Reply to "Comments on 'On the Choice of Average Solar Zenith Angle'"

TIMOTHY W. CRONIN

Department of Earth, Atmospheric, and Planetary Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

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ABSTRACT

The solar zenith angle controls local insolation and also affects the partitioning of insolation into planetary reflection, atmospheric absorption, and surface absorption. Because of this role of the solar zenith angle in modulating albedo, Cronin and others have proposed that insolation weighting should be used to determine the solar zenith angle when a single-angle calculation is used to represent a spatial or temporal average of solar fluxes. A comment by Li claims instead that daytime weighting is optimal, and that insolation weighting leads to serious errors, but this claim is based on a severe misinterpretation of the method proposed by Cronin. With any method of zenith angle averaging, both the solar constant and the zenith angle are free parameters, but their product—the mean insolation—must be held constant. Li fails to hold insolation constant when comparing different methods of zenith angle averaging and, thus, obtains large but spurious "errors." This paper attempts to clarify the method proposed by Cronin and tabulates the insolation-weighted solar zenith angle and solar constant that should be used as a function of latitude for annual-average radiative transfer on a planet with Earth's obliquity and a circular orbit.

1. Introduction

Planetary albedo α exerts a fundamental control on climate. For certain atmospheric profiles, especially those with optically thin clouds, the planetary albedo decreases near linearly with the cosine of the solar zenith angle μ. That is to say, when the sun lies higher in the sky, a smaller fraction of sunlight is reflected than when the sun lies lower in the sky. Because of this covariance of albedo and solar zenith angle, choosing a simple time-mean solar zenith angle may not give the correct time-mean solar flux, even if the insolation $\overline{I}$, is held constant. Cronin (2014) proposed a simple correction: to use the insolation-weighted solar zenith angle, which accounts for covariance of albedo and zenith angle by weighting the sun more strongly when it is high in the sky. This method was proposed particularly for idealized models, which often use a single representative solar zenith angle to eliminate annual or diurnal cycles of solar forcing. The method is exact when α is a linear function of μ and was shown by Cronin (2014) to outperform simple daytime weighting for cloudy skies in both the RRTMG model and a simpler pure-scattering atmosphere. In essence, the proposed approach considers the zenith angle corresponding to the average photon incident on a point or region, rather than the zenith angle corresponding to the average daylight hour.

The comment of Li (2017) claims that the insolation-weighted zenith angle leads to large errors when compared to the daytime-weighted zenith angle. The criticism of Li (2017), however, is based on a serious misinterpretation of the method proposed by Cronin (2014). Specifically, Li (2017) repeatedly fails to modify the solar constant to hold the insolation constant when comparing different choices of average zenith angle. The result is predictably large but spurious "errors" in fluxes calculated with the insolation-weighted zenith angle, because the sun is higher in the sky than it is in the time mean, but has not been turned down in strength to compensate and hold insolation constant.

To summarize this reply: Eq. (3) of Li (2017) is indeed the correct benchmark, and his calculations performed with the daytime-weighted zenith angle also appear to be correct, but his Eq. (5) is not the implementation of insolation-weighting advocated for in Cronin (2014). As a consequence, the rest of Li (2017) is an invalid critique, as it repeatedly compares the correct approach to daytime weighting with an incorrect approach to insolation weighting.

Corresponding author e-mail: Timothy W. Cronin, twcronin@mit.edu

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2. Hold the insolation constant

The importance of keeping insolation constant was intended to be a major point in Cronin (2014, p. 2994):

> A global-average radiative transfer calculation requires specifying both an effective cosine of solar zenith angle μ^* and an effective solar constant S_0^* such that the resulting insolation matches the planetary-mean insolation. . . . Matching the mean insolation constrains only the product $S_0^*\mu^*$, and not either parameter individually, so additional assumptions are needed.

See also Fig. 1 of Cronin (2014) for a depiction of how the solar constant must be reduced when the sun is put higher in the sky.

The insolation-weighted ''benchmark'' Eq. (5) of Li (2017) does not hold insolation fixed when compared to the correct benchmark [his Eq. (3)]. Suppose albedo is constant and does not depend on zenith angle, so that $\mathscr{R} = \alpha\mu S_0$. Then Eq. (5) of Li (2017) gives $\overline{\mathscr{R}} = 2/3\alpha S_0$: the mean reflected flux corresponds to mean insolation of $2/3S_0$, larger by 33% than the correct value of $1/2S_0$ [from, e.g., Li (2017), his Eq. (3)]. The key step of holding insolation fixed has been lost and is never recovered.

Figures 1 and 3 of Li (2017) also fail to hold insolation constant and obtain large ''errors'' as a result for the insolation-weighted zenith angle. These ''errors'' can easily be predicted by comparing to a correct time-mean calculation with time-varying zenith angle and fluxes. If $\mu_I^* = 2/3$ is used instead of $\mu_D^* = 1/2$, but the solar constant is not modified to hold the insolation fixed, then insolation weighting will make all fluxes too high by a factor of $\mu_I^*/\mu_D^* = 4/3$. This is exactly the result in Fig. 1a of Li (2017)—relative ''errors'' for insolation weighting are centered around +33.3%. But these large ''errors'' are spurious and would largely disappear if insolation were held fixed in the comparison of different zenith angles.

Li (2017) attempts to address this point in his Fig. 2a, using effective solar constants from Cronin (2014), yet still fails to perform comparisons at fixed insolation. Li's ''benchmark calculation'' in his Eq. (3) is performed only for the daylit half of the planet, whereas effective solar constants $S_{0D}^* = S_0/2$ and $S_{0I}^* = 3S_0/8$ from Cronin (2014) apply to the whole planet, including the dark half. Thus, the −50% relative ''error'' for all curves in Li's Fig. 2a is again introduced by failing to hold insolation fixed. Li (2017) expresses confusion about ''why the solar constant should be different in the benchmark and approximation calculations related to μ_I^*''—I hope that the reader is clear at this point that the rationale is simple: to hold insolation constant in the comparison. Taking the −50% relative ''error'' from Fig. 2a of Li (2017) as a zero line instead, we see that insolation-weighted and daytime-weighted zenith angles incur nearly equal and opposite true biases [these true biases are the focus of Cronin (2014)]. Near-equal biases for insolation-weighted and daytime-weighted approaches occur in this situation because clear-sky conditions are used—similar to the result from Table 1 of Cronin (2014), for the line marked ''RRTM: *U.S. Standard Atmosphere, 1976*—clear.'' A central result of Cronin (2014) was that insolation-weighted calculations are superior to daytime-weighted calculations primarily in cloudy skies.

Li (2017) also claims that use of albedo rather than reflected flux in Cronin (2014) minimizes errors associated with insolation weighting, because albedo is a compound variable, so compensating errors in insolation and reflection cancel. This claim again arises from a failure to keep insolation fixed. If insolation is constant when comparing different choices of average zenith angle [as in Cronin (2014)], the mean albedo $\overline{\alpha}$ is not a compound variable—it is simply the ratio of time-mean reflected radiation $\overline{\mathscr{R}}$ to insolation $\overline{I}$:

$$\overline{\alpha} = \frac{\overline{\mathscr{R}}}{\overline{I}} = \frac{\int f_\alpha(\mu)P(\mu)\mu S_0\, d\mu}{\overline{I}}. \tag{1}$$

Although this expression is consistent with the first two lines of Li's Eq. (6)—if insolation is held constant—the final line of Li's Eq. (6) introduces two errors. First, the denominator is assumed to imply choice of a specific zenith angle rather than merely constant insolation—as elaborated above. Second, the time-mean reflected flux is assumed equal to the reflected flux at the daytime-weighted zenith angle; it was explicitly shown in Cronin (2014) that this is not generally the case, owing to the covariance of albedo and zenith angle noted at the outset.

Assuming that Li's confusion resulted from lack of clarity in Cronin (2014), it is perhaps worth stating once more, decisively: When the zenith angle is modified from its time-mean value, the solar constant must also be adjusted to keep the insolation fixed.

Li (2017) also makes a peculiar false assertion toward the end of the paper, which deserves comment: "There is no physical meaning to study the averaged SZA over cloudy sky since clouds change from time to time." Regions with near-permanent clouds, such as stratocumulus-covered subtropical eastern ocean basins, are one obvious counterexample to this point—and a situation in which the daytime-weighted zenith angle has been shown to lead to overestimation of diurnal-average reflection by ~20 W m^{-2} (Bretherton et al. 2013). More importantly, from a standpoint of climate, the temporal variation in cloud cover at a fixed location is immaterial (so long as it does not covary with zenith angle). Over long time scales,

TABLE 1. Annual-average normalized insolation, insolation-weighted and daytime-weighted zenith angles, and effective solar constants, as a function of latitude, for a planet with a circular orbit and obliquity 23.5°.

Latitude (°)	Insolation $\overline{I}/S_0$	μ_I^*	S_{0I}^*/S_0	μ_D^*	S_{0D}^*/S_0
0.0	0.305	0.754	0.405	0.611	0.500
7.5	0.303	0.749	0.404	0.606	0.500
15.0	0.296	0.735	0.402	0.592	0.500
22.5	0.284	0.712	0.399	0.568	0.500
30.0	0.268	0.680	0.394	0.537	0.500
37.5	0.249	0.642	0.387	0.497	0.500
45.0	0.226	0.598	0.377	0.451	0.500
52.5	0.200	0.551	0.363	0.400	0.500
60.0	0.174	0.503	0.345	0.348	0.500
67.5	0.150	0.450	0.334	0.301	0.500
75.0	0.137	0.384	0.355	0.273	0.500
82.5	0.129	0.333	0.389	0.259	0.500
90.0	0.127	0.313	0.405	0.254	0.500

different cloud scenes at a point are sampled at many different times of day, and all-sky radiative transfer under suitably averaged illumination is far more relevant than the clear-sky radiative transfer in determining long-term mean radiative fluxes.

3. Implementation

Practically speaking, how should the solar constant be modified in order to actually use the insolation-weighted zenith angle? In general, if the probability density function of the cosine of solar zenith angle is given by $P(\mu)$, then the insolation $\overline{I}$, insolation-weighted zenith angle μ_I^*, and effective solar constant S_{0I}^*, are given by

$$\overline{I} = S_0 \int \mu P(\mu)\, d\mu = \mu_I^* S_{0I}^*, \qquad (2)$$

$$\mu_I^* = \frac{\int \mu^2 P(\mu)\, d\mu}{\int \mu P(\mu)\, d\mu}, \quad \text{and} \qquad (3)$$

$$S_{0I}^* = S_0 \frac{\left[\int \mu P(\mu)\, d\mu\right]^2}{\int \mu^2 P(\mu)\, d\mu}. \qquad (4)$$

Over the daylit hemisphere, $P(\mu) = 1$, whereas over the whole globe, $P(\mu) = 1/2 + 1/2\delta(\mu = 0)$—a delta function must be used to account for the dark half of the planet where $\mu = 0$. Note that this is a correction from Cronin (2014), which did not include the delta function weight at $\mu = 0$.

For the average over the diurnal cycle on the equinox at latitude ϕ, $\overline{I} = S_0 \cos\phi/\pi$, and $P(\mu)$ is given by

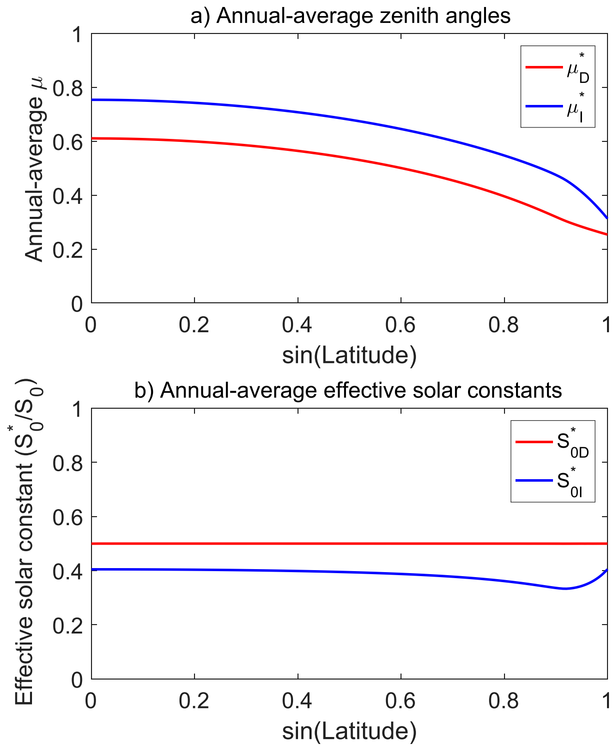

FIG. 1. (a) Annual-average cosine zenith angles and (b) annual-average normalized effective solar constants for a planet in a circular orbit, for daytime weighting (red) and insolation weighting (blue). The insolation, $\overline{I} = \mu^* S_0^*$, is identical in both cases as a function of latitude. The effective solar constants are normalized by the actual solar constant S_0. Note that $S_{0D}^* = S_0/2$ is a constant, but S_{0I}^* depends on latitude.

$$P(\mu) = \frac{1}{\pi\sqrt{\cos^2\phi - \mu^2}} + \frac{1}{2}\delta(\mu = 0); \quad 0 \le \mu \le \cos\phi. \qquad (5)$$

This is equivalent to Cronin (2014), Eq. (15), except that half of the weight of $P(\mu)$ has been corrected to occur at $\mu = 0$ (night). With this form for $P(\mu)$, we find that $\int \mu P(\mu)\, d\mu = (\cos\phi)/\pi$, and that $\int \mu^2 P(\mu)\, d\mu = (\cos^2\phi)/4$, so that $\mu_I^* = (\pi/4)\cos\phi$ and $S_{0I}^* = (4/\pi^2)S_0$. For comparison, the daytime-weighted values for the equinox are $\mu_D^* = (2/\pi)\cos\phi$ and $S_{0D}^* = S_0/2$.

Averages over the full annual and diurnal cycles of insolation cannot be computed analytically, but we can easily tabulate and plot them (Table 1 and Fig. 1). As in Cronin (2014), Fig. 1 shows that the insolation-weighted cosine zenith angle is larger than the daytime-weighted cosine zenith angle by about 20% across latitudes. Figure 1 also includes the appropriate effective solar constant that should be used in the case of insolation weighting. As recognized by Li (2017), fits to the annual-mean profiles are simpler for daytime weighting—$\mu_D^* \approx 0.5[1 - 0.482P_2(\sin\phi)]$ and

$S_{0D}^* = S_0/2$. Daytime weighting, however, is likely to overestimate the albedo in cloudy-sky conditions. For any readers interested in using insolation weighting in an idealized model, Table 1 gives the same information as Fig. 1.

I hope this comment clarifies and makes more accessible the approach of using the insolation-weighted zenith angle. Although not superior in all cases to the daytime-weighted zenith angle, it is likely a far better approach when clouds lead to a substantial fraction of shortwave reflection—as is the case for most skies on Earth.

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