alternatively, the dissipation due to falling precipitation (Pauluis and Dias 2012) is comparable to the estimate of the dissipation of kinetic energy in the general circulation (Lorenz 1967). Quantifying the material entropy production will lead to valuable new insights into the climate system.

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