A New Radiative Transfer Method for Solar Radiation in a Vertically Internally Inhomogeneous Medium

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ABSTRACT

The problem of solar spectral radiation is considered in a layer-based model, with scattering and absorption parallel to the plane for each medium (cloud, ocean, or aerosol layer) and optical properties assumed to be vertically inhomogeneous. A new radiative transfer (RT) method is proposed to deal with the variation of vertically inhomogeneous optical properties in the layers of a model for solar spectral radiation. This method uses the standard perturbation method to include the vertically inhomogeneous RT effects of cloud and snow. The accuracy of the new inhomogeneous RT solution is investigated systematically for both an idealized medium and realistic media of cloud and snow. For the idealized medium, the relative errors in reflection and absorption calculated by applying the homogeneous solution increase with optical depth and can exceed 20%. However, the relative errors when applying the inhomogeneous RT solution are limited to 4% in most cases. Observations show that stratocumulus clouds are vertically inhomogeneous. In the spectral band of 0.25–0.69 μm , the relative error in absorption with the inhomogeneous solution is 1.4% at most, but that with the homogeneous solution can be up to 7.4%. The effective radius of snow varies vertically. In the spectral band of 0.25–0.69 μm , the relative error in absorption with the homogeneous solution can be as much as 72% but is reduced to less than 40% by using the inhomogeneous solution. At the spectral wavelength of 0.94 μm , the results for reflection and absorption with the inhomogeneous solution are also more accurate than those with the homogeneous solution.

1. Introduction

Radiative transfer (RT) is a key issue in climate modeling and remote sensing. In most numerical radiative transfer algorithms, the atmosphere is divided into many homogeneous layers. The inherent optical properties (IOPs) are then fixed within each layer, and variations of IOPs inside each layer are ignored, effectively regarding each layer as internally homogeneous. The standard RT solutions are based on this assumption of internal homogeneity (Lenoble 1985; Toon et al. 1989; Fu et al. 1997; Li et al. 2005; Zhang and Li 2013), which cannot resolve within-layer vertical inhomogeneity.

It has been well established by observation that cumulus and stratocumulus clouds (hereinafter collectively referred to as cumulus clouds) are inhomogeneous, both horizontally and vertically (Vane et al. 2006; Boutle et al. 2014; Young et al. 2013; Luo et al. 2014; Cheng et al. 2015).

Inside a cumulus cloud, the liquid water content (LWC) and the cloud droplet size distribution vary with height, so the IOPs of cloud droplets depend on vertical height. For snow, the mean grain size of snow packs generally increases with time (Nakaya 1954; LaChapelle 1969; Flanner and Zender 2006). A wide variety of research (Arai 1965; Colbeck 1975; Gow 1969; Hobbs 1974; Mellor 1977; Sommerfeld and LaChapelle 1970) suggests that the average grain radius is in the range of 20–100 μm for new snow, 100–300 μm for fine-grain older snow, and 1.0–1.5 mm for old snow near the melting point. Therefore, snow grain size varies with snow depth, and the IOPs of snow are vertically inhomogeneous. In addition, the aerosol concentrations in the atmosphere are vertically inhomogeneous (Li et al. 2008; von Salzen et al. 2013; Guo et al. 2016; Qin et al. 2016, 2017; Wang et al. 2015).

How to deal with vertical internal inhomogeneity in RT models is an interesting topic for researchers. Li et al. (1994) developed a Monte Carlo cloud model that can be used to investigate photon transport in

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inhomogeneous clouds by considering internal variation of the optical properties. Their model showed that when overcast clouds become broken clouds, the difference in reflectance at large solar zenith angles between vertically inhomogeneous clouds and their plane-parallel counterparts can be as much as 10%. However, the Monte Carlo method is time consuming and not applicable to climate models or remote sensing (Liou 2002). The albedo of inhomogeneous mixed-phase clouds at visible wavelengths could be obtained by using a Monte Carlo method to compare such clouds with plane-parallel homogeneous clouds (Macke et al. 1998).

In principle, the vertical-inhomogeneity problem of the RT process can be solved by increasing the number of layers of the climate model. However, it is time consuming to increase the vertical resolution of a climate model. Typically, there are only 30–100 layers in a climate model (von Salzen et al. 2013), which is not high enough to resolve the cloud vertical inhomogeneity. For snow-covered ground, most climate models consider only the change in surface albedo due to the snow. Recently, the RT process has been extended to snow by treating it as either a single layer or a double layer (Namazi et al. 2015), even though the parameterization of snow-layer IOPs was proposed a long time ago (Wiscombe and Warren 1980; Warren and Wiscombe 1980). To completely address the problem of vertical inhomogeneity with only a limited number of layers in a climate model, the standard RT method must be extended to deal with the vertical inhomogeneity internal to each model layer. The main purpose of this study is to develop a new inhomogeneous RT solution. This solution follows a perturbation method: the zeroth-order solution is the Eddington approximation for homogeneous layers, with a first-order perturbation to account for the inhomogeneity effect. In section 2, the basic theory of radiative transfer is introduced, and the new inhomogeneous RT solution is presented. In section 3, the accuracy of the new solution is examined for a wide range of idealized cases. Furthermore, this inhomogeneous RT solution is applied to cloud and snow as realistic examples to demonstrate the practicality of this new method. A summary is given in section 4.

2. RT solution for an inhomogeneous layer

The azimuthally averaged solar radiative transfer equation is

$$\mu \frac{dI(\tau, \mu)}{d\tau} = I(\tau, \mu) - \frac{\omega(\tau)}{2} \int_{-1}^1 I(\tau, \mu') P(\tau, \mu, \mu') d\mu' - \frac{\omega(\tau)}{4\pi} F_s e^{-\tau/\mu_0} P(\tau, \mu, -\mu_0), \quad (1)$$

where μ is the cosine of the zenith angle ($\mu > 0$ and $\mu < 0$ refer to upward and downward radiation, respectively), $P(\tau, \mu, \mu')$ is the scattering phase function, τ is the optical depth ($\tau = 0$ and $\tau = \tau_0$ refer to the top and bottom of the medium, respectively), $\omega(\tau)$ is the single-scattering albedo, and F_s is the incoming solar flux. For the Eddington approximation, $P(\tau, \mu, \mu') = 1 + 3g(\tau)\mu\mu'$ for $-1 \leq \mu \leq 1$. For the scattering atmosphere, the irradiance fluxes in the upward and downward directions can be written as

$$F^\pm(\tau) = 2\pi \int_0^{\pm 1} I(\tau, \mu) \mu d\mu. \quad (2)$$

To simulate a realistic medium such as cloud or snow, we consider $\omega(\tau)$ and $g(\tau)$ to vary with τ , and we use exponential expressions here to simplify the process. The single-scattering albedo and asymmetry factor are written as

$$\omega(\tau) = \hat{\omega} + \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}), \quad (3a)$$

$$g(\tau) = \hat{g} + \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}), \quad (3b)$$

where τ_0 is the optical depth of the layer, $\hat{\omega}$ is the single-scattering albedo at $\tau_0/2$, and $\hat{g}$ is the asymmetry factor at the same place. Both ε_g and ε_ω are small parameters that are far less than $\hat{g}$ and $\hat{\omega}$, respectively, in a realistic medium.

By perturbation theory (Kato 1966), the corresponding flux can also be expanded by using the perturbation coefficients ε_ω and ε_g :

$$F^+ = F_0^+ + \varepsilon_\omega F_1^+ + \varepsilon_g F_2^+, \quad (4a)$$

$$F^- = F_0^- + \varepsilon_\omega F_1^- + \varepsilon_g F_2^-. \quad (4b)$$

By using the Eddington approximation, we can obtain differential equations for F^+ and F^- ; a detailed derivation is presented in appendix A. These equations [see Eq. (A4) for the precise formulation] can be rewritten as separate equations for $F_0^\pm$, $F_1^\pm$, and $F_2^\pm$. We obtain the following equations for the scattered flux $F_0^\pm$:

$$\frac{dF_0^+}{d\tau} = \gamma_1^0 F_0^+ - \gamma_2^0 F_0^- - \gamma_3^0 \hat{\omega} F_s e^{-\tau/\mu_0}, \quad (5a)$$

$$\frac{dF_0^-}{d\tau} = \gamma_2^0 F_0^+ - \gamma_1^0 F_0^- + \gamma_4^0 \hat{\omega} F_s e^{-\tau/\mu_0}, \quad (5b)$$

$$F_0^-(\tau = 0) = 0, \quad (5c)$$

$$F_0^+(\tau = \tau_0) = R_{\text{dir}} F_0^-(\tau = \tau_0) + R_{\text{dir}} \mu_0 F_s e^{-\tau_0/\mu_0}, \quad (5d)$$

where $\gamma_{1,2,3,4}^0$ is defined in the appendix A. Equation (5) is the standard RT equation for a homogeneous layer (Meador and Weaver 1980) and has the following solution:

$$F_0^+ = K_1 e^{k\tau} + \Gamma K_2 e^{-k\tau} + G_1 e^{-\tau/\mu_0}, \quad (6a)$$

$$F_0^- = \Gamma K_1 e^{k\tau} + K_2 e^{-k\tau} + G_2 e^{-\tau/\mu_0}, \quad (6b)$$

where $K_1, K_2, G_1, G_2, \Gamma$, and k are defined in [appendix B](#). According to Eq. (A4), the equations for the perturbation terms $F_i^\pm$ ($i = 1, 2$) are

$$\frac{dF_i^+}{d\tau} = \gamma_1^0 F_i^+ - \gamma_2^0 F_i^- + (e^{-a_i\tau} - e^{-a_i\tau_0/2})(\gamma_1^i F_0^+ - \gamma_2^i F_0^-) - \gamma_3^{i-1} F_s(e^{-a_i\tau} - e^{-a_i\tau_0/2})e^{-\tau/\mu_0}, \quad (7a)$$

$$\frac{dF_i^-}{d\tau} = \gamma_2^0 F_i^+ - \gamma_1^0 F_i^- + (e^{-a_i\tau} - e^{-a_i\tau_0/2})(\gamma_2^i F_0^+ - \gamma_1^i F_0^-) + \gamma_4^{i-1} F_s(e^{-a_i\tau} - e^{-a_i\tau_0/2})e^{-\tau/\mu_0}, \quad (7b)$$

$$F_i^-(\tau = 0) = 0, \quad F_i^+(\tau = \tau_0) = R_{\text{diff}} F_i^-(\tau = \tau_0), \quad (7c)$$

where $\gamma_3^1 = -\gamma_4^1 = \hat{\omega}\gamma_3^2$, $\gamma_{1,2}^1$, $\gamma_{1,2}^2$, and γ_3^2 are defined in [appendix A](#). The solutions (the details are in [appendix C](#)) are

$$M_i = A_i^+ e^{-k\tau} + B_i^+ e^{k\tau} + P_i^+ e^{-(k+a_i)\tau} + Q_i^+ e^{(k-a_i)\tau} + R_i^+ e^{-(a_i+1/\mu_0)\tau} - \frac{\eta_{4i}^+}{2k} e^{-k\tau} + \frac{\eta_{5i}^+}{2k} e^{k\tau} + \frac{\eta_{6i}^+ \mu_0^2}{1 - \mu_0^2 k^2} e^{-\tau/\mu_0}, \quad (8a)$$

$$N_i = C_i^+ e^{-k\tau} + D_i^+ e^{k\tau} + P_i^- e^{-(k+a_i)\tau} + Q_i^- e^{(k-a_i)\tau} + R_i^- e^{-(a_i+1/\mu_0)\tau} - \frac{\eta_{4i}^-}{2k} e^{-k\tau} + \frac{\eta_{5i}^-}{2k} e^{k\tau} + \frac{\eta_{6i}^- \mu_0^2}{1 - \mu_0^2 k^2} e^{-\tau/\mu_0}. \quad (8b)$$

Finally, we can obtain F_i^+ and F_i^- as

$$F_i^+ = D_{1i}^+ e^{-k\tau} + D_{2i}^+ e^{k\tau} + \phi_{1i}^+ e^{-(k+a_i)\tau} + \phi_{2i}^+ e^{(k-a_i)\tau} + \phi_{3i}^+ e^{-(a_i+1/\mu_0)\tau} + \phi_{4i}^+ \tau e^{-k\tau} + \phi_{5i}^+ \tau e^{k\tau} + \phi_{6i}^+ e^{-\tau/\mu_0}, \quad (9a)$$

$$F_i^- = D_{1i}^- e^{-k\tau} + D_{2i}^- e^{k\tau} + \phi_{1i}^- e^{-(k+a_i)\tau} + \phi_{2i}^- e^{(k-a_i)\tau} + \phi_{3i}^- e^{-(a_i+1/\mu_0)\tau} + \phi_{4i}^- \tau e^{-k\tau} + \phi_{5i}^- \tau e^{k\tau} + \phi_{6i}^- e^{-\tau/\mu_0}, \quad (9b)$$

where $D_{1i}, D_{2i}, \phi_{1i}, \phi_{2i}, \phi_{3i}, \phi_{4i}, \phi_{5i}$, and ϕ_{6i} are defined in [appendix D](#).

3. Results and discussion

a. Idealized medium

We analyze the accuracy of the new inhomogeneous RT solution by applying it to a single-layer inhomogeneous scattering medium. In the benchmark calculations, the inhomogeneous medium is divided into 100 sublayers, and the IOPs vary from layer to layer to resolve their vertical variation. The Eddington approximation is used in the benchmark calculation. The optical depth of the medium τ_0 varies from 0.1 to 50.

First, we consider the case of vertical variation in single-scattering albedo with $\omega(\tau) = 0.9 - 0.05(e^{-a\tau} - e^{-a\tau_0/2})$. The asymmetry factor is constant, with $g(\tau) = 0.75$. Different values of a control the degree of vertical variation in $\omega(\tau)$. The results of reflection are shown in the top two rows of [Fig. 1](#) for $a = 0.01, 0.05$, and 0.25 : the larger the value of a , the stronger the vertical variation of $\omega(\tau)$. [Figure 1](#) shows that the reflection calculated with the inhomogeneous solution is always much more accurate than that with the homogeneous solution. For $a = 0.01$, the relative error in the reflection with the homogeneous solution is about 5.8% at $\tau_0 = 50$, whereas the error with the inhomogeneous solution is less than 0.4%. For $a = 0.25$, the relative error in the reflection with the homogeneous solution increases to 20%, but it is limited to 4.5% with the inhomogeneous solution. All the results show that the new algorithm is significantly more accurate than the homogeneous RT solution, especially for large optical depths. There is an interesting phenomenon whereby the reflectance decreases after reaching its maximum. This is because the single-scattering albedo in layers having nearly the same optical depth decreases with the optical depth τ_0 . The absorptance results are shown in the bottom two rows of [Fig. 1](#). For $a = 0.01$, the relative error with the homogeneous RT solution is 2.3% at $\tau_0 = 50$, whereas the error is less than 0.2% with the inhomogeneous solution. For $a = 0.25$, the relative error with the homogeneous solution increases to 7%, whereas the relative error with the inhomogeneous solution does not exceed 1.5%. All results with the homogeneous solution underestimate the solar absorption.

Second, we consider the case of vertical variation in the asymmetry factor, with $g(\tau) = 0.8 - 0.1(e^{-b\tau} - e^{-b\tau_0/2})$, while the single-scattering albedo is constant at $\omega(\tau) = 0.99$. Different values of b represent different degrees of vertical variation in $g(\tau)$; we set $b = 0.01, 0.05$, and 0.25 . The reflection results are shown in the top two rows of [Fig. 2](#). The maximum relative error in reflection with the homogeneous solution can be over 4%, whereas the error is not more than 0.6% with the inhomogeneous solution. The corresponding absorptance results are shown in the bottom two rows of [Fig. 2](#). The maximum relative error with the homogeneous solution can reach 8%, whereas the relative error with the inhomogeneous solution is roughly 1% at large optical depths. Overall, the inhomogeneous solution can dramatically improve the accuracy in both reflection and absorption from that given by the homogeneous solution.

Third, we examine the sensitivity to the two other parameters, ε_ω and ε_g . In the first and second rows of [Fig. 3](#), we take the case of $g = 0.75$, $\omega(\tau) = 0.9 + \varepsilon_\omega(e^{-a\tau} - e^{-a\tau_0/2})$, and $a = 0.01$, where τ_0 varies from 0.05

FIG. 1. (first row) Reflectance vs optical depth and (second row) the corresponding relative errors with the inhomogeneous and homogeneous solutions. The asymmetry factor g is held constant, and the single-scattering albedo ω is a function of optical depth. (third row),(fourth row) As in the first and second rows, respectively, but for absorbance.

to 50 and ε_ω varies from -0.2 to -0.002 . For the reflectance (first row), the relative error with the homogeneous solution can be as high as 20% for large optical depths with large ε_ω but only around 4% with the

inhomogeneous solution at the same optical depth and value of ε_ω . The same applies to the absorbance (second row): the relative error can be up to 7% with the homogeneous solution but is limited to 1.5% with the

FIG. 2. As in Fig. 1, but the single-scattering albedo ω is held constant, and the asymmetry factor g is a function of optical depth.

FIG. 3. Relative errors with the (left) homogeneous solution and (right) inhomogeneous solution for the (first row), (third row) reflectance and (second row), (fourth row) absorbance vs ϵ_ω , ϵ_g , and optical depth τ_0 .

inhomogeneous solution. The accuracy is much higher with the inhomogeneous solution than with the homogeneous solution.

In the third and fourth rows of Fig. 3, we take the case of $\omega = 0.9$, $g(\tau) = 0.75 + \varepsilon_g(e^{-b\tau} - e^{-b\tau_0/2})$, and $b = 0.01$, where τ_0 varies from 0.05 to 50 and ε_g varies from -0.2 to -0.002 . The relative errors with the inhomogeneous solution are more than one order of magnitude smaller than those with the homogeneous solution for both reflectance and absorptance. The relative errors in reflectance (Fig. 3, third row) with the homogeneous solution are greater than 0.5%, whereas with the inhomogeneous solution, they are less than 0.3% in most regions. The relative errors in absorptance (Fig. 3, fourth row) with the inhomogeneous solution are less than 0.1% in most regions, whereas with the homogeneous solution, they are greater than 0.1%.

b. Cloud

The optical properties of ice clouds depend not only on the effective size but also on the complex particle habits (Letu et al. 2012, 2016; Yang et al. 2015). Thus, the ice cloud is not to be considered in this study. We just discuss the water cloud. Observations show that the cloud LWC and droplet radius tend to increase with height above the cloud base (Noonkester 1984). This phenomenon is attributed to the process of adiabatic water vapor condensation. To take into account the internal inhomogeneity of cloud optical properties, we assume that LWC (g m^{-3}) and droplet cross-sectional area (DCA; $\text{cm}^{-2} \text{m}^{-3}$) increase linearly from the cloud base (0 m) to a position near the cloud top (1000 m):

$$\text{LWC} = 0.22 + 0.00008z, \quad (10a)$$

$$\text{DCA} = 100 + z, \quad (10b)$$

where $0 < z < z_0$, z is the height in meters from the cloud base, and z_0 is the height of the cloud top. We can obtain the cloud effective radius r_e (μm) from the given LWC and DCA:

$$r_e(z) = \frac{3}{4\rho} \frac{\text{LWC}}{\text{DCA}} 10^{10}, \quad (11)$$

where ρ (g m^{-3}) is the liquid water density. Therefore, the LWC varies from 0.22 to 0.30 g m^{-3} , and r_e varies from 2.06 to $16.50 \mu\text{m}$. This is consistent with observations (Chen et al. 2008). The liquid water path (LWP) is defined as $\text{LWP} = \int_0^{z_0} \text{LWC} dz$. In this case, the LWP is 260 g m^{-2} , which represents low cloud (Fu et al. 1997). In the benchmark calculations, z_0 is divided into 100 internal homogeneous sublayers, although other numbers can be chosen (e.g., 200). In principle, more internal

sublayers should result in more accurate results. We use 100 internal sublayers throughout this study because having any more makes little difference to the calculated results. Using 100 sublayers is sufficiently accurate to resolve the vertical internal inhomogeneity of the medium. We use the optical properties of a water cloud in the spectral band of $0.25\text{--}0.69 \mu\text{m}$ and at $0.94 \mu\text{m}$. We choose the $0.25\text{--}0.69\text{-}\mu\text{m}$ band because it contains roughly 46% of the solar radiation energy; the $0.94\text{-}\mu\text{m}$ wavelength is at the center of channel 19 of the Moderate Resolution Imaging Spectroradiometer (MODIS), which is used in this paper to illustrate the effect of vertical internal inhomogeneity related to remote sensing. The surface albedo is set to zero. We calculate the asymmetry factor and single-scattering albedo of each sublayer as the benchmark values, which we then use in Eq. (3) to obtain the relevant parameters.

In Figs. 4a and 4b, the benchmark values of the inhomogeneous IOPs and the parameterized results for the spectral band of $0.25\text{--}0.69 \mu\text{m}$ are shown. The parameterized inhomogeneous IOPs are $1 - \omega(\tau) = 3.979 \times 10^{-7} - 1.897 \times 10^{-6}(e^{-0.1539\tau} - e^{-0.1539\tau_0/2})$ and $g(\tau) = 0.8359 + 0.0289(e^{-0.1539\tau} - e^{-0.1539\tau_0/2})$, where $\tau_0 = 110.84$. The corresponding results for reflection and absorption are shown in Figs. 4c–f. For reflection, the relative error with the homogeneous solution increases from 0.25% to 0.71% as μ_0 increases from 0.01 to 1, whereas the relative error with the inhomogeneous solution increases from 0.05% to 0.14%. For absorption, the relative error is not sensitive to μ_0 ; it is around 7.4% with the homogeneous solution but around only 1.4% with the inhomogeneous solution.

In Figs. 5a and 5b, the benchmark values of the inhomogeneous IOPs and the parameterized results for the wavelength $0.94 \mu\text{m}$ are shown. The parameterized inhomogeneous IOPs are $1 - \omega(\tau) = 1.936 \times 10^{-4} - 5.263 \times 10^{-4}(e^{-0.0357\tau} - e^{-0.0357\tau_0/2})$ and $g(\tau) = 0.8321 - 0.0403(e^{0.0218\tau} - e^{0.0218\tau_0/2})$, where $\tau_0 = 54.46$. For both $g(\tau)$ and $1 - \omega(\tau)$, the parameterized results match well with the corresponding benchmark values. Figures 5c–f show the corresponding results for reflection and absorption. For reflection, the relative error with the homogeneous solution increases from 1.1% to 3.0% as μ_0 increases from 0.01 to 1, whereas the relative error with the inhomogeneous solution increases from 0.7% to 2.0%. For absorption, the relative error is not sensitive to μ_0 ; it is around 10% with the homogeneous solution but around only 5.7% with the inhomogeneous solution.

c. Snow

Generally, the effective snow grain size r_e (μm) and the equivalent depth of liquid water in a snowpack L (g cm^{-2}) can be assumed to increase linearly with depth:

FIG. 4. For the band of $0.25\text{--}0.69\ \mu\text{m}$, (a) cloud asymmetry factor and (b) single-scattering albedo vs cloud optical depth; (c) reflectance and (e) absorptance vs optical depth; and (d), (f) relative errors with the inhomogeneous and homogeneous solutions.

$$r_e = 50 + 5h, \quad (12a)$$

$$L = \rho_{\text{snow}} h, \quad (12b)$$

where h is the snow thickness in centimeters from the snow top ($0 < h < h_0$, with h_0 the thickness of the whole snow layer) and $\rho_{\text{snow}} = 0.15\ \text{g cm}^{-3}$ is the density of snow. We assume h_0 is equal to 100 cm; thus, the effective size of snow varies from 50 to 550 μm . The snow layer is divided into 100 internal homogeneous sublayers in the benchmark calculations.

We focus on reflection and absorption by snow in the spectral band of $0.25\text{--}0.69\ \mu\text{m}$ and at the wavelength $0.94\ \mu\text{m}$. The surface albedo is set at zero. We calculate the asymmetry factor and single-scattering albedo of each sublayer as the benchmark values (Wiscombe and

Warren 1980) and then use these in Eq. (3) to obtain the relevant parameters.

Figures 6a and 6b show the benchmark values of the inhomogeneous IOPs and the parameterized results for the spectral band $0.25\text{--}0.69\ \mu\text{m}$. The parameterized inhomogeneous IOPs are $g(\tau) = 0.8896 - 0.0061(e^{-0.0039\tau} - e^{-0.0039\tau_0/2})$ and $\omega(\tau) = 0.99998 - 0.0000039(e^{0.0040\tau} - e^{0.0040\tau_0/2})$, with $\tau_0 = 776.31$. Figures 6c–f show the results for reflection and absorption with the homogeneous and inhomogeneous solutions and their relative errors against the benchmark calculations. For reflection, the relative error with the homogeneous solution is 0.8% for $\mu_0 = 0.01$, rising to 1.9% for $\mu_0 = 1$. The error is reduced by using the inhomogeneous solution, with the relative error increasing

FIG. 5. As in Fig. 4, but for the wavelength $0.94 \mu\text{m}$.

from only 0.4% to only 1.0%. For absorption, the relative error with the homogeneous solution can be in excess of 72%, whereas it is no more than 40% with the inhomogeneous solution. However, because the absorption is very small in this case, the absolute error with the inhomogeneous solution is less than 0.008.

Figures 7a and 7b show the benchmark values of the inhomogeneous IOPs and the parameterized results for the wavelength $0.94 \mu\text{m}$. The parameterized inhomogeneous IOPs are $g(\tau) = 0.8932 - 0.0117(e^{-0.0032\tau} - e^{-0.0032\tau_0/2})$ and $\omega(\tau) = 0.9987 - 0.00028(e^{0.0040\tau} - e^{0.0040\tau_0/2})$, with $\tau_0 = 779.08$. Figures 7c–f show the results for reflection and absorption with the homogeneous and inhomogeneous solutions. For reflection, the relative error with the homogeneous solution is 10% for $\mu_0 = 0$, rising to 27.5% for $\mu_0 = 1$. The inhomogeneous solution produces

smaller relative errors, ranging from 1.2% to 3.7%. For absorption, the relative error with the homogeneous solution can be in excess of 120%, whereas the relative error with the inhomogeneous solution is not more than 15%. Although the relative error with the inhomogeneous solution can be as high as 15%, the absorption is very small in this case, with the absolute error with the inhomogeneous solution being less than 0.04.

4. Summary and conclusions

In the above, we have considered the vertically inhomogeneous structures of only cloud and snow, whereas all physical quantities in the atmosphere are vertically inhomogeneous (e.g., the concentrations of all types of gases and aerosols). In current climate models,

FIG. 6. For the band of $0.25\text{--}0.69\ \mu\text{m}$, (a) snow asymmetry factor (b) single-scattering albedo vs cloud optical depth; (c) reflectance and (e) absorptance vs optical depth; and (d), (f) relative errors with the inhomogeneous and homogeneous solutions.

the vertical-layer resolution is far from that required to resolve such vertical inhomogeneity. In this study, we have proposed a new inhomogeneous RT solution to address the vertical inhomogeneity by introducing an internal variation of IOPs inside each model layer. This scheme is based on standard perturbation theory and allows us to use the standard RT solution for homogeneous layers to identify a zeroth-order equation and a first-order equation that includes the inhomogeneous effect. The new RT solution can accurately express the inhomogeneous effect in each model layer, and it reduces to the standard Eddington solution when the medium is homogeneous.

The accuracy of the new inhomogeneous RT solution has been investigated systematically for both an

idealized medium and realistic samples of cloud and snow. For the idealized medium, the relative errors in reflection and absorption calculated with the inhomogeneous RT solution are significantly smaller than those obtained with the standard homogeneous RT solution. Generally, the relative error with the homogeneous RT solution increases with optical depth, and the relative error can be over 20%. However, the relative error with the inhomogeneous RT solution is not particularly sensitive to optical depth, and the relative error is limited to 4% in most cases.

The new inhomogeneous RT solution is a good way to resolve cloud vertical inhomogeneity. In the spectral band of $0.25\text{--}0.69\ \mu\text{m}$, the relative error in the inhomogeneous RT solution is no more than 1.4%,

FIG. 7. As in Fig. 6, but for the wavelength $0.94 \mu\text{m}$.

whereas the error with the homogeneous RT solution can be up to 7.4%. At the specific wavelength of $0.94 \mu\text{m}$, the relative error with the inhomogeneous solution is not more than 5.7% but can be up to 10% with the homogeneous solution.

The new inhomogeneous RT solution can also resolve snow vertical inhomogeneity. In the spectral band of $0.25\text{--}0.69 \mu\text{m}$, the relative error in reflection with the homogeneous solution can be up to 72%, but the error is reduced to less than 40% by using the inhomogeneous solution. At the specific wavelength of $0.94 \mu\text{m}$, the relative error in the inhomogeneous solution is not more than 15% but can be up to 120% in the homogeneous solution.

In specific spectral bands or at specific wavelengths, the vertical variations in IOPs can typically be fitted easily into Eq. (3) to obtain the required parameters. A simple fitting program can be easily incorporated into a

climate model to produce the inhomogeneous IOPs of stratocumulus clouds. If no such cloud-inhomogeneity information is available in the current climate models, the vertical variation rates of cloud LWC and DCA can be derived empirically from observations, which show that the vertical variation rates of LWC and DCA in stratocumulus clouds are not very different (Vane et al. 2006; Boutle et al. 2014; Young et al. 2013; Luo et al. 2014; Zhang et al. 2017). For snow, the current climate models typically can calculate the snow age (Namazi et al. 2015). In principle, the vertical variation information for snow could be obtained to fit the IOPs in Eq. (3).

In this study, we presented only a single-layer inhomogeneous RT solution, which corresponds to the standard Eddington-approximation solution. To implement the new solution in a climate model, the process of adding layer-to-layer connections has to be resolved

(Hansen 1971; de Haan et al. 1987; Nakajima et al. 2000; Zhang et al. 2013; Zhang and Li 2013). Under the homogeneous condition, the single-layer result in reflection and transmission is the same for an upward path and a downward path, but this is not true for an inhomogeneous layer. Therefore, the adding process has to be modified. We will present an algorithm for this multilayer adding process in our next study, in which the climatic impact of inhomogeneous clouds and inhomogeneous snows will be explored. The code base for the inhomogeneous RT solution is available from the authors upon request.

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APPENDIX A

Perturbation Equations

According to the Eddington approximation, the radiative intensity $I(\tau, \mu)$ can be written as

$$I(\tau, \mu) = I_0(\tau) + I_1(\tau)\mu, \quad -1 \leq \mu \leq 1. \quad (\text{A1})$$

Using Eqs. (1), (2), and (A1), we obtain

$$\begin{aligned} \frac{dF^+}{d\tau} = & [\gamma_1^0 + \gamma_1^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \gamma_1^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2})] (F_0^+ + \varepsilon_\omega F_1^+ + \varepsilon_g F_2^+) - [\gamma_2^0 + \gamma_2^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) \\ & + \gamma_2^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2})] (F_0^- + \varepsilon_\omega F_1^- + \varepsilon_g F_2^-) - [\hat{\omega} \gamma_3^0 + \gamma_3^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \hat{\omega} \gamma_3^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2})] F_s e^{-\tau/\mu_0}, \end{aligned} \quad (\text{A4a})$$

$$\begin{aligned} \frac{dF^-}{d\tau} = & [\gamma_2^0 + \gamma_2^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \gamma_2^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2})] (F_0^+ + \varepsilon_\omega F_1^+ + \varepsilon_g F_2^+) - [\gamma_1^0 + \gamma_1^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) \\ & + \gamma_1^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2})] (F_0^- + \varepsilon_\omega F_1^- + \varepsilon_g F_2^-) + [\hat{\omega} \gamma_4^0 + \gamma_4^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) - \hat{\omega} \gamma_3^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2})] F_s e^{-\tau/\mu_0}, \end{aligned} \quad (\text{A4b})$$

where $\gamma_4^0 = (1 - \gamma_3^0)$.

$$\frac{dF^+}{d\tau} = \gamma_1(\tau)F^+(\tau) - \gamma_2(\tau)F^-(\tau) - \gamma_3(\tau)\omega(\tau)F_s e^{-\tau/\mu_0}, \quad (\text{A2a})$$

$$\begin{aligned} \frac{dF^-}{d\tau} = & \gamma_2(\tau)F^+(\tau) - \gamma_1(\tau)F^-(\tau) \\ & + [1 - \gamma_3(\tau)]\omega(\tau)F_s e^{-\tau/\mu_0}, \end{aligned} \quad (\text{A2b})$$

$$F^-(\tau = 0) = 0,$$

$$F^+(\tau = \tau_0) = R_{\text{dif}} F^-(\tau = \tau_0) + R_{\text{dir}} \mu_0 F_s e^{-\tau_0/\mu_0}, \quad (\text{A2c})$$

where $\gamma_1(\tau) = (1/4)\{7 - [4 + 3g(\tau)]\omega(\tau)\}$, $\gamma_2(\tau) = -(1/4)\{1 - [4 - 3g(\tau)]\omega(\tau)\}$, and $\gamma_3(\tau) = (1/4)[2 - 3g(\tau)\mu_0]$; τ_0 is the optical depth of the single layer; and R_{dif} (R_{dir}) is the diffuse (direct) reflection from the layer below or the diffuse (direct) surface albedo.

Substituting γ_1 , γ_2 , and γ_3 into Eq. (3) and ignoring the small second-order parameters ε_g^2 , ε_ω^2 , and $\varepsilon_g \varepsilon_\omega$, we get

$$\gamma_1(\tau) = \gamma_1^0 + \gamma_1^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \gamma_1^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}), \quad (\text{A3a})$$

$$\gamma_2(\tau) = \gamma_2^0 + \gamma_2^1 \varepsilon_\omega (e^{-a\tau} - e^{-a\tau_0/2}) + \gamma_2^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}), \quad (\text{A3b})$$

$$\gamma_3(\tau) = \gamma_3^0 + \gamma_3^2 \varepsilon_g (e^{-b\tau} - e^{-b\tau_0/2}), \quad (\text{A3c})$$

where $\gamma_1^0 = (1/4)[7 - (4 + 3\hat{g})\hat{\omega}]$, $\gamma_2^0 = -(1/4)[1 - (4 - 3\hat{g})\hat{\omega}]$, $\gamma_3^0 = (1/4)(2 - 3\hat{g}\mu_0)$, $\gamma_1^1 = -(1/4)(4 + 3\hat{g})$, $\gamma_1^2 = -(3/4)\hat{\omega}$, $\gamma_2^1 = (1/4)(4 - 3\hat{g})$, $\gamma_2^2 = -(3/4)\hat{\omega}$, and $\gamma_3^2 = -(3/4)\mu_0$.

Substituting Eqs. (4) and (A3) into Eq. (A2) yields

APPENDIX B

Coefficients of Eq. (6)

The terms K_1 , K_2 , G_1 , G_2 , Γ , and k in Eq. (6) are

$$\begin{aligned} K_1 &= \frac{(\Gamma - R_{\text{dif}})\Gamma G_2 e^{-k\tau_0} - (G_1 - R_{\text{dif}}G_2 - R_{\text{dir}}\mu_0 F_s)e^{-\tau_0/\mu_0}}{(1 - R_{\text{dif}}\Gamma)e^{k\tau_0} - (\Gamma - R_{\text{dif}})\Gamma e^{-k\tau_0}}, \\ K_2 &= \frac{(G_1 - R_{\text{dif}}G_2 - R_{\text{dir}}\mu_0 F_s)e^{-\tau_0/\mu_0} - (1 - R_{\text{dif}})\Gamma G_2 e^{-k\tau_0}}{(1 - R_{\text{dif}}\Gamma)e^{k\tau_0} - (\Gamma - R_{\text{dif}})\Gamma e^{-k\tau_0}}, \quad G_1 = \left[\gamma_3^0 \left(\frac{1}{\mu_0} - \gamma_1^0 \right) - \gamma_2^0 \gamma_4^0 \right] \frac{\mu_0^2 \hat{\omega} F_0}{1 - \mu_0^2 k^2}, \\ G_2 &= - \left[\gamma_4^0 \left(\frac{1}{\mu_0} + \gamma_1^0 \right) + \gamma_2^0 \gamma_3^0 \right] \frac{\mu_0^2 \hat{\omega} F_0}{1 - \mu_0^2 k^2}, \quad \Gamma = 1 - \frac{2k}{\gamma_1^0 + \gamma_2^0 + k}, \quad \text{and} \quad k^2 = (\gamma_1^0 + \gamma_2^0)(\gamma_1^0 - \gamma_2^0). \end{aligned}$$

APPENDIX C

Solution of Eq. (7)

Letting $M_i = F_i^+ + F_i^-$ and $N_i = F_i^+ - F_i^-$, Eqs. (7a) and (7b) yield

$$\begin{aligned} \frac{dM_i}{d\tau} &= (\gamma_1^0 + \gamma_2^0)N_i + (\Psi_i^+ + \Psi_i^-)e^{-(k+a_i)\tau} + (\zeta_i^+ + \zeta_i^-)e^{(k-a_i)\tau} \\ &\quad + (\chi_i^+ + \chi_i^-)e^{-(a_i+1/\mu_0)\tau} - e^{-a_i\tau_0/2}[(\Psi_i^+ + \Psi_i^-)e^{-k\tau} \\ &\quad + (\zeta_i^+ + \zeta_i^-)e^{k\tau} + (\chi_i^+ + \chi_i^-)e^{-\tau/\mu_0}], \end{aligned} \quad (\text{C1a})$$

$$\begin{aligned} \frac{dN_i}{d\tau} &= (\gamma_1^0 - \gamma_2^0)M_i + (\Psi_i^+ - \Psi_i^-)e^{-(k+a_i)\tau} + (\zeta_i^+ - \zeta_i^-)e^{(k-a_i)\tau} \\ &\quad + (\chi_i^+ - \chi_i^-)e^{-(a_i+1/\mu_0)\tau} - e^{-a_i\tau_0/2}[(\Psi_i^+ - \Psi_i^-)e^{-k\tau} \\ &\quad + (\zeta_i^+ - \zeta_i^-)e^{k\tau} + (\chi_i^+ - \chi_i^-)e^{-\tau/\mu_0}], \end{aligned} \quad (\text{C1b})$$

where $\Psi_i^+ = K_2(\gamma_1^i \Gamma - \gamma_2^i)$, $\Psi_i^- = K_2(\gamma_2^i \Gamma - \gamma_1^i)$, $\zeta_i^+ = K_1(\gamma_1^i - \gamma_2^i \Gamma)$, $\zeta_i^- = K_1(\gamma_2^i - \gamma_1^i \Gamma)$, $\chi_i^+ = \gamma_1^i G_1 - \gamma_2^i G_2 - \gamma_3^{i-1} F_s$, and $\chi_i^- = \gamma_2^i G_1 - \gamma_1^i G_2 + \gamma_4^{i-1} F_s$.

From Eq. (C1), we obtain

$$\begin{aligned} \frac{d^2 M_i}{d\tau^2} &= k^2 M_i + \eta_{1i}^+ e^{-(k+a_i)\tau} + \eta_{2i}^+ e^{(k-a_i)\tau} + \eta_{3i}^+ e^{-(a_i+1/\mu_0)\tau} \\ &\quad + \eta_{4i}^+ e^{-k\tau} + \eta_{5i}^+ e^{k\tau} + \eta_{6i}^+ e^{-\tau/\mu_0}, \end{aligned} \quad (\text{C2a})$$

$$\begin{aligned} \frac{d^2 N_i}{d\tau^2} &= k^2 N_i + \eta_{1i}^- e^{-(k+a_i)\tau} + \eta_{2i}^- e^{(k-a_i)\tau} + \eta_{3i}^- e^{-(a_i+1/\mu_0)\tau} \\ &\quad + \eta_{4i}^- e^{-k\tau} + \eta_{5i}^- e^{k\tau} + \eta_{6i}^- e^{-\tau/\mu_0}, \end{aligned} \quad (\text{C2b})$$

where $\eta_{1i}^\pm = (\gamma_1^0 \pm \gamma_2^0)(\Psi_i^+ \mp \Psi_i^-) - (k + a_i)(\Psi_i^+ \pm \Psi_i^-)$, $\eta_{2i}^\pm = (k - a_i)(\zeta_i^+ \pm \zeta_i^-) + (\gamma_1^0 \pm \gamma_2^0)(\zeta_i^+ \mp \zeta_i^-)$, $\eta_{3i}^\pm = (\chi_i^+ \mp \chi_i^-)(\gamma_1^0 \pm \gamma_2^0) - (a_i + 1/\mu_0)(\chi_i^+ \pm \chi_i^-)$, $\eta_{4i}^\pm = -e^{-a_i\tau_0/2}[(\gamma_1^0 \pm \gamma_2^0)(\Psi_i^+ \mp \Psi_i^-) - k(\Psi_i^+ \pm \Psi_i^-)]$, $\eta_{5i}^\pm = -e^{-a_i\tau_0/2}[k(\zeta_i^+ \pm \zeta_i^-) + (\gamma_1^0 \pm \gamma_2^0)(\zeta_i^+ \mp \zeta_i^-)]$, and $\eta_{6i}^\pm = -e^{-a_i\tau_0/2}[(\chi_i^+ \mp \chi_i^-)(\gamma_1^0 \pm \gamma_2^0) - (1/\mu_0)(\chi_i^+ \pm \chi_i^-)]$.

APPENDIX D

Coefficients of Eq. (9)

The terms D_{1i} , D_{2i} , φ_{1i} , φ_{2i} , φ_{3i} , φ_{4i} , φ_{5i} , and φ_{6i} in Eq. (9) are $D_{1i}^\pm = A_i \alpha^\pm \pm X_i$, $D_{2i}^\pm = B_i \alpha^\pm \pm Y_i$, $\alpha^\pm = (1/2)[1 \pm k/(\gamma_1^0 + \gamma_2^0)]$, $X_i = [e^{-a_i\tau_0/2}(\Psi_i^+ + \Psi_i^-)]/[2(\gamma_1^0 + \gamma_2^0)] - \eta_{4i}^+/[4k(\gamma_1^0 + \gamma_2^0)]$, $Y_i = [e^{-a_i\tau_0/2}(\zeta_i^+ + \zeta_i^-)]/[2(\gamma_1^0 + \gamma_2^0)] + \eta_{5i}^+/[4k(\gamma_1^0 + \gamma_2^0)]$, $\varphi_{1i}^\pm = (1/2)(P_i^+ \pm P_i^-)$, $\varphi_{2i}^\pm = (1/2)(Q_i^+ \pm Q_i^-)$, $\varphi_{3i}^\pm = (1/2)(R_i^+ \pm R_i^-)$, $\varphi_{4i}^\pm = -(\eta_{4i}^+ \pm \eta_{4i}^-)/4k$, $\varphi_{5i}^\pm = (\eta_{5i}^+ \pm \eta_{5i}^-)/4k$, $\varphi_{6i}^\pm = [(\eta_{6i}^+ \pm \eta_{6i}^-)\mu_0^2]/(1 - \mu_0^2 k^2)$, $P_i^\pm = \eta_{1i}^\pm/[(k + a_i)^2 - k^2]$, $Q_i^\pm = \eta_{2i}^\pm/[(k - a_i)^2 - k^2]$, and $R_i^\pm = \eta_{3i}^\pm/[(a_i + 1/\mu_0)^2 - k^2]$. We determine B_i and A_i by the boundary conditions as

$$\begin{aligned}
B_i = & -\frac{(\varphi_{1i}^- + \varphi_{2i}^- + \varphi_{3i}^- + \varphi_{6i}^-)(\alpha^- - R_{\text{dif}}\alpha^+)e^{-k\tau_0}}{\alpha^-(\alpha^- - R_{\text{dif}}\alpha^+)e^{-k\tau_0} - \alpha^+(\alpha^+ - R_{\text{dif}}\alpha^-)e^{k\tau_0}} \\
& + \frac{\alpha^+[(\varphi_{1i}^+ - R_{\text{dif}}\varphi_{1i}^-)e^{-(k+a_i)\tau_0} + (\varphi_{2i}^+ - R_{\text{dif}}\varphi_{2i}^-)e^{(k-a_i)\tau_0} + (\varphi_{3i}^+ - R_{\text{dif}}\varphi_{3i}^-)e^{-(a_i+1/\mu_0)\tau_0}]}{\alpha^-(\alpha^- - R_{\text{dif}}\alpha^+)e^{-k\tau_0} - \alpha^+(\alpha^+ - R_{\text{dif}}\alpha^-)e^{k\tau_0}} \\
& + \frac{\alpha^+[(\varphi_{4i}^+ - R_{\text{dif}}\varphi_{4i}^-)\tau_0 e^{-k\tau_0} + (\varphi_{5i}^+ - R_{\text{dif}}\varphi_{5i}^-)\tau_0 e^{k\tau_0} + (\varphi_{6i}^+ - R_{\text{dif}}\varphi_{6i}^-)e^{-\tau_0/\mu_0}]}{\alpha^-(\alpha^- - R_{\text{dif}}\alpha^+)e^{-k\tau_0} - \alpha^+(\alpha^+ - R_{\text{dif}}\alpha^-)e^{k\tau_0}} \\
& + \frac{(X_i + Y_i)(\alpha^- - R_{\text{dif}}\alpha^+)e^{-k\tau_0} + (X_i + R_{\text{dif}}X_i)\alpha^+e^{-k\tau_0} + (Y_i + R_{\text{dif}}Y_i)\alpha^+e^{k\tau_0}}{\alpha^-(\alpha^- - R_{\text{dif}}\alpha^+)e^{-k\tau_0} - \alpha^+(\alpha^+ - R_{\text{dif}}\alpha^-)e^{k\tau_0}}, \tag{D1a}
\end{aligned}$$

$$A_i = \frac{1}{\alpha^+}(X_i + Y_i - B_i\alpha^- - \varphi_{1i}^- - \varphi_{2i}^- - \varphi_{3i}^- - \varphi_{6i}^-). \tag{D1b}$$

REFERENCES

- Arai, T., 1965: On the relationship between albedo and the properties of snow cover. Research Institute for Natural Resources Miscellaneous Rep. 64, 12–19.
- Boutle, I. A., S. J. Abel, P. G. Hill, and C. J. Morcrette, 2014: Spatial variability of liquid cloud and rain: Observations and microphysical effects. *Quart. J. Roy. Meteor. Soc.*, **140**, 583–594, <https://doi.org/10.1002/qj.2140>.
- Chen, R., R. Wood, Z. Li, R. Ferraro, and F.-L. Chang, 2008: Studying the vertical variation of cloud droplet effective radius using ship and space-borne remote sensing data. *J. Geophys. Res.*, **113**, D00A02, <https://doi.org/10.1029/2007JD009596>.
- Cheng, M. N., C. S. Lu, and Y. G. Liu, 2015: Variation in entrainment rate and relationship with cloud microphysical properties on the scale of 5 m. *Sci. Bull.*, **60**, 707–717, <https://doi.org/10.1007/s11434-015-0737-8>.
- Colbeck, S. C., 1975: Grain and bond growth in wet snow. *Snow Mechanics: Proceedings of the Grindelwald Symposium*, IAHS Publ. **114**, 51–61.
- de Haan, J. F., P. B. Bosma, and J. W. Hovenier, 1987: The adding method for multiple scattering calculations of polarized light. *Astron. Astrophys.*, **183**, 371–391.
- Flanner, M. G., and C. S. Zender, 2006: Linking snowpack microphysics and albedo evolution. *J. Geophys. Res.*, **111**, D12208, <https://doi.org/10.1029/2005JD006834>.
- Fu, Q., K. N. Liou, M. C. Cribb, T. P. Charlock, and A. Grossman, 1997: Multiple scattering parameterization in thermal infrared radiative transfer. *J. Atmos. Sci.*, **54**, 2799–2812, [https://doi.org/10.1175/1520-0469\(1997\)054<2799:MSPITI>2.0.CO;2](https://doi.org/10.1175/1520-0469(1997)054<2799:MSPITI>2.0.CO;2).
- Gow, A. J., 1969: On the rates of growth of grains and crystals in South Polar firn. *J. Glaciol.*, **8**, 241–252, <https://doi.org/10.3189/S0022143000031233>.
- Guo, J. P., and Coauthors, 2016: Three-dimensional structure of aerosol in China: A perspective from multi-satellite observations. *Atmos. Res.*, **178–179**, 580–589, <https://doi.org/10.1016/j.atmosres.2016.05.010>.
- Hansen, J. E., 1971: Multiple scattering of polarized light in planetary atmospheres. Part I. The doubling method. *J. Atmos. Sci.*, **28**, 120–125, [https://doi.org/10.1175/1520-0469\(1971\)028<0120:MSOPLI>2.0.CO;2](https://doi.org/10.1175/1520-0469(1971)028<0120:MSOPLI>2.0.CO;2).
- Hobbs, P. V., 1974: *Ice Physics*. Clarendon Press, 837 pp.
- Kato, T., 1966: *Perturbation Theory for Linear Operators*. Springer-Verlag, 592 pp.
- LaChapelle, E. R., 1969: Field guide to snow crystals. *Arct. Alp. Res.*, **207**, 464–465.
- Lenoble, J., 1985: *Radiative Transfer in Scattering and Absorbing Atmospheres: Standard Computational Procedures*. A. Deepak Publishing, 314 pp.
- Letu, H., T. Y. Nakajima, and T. N. Matsui, 2012: Development of an ice crystal scattering database for the Global Change Observation Mission/Second Generation Global Imager Satellite Mission: Investigating the refractive index grid system and potential retrieval error. *Appl. Opt.*, **51**, 6172–6178, <https://doi.org/10.1364/AO.51.006172>.
- , H. Ishimoto, J. Riedi, T. Y. Nakajima, L. C. Labonnote, A. Baran, T. M. Nagao, and M. Sekiguchi, 2016: Investigation of ice particle habits to be used for ice cloud remote sensing for the GCOM-C satellite mission. *Atmos. Chem. Phys.*, **16**, 12 287–12 303, <https://doi.org/10.5194/acp-16-12287-2016>.
- Li, J., D. J. W. Geldart, and P. Chylek, 1994: Solar radiative transfer in clouds with vertical internal homogeneity. *J. Atmos. Sci.*, **51**, 2542–2552, [https://doi.org/10.1175/1520-0469\(1994\)051<2542:SRTICW>2.0.CO;2](https://doi.org/10.1175/1520-0469(1994)051<2542:SRTICW>2.0.CO;2).
- , S. Dobbie, P. Räisänen, and Q. Min, 2005: Accounting for unresolved clouds in a 1-D solar radiative-transfer model. *Quart. J. Roy. Meteor. Soc.*, **131**, 1607–1629, <https://doi.org/10.1256/qj.04.31>.
- , X. Ma, K. von Salzen, and S. Dobbie, 2008: Parameterization of sea-salt optical properties and physics of the associated radiative forcing. *Atmos. Chem. Phys.*, **8**, 4787–4798, <https://doi.org/10.5194/acp-8-4787-2008>.
- Liou, K. N., 2002: *An Introduction to Atmospheric Radiation*. 3rd ed. Academic Press, 583 pp.
- Luo, Z. J., J. Jeyaratnam, S. Iwasaki, H. Takahashi, and R. Anderson, 2014: Convective vertical velocity and cloud internal vertical structure: An A-Train perspective. *Geophys. Res. Lett.*, **41**, 723–729, <https://doi.org/10.1002/2013GL058922>.
- Macke, A., D. L. Mitchell, and L. V. Bremen, 1998: Monte Carlo radiative transfer calculations for inhomogeneous mixed phase clouds. *Phys. Chem. Earth*, **24B**, 237–241, [https://doi.org/10.1016/S1464-1909\(98\)00044-6](https://doi.org/10.1016/S1464-1909(98)00044-6).
- Meador, W. E., and W. R. Weaver, 1980: Two-stream approximations to radiative transfer in planetary atmospheres: A unified description of existing methods and a new improvement. *J. Atmos. Sci.*, **37**, 630–643, [https://doi.org/10.1175/1520-0469\(1980\)037<0630:TSATRT>2.0.CO;2](https://doi.org/10.1175/1520-0469(1980)037<0630:TSATRT>2.0.CO;2).
- Mellor, M., 1977: Engineering properties of snow. *J. Glaciol.*, **19**, 15–66, <https://doi.org/10.3189/S002214300002921X>.

- Nakajima, T., M. Tsukamoto, Y. Tsushima, A. Numaguti, and T. Kimura, 2000: Modeling of the radiative process in an atmospheric general circulation model. *Appl. Opt.*, **39**, 4869–4878, <https://doi.org/10.1364/AO.39.004869>.
- Nakaya, U., 1954: *Snow Crystals: Natural and Artificial*. Harvard University Press, 510 pp.
- Namazi, N., K. von Salzen, and J. N. S. Cole, 2015: Simulation of black carbon in snow and its climate impact in the Canadian Global Climate Model. *Atmos. Chem. Phys.*, **15**, 10 887–10 904, <https://doi.org/10.5194/acp-15-10887-2015>.
- Noonkester, V. R., 1984: Droplet spectra observed in marine stratus cloud layers. *J. Atmos. Sci.*, **41**, 829–845, [https://doi.org/10.1175/1520-0469\(1984\)041<0829:DSOIMS>2.0.CO;2](https://doi.org/10.1175/1520-0469(1984)041<0829:DSOIMS>2.0.CO;2).
- Qin, K., and Coauthors, 2016: Trans-boundary aerosol transport during a winter haze episode in China revealed by ground-based lidar and CALIPSO satellite. *Atmos. Environ.*, **141**, 20–29, <https://doi.org/10.1016/j.atmosenv.2016.06.042>.
- , L. Wang, L. Wu, J. Xu, L. Rao, H. Letu, T. Shi, and R. Wang, 2017: A campaign for investigating aerosol optical properties during winter hazes over Shijiazhuang, China. *Atmos. Res.*, **198**, 113–122, <https://doi.org/10.1016/j.atmosres.2017.08.018>.
- Sommerfeld, R. A., and E. LaChapelle, 1970: The classification of snow metamorphism. *J. Glaciol.*, **9**, 3–18, <https://doi.org/10.3189/S0022143000026757>.
- Toon, O. B., C. P. McKay, T. P. Ackerman, and K. Santhanam, 1989: Rapid calculation of radiative heating rates and photodissociation rates in inhomogeneous multiple scattering atmospheres. *J. Geophys. Res.*, **94**, 16 287–16 301, <https://doi.org/10.1029/JD094iD13p16287>.
- Vane, D., N. Tourville, G. Stephens, and A. Kankiewicz, 2006: New observations of hurricanes from the CloudSat radar. *Eos, Trans. Amer. Geophys. Union*, **87** (Fall Meeting Suppl.), Abstract A13A-0885.
- von Salzen, K., and Coauthors, 2013: The Canadian Fourth Generation Atmospheric Global Climate Model (CanAM4). Part I: Representation of physical processes. *Atmos.–Ocean*, **51**, 104–125, <https://doi.org/10.1080/07055900.2012.755610>.
- Wang, F., and Coauthors, 2015: Multi-sensor quantification of aerosol-induced variability in warm cloud properties over eastern China. *Atmos. Environ.*, **113**, 1–9, <https://doi.org/10.1016/j.atmosenv.2015.04.063>.
- Warren, S. G., and W. J. Wiscombe, 1980: A model for the spectral albedo of snow. II: Snow containing atmospheric aerosols. *J. Atmos. Sci.*, **37**, 2734–2745, [https://doi.org/10.1175/1520-0469\(1980\)037<2734:AMFTSA>2.0.CO;2](https://doi.org/10.1175/1520-0469(1980)037<2734:AMFTSA>2.0.CO;2).
- Wiscombe, W. J., and S. G. Warren, 1980: A model for the spectral albedo of snow. I: Pure snow. *J. Atmos. Sci.*, **37**, 2712–2733, [https://doi.org/10.1175/1520-0469\(1980\)037<2712:AMFTSA>2.0.CO;2](https://doi.org/10.1175/1520-0469(1980)037<2712:AMFTSA>2.0.CO;2).
- Yang, P., K. Liou, L. Bi, C. Liu, B. Yi, and B. Baum, 2015: On the radiative properties of ice clouds: Light scattering, remote sensing, and radiation parameterization. *Adv. Atmos. Sci.*, **32**, 32–63, <https://doi.org/10.1007/s00376-014-0011-z>.
- Young, A. H., J. Bates, and J. Curry, 2013: Application of cloud vertical structure from CloudSat to investigate MODIS-derived cloud properties of cirriform, anvil, and deep convective clouds. *J. Geophys. Res. Atmos.*, **118**, 4689–4699, <https://doi.org/10.1002/jgrd.50306>.
- Zhang, F., and J. Li, 2013: Doubling–adding method for delta-four-stream spherical harmonic expansion approximation in radiative transfer parameterization. *J. Atmos. Sci.*, **70**, 3084–3101, <https://doi.org/10.1175/JAS-D-12-0334.1>.
- , Z. Shen, J. Li, X. Zhou, and L. Ma, 2013: Analytical delta-four-stream doubling–adding method for radiative transfer parameterizations. *J. Atmos. Sci.*, **70**, 794–808, <https://doi.org/10.1175/JAS-D-12-0122.1>.
- Zhang, J. L., J. Shang, and G. Y. Zhang, 2017: Verification for different contrail parameterizations based on integrated satellite observation and ECMWF reanalysis data. *Adv. Meteor.*, **2017**, 8707234, <https://doi.org/10.1155/2017/8707234>.