Origin and Flow History of Air Parcels in Orographic Banner Clouds

SEBASTIAN SCHAPPERT AND VOLKMAR WIRTH
University of Mainz, Mainz, Germany

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ABSTRACT

Banner clouds are clouds in the lee of steep mountains or sharp ridges. Previous work suggests that the main formation mechanism is vertical uplift in the lee of the mountain. On the other hand, little is known about the Lagrangian behavior of air parcels as they pass the mountain, which motivates the current investigation. Three different diagnostics are applied in the framework of large-eddy simulations of airflow past an isolated pyramid-shaped obstacle: Eulerian tracers indicating the initial positions of the parcels, streamlines along the time-averaged wind field, and online trajectories computed from the instantaneous wind field.

All three methods diagnose a plume of large vertical uplift in the immediate lee of the mountain. According to the time-mean Eulerian tracers, the cloudy parcels originated within a fairly small coherent area at the inflow boundary. In contrast, the time-mean streamlines indicate a bifurcation into two distinct classes of air parcels with very different characteristics. The parcels in the first class originate at intermediate altitudes, pass the obstacle close to its summit, and proceed directly into the cloud. By contrast, the parcels in the second class start at low altitude and take a fairly long time before they reach the cloud on a spiraling path. A humidity tracer quantifies mixing, revealing partial moistening for the first class of parcels and drying for the second class of parcels. For the online trajectories, the originating location of parcels is more scattered, but the results are still consistent with the basic features revealed by the other two diagnostics.

1. Introduction

Banner clouds are cloud plumes in the lee of isolated mountain peaks or ridges, which often occur on otherwise cloud-free days (Glickman 2000; Ahrens 2009). They are frequently observed, for instance, at Mount Matterhorn in the Swiss Alps or at Mount Zugspitze in the Bavarian Alps. Based on an extended set of time-lapse movies taken at Mount Zugspitze, Schween et al. (2007) provided a comprehensive definition, suggesting that a banner cloud has to satisfy four key criteria: 1) the cloud must be attached exclusively to the mountain’s immediate lee; 2) the cloud must consist of condensate that originates from water vapor contained in the air (rather than blowing snow); 3) the cloud must be persistent; and 4) the cloud should not be of convective character.

Recently, there have been a few studies trying to deepen our scientific understanding of this beautiful phenomenon. The paper by Wirth et al. (2012) reports systematic observations of banner clouds at Mount Zugspitze, including statistical information about the duration of banner cloud events, the distribution of wind speed and direction during banner cloud events, and the diurnal and seasonal characteristics. Early simulations of banner clouds were carried out by Geerts (1992) with a focus on the role of surface friction. More systematic large-eddy simulations (LESs) were presented by Reinert and Wirth (2009) and Voigt and Wirth (2013), who considered turbulent flow of dry air past an isolated pyramid. These simulations revealed the key importance of Lagrangian uplift and associated adiabatic cooling for banner cloud formation. The vertical uplift turns out to have a pronounced windward–leeward asymmetry, with much larger values in the immediate lee of the obstacle compared to its windward side. This striking asymmetry is associated with an arch-shaped vortex that exists in the lee of the obstacle. Reinert and Wirth (2009) compared simulations of dry versus moist airflow in order to investigate the relative importance of dry dynamics versus moist thermodynamics. Although the release of latent heat within the banner cloud leads to some modifications, the overall behavior was quite similar in both runs. This led to the conclusion that banner clouds can, to a first
approximation, be understood in the framework of dry dynamics, with the location of the cloud given through that volume of air where vertical uplift exceeds a certain threshold.

In the current paper, we will adopt a Lagrangian perspective for analyzing the highly complex flow associated with banner cloud occurrence, using again an idealized setup. We will investigate the origin of those air parcels that experience strong uplift in the lee, as well as the path along which these parcels pass the mountain. To this end, we will implement and apply three different diagnostic tools: 1) three Eulerian tracers providing information about the displacement in the streamwise, lateral, and vertical directions; 2) streamlines following the time-averaged flow; and 3) trajectories following the instantaneous, fully turbulent flow. Eulerian tracers are straightforward to implement and have been used before in the context of banner cloud simulations in order to diagnose vertical parcel displacements (Reinert and Wirth 2009; Voigt and Wirth 2013). Repeating this analysis in the current paper allows us to connect our new results from trajectory analyses to previous results. In addition, we go beyond the previous analysis by considering two Eulerian tracers for the spanwise and streamwise directions, respectively, which provides us with novel information. Regarding the trajectories, we deliberately compute them both from the time-averaged flow and from the time-dependent fully turbulent flow. Of course, the latter trajectories are more realistic, while the former give a somewhat idealized picture. Yet the streamlines from the time-averaged flow turn out to be very interesting: they indicate certain singular behavior that could not be seen from the fully turbulent trajectories but that helps with interpreting specific features of the latter. Although the results from our three diagnostic tools differ in a number of ways, they nevertheless provide a consistent general picture. We will also quantify the strength of turbulent mixing, which turns out to play an important role.

The paper is organized as follows. A description of the numerical model and its configuration is given in section 2. In section 3, we introduce the different diagnostic tools and use them to study vertical uplift and the potential for cloud formation. Section 4 then turns to the origin and flow history of the air parcels, with the interesting result showing the existence of two distinct classes in the case of the time-averaged flow. Finally, section 5 provides a summary and our main conclusions.

2. Model and setup

In this study, we simulate turbulent three-dimensional flow of dry air past idealized orography in a nonrotating atmosphere. We use the nonhydrostatic anelastic version of the EULAG model in LES mode [for details, see Prusa et al. (2008)]. The validity of the anelastic approximation for small- and mesoscale atmospheric applications is now firmly established (Smolarkiewicz et al. 2014; Kuroski et al. 2014). Model variables are the Cartesian components of the three-dimensional wind \( \mathbf{u} = (u, v, w) \), perturbation pressure \( p^* \), and perturbation potential temperature \( \theta^* \). Perturbations are defined as deviations from the inflow profiles (to be defined below). The continuity equation \( \mathbf{V} \cdot (\rho_0 \mathbf{u}) = 0 \) involves a reference density \( \rho_0(z) \). The Eulerian equations are discretized on a Cartesian grid, and orography is represented through an immersed boundary approach (Mittal and Iaccarino 2005; Smolarkiewicz et al. 2007). Subgrid-scale fluxes of momentum and heat are parameterized through a turbulent kinetic energy (TKE) closure (Smolarkiewicz et al. 1997). The model domain is decomposed and distributed to 512 processors on an IBM Power6 computer.

Regarding the model configuration, we essentially follow Voigt and Wirth (2013). The model domain (Fig. 1) covers 8 km in the streamwise direction \( x \), 4.8 km in the spanwise direction \( y \), and approximately 3 km in the vertical direction \( z \). This domain turns out to be large enough for our purposes. We tested the corresponding sensitivity by doubling the model extent in either direction, and this was associated with only marginal changes of our results. The grid spacing is equidistant in all three directions with \( \delta x = \delta y = \delta z = 25 \) m, which turns out to be sufficient to resolve the majority of the most energetic part of the spectrum. Our orography is a square-based pyramid, which is located 3 km downstream of the inflow boundary and is centered in the spanwise direction. The center of its base marks the origin of the coordinate system. With a height of \( H = 975 \) m and a width of \( L = 1000 \) m, the pyramid has a slope angle of \( \alpha = 63^\circ \). Note that our plots below show only the inner part of the model domain.

The inflow boundary is located at \( x = x_{\text{in}} \approx -3000 \) m. At this location, we specify profiles of wind and temperature. The wind is prescribed as \( \mathbf{u} = (U, 0, 0) \), with \( U = 9 \text{ m s}^{-1} \) being a constant (i.e., independent of time and space); potential temperature is specified as a function of altitude such that the stratification is neutral \( (\partial \theta / \partial z = 0) \) for \( z \leq 1 \) km and stably stratified above \( (\partial \theta / \partial z = 4 \text{ K km}^{-1}) \). Temperature is related to potential temperature through \( T = \theta(p/p_0)^\kappa \), where \( p_0 \) denotes pressure, \( p_0 = 1000 \) hPa is a constant reference pressure, \( \kappa = R/c_p \), \( R \) is the gas constant for dry air, and \( c_p \) is the specific heat at constant pressure. We apply open conditions at the streamwise and spanwise boundaries and a rigid-lid condition at the upper boundary. A sponge
layer in the upper part of the domain \((z \geq 2 \text{ km})\) relaxes the wind toward the inflow profile and avoids wave reflections from the upper boundary. In the flat portion of the lower boundary, we formulate the surface stress as
\[ t = r C_D \rho v_0, \]
where \(v_0\) is the horizontal wind at the surface, \(V_0 = |v_0|\), \(\rho\) is air density, and \(C_D = 0.01\) is a drag parameter. Next to the obstacle, the immersed boundary approach effectively represents a no-slip condition (Goldstein et al. 1993; Smolarkiewicz et al. 2007). There are no surface heat fluxes in our model configuration.

In addition to the original model variables, we implemented four tracers \(f, \eta, \chi, \text{ and } q\). They evolve according to
\[ \frac{D \phi}{D t} = M_\phi, \]
where \(D/Dt\) denotes the material rate of change following the turbulent flow, and \(M_\phi\) represents material nonconservation owing to the parameterized subgrid turbulence. Material nonconservation is considered to be weak as long as the TKE from subgrid-scale motions is small compared to the TKE from explicitly resolved motions; this condition is satisfied throughout the domain except close to the surface and in regions of extreme shear (like the separated boundary layer). Note that our tracers depend on all three space dimensions as well as time, \(\phi(x, y, z, t)\), thus representing a fully turbulent three-dimensional scalar field. For the purpose of presentation, we show in our plots only vertical sections of the time-averaged fields.

Advection in all model equations is implemented numerically through the multidimensional positive definite advection transport algorithm (MPDATA; Smolarkiewicz and Margolin 1998). MPDATA contains just enough numerical diffusion so as to keep the numerics stable (Smolarkiewicz 1984; Smolarkiewicz et al. 2007; Beaudoin et al. 2013). This suggests that, in our simulations, the majority of subgrid mixing is due to the parameterized sub-scale stresses rather than the numerical diffusion.

The tracers \(\xi, \eta, \chi\) are specified at the initial time of the integration \((t = 0)\) as
\[ (\xi, \eta, \chi) = (x + U t_s, y, z), \]
throughout the model domain, where \(t_s\) denotes the length of the spin-up interval (see below). At the inflow boundary, the tracers are prescribed as
\[ (\xi, \eta, \chi) = [x_{in} - U(t - t_s), y, z]. \]
These specifications guarantee that, at \(t = t_s\), the values of \(\xi, \eta, \chi\) at any grid point represent the coordinates \(x, y, \) and \(z\) of that grid point. The tracers will subsequently be used to provide Eulerian estimates of the streamwise, lateral, and vertical parcel displacements for \(t > t_s\).

The tracer \(q\) is meant to represent moisture. Assuming that relative humidity is constant throughout the domain at initial time (with a value \(RH_0\)), \(q\) is initialized so as to represent the corresponding specific humidity. To a good approximation, this results in the following relation (e.g., Bohren and Albrecht 1998):
\[ q = RH_0 \frac{e(T)}{p}, \]
where \(e_c(T)\) is the saturation vapor pressure at temperature \(T\), and \(\epsilon = 0.622\) is the ratio between the molecular
mass of water and dry air. The initial relative humidity is chosen to be RH$_0 = 72\%$, which results in a surface specific humidity of $q = 7 \text{ g kg}^{-1}$. Figure 2 shows the corresponding profile, which is used at the same time as inflow condition. Note that, in contrast with potential temperature, specific humidity is not constant with altitude, implying that our inflow conditions do not represent a well-mixed boundary layer. This assumption is consistent with the work of Reinert and Wirth (2009) and the observations by Wirth et al. (2012), both indicating that deviations from a well-mixed boundary layer are essential for the occurrence of banner clouds.

The tracer $q$ allows us to quantify relative humidity and, hence, the potential for cloud formation throughout the evolution. Having said this, we want to stress again that we simulate the flow of dry air, which means that we neglect the effect of latent heat release from condensation of water vapor. As mentioned in the introduction, this effect can be considered small and does not have an impact on the overall flow geometry or the basic mechanism. Therefore, in order to keep the interpretation simple, we decided to refrain from the simulation of moist air in the current study.

The model is run for 90 min altogether, with a time step $\Delta t = 0.25 \text{ s}$. The first 30 min are considered to be a spin-up period (i.e., $t_s = 30 \text{ min}$), during which the flow and the tracer distributions reach a statistically stationary state. The appropriate value for $t_s$ was found by explicitly diagnosing time series of relevant quantities at selected grid points. The end of the spin-up run ($t = t_s$) is identical with the initial time for the main run, which consists of the remaining 60 min. When referring to the initial time in the following, we mean the initial time of the main run. The data from the main run are used to accumulate flow statistics and to compute trajectories.

Following Fröhlich (2006), time averages of all model variables $\psi$ during the main run are computed recursively as

$$
\langle \psi \rangle_{n+1} = \alpha \psi_{n+1} + (1 - \alpha) \langle \psi \rangle_n,
$$

where angle brackets with a subindex $n$ denote the time average at time $t_n > t_s$, corresponding to time step $n$, and the parameter $\alpha$ is defined as $\alpha = \Delta t / (t_n - t_s)$. Angle brackets without a number, as well as the term “time average,” will be used in the following to denote the time average over the entire main model run.

For illustration and further reference, we show in Fig. 3 the streamlines of the time-averaged flow field. One can clearly distinguish two vortices with clockwise rotation: first, the so-called horseshoe vortex (e.g., Fröhlich 2006) on the windward side below the stagnation point; second, the bow vortex [see Fig. 4 of Voigt and Wirth (2013)] on the leeward side below the top of the pyramid. Note that the flow is practically laminar on the windward side; most of the turbulence is found on the leeward side [see Fig. 6 of Voigt and Wirth (2013)] and can be considered as self-induced by airflow past the pyramid.

3. Diagnosing parcel displacements

In this section, we introduce basically three methods to diagnose Lagrangian information. Two of these methods can be applied in two variants each (see Table 1 for an overview). Our focus will be on the upward vertical displacement, also called uplift, because of its relevance for cloud formation.
We generalize the approach of Voigt and Wirth (2013) and define

\[ D_x(x, y, z, t) = x - \xi(x, y, z, t), \]
\[ D_y(x, y, z, t) = y - \eta(x, y, z, t), \]
\[ D_z(x, y, z, t) = z - \chi(x, y, z, t). \]

Recognizing that \( \xi, \eta, \) and \( \chi \) denote the streamwise, lateral, and vertical position at the initial time, and assuming that the Eulerian tracers \( \xi, \eta, \) and \( \chi \) are materially conserved, the fields \( D_x, D_y, \) and \( D_z \) represent the displacement in the streamwise, lateral, and vertical direction since the initial time at each grid point. We thus obtain Lagrangian information (about parcel displacements) from Eulerian variables. Obviously, this interpretation is only true to the extent that \( \xi, \eta, \) and \( \chi \) are materially conserved. As noted in the previous section, we expect nonconservation in areas of strong shear.

The time-averaged streamwise displacement \( \langle D_x \rangle \) is shown in Fig. 4. In the absence of surface friction and without any obstacle, the parcels would proceed at a speed of \( U = 9 \text{ m s}^{-1} \) in the streamwise direction, yielding \( \Delta x = 32.4 \times 10^3 \text{ m} \) after 1 h. Since, in this case, \( \Delta x \) would increase linearly with time (starting with \( \Delta x = 0 \) at initial time), the time average value would be \( \langle \Delta x \rangle = 16.2 \times 10^3 \text{ m} \). The figure indicates that the parcels close to the ground and, especially, in the lee of the pyramid have traveled less than \( 16.2 \times 10^3 \text{ m} \) on average. This is because of surface friction and deviations of trajectories from straight lines. However, throughout the domain, \( \langle \Delta x \rangle \) is much larger than the streamwise distance from the inflow boundary. This, together with the fact that the flow is purely streamwise upstream of the inflow boundary, allows us to interpret the fields \( \langle D_y \rangle \) and \( \langle D_z \rangle \) as Lagrangian lateral and vertical displacements since the air parcel entered the model domain at the inflow boundary.

Figure 5 shows the time-averaged vertical displacement \( \langle D_z \rangle \). As noted before by Reinert and Wirth (2009) and Voigt and Wirth (2013), \( \langle D_z \rangle \) exhibits a pronounced windward–leeward asymmetry with a plume of large uplift attached to the immediate lee of the mountain. Qualitatively, this plume of uplift resembles the contours in a plot of relative humidity (not shown here; see Voigt and Wirth (2013), their Figs. 3b and 3d), although the details surely depend on the specified inflow conditions for humidity. Here, we aim to make some general statements about the possible existence of a cloud without much focus on the details of the moisture field.
We therefore define in this paper the cloud as that volume in space where $\Delta z$ exceeds a certain threshold. The rationale behind this definition is that vertical uplift is a key process for cloud formation in an unsaturated atmosphere. In our case, the value $\Delta z = 570$ m turns out to be appropriate (white contour in Fig. 5). In a simulation of moist airflow past a mountain, the occurrence and size of a cloud would depend on the humidity profile specified at the inflow boundary of the domain. The moisture profile could be specified so as to obtain a sizeable cloud in the lee and practically cloud-free conditions on the windward side. This freedom of choice corresponds to the freedom of choice of an appropriate contour of $\Delta z$ in our dry simulation. (The particular threshold that we chose will become more plausible below in connection with Fig. 8.)

We will use this definition of cloud later in section 4 in connection with time-mean streamlines and online trajectories. There, we will initialize streamlines/trajectories on all grid points within the cloud and integrate backward in time toward the inflow boundary. This results in diagnostics that are hybrid in character (last column in Table 1), because the cloud is defined through an Eulerian tracer, while the streamline/trajectory is a Lagrangian concept.

b. Time-mean streamlines

The obvious method to obtain Lagrangian information from the turbulent flow field is through the computation of trajectories, and this will indeed be done in the next subsection. However, in an attempt to extract some generic features, in this subsection we consider trajectories from the time-averaged flow field. Owing to the stationarity of the time-mean flow, trajectories are equal to streamlines. We will refer to them as time-mean streamlines in the following in order to distinguish them from the online trajectories to be described in the next subsection.

The time-mean streamlines are initialized either at the inflow boundary (resulting in so-called forward streamlines) or inside the cloud volume (resulting in so-called backward streamlines). The forward streamlines are initialized at the inflow boundary with eight initial seeds per grid point distributed within the area $|y| \leq 412.5$ m and $z < 800$ m. This results in some 9000 streamlines. The backward streamlines are initialized within the cloud volume, with one initial seed per grid point, yielding close to 4000 streamlines. The integration is performed using a simple Euler forward scheme forward or backward in time, with a time step of $\Delta t = 1$ s. Sensitivity studies indicated that our integration method is sufficiently accurate: using $\Delta t = 0.25$ s instead of 1 s leads to no significant change of our results. Similarly, different integration schemes (such as the Heun scheme or classical Runge–Kutta fourth order) lead to almost identical results.

The integration of a forward streamline is terminated when the streamline exits the domain. Over 99% of all streamlines exit the domain within an integration period of 4 h. The integration of the backward streamlines is continued until the streamlines hit the inflow boundary. At this time, the location in the $y$ and $z$ directions is noted, as well as the time it has taken for the parcel to go (backward in time) from the grid point within the cloud to the inflow boundary. Some of our backward streamlines never reach the inflow boundary within the integration time of 5 h, because they either get caught in a quasi-infinite loop inside the lee vortex, or they run into the mountain owing to numerical inaccuracies in the immediate neighborhood of the mountain. This leads to a loss of about 30% of all backward streamlines.

Stationary flow past a symmetric mountain can be expected to obey mirror symmetry about the center plane $y = 0$. Obviously, our instantaneous flow field deviates from this left–right symmetry owing to the turbulence. But even after time averaging, our mean flow field turns out to deviate from this symmetry to a certain extent. The reason lies in the fact that the actual flow field is time dependent on various time scales: in addition to fast fluctuation from small-scale turbulence, the flow features quasi-periodic vortex shedding in the lee, which operates on a much slower time scale. It follows that even a very long time average does not completely eliminate the left–right asymmetry; the remaining asymmetry is coincidental, because it depends on how many periods of vortex shedding it includes on the right side versus the left side of the obstacle. We found these asymmetries annoying, because they distract attention from the generic features that we are interested in. We, therefore, decided to symmetrize the time-mean flow field through replacing $(\mathbf{u})$ by $(\mathbf{\hat{u}})$ with $(\mathbf{\hat{u}})(x, y, z) = \frac{1}{2} \left[ (\mathbf{u})(x, y, z) + (\mathbf{u})(x, -y, z) \right], \quad (9)$

$(\mathbf{\hat{v}})(x, y, z) = \frac{1}{2} \left[ (\mathbf{v})(x, y, z) - (\mathbf{v})(x, -y, z) \right], \quad \text{and} \quad (10)$

$(\mathbf{\hat{w}})(x, y, z) = \frac{1}{2} \left[ (\mathbf{w})(x, y, z) + (\mathbf{w})(x, -y, z) \right], \quad (11)$

before computing the streamlines. Similarly, we symmetrized all time-averaged tracer fields. In the following, we omit the hat, because we exclusively consider the symmetrized variables in connection with the time-averaged fields.

For illustration, we show in Fig. 6 a single time-mean streamline passing through a point in the vicinity of the
center of the cloud. (This streamline was integrated both backward and forward from its initial condition.) Although this streamline only represents the time-mean flow, it is very convoluted with a complex geometry, which could not be gleaned from the time-averaged flow field itself. As we will see later, this complexity gives rise to interesting behavior.

The forward streamlines offer a simple and intuitive method to diagnose vertical uplift and, hence, an estimate for the location of the cloud. At each point on the streamline, we note the vertical displacement as the difference between the actual altitude and the altitude at the inflow boundary. We then mark only those parts of the streamline where the vertical displacement exceeds a threshold. Figure 7a shows the result for a large number of streamlines with the threshold chosen to be 770 m. The marked region (red) is quite similar to the cloud from Fig. 5 (white isoline). Interestingly, the threshold had to be somewhat larger here than in Fig. 5 in order to obtain a similar-size cloud in both cases. As we discuss in more detail below, we believe that this difference is due to the fact that the Eulerian tracer is subject to mixing from subscale processes.

c. Online trajectories

Obviously, our flow is turbulent and highly nonstationary. Air parcels move with the instantaneous wind at the location of the parcel, rather than with some hypothetical time-averaged wind. It is not evident to what extent our time-mean streamlines from the previous subsection provide a reliable estimate for the Lagrangian behavior of individual air parcels. We, therefore, computed true forward trajectories following air parcels using the instantaneous wind at the location of the parcel. This was done using, again, a simple Euler forward scheme with the time step \( \Delta t = 0.25 \text{s} \) corresponding to the model time step.

For each grid point at the inflow boundary, we initialize eight trajectories; the initial locations of these trajectories are randomly distributed in the neighborhood of the respective grid point. The grid points used are restricted to the area \( y = [-400, 400] \text{m} \) and \( z < 900 \text{m} \), resulting in a total of some 9000 initial points. The trajectories are integrated forward in time for 60 min. Interpolation of the wind field (and other model variables) to the location of the air parcel is done by trilinear interpolation in the inner part of the domain, with some refined treatment close to the pyramid. Trajectories that intersect with the surface of the obstacle because of numerical inaccuracies are terminated, and their final position is noted; less than 2% of all trajectories are lost because of this reason.

We make use of the online trajectories in two different ways. First, we estimate the location of the cloud by a technique similar to that used before in connection with the time-mean streamlines. Whenever the vertical displacement of a parcel on an online trajectory is larger than a predefined threshold (again, 770 m in our case), this part of the trajectory is marked by a flag. Figure 7b shows all flagged locations. Again, there is a region of large uplift in the lee, extending downstream like a banner. The region of flagged locations is much more elongated in the streamwise direction than in the
corresponding plot from the time-mean streamlines (Fig. 7a). The underlying reason is that online trajectories are affected by the turbulence, while the time-mean streamlines follow a somewhat artificial laminar time-mean flow. The latter is characterized by general downwelling at cloud level in the lee of the mountain (except very close to the mountain; see Fig. 3), which explains the relatively small streamwise extent of the flagged region in Fig. 7a. However, this time-mean downwelling is counteracted by the turbulent part of the flow in case of the online trajectories, which explains why some (albeit by far not all) trajectories manage to exceed the uplift threshold even far downstream in Fig. 7b. Note that this method is likely to overestimate the streamwise extent of the cloud, because it does not allow for any mixing. On the other hand, moisture in the real atmosphere is subject to mixing. Our diagnostic in Fig. 7b marks a few trajectories as cloudy far downstream, but these are surrounded by drier air that has undergone less lifting; mixing that occurs in reality would, therefore, reduce the moisture along the cloudy trajectories. In other words, mixing produces a tendency for the cloud to be shorter in the streamwise direction than suggested in Fig. 7b.

Another way to make use of the online trajectories (to be elaborated on in the next section) is to select only those that pass through a predefined cloud volume at some time during their life. The cloud volume was, again, defined through \( h \) and \( x \), as described in the previous section (see Fig. 5). With this application, somewhat over 90% of our trajectories are discarded, as they never pass through the cloud. The time of transit of an air parcel along the trajectory between the inflow boundary of the model domain and the cloud is defined as

\[
T_{\text{trans}} = \frac{1}{2} (T_{\text{in first}} + T_{\text{out last}}),
\]

where \( T_{\text{in first}} \) is the time of the first entry of the parcel into the cloud, and \( T_{\text{out last}} \) is the time of the last exit of the parcel out of the cloud. This definition accounts for the fact that a parcel may loop through the cloud several times (see Fig. 6), although in practice this happens only rarely for our online trajectories. Our subsequent analysis focuses, again, on the statistical properties of the selected trajectories.

4. Origin and flow history of air parcels

In this section, we study the origin and flow history of those air parcels that are located within the cloud volume at some time during the evolution. The cloud volume is defined through \( \langle \Delta z \rangle \) exceeding 570 m, as described in connection with Fig. 5. For our investigation, we will use all three methods introduced above: Eulerian estimates of the Lagrangian displacements, time-mean streamlines, and online trajectories.

a. Eulerian estimate of Lagrangian displacement

It has been pointed out before that the Lagrangian uplift plays a key role for banner cloud formation. Combining our two Eulerian diagnostics \( \eta \) and \( \chi \), we are now able to extend the picture of the Lagrangian history of air parcels to two dimensions (i.e., both the lateral and the vertical directions). At each grid point, the values of \( \eta \) and \( \chi \) represent the initial spanwise and vertical positions of the air parcel at that grid point. This diagnostic accounts for the fact that a parcel may loop through the cloud several times (see Fig. 6), although in practice this happens only rarely for our online trajectories. Our subsequent analysis focuses, again, on the statistical properties of the selected trajectories.
Interestingly, the cloud of points plotted in Fig. 8 seems to be split into two subclouds on either side of the plane of symmetry, with only very few points right on \( y = 0 \). This can be readily explained: some of the parcels that enter the domain right on the plane of symmetry are caught in the horseshoe vortex during their approach toward the pyramid, which advects them backward toward the inflow boundary (see Fig. 3). Parameterized subscale mixing, as well as numerical diffusion, should be able to help parcels to escape this “horseshoe trapping.” As a test, we carried out an additional run in which we switched off the subscale mixing for the tracers by setting \( M_\phi = 0 \) in (1) for \( \phi = \xi, \eta, \chi \). The corresponding plot for the origin of cloudy air parcels (not shown) is very similar to Fig. 8, except that the separation between the two subclouds of points is somewhat more distinct. This is consistent with the interpretation that subscale mixing facilitates air parcels to escape the horseshoe trapping right on the plane of symmetry.

Although the overall picture that transpires from Fig. 8 appears reasonable, it is not clear to what extent it can be trusted quantitatively. The reason lies in the fact that our Eulerian tracers are subject to mixing, especially in regions where strong stirring from turbulence coincides with large tracer gradients. The amount of mixing and its impact on the derived Lagrangian information is unclear at this point. We, therefore, proceed to obtain Lagrangian information about the history of air parcels through time-mean streamlines and, later, through online trajectories.

b. Time-mean backward streamlines

Figure 9 shows the intersection of time-mean backward streamlines with the inflow boundary, thus indicating the origin of those air parcels that, at some later time, pass through the cloud. Again, by construction, the parcel locations are mirror symmetric about \( y = 0 \). In contrast with the Eulerian estimate of Lagrangian displacement, we recognize a pattern of two distinct classes of streamlines. The parcels from the first class originate at altitudes between 150 and 600 m, with a spread of less than 100 m in the \( y \) direction. The parcels from the second class originate from below 150-m altitude. This class can, in turn, be divided into two subclasses, one at positive values of \( y \) and one at negative values of \( y \).

The time-mean diagnostics displayed in Figs. 8 and 9 both indicate substantial lifting of air parcels that end up within the cloud. Nevertheless, there are striking differences. In particular, on average, the mean streamlines in Fig. 9 imply a larger vertical uplift than the Eulerian tracers in Fig. 8 (see the yellow diamonds in both figures). As mentioned before, we interpret this as a result of material nonconservation of the Eulerian tracers.

This affects especially the parcels in the second class, originating from close to the bottom surface. These parcels get lifted by some 1000 m until they reach the cloud. During this process, they are engulfed into an environment with very different characteristics. This gives rise to large gradients in an Eulerian representation and, hence, to strong mixing. In addition, according to Fig. 9, some of these parcels originate from lateral positions that are quite far away (a few hundred meters) from the plane of symmetry, but this is not at all reflected in the corresponding plot in Fig. 8. Again, we suggest that this difference is because of a combination of parameterized and numerical material nonconservation of the Eulerian tracers. We will come back to this issue later.

Although the originating locations from the time-mean backward streamlines suffer much less from mixing than the Eulerian tracers, they do not represent the truth either. This is simply because parcels in a turbulent flow do not follow the time-mean wind. However, before we go on to study the true online trajectories, we aim to analyze the structures found in Fig. 9 in more detail.

In Fig. 10a, we plot the modulus of the inflow \( y \) position of the parcels against the time they require to travel between the inflow boundary and the grid point within the cloud. As in Fig. 9, there are two distinct classes of parcels. The first class consists of fast parcels, which originate close to the plane of symmetry \((y = 0)\) at the inflow boundary. The second class consists of much
slower parcels, which have a much larger value of $|y|$ at the inflow boundary. Yet another view is provided in Fig. 10b, where we plot the height of the streamlines at the inflow boundary against their maximum displacement in the lateral direction. Again, there are two distinct classes of air parcels: parcels starting at low altitude suffer a large lateral displacement, while parcels that enter the domain at a higher altitude are displaced significantly less in the lateral direction.

Figures 9 and 10 together suggest a consistent interpretation in terms of two distinct classes of streamlines. One of the two classes corresponds to the upper-right cloud of points in Fig. 10a and the lower-right cloud of points in Fig. 10b. These parcels start at a low altitude, but at some distance away from $y = 0$; they are slow in the sense that the parcels take a long time to proceed from the inflow boundary to the cloud. The slowness stems partly from the fact that the parcels start at low altitude, where the wind speed is reduced owing to surface friction; however, more importantly, they are slow because they are caught in a spiraling motion in the lee vortex, rendering their path very long before they connect to the respective grid point within the cloud (see Fig. 6). The other class of streamlines corresponds to the lower-left cloud of points in Fig. 10a and the upper-left cloud of points in Fig. 10b. These parcels enter the domain at some intermediate altitude but close to $y = 0$. They are much faster, because (i) their speed is not reduced by surface friction, and (ii) they directly pass over or around the top of the pyramid without any spiraling motion. The most striking result is the fact the two classes of streamlines are so well separated. As we will see later, this is at least partly an artifact of the fact that these streamlines are based on the time-averaged flow field.

We corroborate our interpretation through a 3D visualization of selected streamlines. More specifically, we selected three streamlines for each class and plotted them in Fig. 11. The initial coordinates are given by $(y, z) = (-8.2, 595.3), (-6.5, 401.7), (-3.1, 202.9), (-125.2, 73.0), (-212.7, 74.5),$ and $(-325.2, 73.8)$ m. Apparently, the fast streamlines proceed directly from the inflow boundary into the cloud by passing over the pyramid or by passing sideways close to its summit. By contrast, the slow streamlines pass the pyramid at a low altitude; subsequently, they are caught in the lee vortex, where they apparently take a long path, along which they spiral upward into the cloud.

The visualization of streamlines in Fig. 11 also helps to understand the strong impact that subscale mixing and numerical diffusion apparently have on the Eulerian tracers. Parcels that enter the domain at the outer edges of the lower two lobes (purple in Fig. 11) go around the pyramid, where they are caught in an upward-spiraling motion because of the bow vortex. It follows that parcels with large and opposite values of $\eta$ (say, a parcel with $\eta = 300$ m and a parcel with $\eta = -300$ m) are brought

![Fig. 11. Three-dimensional visualization of six selected streamlines: three for each of the two distinct classes. The two classes are distinguished by color.](image)
into close proximity to each other in the lee of the pyramid for a substantial amount of time. Even a small amount of material nonconservation then modifies each parcel’s $\eta$ value to its algebraic mean value, which is zero. As a consequence, both parcels appear to have originated from (close to) the axis of symmetry, in distinct contrast to their true originating positions, as diagnosed in Fig. 9.

The existence of two distinct classes of streamlines suggests some form of bifurcation to be at the heart of the phenomenon. As air parcels approach the pyramid, they are deflected either upward or downward (see Fig. 3). Those parcels that are deflected downward get engulfed into the horseshoe vortex. Obviously, this upward or downward deflection is tantamount to a bifurcation (i.e., parcels that enter the horseshoe vortex experience a completely different subsequent evolution than the remaining parcels). Correspondingly, the first class consists of those parcels that manage to escape the horseshoe vortex; they get lifted right away and proceed directly into the cloud. The second class consists of those parcels that are engulfed into the horseshoe vortex; they descend but then manage to flow around the pyramid. Subsequently, they enter the recirculation region in the lee, where they experience strong lifting that overcompensates for the initial downwelling.

The foregoing implies that the cloud volume is filled with two different sorts of parcels corresponding to the two classes of streamlines. We depict the two classes with two different colors in Fig. 12. Apparently, the class of fast parcels resides in the upper part of the cloud, while the class of slow parcels resides in the lower part of the cloud.

It is also interesting to study the change of time-mean specific humidity $\langle q \rangle$ along the time-mean streamlines. We interpret this change as an indicator for “mixing,” where mixing here represents the combination of the impact of parameterized subscale turbulence $M_q$ and the impact of explicitly resolved turbulence on the time-mean variable [see (12) in Voigt and Wirth (2013)]. More specifically, we diagnose whether a parcel’s moisture increases or decreases on its way from the inflow boundary into the cloud. Dried parcels are depicted in red, while moistened parcels are depicted blue in Fig. 9. Apparently, there is a tendency for high-starting parcels to get moistened and for low-starting parcels to get dried.

The mixing behavior is diagnosed in more detail in Fig. 13, where, again, the two classes of parcels can clearly be distinguished as two clusters of points. The class of low-starting parcels (lower-left cluster) experiences drying on the way into the cloud. On the other hand, for the class of high-starting parcels (upper-right cluster), some parcels suffer drying and some parcels suffer moistening. For these parcels, the plot indicates a good correlation between the change in specific humidity $\langle \Delta q \rangle$ and the starting altitude. Combining this correlation with the color information from Fig. 9 suggests a simple interpretation. Parcels entering the domain at low or intermediate altitudes initially have a rather large specific humidity. As they are lifted on their way into the cloud, they get into an environment that has generically a lower specific humidity (see Fig. 2). Mixing between the moist parcel and the dry environment then leads to drying of the air parcel. On the other hand,
parcels that enter the domain at a higher altitude have relatively low specific humidity initially. As they proceed into the cloud, they are embedded in a generally moister environment, which leads to moistening. Note also that the amount of mixing as quantified in Fig. 13 is significantly stronger for the second class of parcels (lower-left cluster) than for the first class of parcels (upper-right cluster). This is consistent with our earlier interpretation arguing that mixing is responsible for some of the difference between Figs. 8 and 9.

The coloring in Fig. 13 indicates the altitude of the parcels as they intersect the $x = 0$ plane. All parcels below 600-m altitude (dark blue color) suffer drying on their way into the cloud. There is no streamline between 600- and 850-m altitude at $x = 0$ (cyan color), resulting once again in a distinct separation of the two classes of streamlines. This separation of streamlines on the spanwise faces of the mountain is also clearly visible in Fig. 11. Parcels passing the mountain between 850 m and summit altitude (red dots in Fig. 13) experience either drying or moistening. A few parcels (brown dots in Fig. 13) go over the top of the mountain, and practically all of these moisten.

c. Online trajectories

We now carry out a similar analysis as in the previous subsection, but this time using the online trajectories. We only consider those trajectories that, at some point during the evolution, pass through the cloud volume. Figure 14 shows the intersection of these trajectories with the inflow boundary. There appears to be a rather concentrated area of parcels around the lateral plane of symmetry ($y = 0$) between 100- and 750-m altitude, and a more scattered area of parcels below $z \approx 500$ m, expanding to a maximum of $y = \pm 380$ m for the lowest parcels. Comparison with the corresponding plot for the time-mean streamlines (Fig. 9) indicates some qualitative agreement: in both plots, the lower parcels show a larger lateral spread than the higher parcels. On the other hand, the parcels from the online trajectories are much more scattered than the parcels from the time-mean streamlines and cannot be classified into two classes on this plot. In addition, the spatial average of all points (yellow diamond) is located some 150 m higher in Fig. 14 than in Fig. 9. This is consistent with the earlier result that the time-mean streamlines that lead through the cloud volume either start close to the ground or get into close proximity to the leading edge of the pyramid during their transit (see Fig. 11). As a consequence, dispersion as a result of turbulence is preferentially upward owing to the geometric constraint from the lower boundary. This explains the upward bias of parcel altitude when integrating in a backward sense.

The scatterplots in Fig. 15 show both similarities and differences with respect to the corresponding plots in Fig. 10. Overall, the scatterplots for the online trajectories show significantly more scatter, and there seems to be much less of a separation into two distinct classes. This is not too surprising, because online trajectories are subject to the simulated turbulence of the flow, while the time-mean streamlines effectively ignore the turbulence. Nevertheless, even for the turbulent online trajectories, the two classes can still be identified. For instance, both histograms quantifying the marginal distributions in Fig. 15 show a bimodal behavior similar to that in Fig. 10a, while the bimodality in Fig. 15b is less pronounced than in Fig. 10b. Similarly, the distribution of points in Fig. 15b suggests the existence of two separate classes of trajectories, although the distinction is not quite as clear as in the analogous plot in Fig. 10. Another striking difference between Figs. 15a and 10a is the fact that the lowest parcels along the time-mean streamlines are much slower than the slowest parcels along the online trajectories. We will discuss this result further in the following section.

Regarding the mixing behavior, the color in Fig. 14 indicates, again, that moistening (blue color) requires a parcel to start high and, hence, relatively dry. On the other hand, drying (red color) seems to be less correlated with starting altitude, in contrast to Fig. 9. Regarding the amount of mixing, the scatterplot in Fig. 16 shows, again, a broad correlation between inflow altitude and $\Delta q$, although this correlation is less pronounced than for the time-mean streamlines in Fig. 13, and the scatter of points is much larger. Again, the difference between the two plots appears plausible in view of the fact that the online trajectories experience the
simulated turbulence of the flow while the time-mean streamlines do not. There is no grouping into two distinct classes visible in Fig. 16, which is consistent with the fact that all starting altitudes at the inflow boundary contribute to trajectories that go through the cloud. We also note that there do exist cyan points in Fig. 16 (in contrast to Fig. 13), indicating that turbulence is strong enough to populate all altitudes right next to the mountain with parcels that end up in the cloud.

5. Summary, discussion, and conclusions

We investigated Lagrangian characteristics of air parcels as they pass an idealized steep mountain in the framework of large-eddy simulations. The focus was on parcels experiencing large vertical uplift, because, given suitable moisture conditions, they can become part of a so-called banner cloud. Three different diagnostics have been applied: Eulerian tracers to estimate Lagrangian displacements, streamlines computed from the time-averaged wind field, and online trajectories obtained from the instantaneous wind field.

As was previously known, the vertical uplift from the Eulerian tracers shows a plume of large values in the immediate lee of the obstacle. A similar leeside plume is obtained when diagnosing the upward displacement of the time-mean streamlines, although this requires a larger threshold. The online trajectories indicate, again, a plume of large uplift in the lee, but this time it is significantly more extended in the streamwise direction. Thus, the basic result is the same for all three methods: a plume of large uplift in the immediate lee of the obstacle. The differences in the details could plausibly be explained by the differences between the methods in accounting for turbulence and mixing.

For the subsequent analysis, a suitable isosurface of the uplift estimated from the Eulerian tracers was taken as a proxy for the banner cloud. All three diagnostics were then used in order to study the origin and flow history of those parcels that pass through this cloud volume at some later time. The Eulerian tracers indicate a compact and rather small source region of air parcels upstream of the mountain. By contrast, the time-mean streamlines indicate two distinct classes of air parcels with very different characteristics regarding their flow history. Parcels from the first class originate at intermediate altitudes; these parcels pass the obstacle close to its summit and immediately proceed into the cloud, leading to a rather short transit time. Parcels from the second class originate at low altitude; these parcels go around the mountain close to its base and need a fairly long time to be lifted into the cloud on a spiraling path through the action of the lee vortex. The existence of two distinct classes can be traced back to a flow

**Fig. 15.** Scatterplots of online trajectories passing through the cloud. Plot conventions are as in Fig. 10.

**Fig. 16.** Mixing behavior of parcels on online trajectories. Plot conventions are as in Fig. 13.
bifurcation on the windward side: the horseshoe vortex close to the bottom surface is associated with a stagnation line on the windward face of the pyramid, which forces the parcels to move either upward or downward, thus leading to a very different subsequent evolution.

Turning to the online trajectories, they show broadly the same features as the time-mean streamlines, although they reveal much more scatter, and the distinction between the two classes of air parcels is partly blurred. This appears to be intuitively right, because the online trajectories are subject to the resolved turbulence, which is strong in the neighborhood of the mountain [e.g., Fig. 6 in Voigt and Wirth (2013)]. Arguably more striking is the fact that the parcels in the slow class take several times longer to proceed from the inflow boundary into the cloud along the time-mean streamlines than along the online trajectories. We suggest that this is consistent from a fundamental dynamical systems point of view (e.g., Ottino 1989): stationary flows allow one to identify invariant sets, which is generally not possible for nonstationary, nonperiodic flows. In this sense, our time-averaged flow field is likely to show singular behavior owing to its stationarity, which gets lost when turbulence is accounted for. Nevertheless, we believe that the analysis of the time-averaged flow is interesting, for instance, when trying to work out key aspects of the flow for different model setups. Obviously this is beyond the scope of the present paper.

We quantified mixing along time-mean streamlines and online trajectories using a quasi-conserved humidity tracer, which decreases upward at the inflow boundary. Parcels that start at low altitude and experience significant vertical uplift are generally dried, because they are advected into a much drier environment. By contrast, some of the parcels that start at a relatively high altitude experience moistening, because they are lifted by a lesser amount, and they are engulfed into the much moister environment of the cloud. The latter, in turn, was generated through the action of other air parcels that originate from low altitudes and experience much stronger uplift.

Regarding the origin of cloudy parcels, our two Lagrangian methods yield more extended and more complex spatial structures for the parcel origins than the Eulerian method; on the other hand, the average over all parcel origins is relatively similar. We interpreted this as due to the fact that the Lagrangian methods effectively suffer much less from mixing than the Eulerian method. The difference between the time-mean streamlines and the online trajectories is small by comparison. At the same time, all three methods yield similar results regarding the diagnosed cloud volume (i.e., the location of those parcels that suffer strongest lifting when following the parcels in a forward sense). Both can be reconciled by noting that the threshold used to define the cloud volume was larger for the two Lagrangian methods than for the Eulerian method. This amounts to effectively removing the effect of mixing through recalibration.

One is tempted to ask which method is the most realistic one. We think that there is no simple answer to this question, as it depends on what one is really interested in. If one tries to follow parcel motions as closely as possible, our online trajectories clearly give the most realistic results. On the other hand, they probably do not provide the most realistic perspective on cloud occurrence, because moisture in the real atmosphere is, indeed, subject to mixing. The latter is effectively ignored in a truly Lagrangian method. For instance, as we discussed in the main text, we think that the visualization in Fig. 7b based on online trajectories is somewhat misleading.

There are some caveats. Quite deliberately, we have chosen a model setup that has been used several times before (e.g., Reinert and Wirth 2009; Voigt and Wirth 2013). This setup was argued to be generic and consistent with available observations (Wirth et al. 2012). Nevertheless, the results may, to some extent, depend on the assumptions inherent in our setup. In this sense, the current paper is considered to be a basis for future investigations. A promising avenue lies in systematically changing the stratification, the inflow profile, and the mountain geometry. This would allow one to address the connection between Lagrangian structures and the dynamics of the flow field. We are currently investigating related sensitivities and plan to report results in a forthcoming publication.

In summary, all three diagnostics have shown a distinct plume of parcels with large uplift in the immediate lee of the mountain. This corroborates our previous findings regarding the importance of strong uplift for banner cloud formation. On the other hand, we have seen significant differences in the details between the diagnostics. The differences could plausibly be explained as resulting from the different impacts that mixing and turbulence have on the respective method. In addition, our discovery of two distinct classes of air parcels is considered conceptually interesting.

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